Bridging the gap between spectroscopy and partonic observables: (lecture 2)

instantons, chiral breaking, confinement

Edward Shuryak

Center for Nuclear Theory, Stony Brook University

Based on summary
of 20+papers
with Ismail Zahed
2019-2026
book e-Print: 2601.15085

LF lectures, March 2026

Instantons are paths describing tunneling events

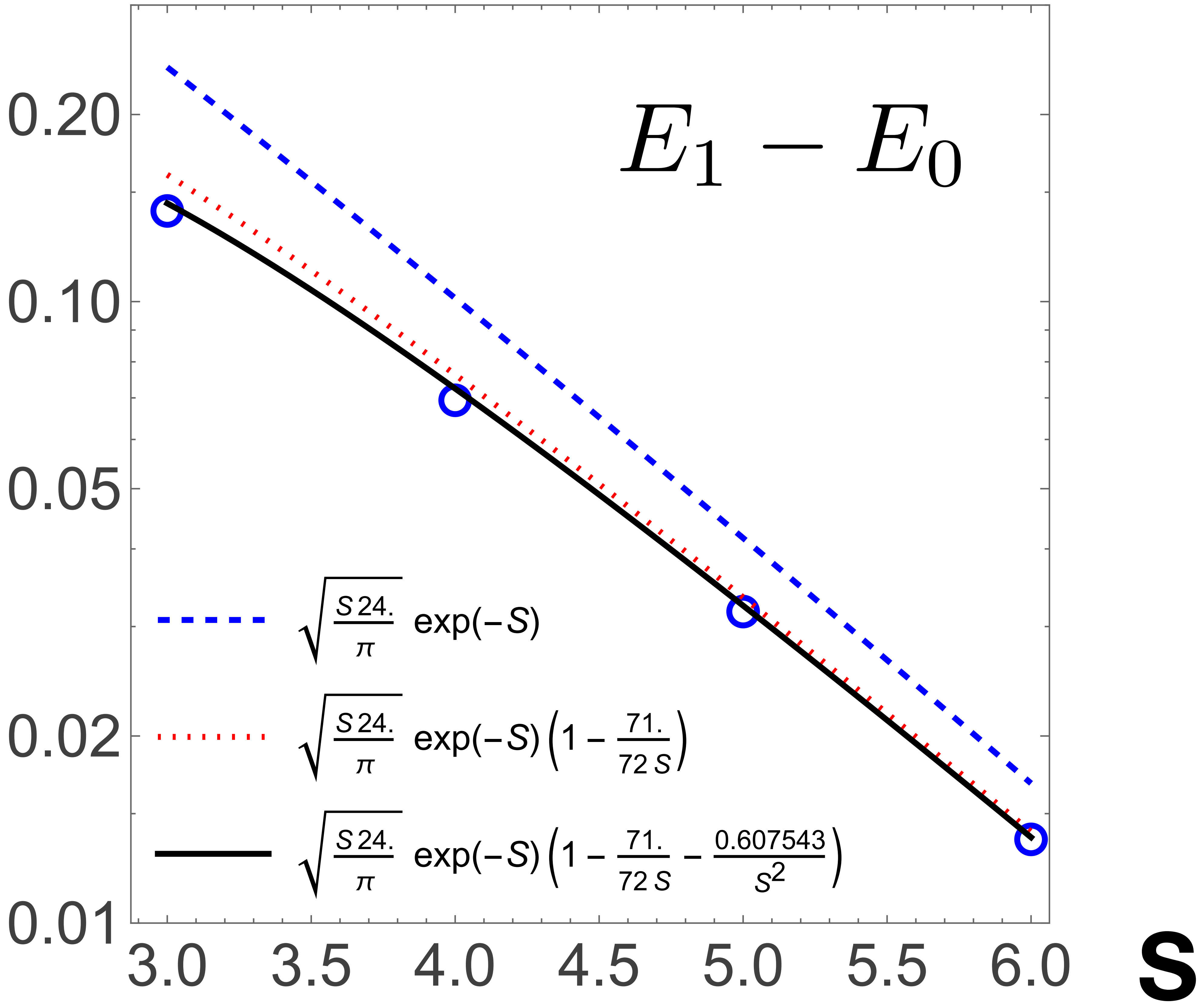
double well potential
EOM with inverted potential

$$V_{DWP} = \lambda(x^2 - f^2)^2,$$
$$x_{inst}(\tau)/f = \tanh\Big[\omega(\tau - \tau_0)/2\Big].$$

instantons break x->-x symmetry

semiclassical amplitude
foun with 1,2, and even
3 loop accuracy

ES, Turbiner 2015

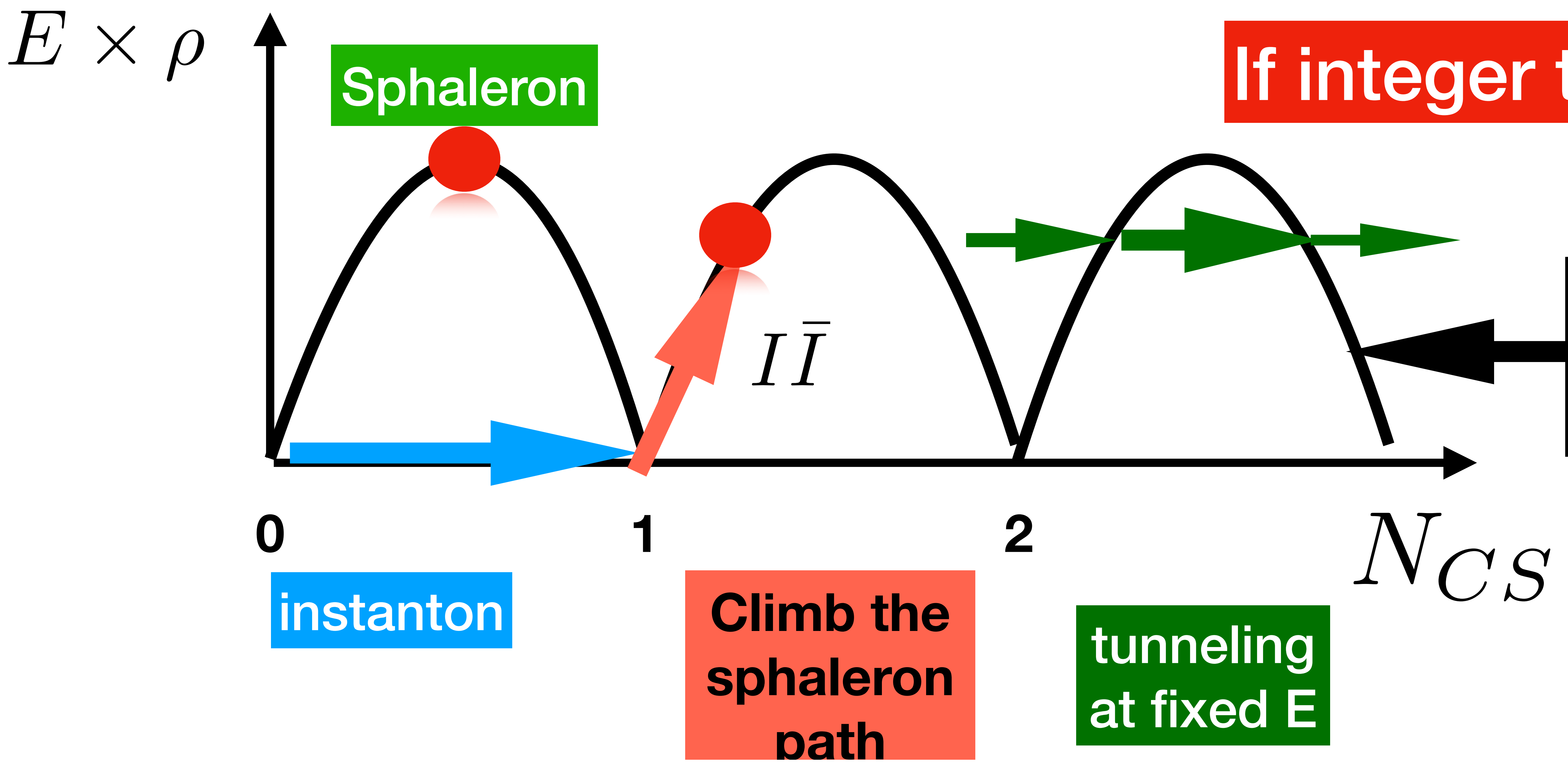


In QCD only one loop, ’t Hooft 1976

Topological landscape of gauge fields

Chern-Simons number, $$N_{CS} \equiv \frac{\epsilon^{\alpha\beta\gamma}}{16\pi^2}\int d^3x \left(A^a_\alpha \partial_\beta A^a_\gamma + \frac{1}{3}\epsilon^{abc}A^a_\alpha A^b_\beta A^c_\gamma\right),$$

If integer then A is pure gauge, Gmunu=0



Sphaleron path, streamline or thimble

lines in parametric form

$$U_{\min}(k,\rho) = (1-k^2)^2 \frac{3\pi^2}{g^2\rho},$$

$$N_{CS}(k) = \frac{1}{4}\text{sign}(k)(1-|k|)^2(2+|k|),$$

Conditional energy minimization Ostrovsky and Shuryak,2002)

Instanton Liquid Model of the QCD vacuum, ES 1982

the typical instanton size $\rho \sim 1/3\,\mathrm{fm}$.

the typical instanton (+antiinstantons) density $n_{inst} \approx 1\, fm^{-4}$

the diluteness parameter $n_{inst}(\pi^2/2)\rho^4 \sim 1/20 \ll 1$

yet it is enough to break chiral symmetry! inputs <Q^2> and <qbar q>

$$\frac{G}{8(N_c^2-1)}\left(\frac{2N_c-1}{2N_c}\left[(\bar\Psi\Psi)^2-(\bar\Psi\tau^a\Psi)^2-(\bar\Psi i\gamma^5\Psi)^2+(\bar\Psi i\gamma^5\tau^a\Psi)^2\right]-\frac{1}{4N_c}\left[(\bar\Psi\sigma_{\mu\nu}\Psi)^2-(\bar\Psi\sigma_{\mu\nu}\tau^a\Psi)^2\right]\right).$$

’t Hooft effective Lagrangian, 1976

Historically, the concept of effective quark masses generated by dynamical chiral symmetry breaking was introduced by Nambu and Jona-Lasinio [Nambu and Jona-Lasinio, 1961b]. The central idea is that an attractive interaction between a quark and an antiquark can be sufficiently strong to open a gap at the surface of the massless Dirac sea, in close analogy with electron-electron attraction producing a spectral gap at the Fermi surface in superconductors. As discussed in the Introduction and in the chapter on the QCD vacuum, the modern formulation [Shuryak, 1982e, Diakonov and Petrov, 1986] of the NJL four-fermion operator is nothing but the instanton-induced ’t Hooft vertex.

The mechanism can be demonstrated most simply in the mean-field approximation, following the NJL approach. The emergent constituent quark mass in the rest frame appears as a solution of the *gap equation* (14.34),

$$M(k) = m + 2g_S\mathcal{F}(k)\int\frac{d^4q}{(2\pi)^4}\frac{4M(q)}{q^2+M^2(q)}\mathcal{F}(q). \tag{3.18}$$

The last term represents a dressed quark loop generated by the four-fermion interaction, with $g_S = G_S/N_c$ denoting the rescaled ’t Hooft coupling. In contrast to the local NJL model, instanton semiclassical theory introduces the form factor $\mathcal{F}(q)$, which reflects the finite size of instantons.

A formal solution of Eq. (3.18) can be written as

$$M(k) = m\,[1-\mathcal{F}(k)] + M\,\mathcal{F}(k) \sim M\,\mathcal{F}(k), \tag{3.19}$$

which reduces Eq. (3.18) to the standard *gap equation* for the constituent mass M,

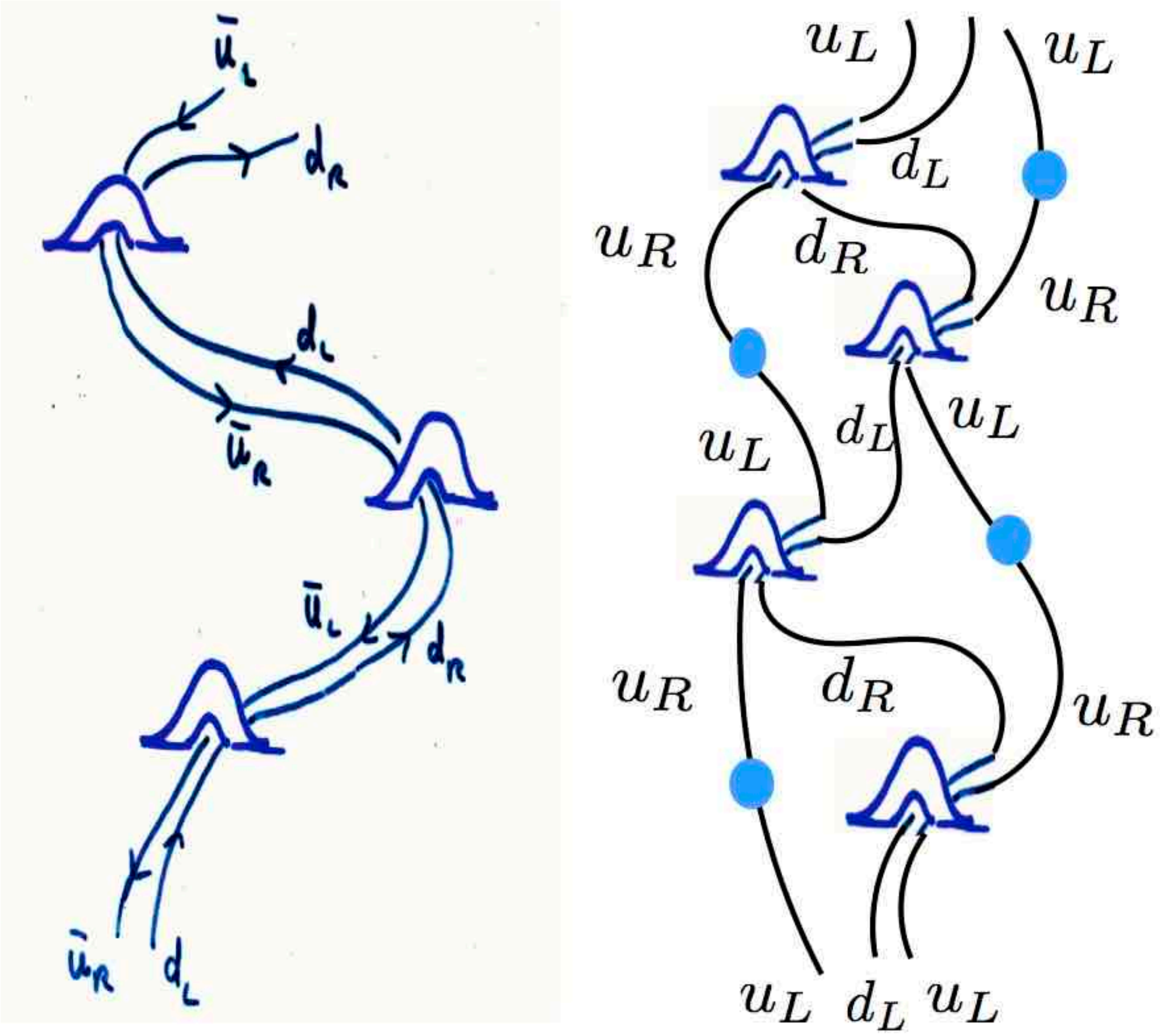
$$\frac{m}{M} = 1 - 8g_S\int\frac{d^4k}{(2\pi)^4}\frac{\mathcal{F}^2(k)}{k^2+M^2}. \tag{3.20}$$

After neglecting the momentum dependence of the running mass in the denominator of Eq. (3.20), as discussed in [Kock and Zahed, 2021], one finds that solutions for M exist only for couplings exceeding a critical value. In the instanton liquid model (ILM), this critical coupling is determined by the instanton-antiinstanton density.

The chiral condensate is obtained in a similar manner,

$$\langle\bar{\psi}\psi\rangle = -\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\,S(k) = -8N_cM\int\frac{d^4k}{(2\pi)^4}\frac{\mathcal{F}(k)}{k^2+M^2}. \tag{3.21}$$

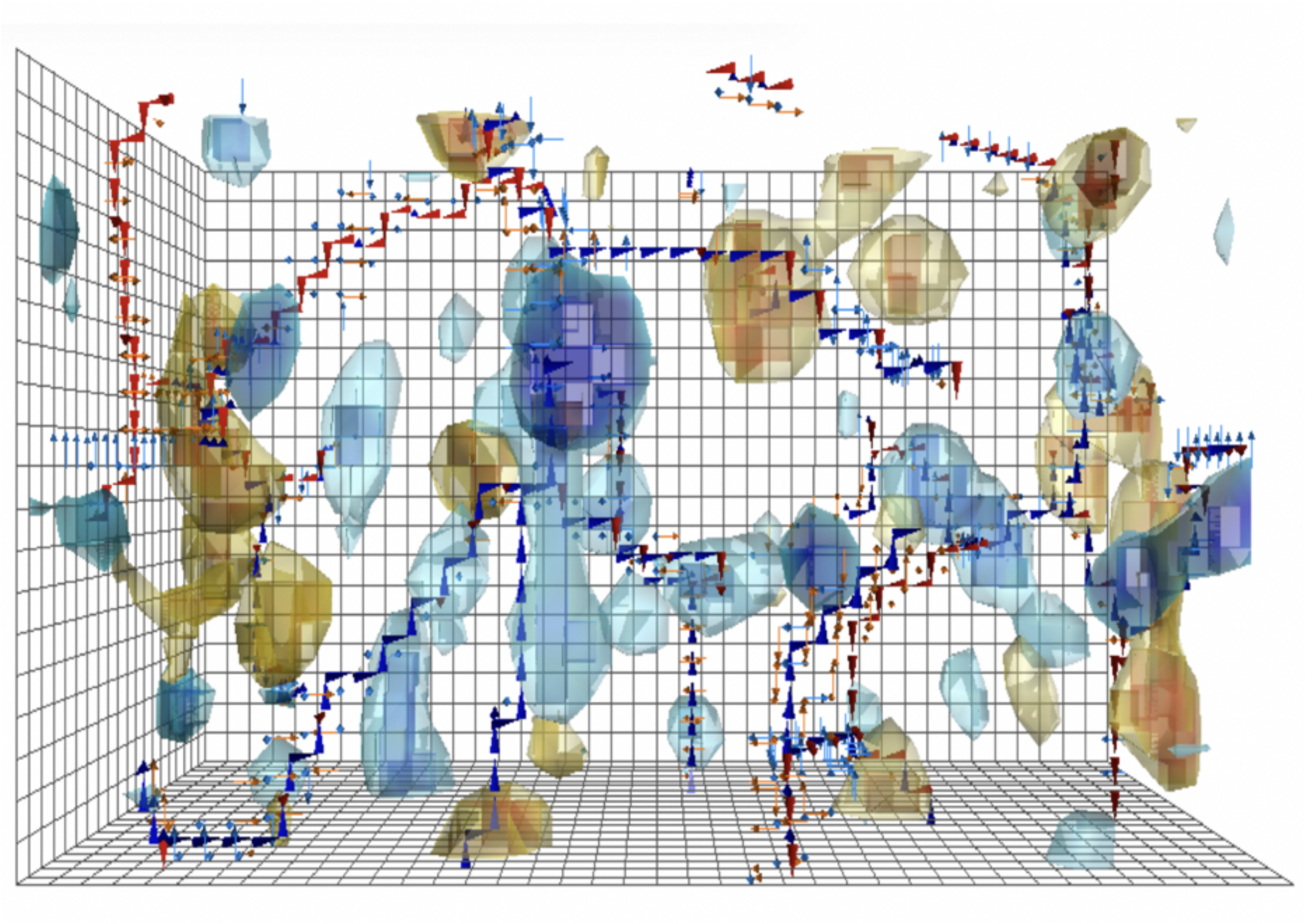
schematic picture for the pion and nucleon propagating in ILM



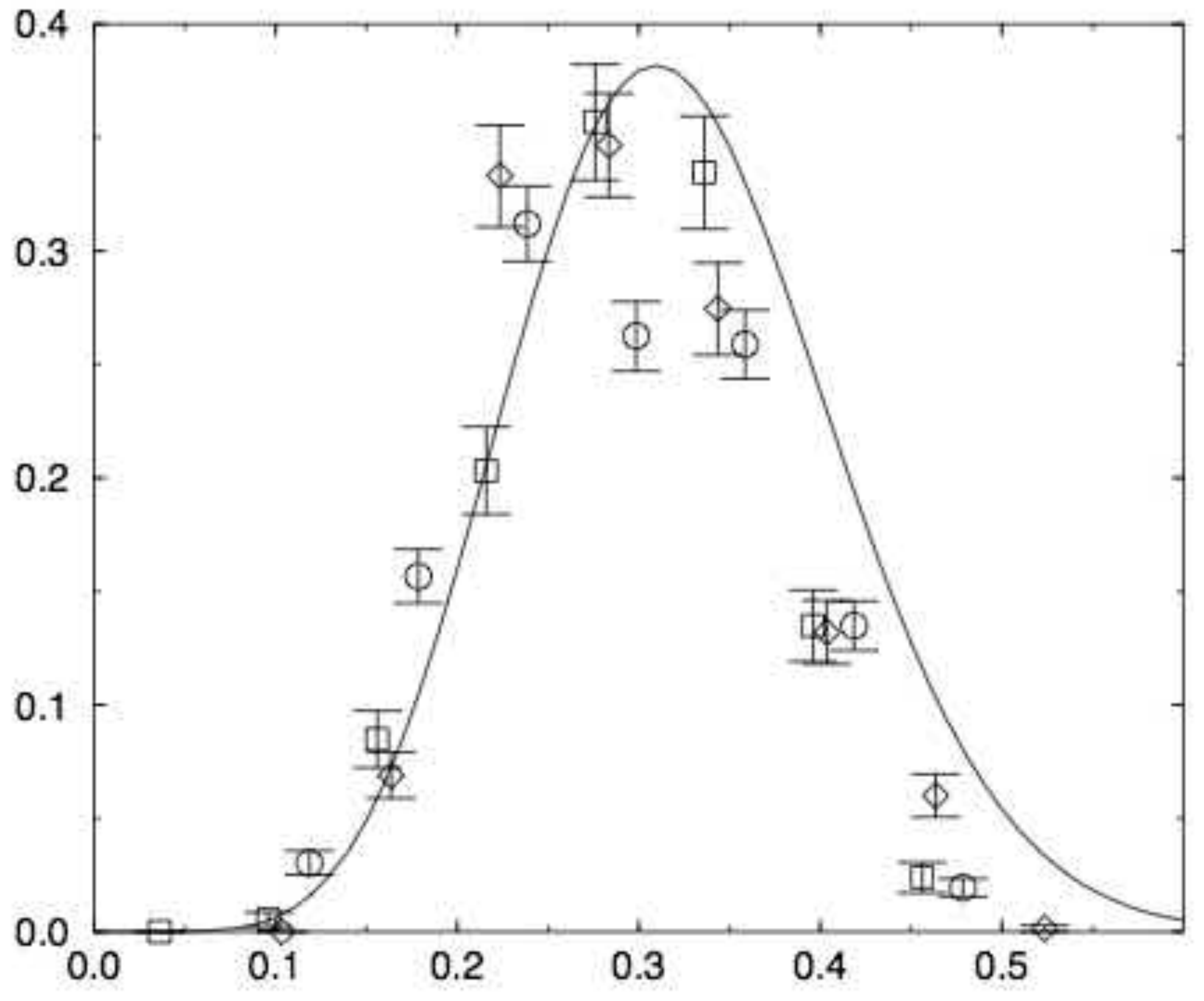
't Hooft 1976
discovered effective interaction of light fermions

which flips chirality
and antisymmetric in flavor

Instantons on the lattice



size distribution



Leinweber et al, 2020

A.Hazenfratz et al, 2000

instanton liquid model (ILM) has two components

$$\rho = 1/3\, fm, R = 1\, fm$$

$$\kappa = \pi^2 (\frac{\rho}{R})^4 \approx 0.12$$ ES, 1982

dilute:
consists of well-separated instantons
their collectivised zero modes = quark condensate

instanton liquid model (ILM) has two components

$$\rho = 1/3\, fm, R = 1\, fm$$

$$\kappa = \pi^2 (\frac{\rho}{R})^4 \approx 0.12$$ ES, 1982

dilute:
consists of well-separated instantons
their collectivised zero modes = quark condensate

"molecular component"
which is denser
because overlapping I and bar-I
has smaller action
but without near-zero Dirac eigenvalues!

instanton liquid model (ILM) has two components

$$\rho = 1/3\, fm, R = 1\, fm$$

$$\kappa = \pi^2 (\frac{\rho}{R})^4 \approx 0.12$$ ES, 1982

dilute:
consists of well-separated instantons
their collectivised zero modes = quark condensate

"molecular component" which is denser because overlapping I and bar-I has smaller action but without near-zero Dirac eigenvalues!

Ilgenfritz,ES 1988,1993

instanton liquid model (ILM) has two components

$$\rho = 1/3\, fm, R = 1\, fm$$

$$\kappa = \pi^2 (\frac{\rho}{R})^4 \approx 0.12$$ ES, 1982

dilute:
consists of well-separated instantons
their collectivised zero modes = quark condensate

"molecular component" which is denser because overlapping I and bar-I has smaller action but without near-zero Dirac eigenvalues!

Ilgenfritz,ES 1988,1993

current lattice studies with G^2,G^3 observables and gradient flow cooling (extrapolated to zero time) suggest kappa=O(1)

instanton liquid model (ILM) has two components

$$\rho = 1/3\, fm, R = 1\, fm$$

$$\kappa = \pi^2 (\frac{\rho}{R})^4 \approx 0.12$$

ES, 1982

dilute:
consists of well-separated instantons
their collectivised zero modes = quark condensate

"molecular component" which is denser because overlapping I and bar-I has smaller action but without near-zero Dirac eigenvalues!

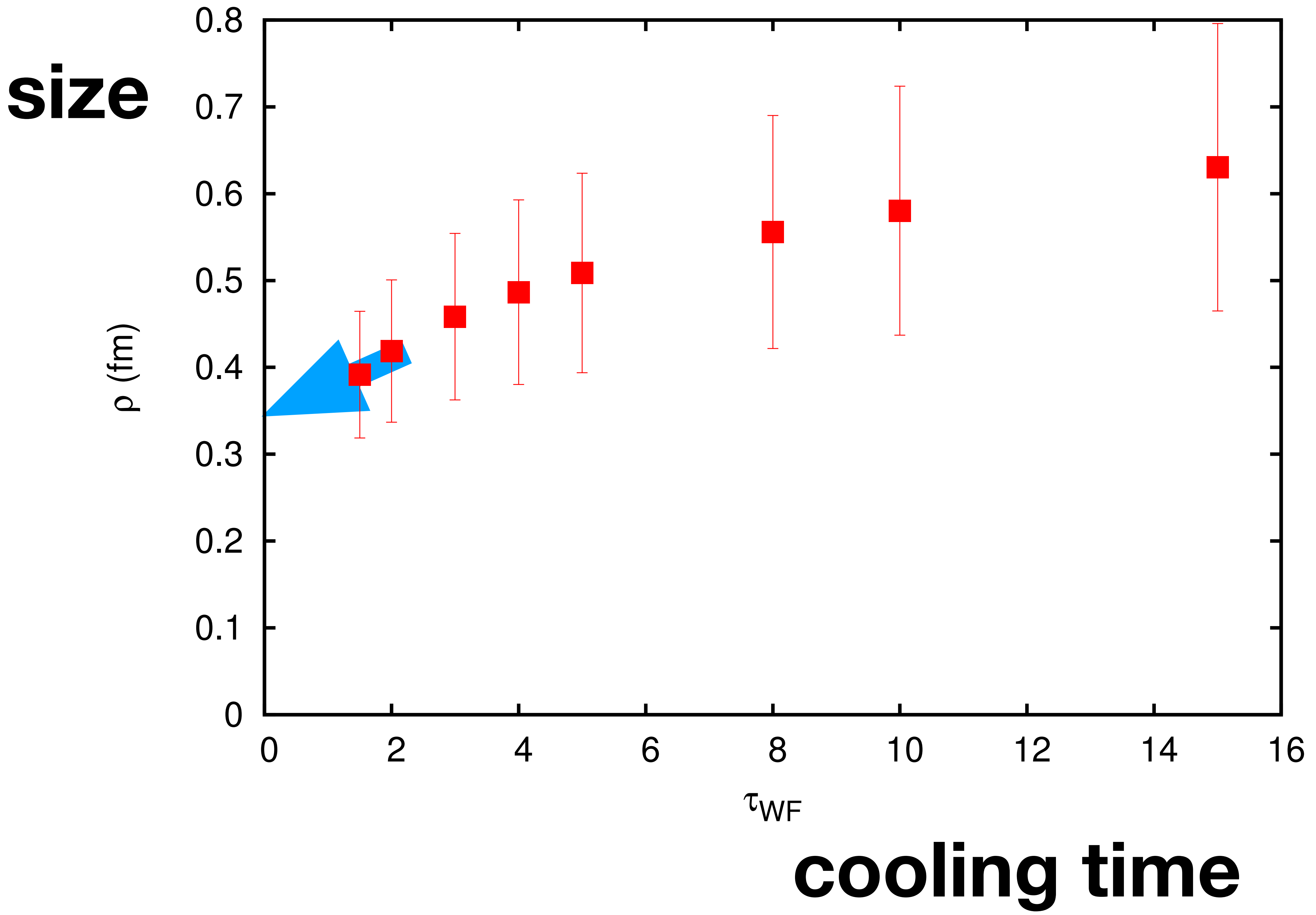
Ilgenfritz,ES 1988,1993

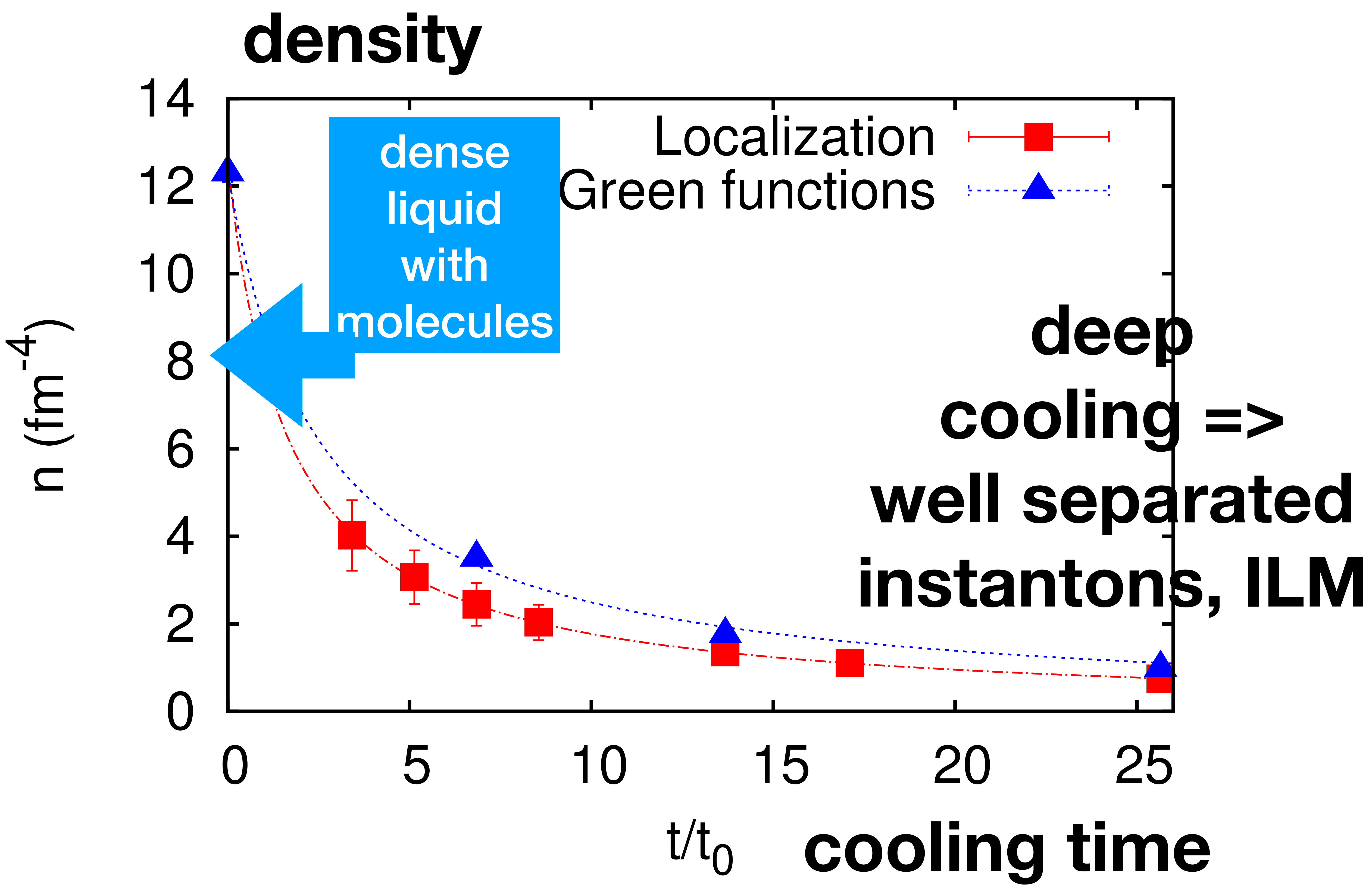
current lattice studies with G^2,G^3 observables and gradient flow cooling (extrapolated to zero time) suggest kappa=O(1)

in the ff plots we used kappa=1 and this gets the data!

- A. Athenodorou, P. Boucaud, F. De Soto, J. Rodríguez-Quintero, and S. Zafeiropoulos JHEP 02, 140 (2018), arXiv:1801.10155 [hep-lat].

- **Careful study of "cooling" by gradient flow**





Strongly correlated pairs or molecules:

remain there at T>Tc

``streamline equation"

solved numerically

used to calculate sphaleron production

[Ilgenfritz and Shuryak, 1989].

[Balitsky and Yung, 1986],

[Verbaarschot, 1991].

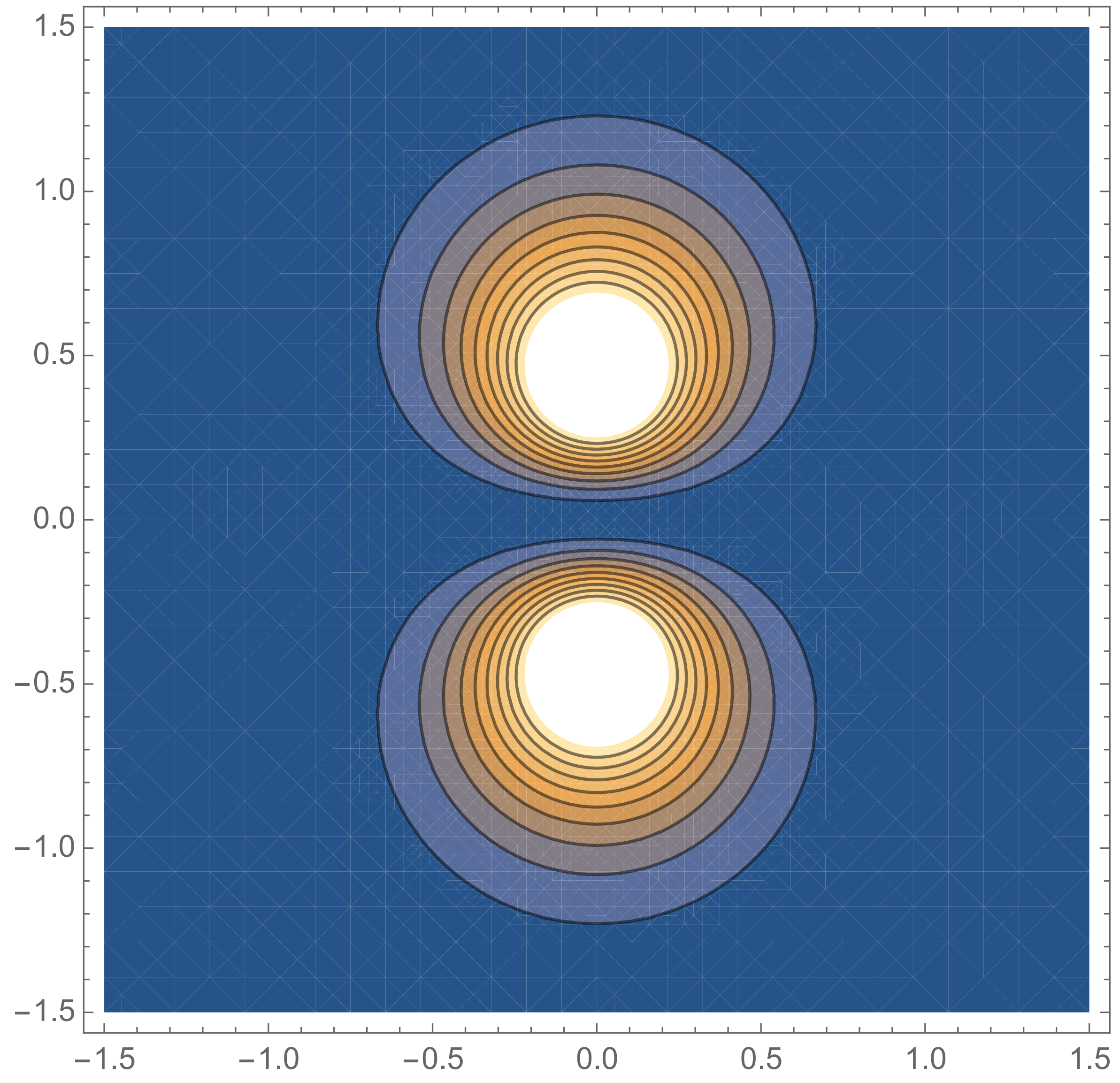
Shuryak and Verbaarschot, 1992]

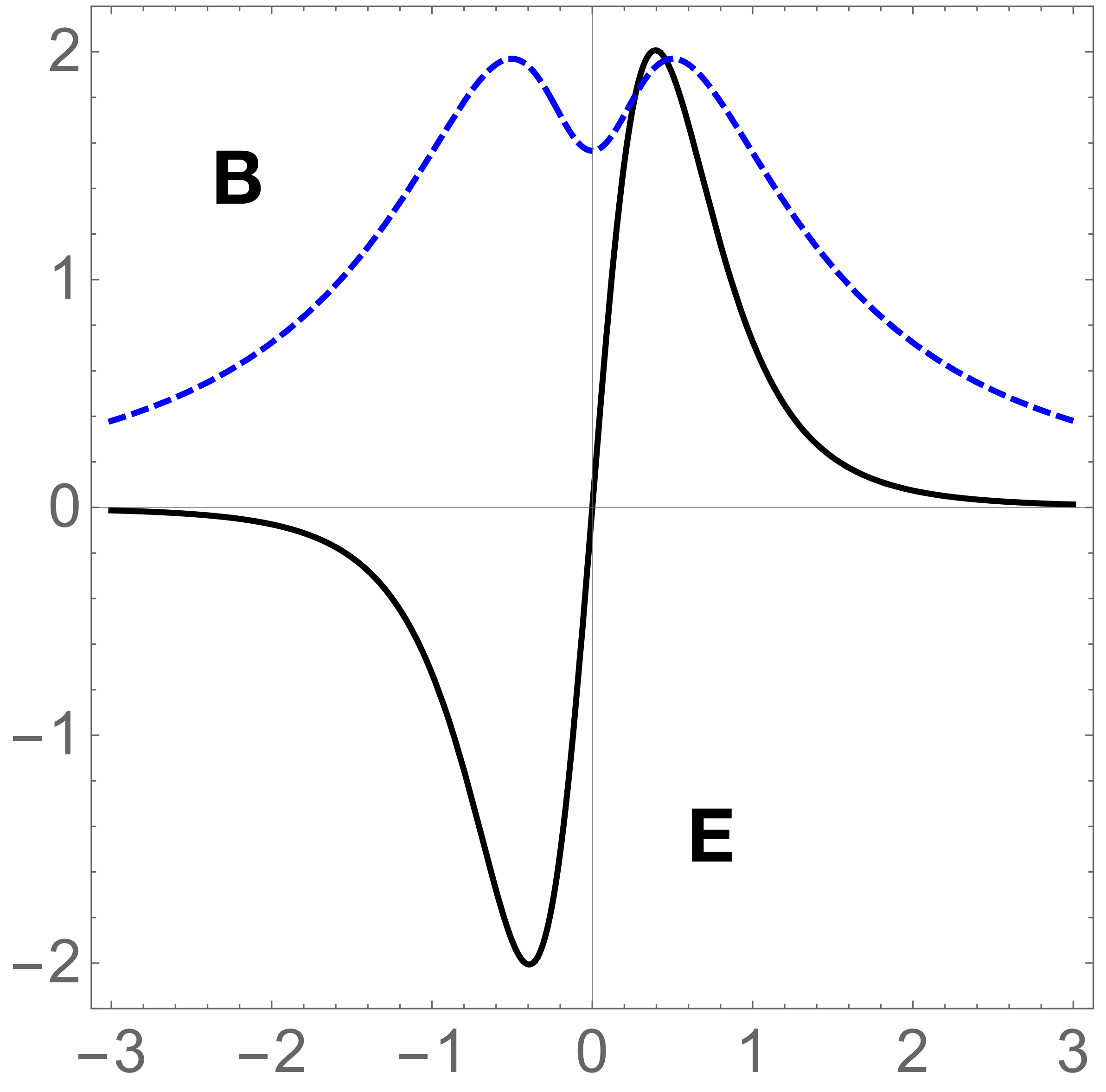
[Shuryak and Zahed, 2021

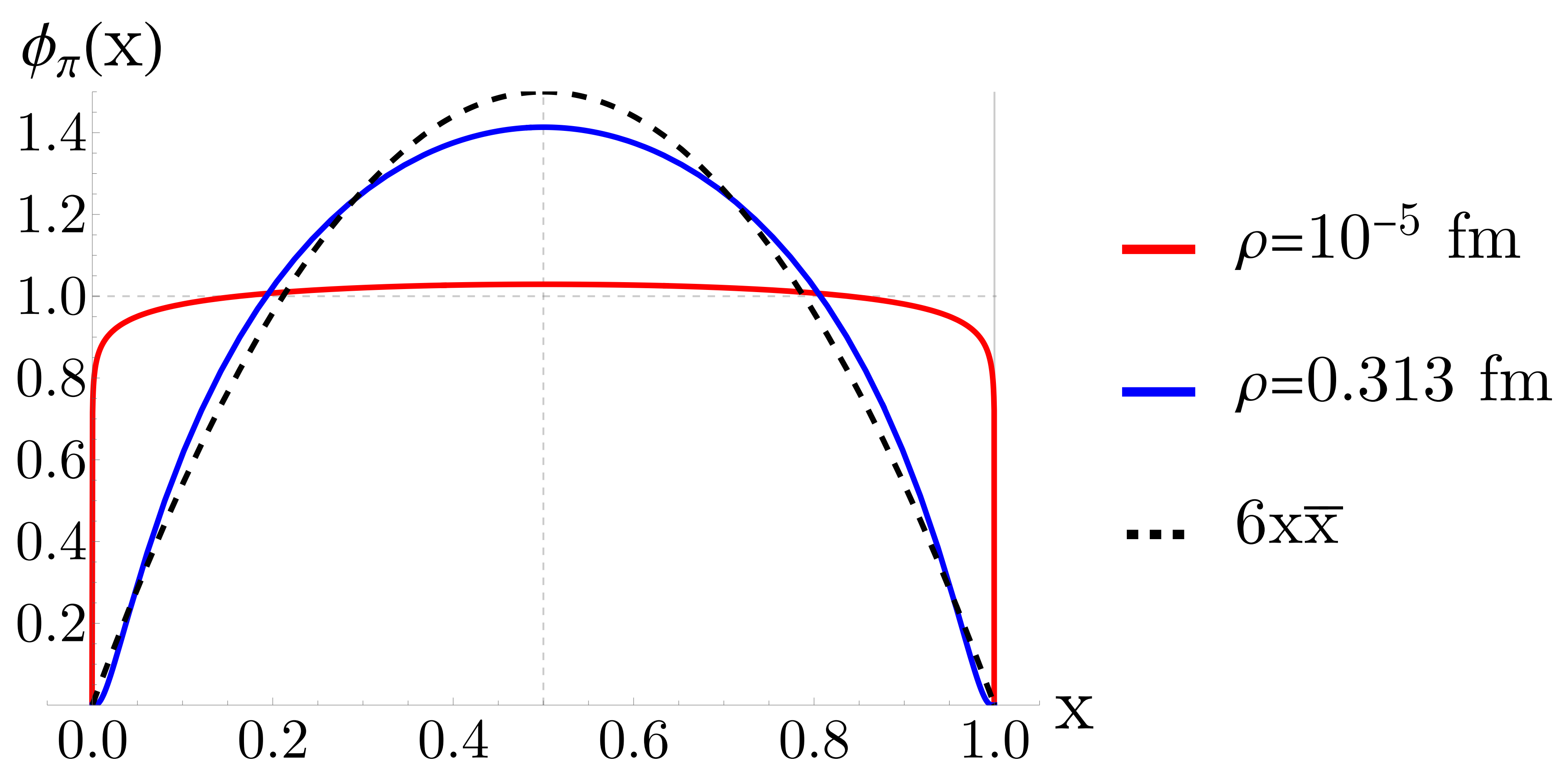
$$A^{\mu a}(x) = \frac{\bar\eta^{a\mu\nu} y_I^\nu \rho^2/Y_I^2 + \eta^{a\mu\nu} y_A^\nu \rho^2/Y_A^2}{1+\rho^2/Y_A+\rho^2/Y_I},$$

strong fields, O(few GeV^2)

action density G^2



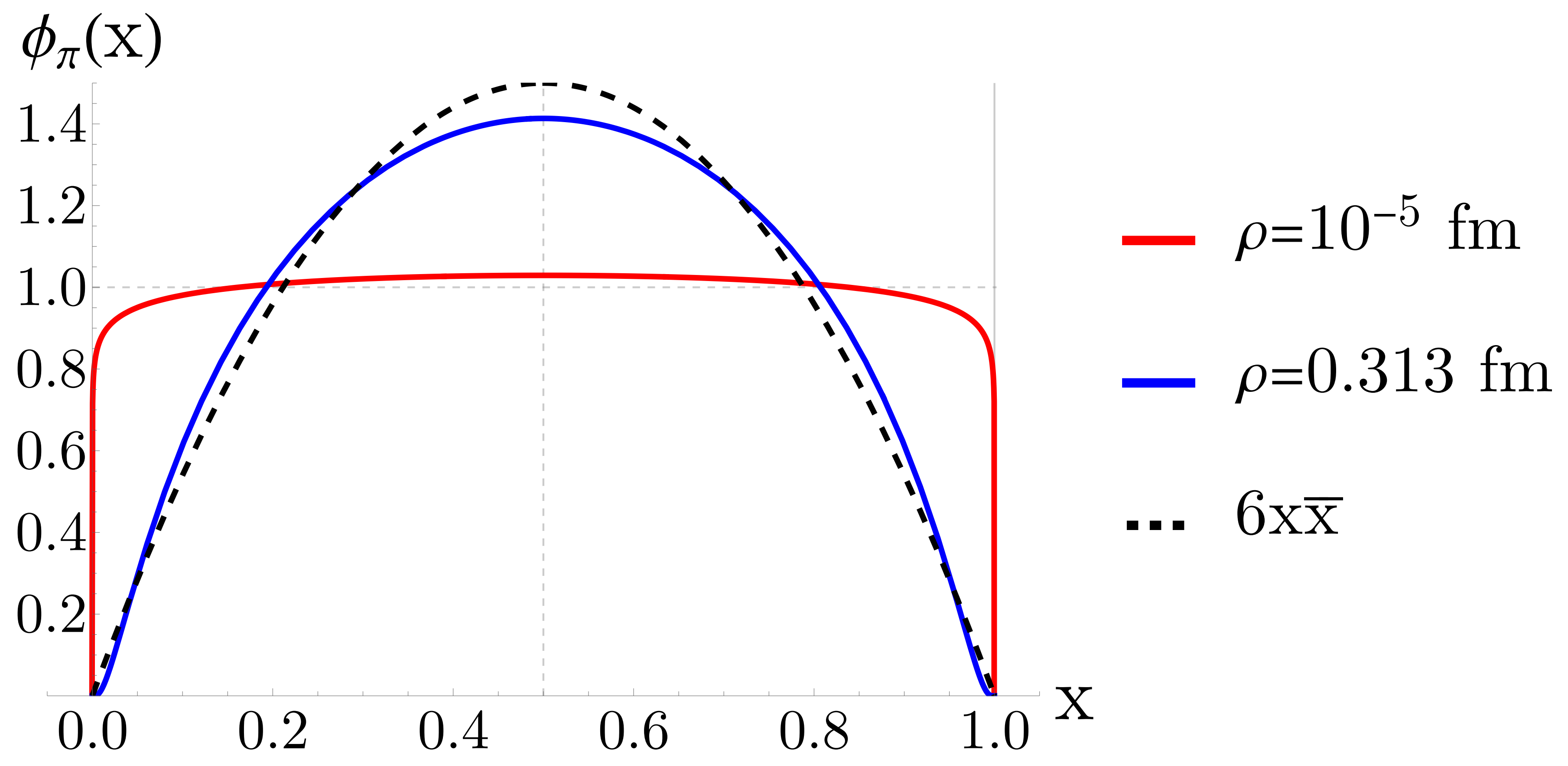




$$\varphi_P(x) = \frac{2N_c}{f_P^2}\int \frac{d^2k_\perp}{(2\pi)^3}\frac{\theta(x\bar{x})}{(k_\perp^2 + M^2(0,m_f) - x\bar{x}m_P^2)} M^2\left(\frac{\sqrt{k_\perp^2 + M^2(0,m_f)}}{\lambda_P\sqrt{x\bar{x}}}\right)$$

$$M(k) = M(0)\left(\left|z\left(I_0K_0 - I_1K_1\right)'\right|^2\right)_{z=\frac{1}{2}\rho k}$$

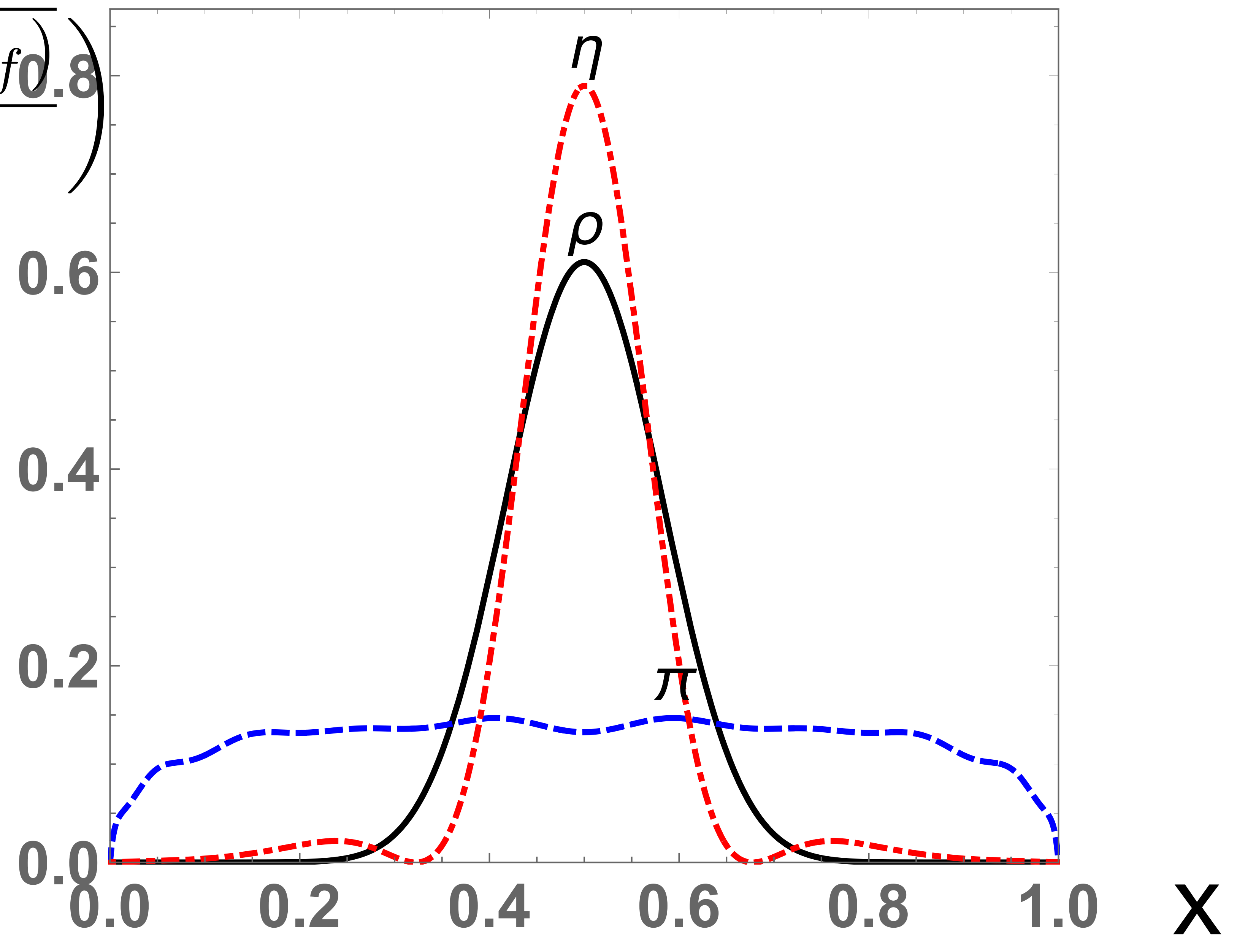
meson distribution amplitudes in ILM



$$\varphi_P(x) = \frac{2N_c}{f_P^2}\int \frac{d^2k_\perp}{(2\pi)^3}\frac{\theta(x\bar{x})}{(k_\perp^2 + M^2(0,m_f) - x\bar{x}m_P^2)}\,M^2\left(\frac{\sqrt{k_\perp^2 + M^2(0,m_f)}}{\lambda_P\sqrt{x\bar{x}}}\right)$$

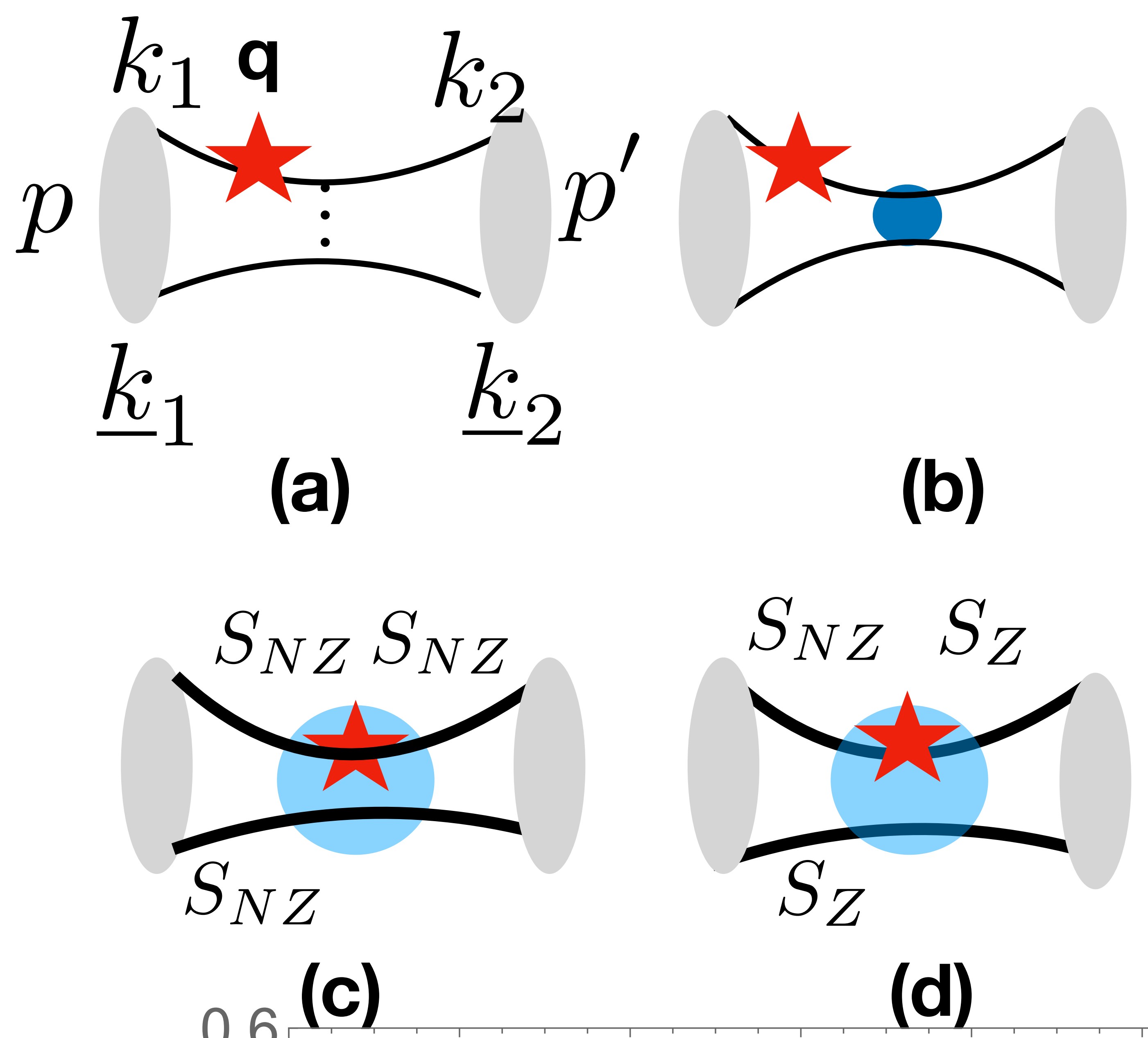
$$M(k) = M(0)\left(\left|z\left(I_0K_0 - I_1K_1\right)'\right|^2\right)_{z=\frac{1}{2}\rho k}$$

meson distribution amplitudes in ILM



The first usage of "molecules" was for hard block in the pion formfactor

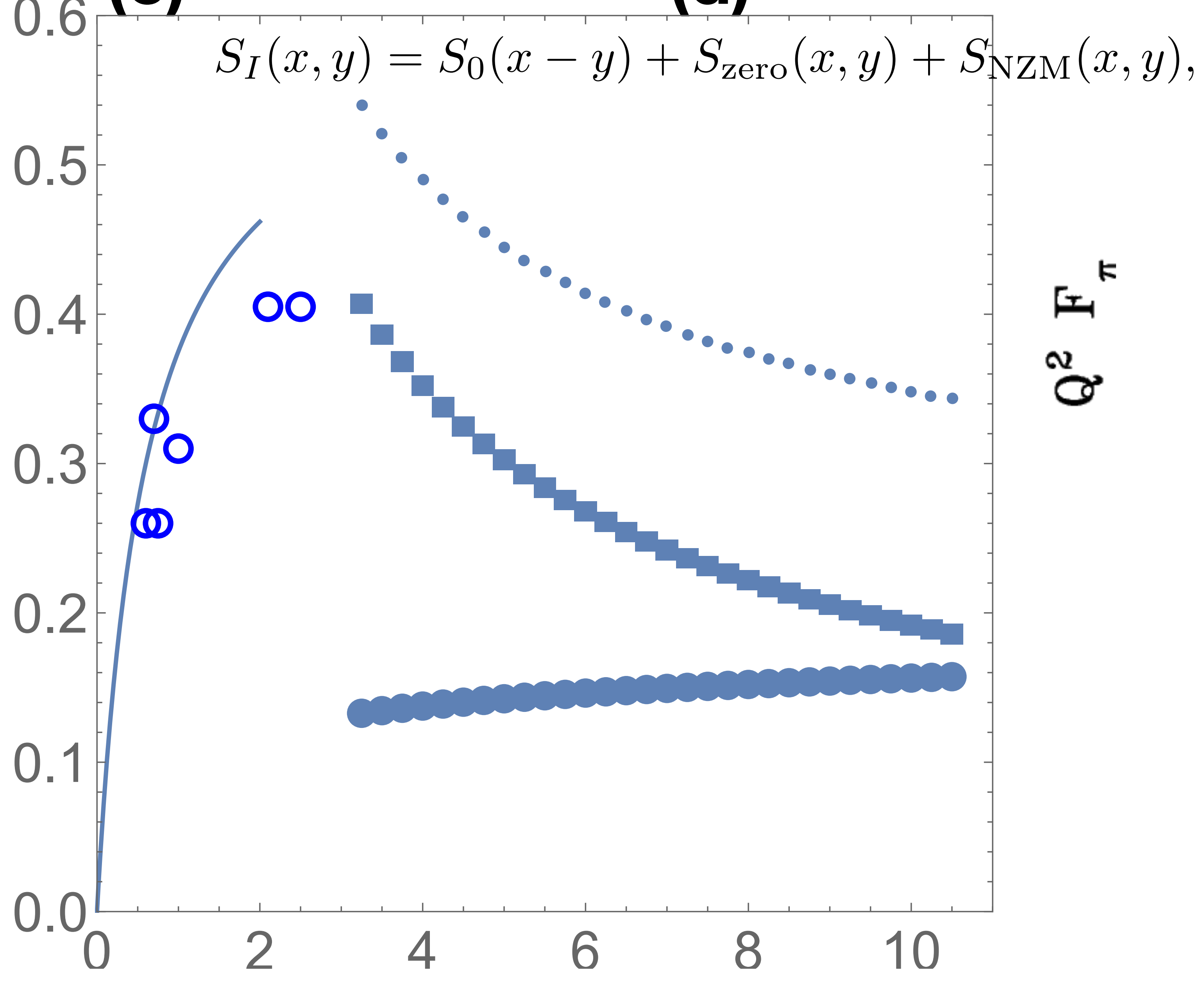
ES+Zahed, 2019



$$\pi\alpha_s(Q/2) \to \kappa \left\langle \mathbb{G}^2(Q\rho\sqrt{\bar{x}_1\bar{x}_2})\right\rangle$$

$$V_b^\pi(Q^2) = \epsilon_\mu(q)(p^\mu + p'^\mu)\,(e_u + e_{\bar{d}})\left(\frac{2C_F\kappa f_\pi^2}{N_c Q^2}\right)\int dx_1 dx_2\,\langle \mathbb{G}^2(Q\rho\sqrt{\bar{x}_1\bar{x}_2})\rangle$$
$$\times\left[\frac{\varphi_\pi(x_1)\varphi_\pi(x_2)}{\bar{x}_1\bar{x}_2 + m^2_{\rm gluon}/Q^2} + \frac{\chi_\pi^2}{Q^2}\left(\frac{\tilde{\varphi}_\pi(x_1)\tilde{\varphi}_\pi(x_2)}{\bar{x}_1\bar{x}_2 + m^2_{\rm gluon}/Q^2}\right)\left(\frac{1}{\bar{x}_1 + E_\perp^2/Q^2} + \frac{1}{\bar{x}_2 + E_\perp^2/Q^2} - 2\right)\right]$$

$$V_c^\pi = \epsilon_\mu(q)(p^\mu + p'^\mu)\,(e_u + e_{\bar{d}})\,\frac{\kappa\pi^2\rho^2 f_\pi^2\chi_\pi^2}{N_c M^2}\langle \mathbb{G}_V(Q\rho)\rangle\int dx\,\tilde{\varphi}_\pi(x)x$$

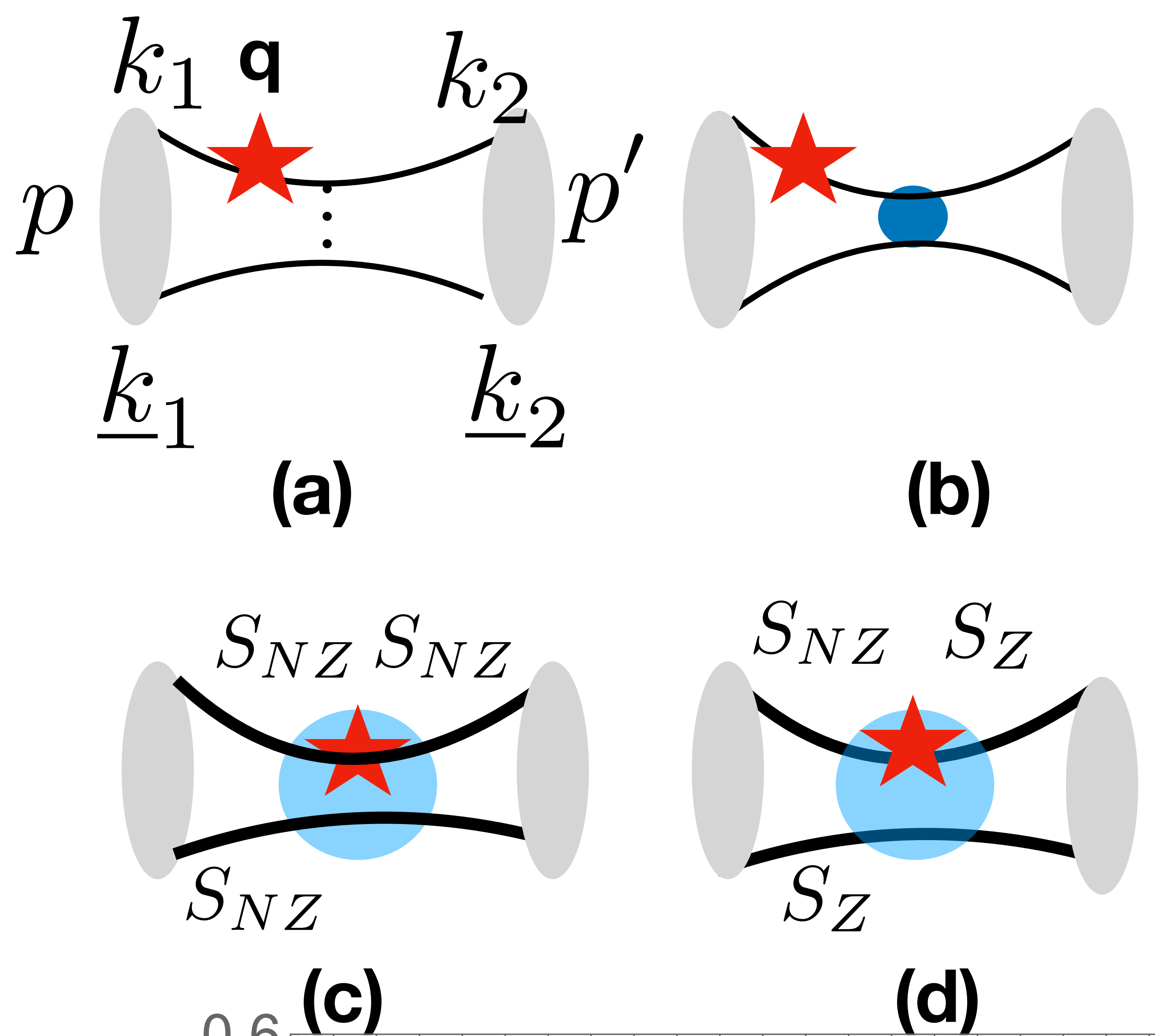


lattice

[Ding et al., 2024]

The first usage of ``molecules" was for hard block in the pion formfactor

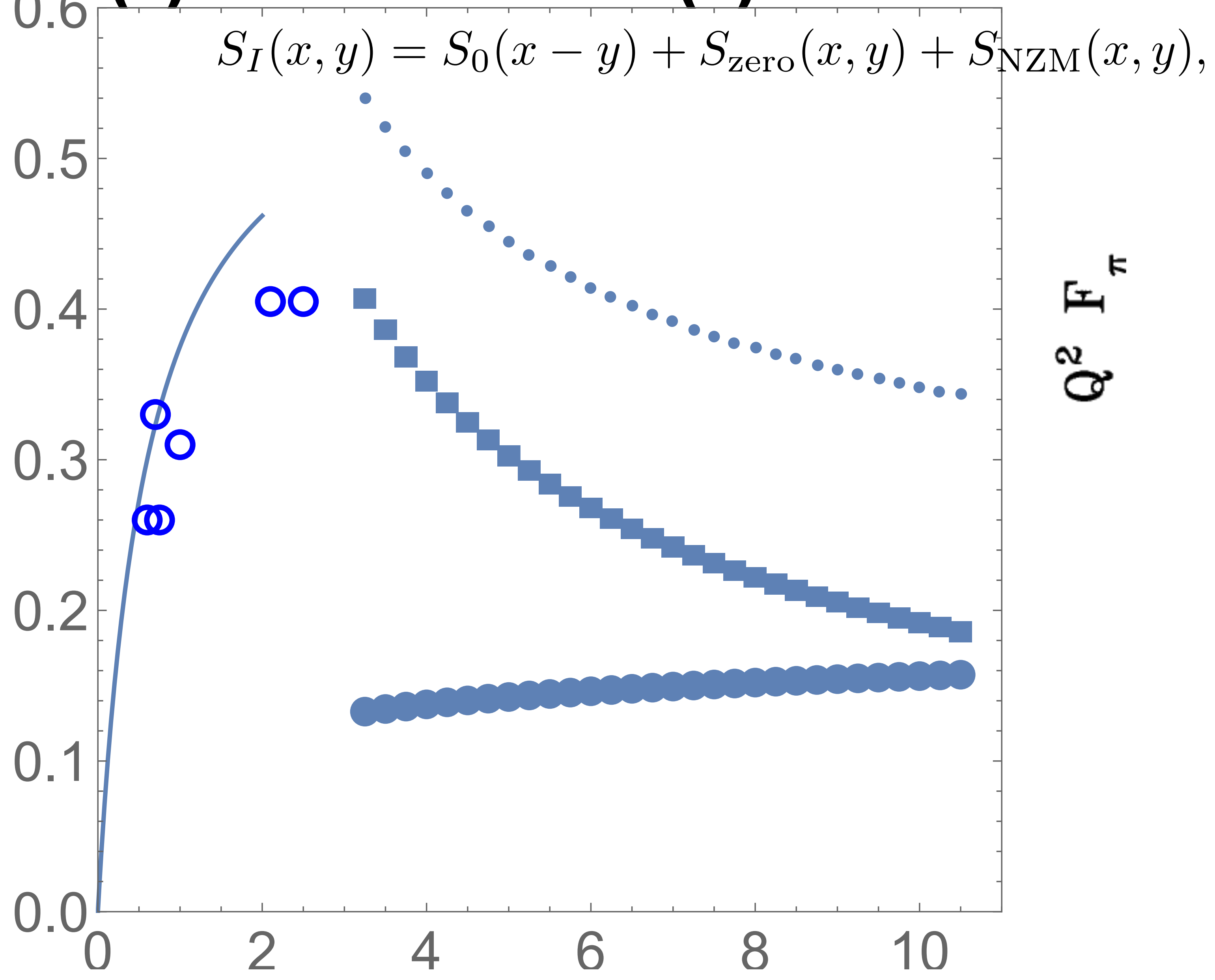
ES+Zahed, 2019

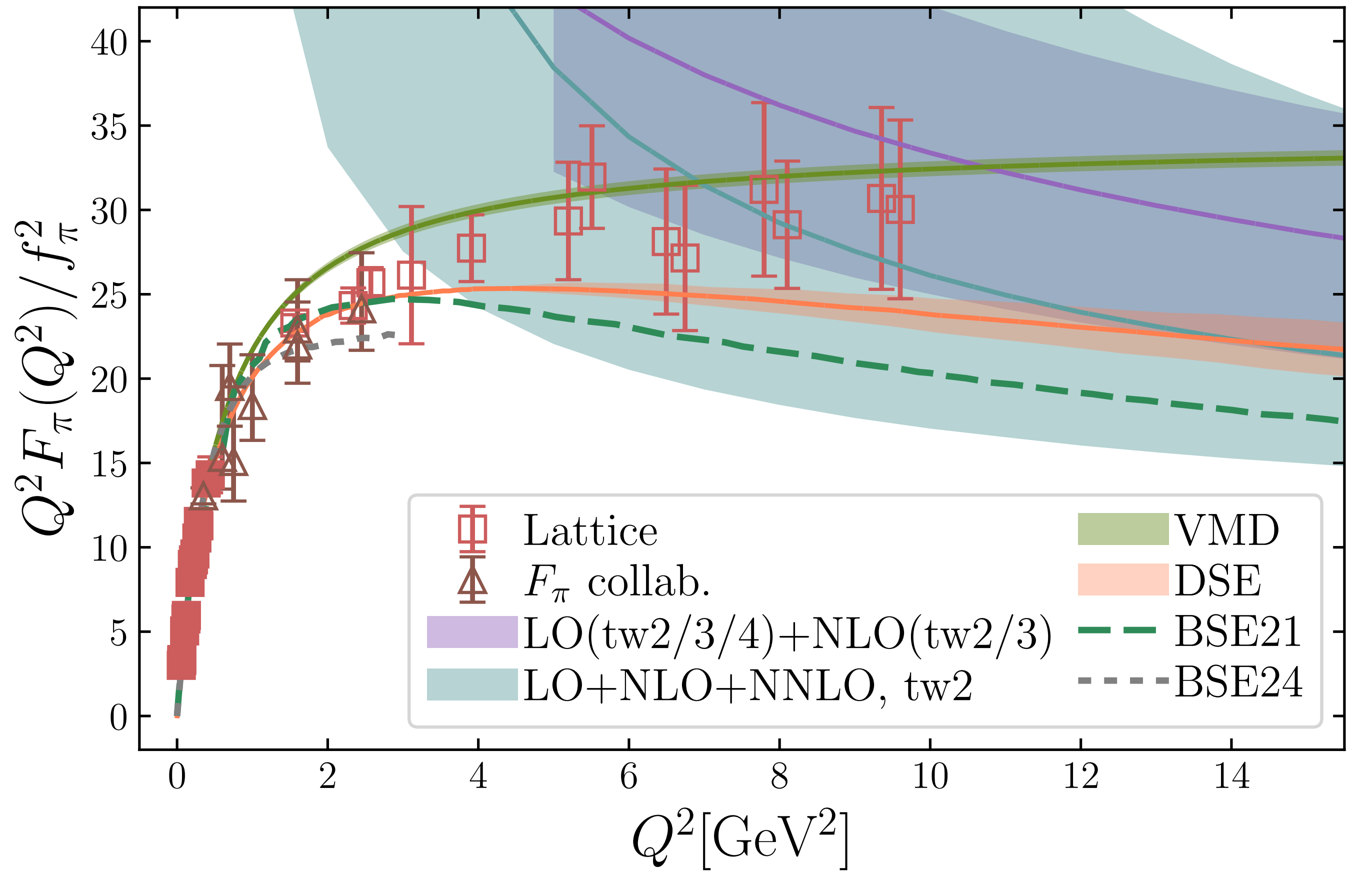


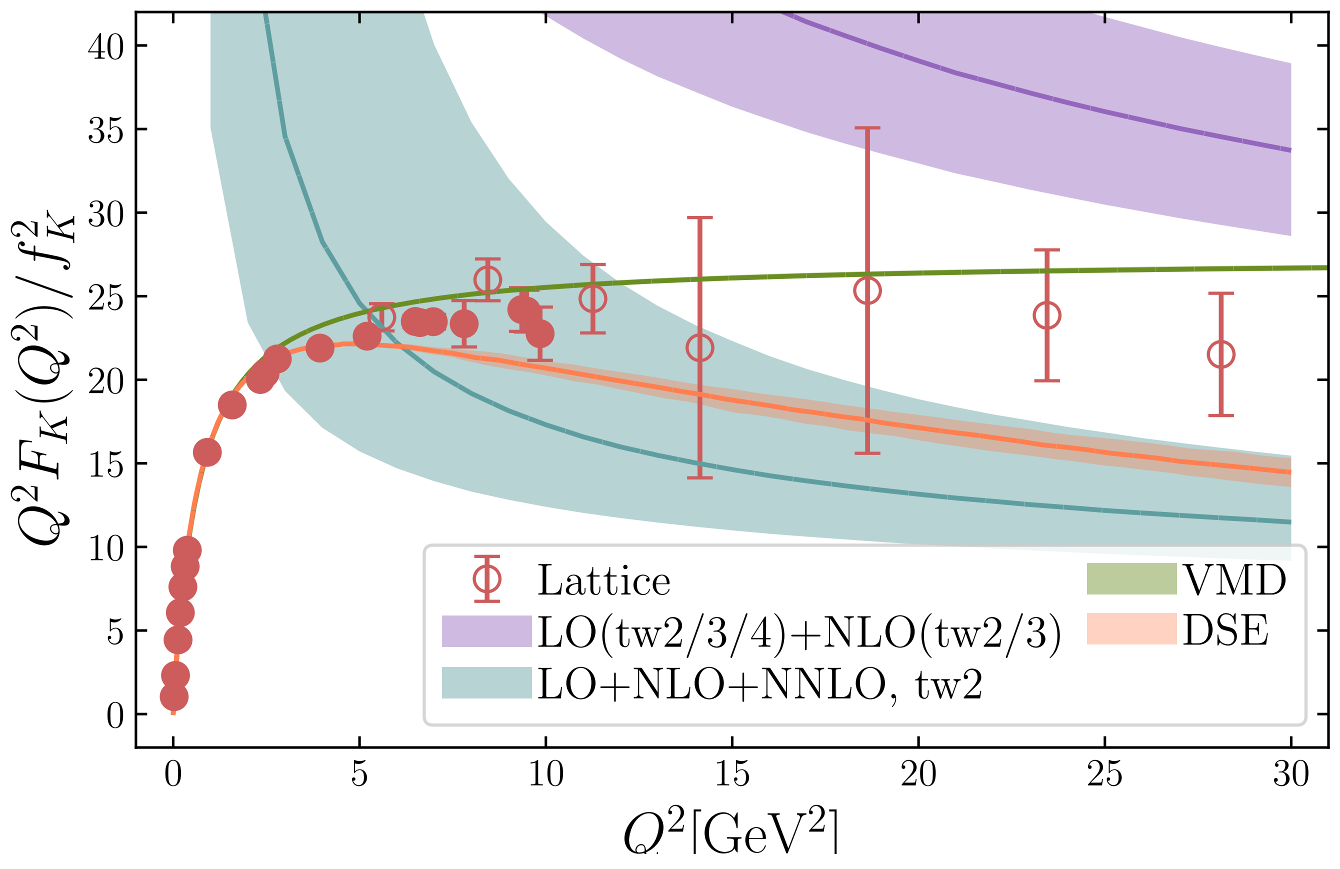
$$\pi\alpha_s(Q/2) \to \kappa\left\langle \mathbb{G}^2(Q\rho\sqrt{\bar{x}_1\bar{x}_2})\right\rangle$$

$$V_b^\pi(Q^2) = \epsilon_\mu(q)(p^\mu + p'^\mu)\,(e_u + e_{\bar{d}})\left(\frac{2C_F\kappa f_\pi^2}{N_c Q^2}\right)\int dx_1 dx_2\,\left\langle \mathbb{G}^2(Q\rho\sqrt{\bar{x}_1\bar{x}_2})\right\rangle$$
$$\times\left[\frac{\varphi_\pi(x_1)\varphi_\pi(x_2)}{\bar{x}_1\bar{x}_2 + m^2_{\rm gluon}/Q^2} + \frac{\chi_\pi^2}{Q^2}\left(\frac{\tilde{\varphi}_\pi(x_1)\tilde{\varphi}_\pi(x_2)}{\bar{x}_1\bar{x}_2 + m^2_{\rm gluon}/Q^2}\right)\left(\frac{1}{\bar{x}_1 + E_\perp^2/Q^2} + \frac{1}{\bar{x}_2 + E_\perp^2/Q^2} - 2\right)\right]$$

$$V_c^\pi = \epsilon_\mu(q)(p^\mu + p'^\mu)\,(e_u + e_{\bar{d}})\,\frac{\kappa\pi^2\rho^2 f_\pi^2\chi_\pi^2}{N_c M^2}\langle \mathbb{G}_V(Q\rho)\rangle\int dx\,\tilde{\varphi}_\pi(x)x$$

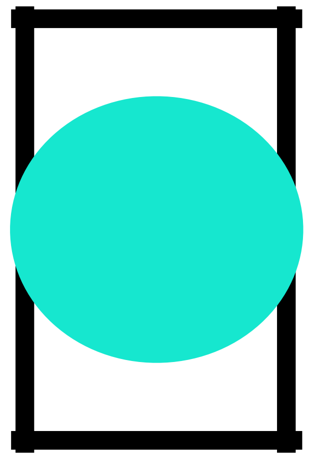






lattice

[Ding et al., 2024]



Instanton effects in central potentials

$$e^{-V_c(r)T} = \langle W(\vec{x}_1) W^+(\vec{x}_2) \rangle$$

spin forces are related to WGWG nonlocals

$$W = Pexp[ig \int dx^\mu A^a_\mu \hat{t}^a]$$

angle of color rotationalong a straight line
is easy to calculatefor instanton fields

Callan et al 1978,
Eichten, Feinberg 1981

$$V_{instanton}(r) = \frac{4\pi n_{\bar{I}+I} \rho^3}{N_c \rho} I(\frac{r}{\rho})$$

$$I(x) = \int_0^\infty dy y^2 \int_{-1}^1 dc [1 - cos(\alpha_1) cos(\alpha_2) - \frac{y + xc}{\sqrt{y^2 + x^2 + 2xyc}} sin(\alpha_1) sin(\alpha_2)]$$

in which $c = cos(\phi)$, and two color rotation angles are

$$\alpha_1 = \pi \frac{y}{\sqrt{y^2+1}}, \quad \alpha_2 = \pi \sqrt{\frac{y^2 + x^2 + 2xyc}{y^2 + x^2 + 2xyc + 1}}$$

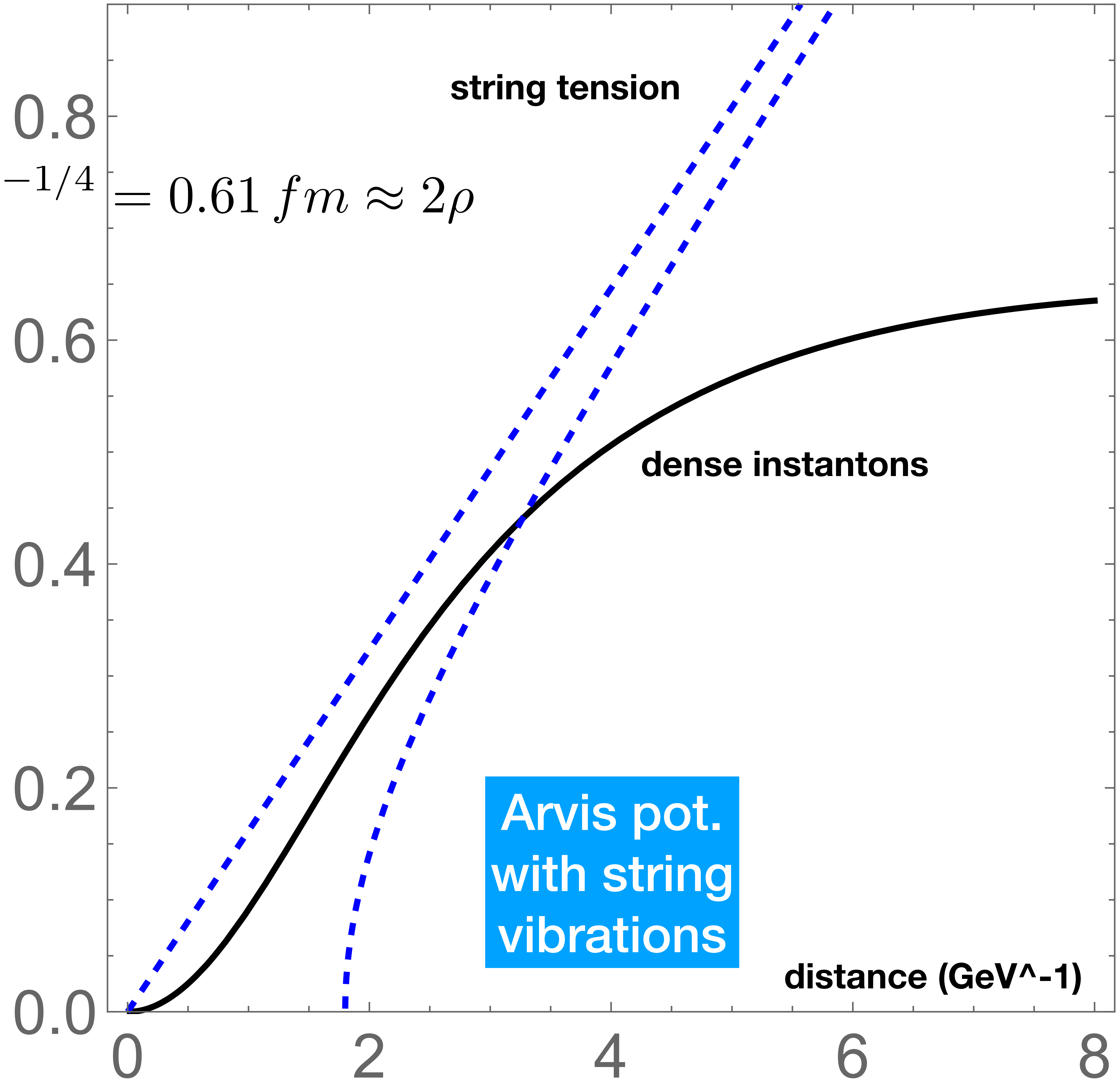
Nonperturbative central potential in ILM

$$n_{mol} + n_I + n_{\bar{I}} = 7.\,\text{fm}^{-4} \quad R_{dense} \equiv n^{-1/4} = 0.61\, fm \approx 2\rho$$

instanton-induced is as good as Cornell potential for many mesons, e.g.

$$\Upsilon[1S], \eta_b[1S], \Upsilon[2S], \Upsilon[3S], \Upsilon[4S],$$

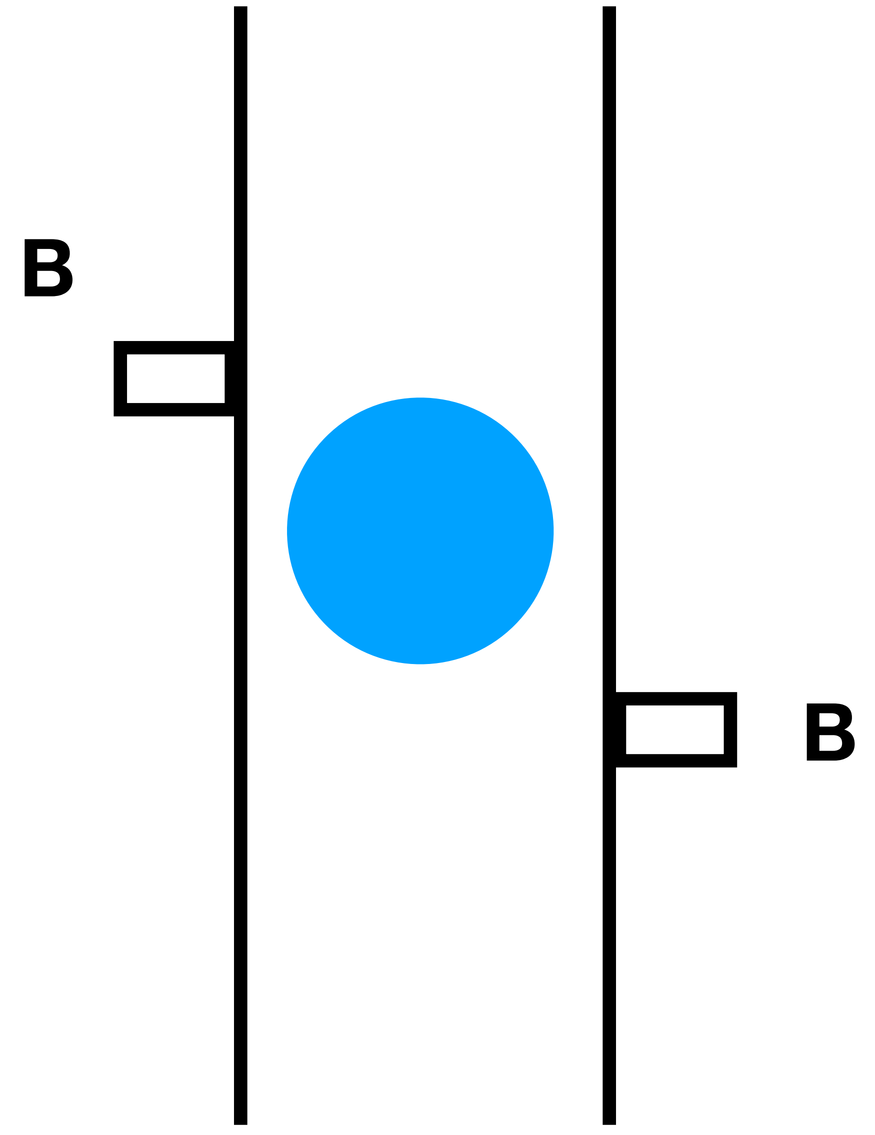
but for bb states with N>4 or ligher quark states, still one has to use linear for r> 1 fm



relativistic corrections to order 1/M^2 have in genral 5 potentials
3 spin-dependent potentials
spin-spin, spin-orbit and tensor

E. Eichten and F. Feinberg, "Spin Dependent Forces in QCD," Phys. Rev. D **23**, 2724 (1981

the corresponding potentials are given by Wilson lines decorated by field strength insertions



Here is the main point: electric flux tubes models do not give predictions for magnetic fields

flux tube is electric
spins interact with magnetic field

instantons on the other hand are selfdual E=B

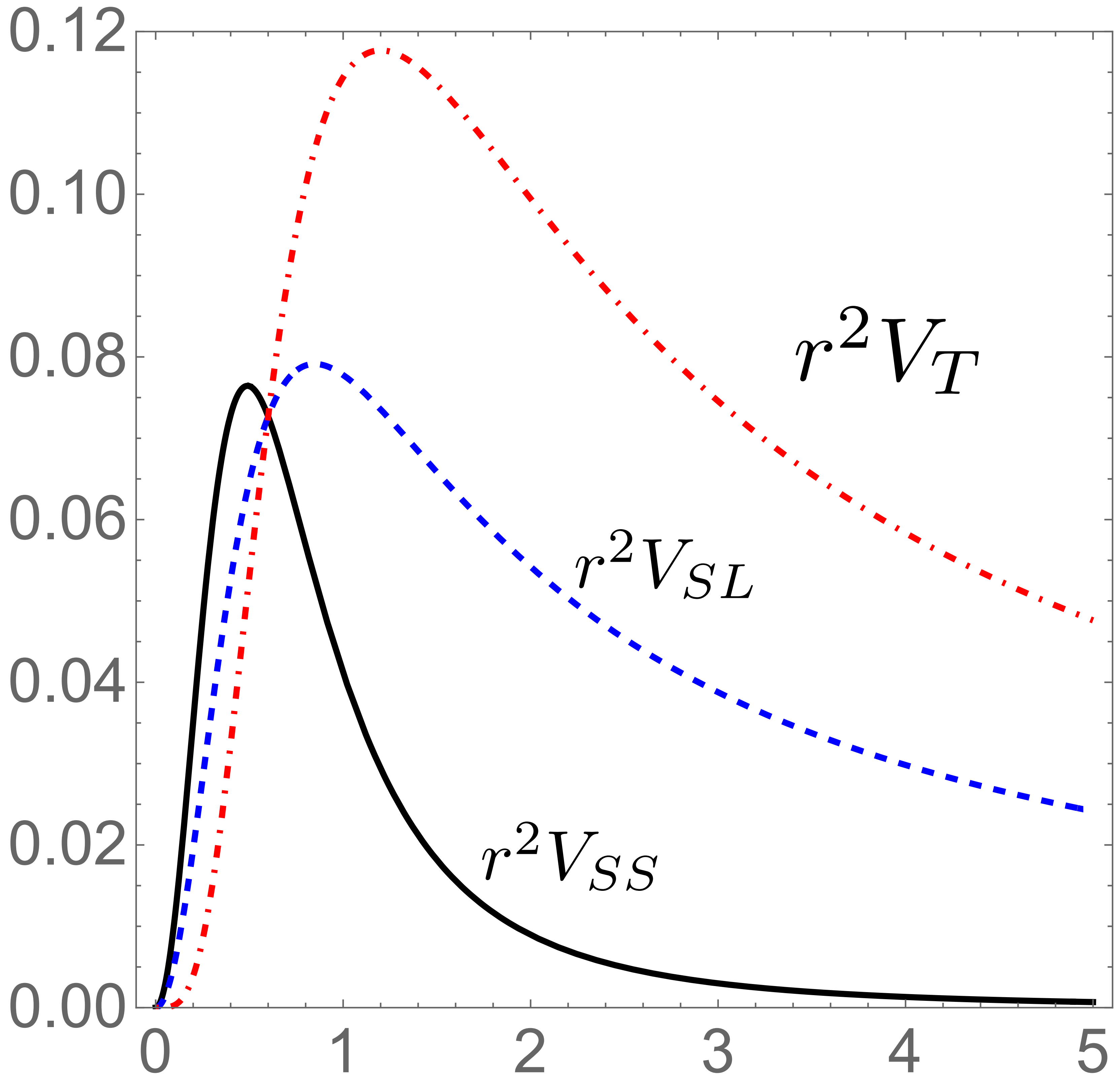
Instanton effects in spin-related potentials

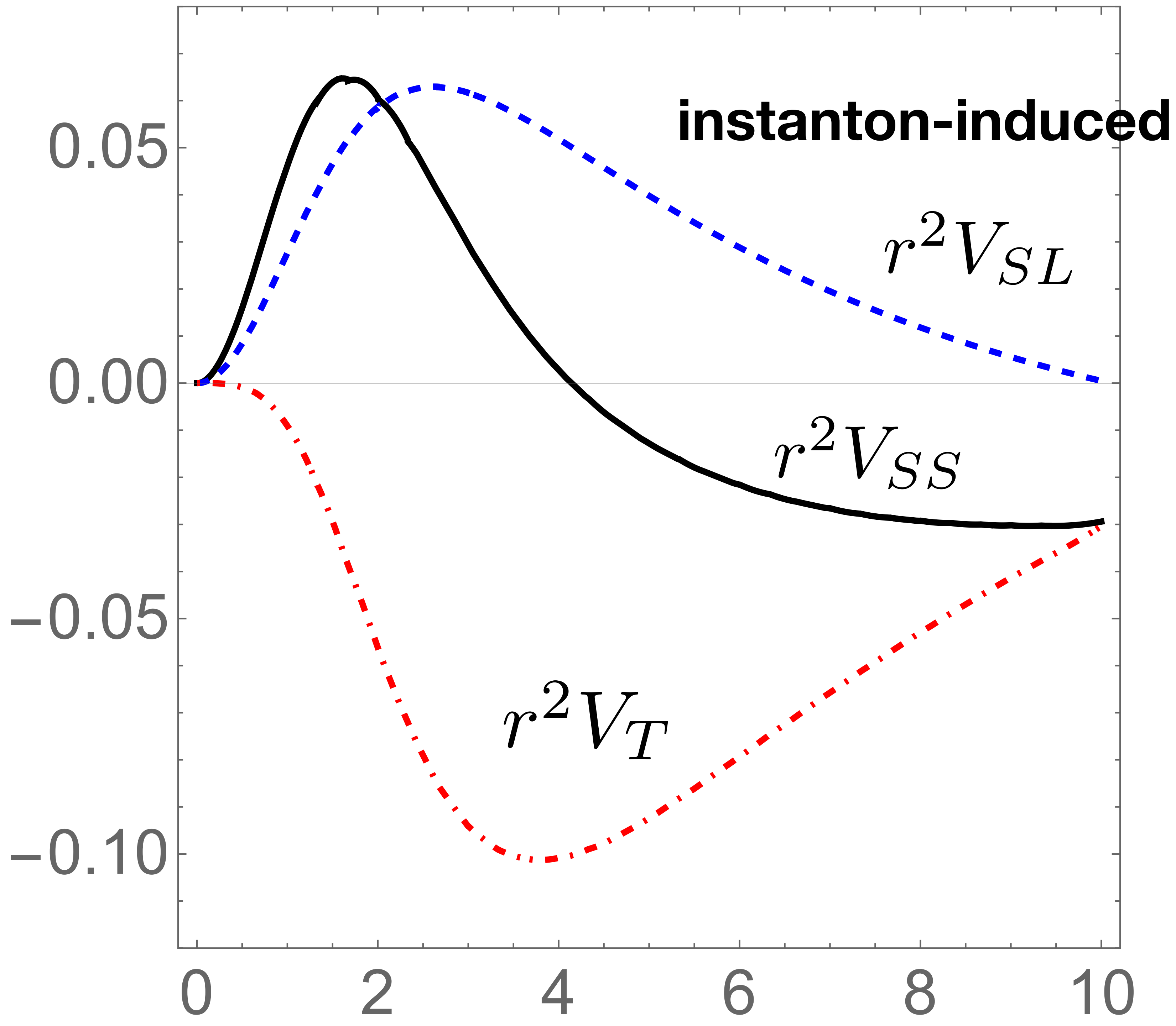
**Wilson lines complemented by two field strengths =>
in general, 5 potentials
for instantons related to Vc**

E. Eichten and F. Feinberg, "Spin Dependent Forces in QCD," Phys. Rev. D **23**, 2724 (1981

$$V_{SD} = \left(\frac{S_Q \cdot L_Q}{2m_Q^2} - \frac{S_{\bar{Q}} \cdot L_{\bar{Q}}}{2m_{\bar{Q}}^2}\right)\left(\frac{1}{r}\frac{d}{dr}(V(r) + 2V_1(r))\right)$$
$$+ \left(\frac{S_{\bar{Q}} \cdot L_Q}{m_Q m_{\bar{Q}}} - \frac{S_Q \cdot L_{\bar{Q}}}{m_{\bar{Q}} m_{\bar{Q}}}\right)\left(\frac{1}{r}\frac{d}{dr}V_2(r)\right)$$
$$+ \frac{(3S_Q \cdot \hat{r} S_{\bar{Q}} \cdot \hat{r} - S_Q \cdot S_{\bar{Q}})}{3m_Q m_{\bar{Q}}} V_3(r) + \frac{1}{3}\frac{S_Q \cdot S_{\bar{Q}}}{m_Q m_{\bar{Q}}} V_4(r)$$

perturbative





r (GeV^-1)

Their sum explains lattice data for Vss and explains spin splittings rather well, except in light-light mesons

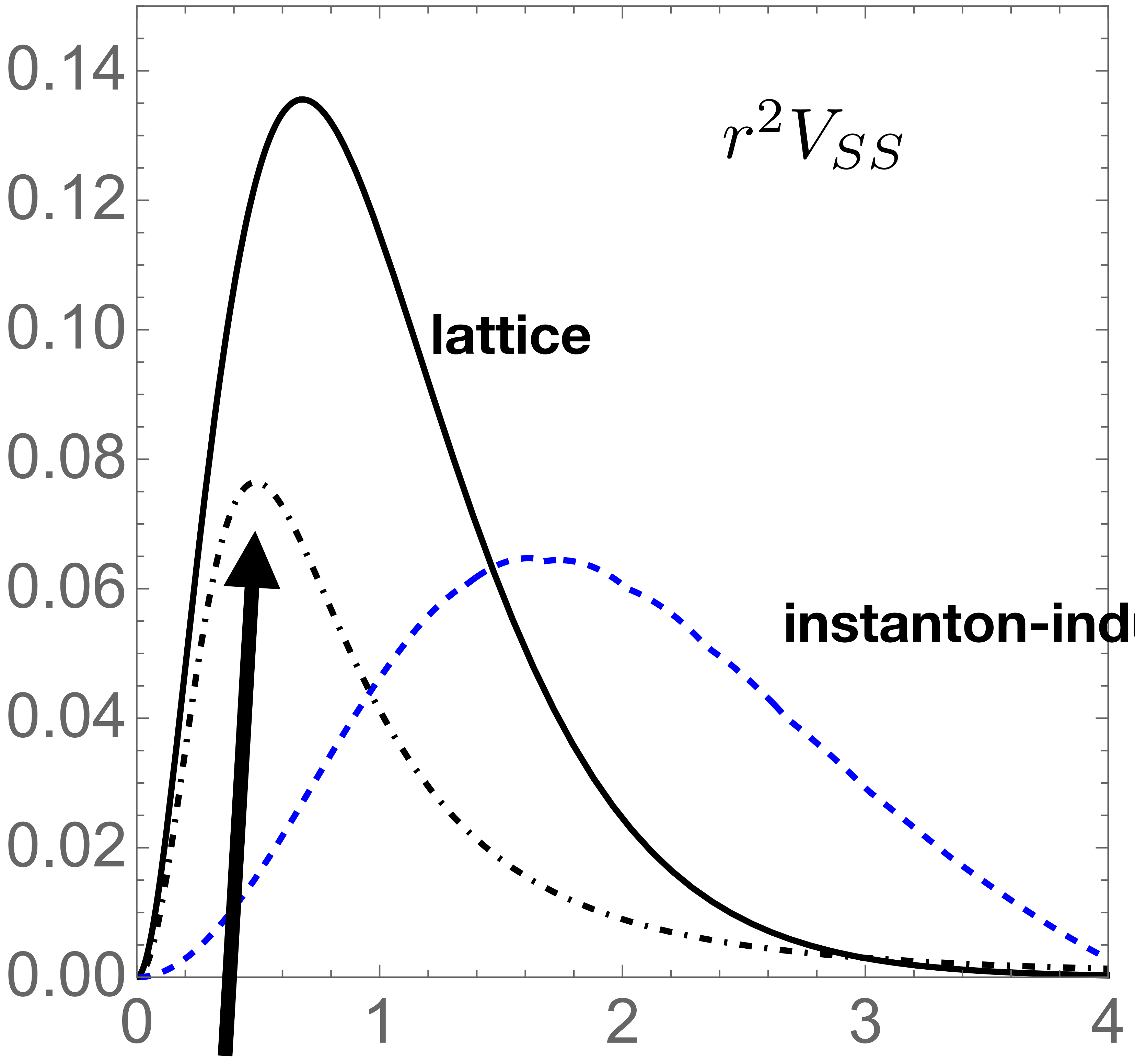


TABLE II. "Hyperfine" splittings of certain $L = 0$ mesons with $J = 1$ and $J = 0$. The first row of numbers shows the experimental values (MeV) (rounded to 1 MeV). The second gives matrix elements of the lattice-based spin-spin potential (19), the next two are those for (regulated) Coulomb and instanton-induced spin-spin forces.

flavors	$M_\Upsilon - M_{\eta_b}$	$M_{J/\psi} - M_{\eta_c}$	$M(D^*) - M(D)$	$M(K^*) - M(K)$	$M(\rho) - M(\pi)$
Exp	61.	116.	137.	398.	636.
$\langle V_{SS}^{lat}/3M_1M_2\rangle$	46.	108.	98.	170.	
$\langle\vec{\nabla}^2 V_C/3M_1M_2\rangle$	28.	58.	48.	82.	
$\langle\vec{\nabla}^2 V_{inst}/3M_1M_2\rangle$	7.	30.	48.	90.	

another type of instanton effect due to t'Hooft Lagrangian

adds 40 MeV all three explain the observed splitting

massless pion is due to zero modes (t' Hooft Lagrangian)

we also studied splittings of 1P states h,chi_0,chi_1,chi_2 and calculated matrix elements of VSS,VSL,VT also from lecture 1

DIQUARK PAIRING IN THE NUCLEONS

as first order in ’t Hooft Lagrangian

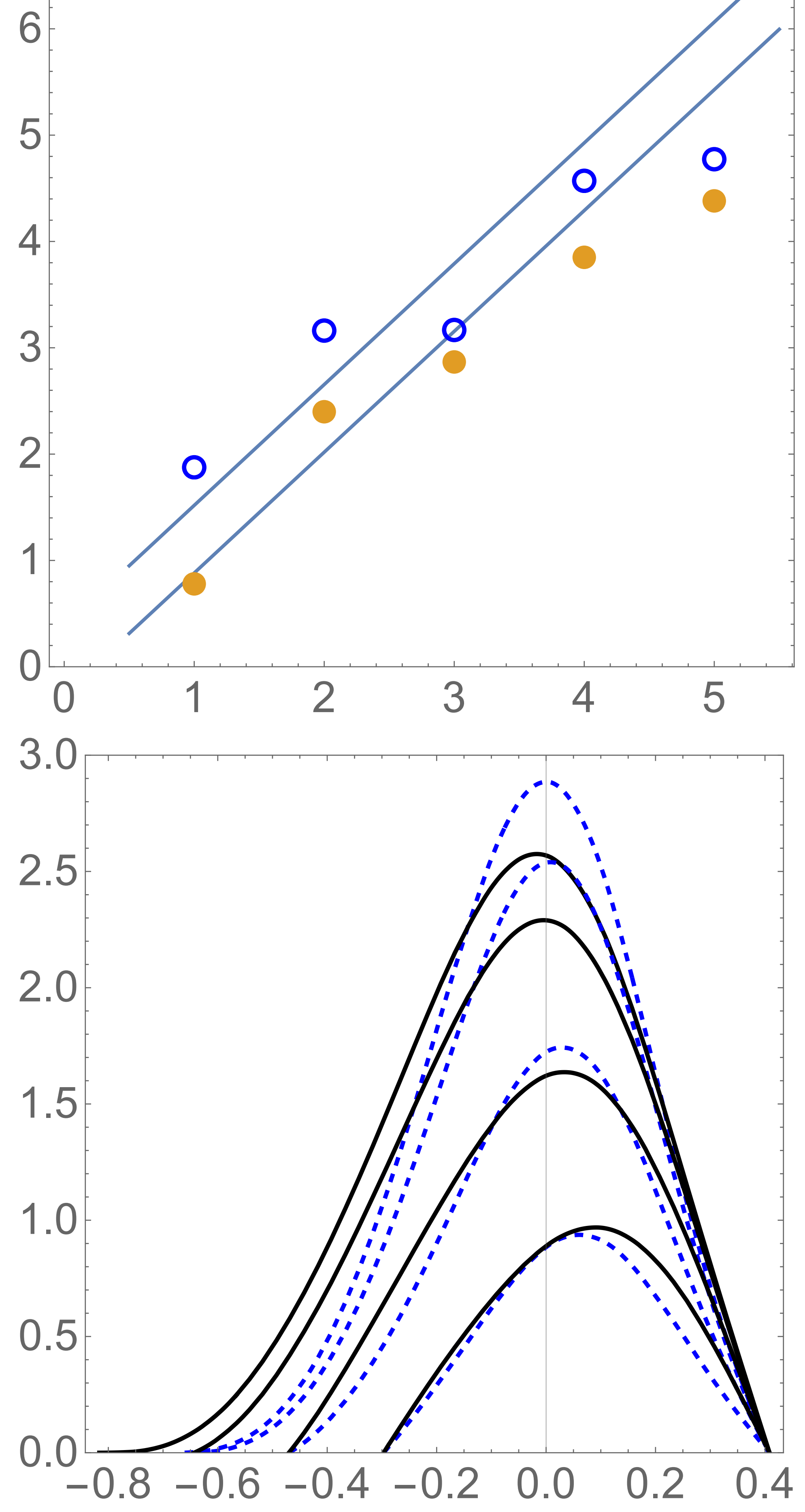


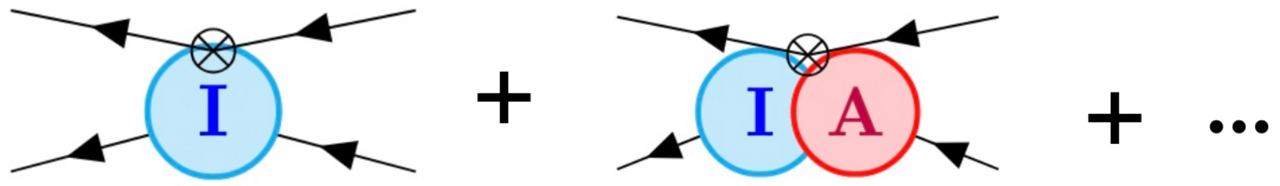
FIG. 6: Upper: Squared masses of the Delta (open points) and N (closed) resonances versus their successive quantum number n. The two straight lines shown for comparison, are the Regge trajectories fitted to the experimental values of $M^2(J)$, versus the total angular momentum J, with the slope $\alpha' = 0.88\, GeV^2$.
Lower: LFWFs for the lowest Delta (dashed lines) and N (solid lines). The plots are shown versus the Jacobi coordinate λ, for fixed $\rho = 0, 0.1, 0.2, 0.3$, top to bottom.

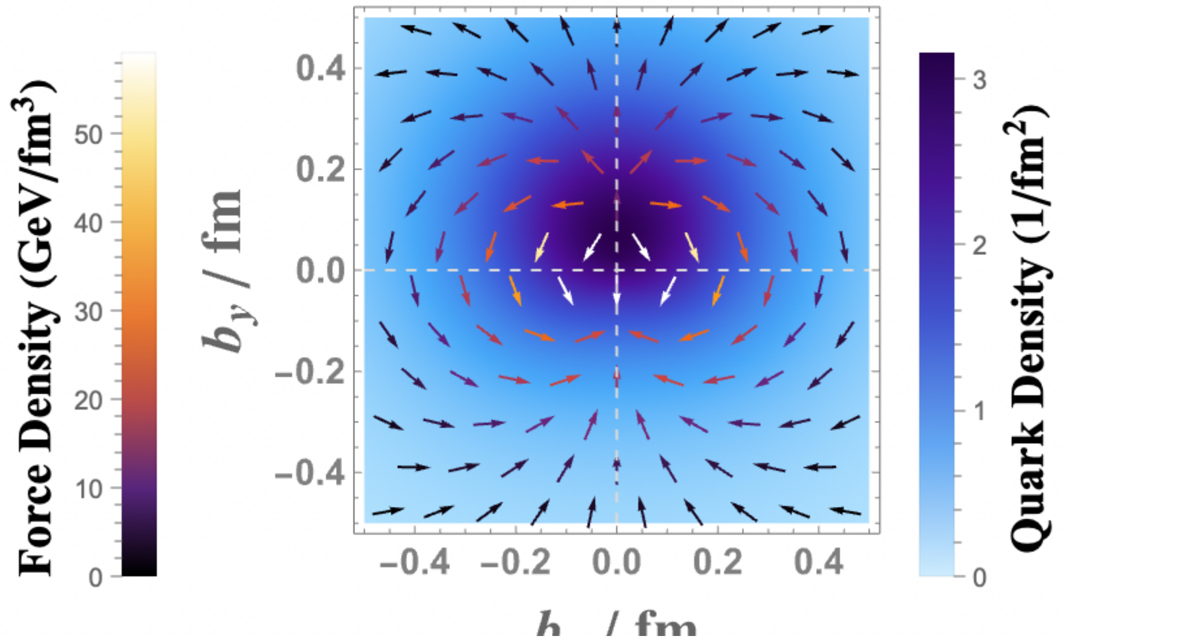
The color force acting on a quark in the pion and nucleon

Wei-Yang Liu,* Edward Shuryak,† and Ismail Zahed‡

The most interesting correlation between partons are those stemming from a polarized target. While the structure function $g_1(x,Q^2)$ starts with the usual twist-2 operators, the structure function $g_2(x,Q^2)$ starts with twist-3. Since in experiments the structure functions can be separated *kinematically*, this fact offers the most direct access to the higher twist physics. For a transversely polarized nucleon, at 90^0 to the incoming momentum, $g_1(x,Q^2)$ vanishes and the remaining DIS amplitude is purely twist-3. The pertinent physics is related with the local operator[1]

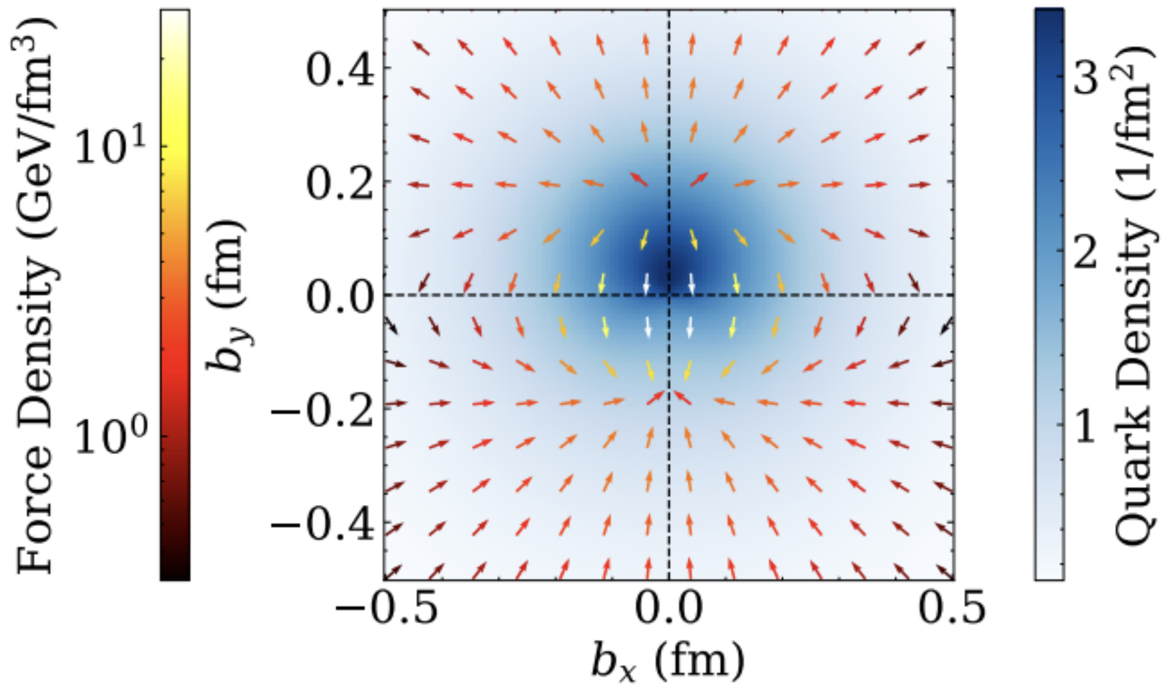
$$O_{\bar{q}Gq} = ig\left(\bar{q}\gamma^\rho G^{\mu\nu} q\right) \tag{1}$$





lattice

J. A. Crawford, K. U. Can, R. Horsley, P. E. L. Rakow, G. Schierholz, H. Stüben, R. D. Young, and J. M. Zanotti, "Transverse force distributions in the proton from lattice QCD," (2024), arXiv:2408.03621 [hep-lat].



Edward V. Shuryak and A. I. Vainshtein, "Theory of Power Corrections to Deep Inelastic Scattering in Quantum Chromodynamics. 2. Q**4 Effects: Polarized Target," Nucl. Phys. B 201, 141 (1982).

Summary

1.Topological structure: energy periodic as a function of Chern-Simons number
2.Instantons are tunneling from one valley to the next,
3.correlated pairs or ``molecules" are events of unsucessful tunneling
4.All of them are well seen on the lattice using e.g. gradient cooling
5.dilute instantons have topological charges and induce what we call a chiral physics
6.molecules are just clusters of strong local field flctuations, important in fformfactors and potentials

What we also worked out but I have no time to speak about:

in 1990's we did extensive numerical simulations of the instanton ensemble
calculated vacuum correlation functions for mesonic and baryonic currents

QCD (but not electroweak) sphalerons can be produced on colliders in diffractive events

we calculated confining potentials for baryons and teraquarks
for baryons it is binary but for teraquarks it is not

on top of electromagnetic formfactors we calculated scalar and gravitational ones, for mesons and baryons

What needs to be done

Complete addition of the gluon sector:

1.glueball spectroscopy with constituent gluons and modified confinement

2.extend it to hybrids e.g. ($\bar q G q$)

3.work out their mixing with mesons and baryons

4.substitute DGLAP (eqn for PDFs) by RG for wave functions -> expanding wave functions with gluons