

# Joint Sparse Deconvolution of Timing and Energy Waveforms for Photon Arrival Reconstruction

Minseok Yi<sup>1,2</sup>, Jae Sung Lee<sup>1,2,3</sup>

<sup>1</sup>Interdisciplinary Program in Bioengineering, College of Engineering, Seoul National University, Seoul, Korea

<sup>2</sup>Integrated Major in Innovative Medical Science, Seoul National University, Seoul, Korea

<sup>3</sup>Brightonix Imaging Inc., Seoul, Korea

Corresponding Author Email: jaes@snu.ac.kr

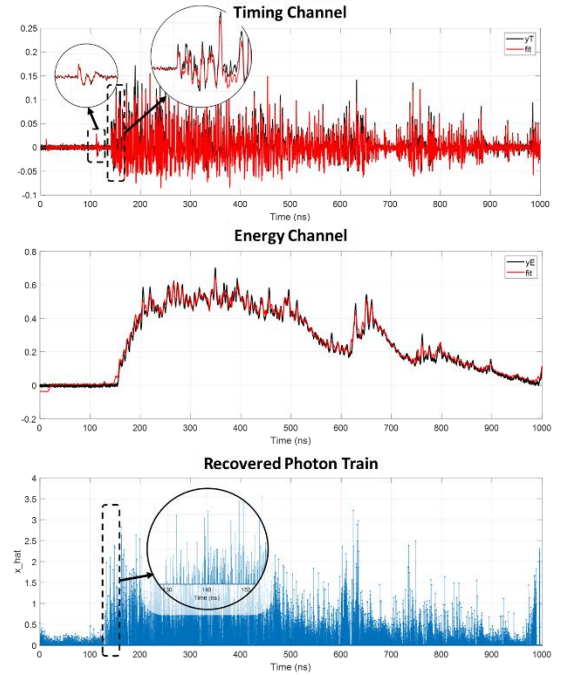
Photon-level knowledge of arrival times can provide rich information for fast-timing scintillation detectors, particularly for resolving early-photon structure and event-by-event timing behavior. Fast-timing scintillation detectors are often read out using multiple waveform representations optimized for different purposes, such as aggressively high-pass-filtered signals for timing pick-off and more slowly shaped signals for energy estimation. Although these waveforms originate from the same photon detection process, their distinct impulse response functions (IRFs) complicate direct reconstruction of the underlying photon arrival structure. In particular, direct deconvolution of the timing waveform is unstable due to undershoot-dominated bipolar IRFs, whereas deconvolution of the energy waveform is limited by slow rise/decay shaping that blurs photon timing.

Here, we propose a joint sparse deconvolution framework that reconstructs a common latent photon arrival train by simultaneously fitting timing and energy waveforms on an event-by-event basis. Each waveform is modeled as a convolution of the shared photon train with a channel-specific IRF plus baseline and noise; the photon arrival train is represented on a fine grid (16 GS/s) and constrained to be non-negative and sparse. Reconstruction is formulated as a joint inverse problem and solved using the alternating direction method of multipliers (ADMM), with convolution operators implemented in the Fourier domain for computational efficiency (16,000 samples/event) and conjugate gradient updates for linear subproblems without explicit matrix construction. The reconstructed photon train is interpreted as a continuous-valued arrival intensity, naturally accommodating sub-sample timing through amplitude spreading across neighboring bins. Integration of the reconstructed coefficients provides a physically meaningful estimate of the detected photon yield; when applied to a BGO-based experimental scintillation detector waveforms, an apparent yield of  $\sim 2900$  photons per 511 keV was obtained, consistent with expected light output (BGO light yield  $\sim 8,200$  photons/MeV [1]) after accounting for partial tail truncation.

This joint deconvolution approach enables stable photon-level reconstruction from complementary waveform representations and provides a flexible foundation for advanced timing analysis in fast scintillation and especially in prompt-photon starved configuration such as Cherenkov-based TOF PET detector systems.

[1] Luxium Solutions, “BGO (Bismuth Germanate) Scintillator, accessed Sep. 2025.

This work was supported by a National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (Grant No. RS-2024-00354123).



(Top) Measured timing (black) and fitted timing signal (red). (Middle) Measured energy (black) and fitted energy signal (red). (Bottom) Reconstructed photon train.