Light-Cone Quantized QCD

and

Heavy Quark Effects

S. Brodsky

Lutsen

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Bound States in QCD

$$P^\mu P_\mu = M_p^2$$

$$H_{LC}^{QCD} |\Psi_p\rangle = M_p^2 |\Psi_p\rangle$$ proton eigenstate

$p^+ i\frac{\partial}{\partial t}$

$$|\Psi_p\rangle = \sum |n\rangle\langle n|\Psi_p\rangle$$

e.states of H_0

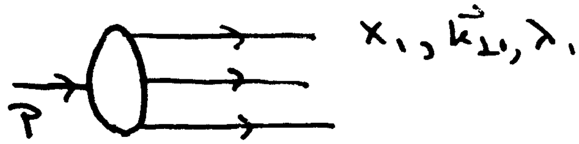
$$= |uud\rangle\langle uud|\Psi_p\rangle$$

$$+ |uudg\rangle\langle uudg|\Psi_p\rangle$$

$$+ \cdots$$

$$\langle uud|\Psi_p\rangle = \psi_{uud/p}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

moving proton



$i = n$

$$x_i = \frac{k_i^+}{P^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}, \quad y_i = \ln x_i$$

$$\sum_{i=1}^{n} x_i = 1, \quad \sum_{i=1}^{n} \vec{k}_{\perp i} = 0$$

Boost invariant description

$$0 < x_i < 1$$

Light-Cone Wavefunctions

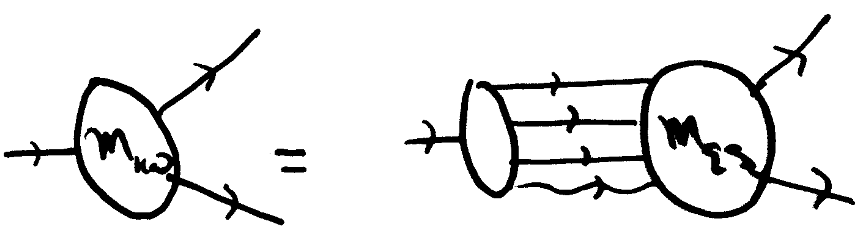
$\Rightarrow$ Hadron and Nuclear Amplitudes

$$H_{LC} |\Psi\rangle = M^2 |\Psi\rangle$$

$$|\Psi\rangle = \sum_n |n\rangle \langle n|\Psi\rangle$$

$$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\mathcal{M}_{Had} = \sum_n \psi_n \otimes \mathcal{M}_{qg}$$



$\therefore \psi_n$ interpolate between q, g deg. of freedom

Form Factors $\qquad \langle p'|j|p\rangle = \psi' \otimes \psi$

Decay Matrix Elements

Structure Functions $\qquad |\psi|^2$

Hard-Scattering Expansion $\qquad \frac{1}{Q^n} \quad (1-x)^n$

PQCD $\quad$ Dim. Counting rules

Evolution

S2B
CP [illegible]

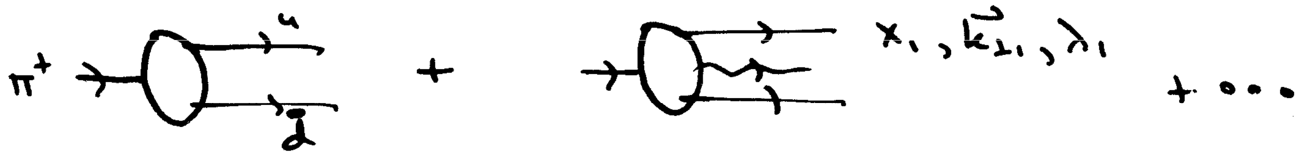
Light-Cone Wavefunctions $\{\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\}$

Interpolate between Hadron and q, g Degrees of Freedom

* $$|\Psi_H\rangle = \sum\!\!\!\!\!\!\int_n |n\rangle\langle n|\Psi_H\rangle$$

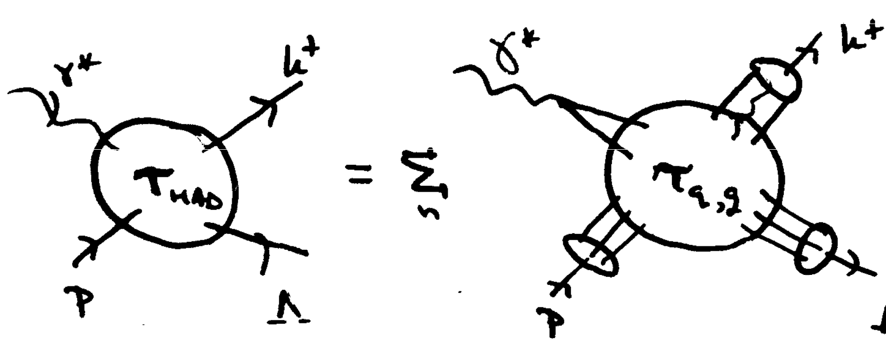
↑ complete set of e.fcts H^0_{QCD}

* $$|\Psi_H\rangle = \sum\!\!\!\!\!\!\int_n |q,\bar{q},g\rangle\, \Psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$$



$$x = \frac{k^+}{p^+} = \frac{k^0 + k^z}{p^0 + p^z} \qquad \sum_{i=1}^{n} x_i = 1 \;, \; \sum \vec{k}_{\perp i} = 0$$

* Fixed $\tau = t + z/c \Rightarrow \Psi_n$: Boost invariant !



Large P_T: minimum (valence) Fock states dominate
($A^+ = 0$ gauge)

SJB
+ G.P. Lepage

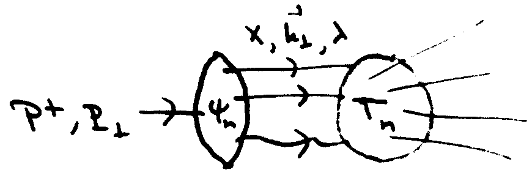
Calculating hadronic matrix elements

$$\sum_n \sum_{\lambda_i} \int \prod_i \frac{dx_i\, d^2k_{\perp i}}{\sqrt{x_i}\, 16\pi^3}\, \psi_n^{(\Lambda)}(x_i, \vec{k}_{\perp i}, \lambda_i)\, T_n^{(\Lambda)}(x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i)$$

determines properties of hadron

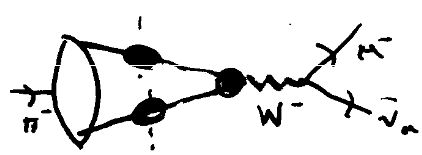
irreducible $H \to |n\rangle$

loops: $k_\perp^2 > \Lambda^2$



e.g.

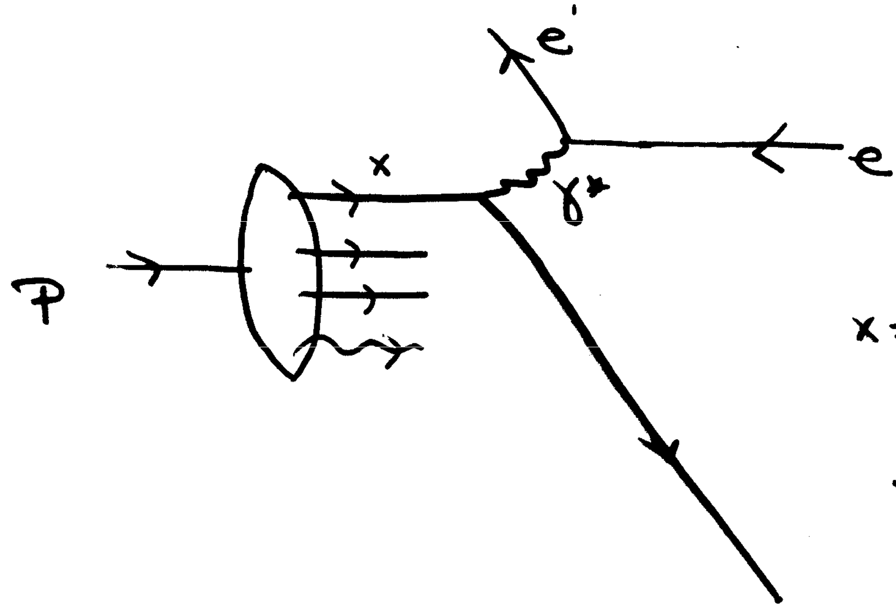
$$\int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3}\, \psi^{(\Lambda)}_{q\bar{q}/\pi}(x, \vec{k}_\perp) = \frac{f_\pi}{2\sqrt{3}}\left(1 + \frac{\mu^2}{\Lambda^2}\right)$$



$\Rightarrow$ Finite probability for $\psi_{q\bar{q}/\pi}$. !

(Zero probability for $\psi_{e/e}$ due to I.R.)

* Huang, SJB, Lepage: $P_{\bar{q}q/\pi} \cong 1/4$ use $\pi^0 \to \gamma\gamma$, $\pi \to \mu\nu$



$$x = \frac{k^+}{P^+} = \frac{k^0+k^3}{p^0+p^3}$$

$$x_i \Rightarrow x_{Bj} = \frac{Q^2}{2p\cdot q}$$

Light-cone description of deep inelastic lepton-scattering $\quad ep \to e' X$

Frame: $\quad q^+ = q^0 + q^3 = 0$

$$q_\perp^2 = Q^2 = -q^2$$

Bj scaling
PQCD Factorization

$$\frac{d\sigma}{dQ^2 dx_{Bj}} = \sum_{q\in p} \overset{\ln Q^2}{q(x,Q^2)} \; \overset{\frac{1}{Q^4}}{\frac{d\sigma}{dq^2}}(eq \to eq)$$

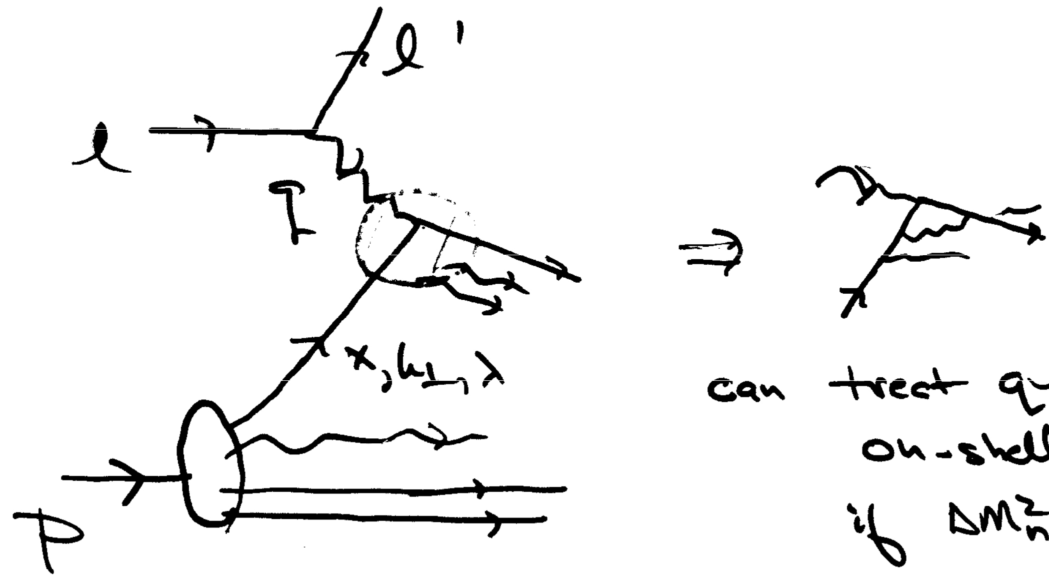
$$* \quad q(x,\Lambda) = \sum_n \int \prod_i dx_i \int \prod_i d^2k_{\perp i} \left|\psi_n^{(\Lambda)}(x_i, k_{\perp i}, \lambda_i)\right|^2$$

Sum over all $n \geq 3$ $\qquad \sum \delta(x_q - x_{Bj})$

SJB + GPL
(Hori + SJB)

Light-Cone Factorization Scheme

* Separate hard and soft dynamics



can treat quark as on-shell if $\Delta M_n^2 << Q^2, W^2$

$$q(x, \Lambda) = \sum_n \int \prod d^2k_\perp dx \left| \psi_n^{(\Lambda)}(x_i, k_{\perp i}) \right|^2 \sum_{q_a = q} \delta(x - x_q)$$

* $\psi_n^{(\Lambda)} = 0$ if $\Delta M_n^2 = \left| M_p^2 - \sum \frac{k_\perp^2 + m^2}{x} \right| > \Lambda^2$

Note: no factorization if $x \to 1$, $\underset{\to -\infty}{k_F^2} \sim \frac{-Q^2}{1-x}$

Properties of L.C. wavefunctions

PQCD evolution

* $\dfrac{\partial}{\partial \ln \Lambda^2} q(x, \Lambda)$ DGLAP evolution.

* $\dfrac{\partial}{\partial \ln \Lambda^2} \phi_H(x_i, \Lambda)$ evolution of distribution amplitudes

$H = M, B, d$

$$\phi(x_i, \Lambda) \equiv \int^{\Lambda} d^2k_\perp \, \psi^{(\Lambda)}_{val}(x_i, \vec{k}_{\perp i}, \lambda)$$

$$\Delta m_n^2 < \Lambda^2$$

$\phi(x_i, \Lambda)$ gauge-invariant ψ_{BS} at $x^+ = c$

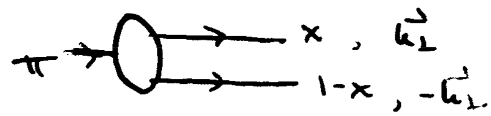
OPE

SJB GPL
Frishman
Sachrajda

Pion Distribution Amplitude

Simplest aspect of hadron w.f.

$$\phi_\pi(x, \tilde{Q}) = \int \frac{d^2k_\perp}{16\pi^3}\, \psi^{(\tilde{Q})}_{q\bar{q}/\pi}(x, \vec{k}_\perp)$$



$$= P_\pi^+ \int \frac{dz^-}{4\pi}\, e^{i x P_\pi^+ z^-/2}$$

$$\langle 0 | \bar{\psi}(0) \frac{\gamma^+ \gamma^5}{2\sqrt{2n_c}} \psi(z) | \pi \rangle^{(\tilde{Q})} \Big|_{z^+ = z_\perp = 0}$$

Line integral

$$\mathcal{P} \exp \int_0^1 ds\, i g A(sz) \cdot z \quad = 1$$

in $A^+ = 0$ gauge

OPE, RGE, Evol. Eqn

GPL + SJB

+ Frishman, Sachrajda.

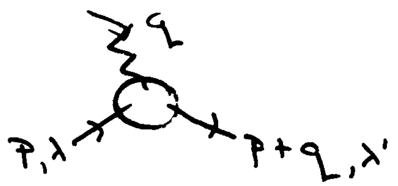
Large $k_\perp$, $x \to 1$ behavior of ψ_{val} controlled by PQCD.

Why Light-Cone Wave Functions ?

$$\boxed{\psi_n(x, \vec{k}_\perp, \lambda)}$$

Form Factors calculable from overlap of L.C. W.F.'s

$$q_\perp^2 = -q^2 = Q^2$$



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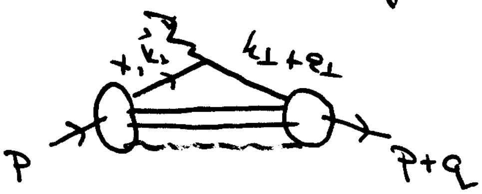


Sum over ψ_n, e_q

$$F_{\lambda\lambda'}(q^2) = \langle p+q, \lambda' | \frac{J^+(0)}{p^+} | p, \lambda \rangle$$

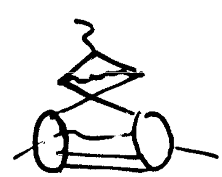
$$= \sum_q e_q \sum_n \int [d^2k_\perp] \int [dx]\, \psi_{n,\lambda'}(x, \vec{k}'_\perp, \lambda')\, \psi_{n,\lambda}(x, \vec{k}_\perp, \ldots$$

Drell-Yan-West
Drell - SJB

$$\vec{k}'_\perp = \begin{cases} \vec{k}_\perp + (1-x)\vec{q}_\perp & \text{struck q.} \\ \vec{k}_\perp - x\,\vec{q}_\perp & \text{spectators} \end{cases}$$

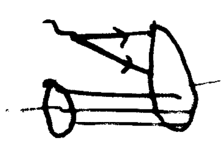
Remarks: $\tau = 0$: $J^+ = J^+$ free current

L.C. : $(k^+ > 0)$



$= 0.$

L.C. : $(q^+ = 0)$



$= 0$

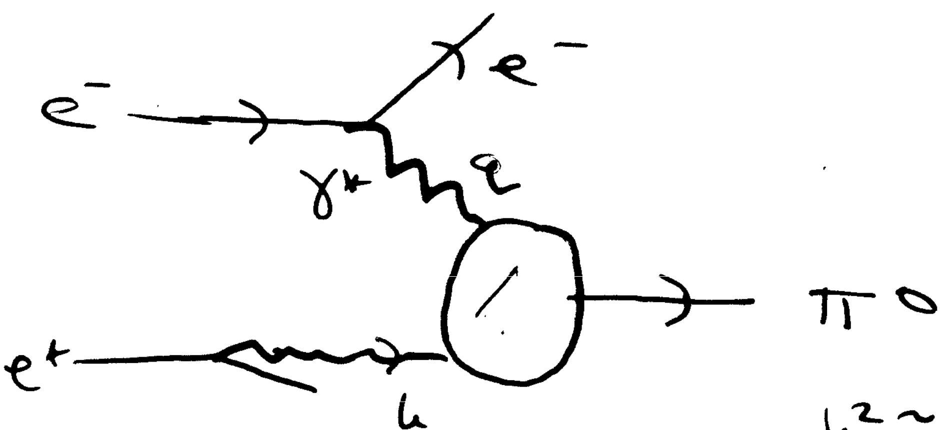
good current J^+

L.C. : $(A^+ = 0)$ no ghosts

Primary test of PQCD

$\gamma \to \pi^0$

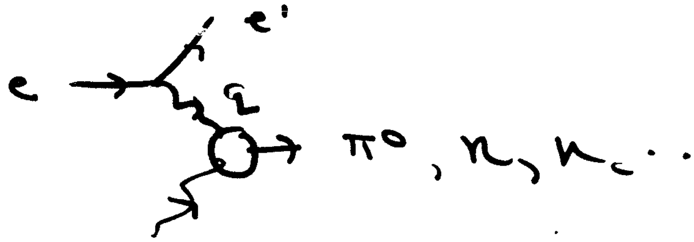
transition form factor



$k^2 \cong 0$

$$\boxed{F_{\gamma\pi^0}(q^2)}$$

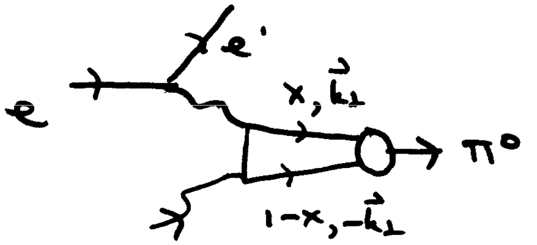
Simplest example of exclusive process



$q^2 = -Q^2$

$$\frac{d\sigma}{dQ^2} \sim \frac{1}{Q^6}$$

$Q^2 \gg \Lambda_{QCD}^2$ $\qquad F_{\gamma\pi^0}(Q^2)$



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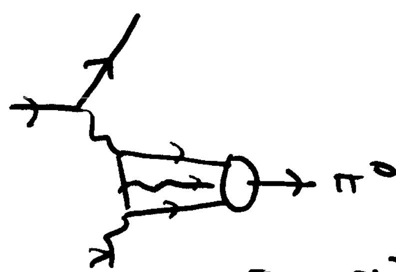


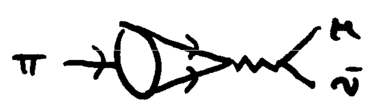
suppressed by $\frac{1}{Q}$

$$F_{\gamma\pi^0}(Q^2) = \frac{1}{Q^2}\, 2\sqrt{N_c}\,(e_u^2 - e_d^2) \int_0^1 \frac{dx}{x(1-x)}\, \phi_\pi(x, \tilde{Q})$$

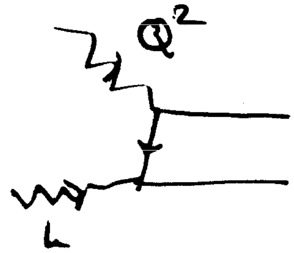
pion distribution amplitude

$$\phi_\pi(x, \tilde{Q}) = \int^{\tilde{Q}} \frac{d^2k_\perp}{16\pi^3}\, \psi_{q\bar{q}}^{(\tilde{Q})}(x, k_\perp)$$

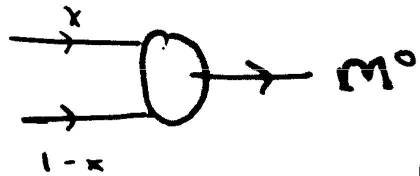
$$\int_0^1 dx\, \phi_\pi(x, Q) = \frac{f_\pi}{2\sqrt{3}}$$



PQCD : $\boxed{F_{\gamma \to M^0}(Q^2) \sim \frac{1}{Q^2}\int_0^1 \frac{dx}{1-x}\, \phi_M(x,\tilde{Q})}$



T_H



$$\phi_M(x,Q) = \int^{k_\perp^2 < \tilde{Q}^2} d^2k_\perp\, \psi_{q\bar{q}}(x,\vec{k}_\perp)$$

* $T_H(\gamma^* \gamma \to \underbrace{q\bar{q}}_{\text{collinear}}) \sim \frac{1}{Q^2(1-x)}$

* Higher Fock states : $\frac{1}{Q^4}$ 

Other diagrams $\mathcal{O}(\alpha_s(Q^2))$!

* $\phi_M(x,Q) = \sum_{n=0}^{\infty} a_n P_n(x) \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n}$ 

log evolution

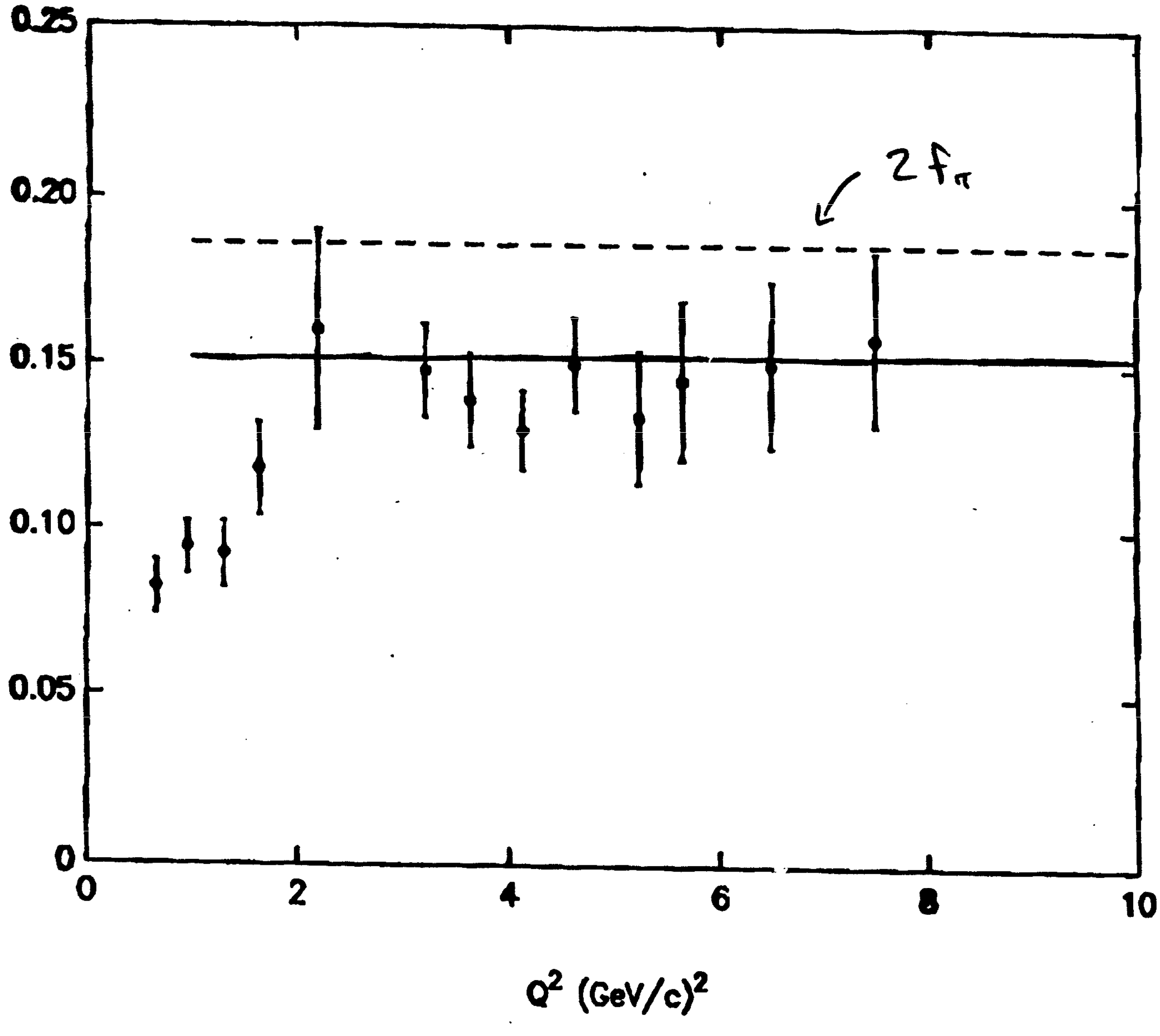
* $\lambda_M = \lambda_q + \lambda_{\bar{q}} = 0.$

** Small part of Fock state dominates

$$\phi_M \sim \psi(x, b_\perp \sim \tfrac{1}{Q})$$

$$\phi = \phi_{asympt} = \sqrt{3}\, x(1-x)\, f_\pi$$

$$Q^2 F_{\pi\gamma}(Q^2) = 2 f_\pi \left[1 - \frac{5}{3\pi}\alpha_V(e^{-3}Q^2)\right]$$



Can we compute Fock State Structure of Hadrons?

$$|p\rangle = \sum_n |n\rangle \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$= |uud\rangle \psi_{uud} + |uudg\rangle \psi_{uudg} + \cdots$$

Light-cone Hamiltonian methods: $\boxed{H^{LC}|p\rangle = M^2|p\rangle}$

$\left\{ \begin{array}{l} \text{Discretized Light-Cone Quantization} \\ \text{Light-Front Tamm Dancoff} \end{array} \right.$

SJB Pauli ...

Wilson Perry

Coster Hamada Junko Susskind

DLCQ: Diagonalize H^{LC} on Fock basis

Complete numerical solutions

Spectrum, $\{\psi_n(x_i, k_{\perp i}, \lambda_i)\}$

for QCD(1+1), QED(1+1) ...

Hornbostel, Pauli, SJB, Tang

Klebanov, Dalley, Demetrifi : adjoint quarks

Burkardt Antonuccio

$\mathcal{L}_{QCD} \Rightarrow H^{LC}_{QCD}$ canonical quantization

$A^+ = 0$

Interactions:

$\bar{u}\gamma^\mu u$

$\dfrac{\bar{u}\gamma^\nu \gamma^+ \gamma^\mu u}{k^+}$

analogous to

"seagull"

$\dfrac{\bar{u}\gamma^+ u \, \bar{u}\gamma^+ u}{(k^+)^2}$

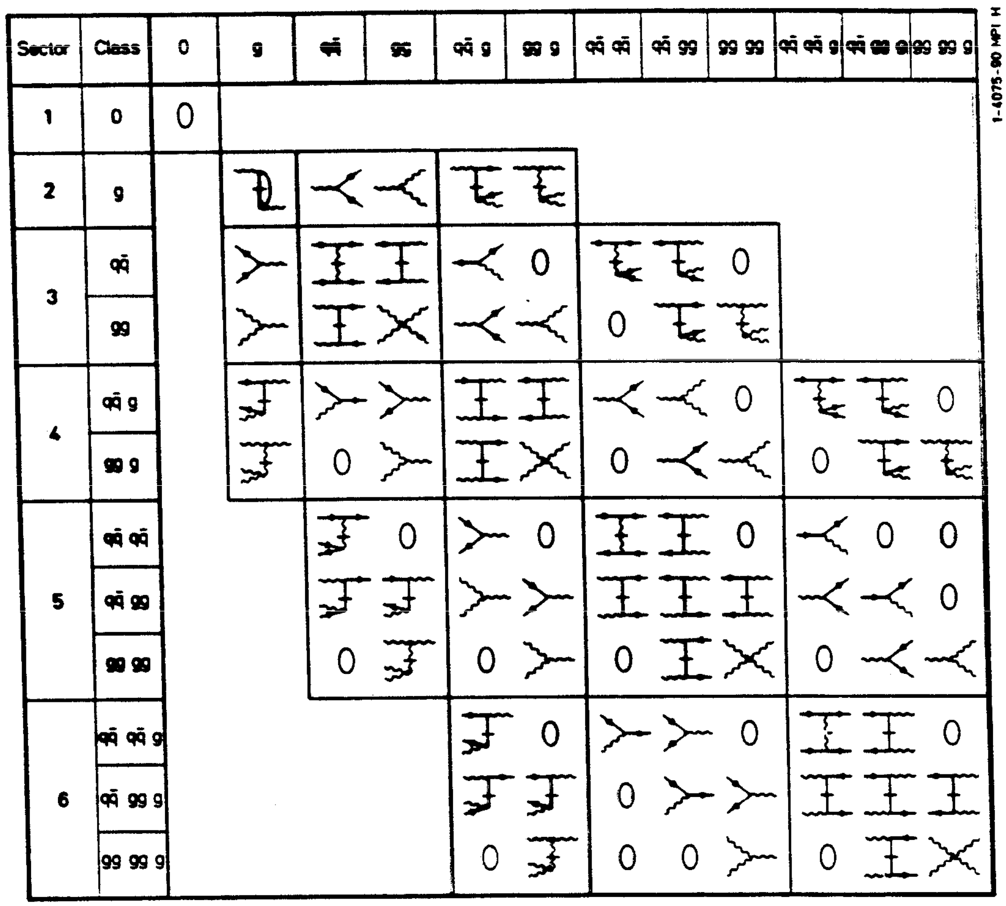
$k^+ = k^0 + k^3$, $\gamma^+ = \gamma^0 + \gamma^3$

Spinors: eigenstates of $\Lambda_\pm = \dfrac{\gamma^0\gamma^\pm}{2}$

$k^+ = 0$ sing. cancel in P.T.

problem in LFTD

QCD (3+1)

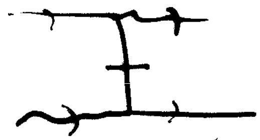


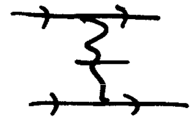
DLCQ: $\langle n | H^{I}_{LC} | m \rangle$

$$\left(M^2 - \sum_n \frac{k_\perp^2 + m^2}{x} \right) \psi_n = \sum_m \langle n | H^{I}_{LC} | m \rangle \psi_m$$

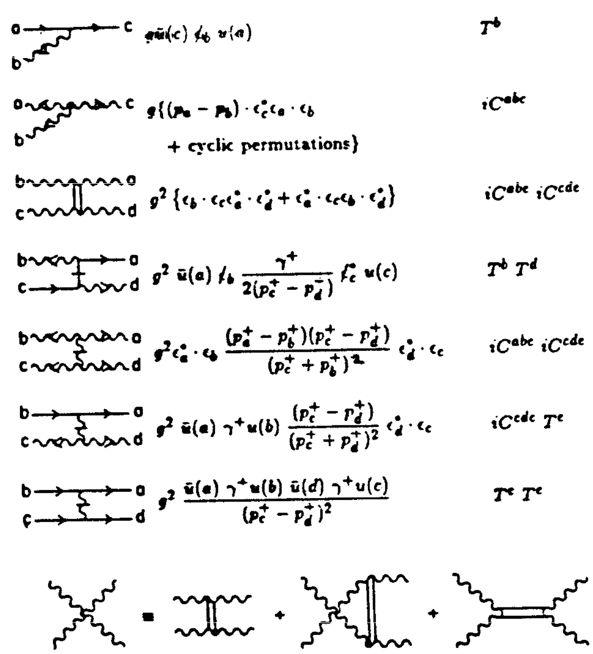
PBC: $k^+ = \frac{2\pi}{L} n$, $\vec{k}_\perp = \frac{2\pi}{L_\perp} \vec{n}_\perp$

H_{LC}^{QCD} : New 4-pt interactions





$\frac{\gamma^+}{k^+}$ $\qquad\qquad$ $\gamma^+ \frac{1}{(k^+)^2} \gamma^+$



	Vertex Factor	Color Factor
	$g\bar{u}(c) \not{\epsilon}_b u(a)$	T^b
	$g\{(p_a - p_b)\cdot\epsilon_c^* \epsilon_a\cdot\epsilon_b$ + cyclic permutations$\}$	iC^{abc}
	$g^2\{\epsilon_b\cdot\epsilon_c\epsilon_a^*\cdot\epsilon_d^* + \epsilon_a^*\cdot\epsilon_c\epsilon_b\cdot\epsilon_d^*\}$	$iC^{abe}\,iC^{cde}$
*	$g^2\,\bar{u}(a)\not{\epsilon}_b \frac{\gamma^+}{2(p_c^+ - p_d^-)} \not{\epsilon}_c^* u(c)$	$T^b T^d$
*	$g^2\epsilon_a^*\cdot\epsilon_b \frac{(p_a^+ - p_b^+)(p_c^+ - p_d^+)}{(p_c^+ + p_b^+)^2} \epsilon_d^*\cdot\epsilon_c$	$iC^{abe}\,iC^{cde}$
*	$g^2\,\bar{u}(a)\gamma^+ u(b) \frac{(p_c^+ - p_d^+)}{(p_c^+ + p_d^+)^2} \epsilon_d^*\cdot\epsilon_c$	$iC^{cde}\,T^e$
*	$g^2 \frac{\bar{u}(a)\gamma^+ u(b)\,\bar{u}(d)\gamma^+ u(c)}{(p_c^+ - p_d^+)^2}$	$T^e T^e$

Instaneous Ferm

Instaneous gluon

3-83 $\qquad$ 4507A25

Figure 54. Graphical rules for QCD in light-cone perturbation theory.

SJB
H.C. Pauli

Program for solving QCD:

* Diagonalize H_{LC}^{QCD}

$$H_{LC}|\Psi\rangle = M^2|\Psi\rangle$$

$$|n\rangle\langle n| = I$$

* $$\langle n|H_{LC}|m\rangle\langle m|\Psi\rangle = M^2\langle n|\Psi\rangle$$

$|n\rangle$: e. state of H^0_{LC}
PBC

$$k_i^+ = \frac{2\pi}{L} n_i$$

$$P^+ = \frac{2\pi}{L} K$$

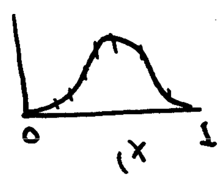
Discretized
Light-Cone
Quantization

$$\sum_i n_i = K \quad , \quad n_i > 0$$

String + M-Theory: Susskind, Klebanov, Antonuccio

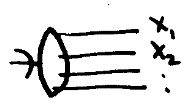
For 1+1 Theories, K cuts off Fock states

$$x_i = \frac{k_i^+}{P^+} = \frac{n_i}{K} = \left\{ \frac{1}{K}, \frac{2}{K}, \dots \frac{K-1}{K} \right\}$$

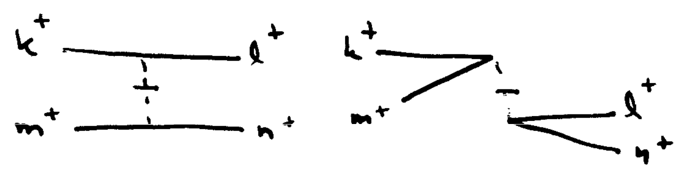
$\psi(x_i)$  sample states at finite resolution

continuum limit: $K \to \infty$.

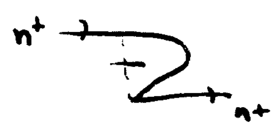
For fixed K: finite # partitions $\sum_i n_i = K$.

 $x_i = \frac{n_i}{K}$

QED (1+1): No dynamical photons in $A^+ = 0$ gauge

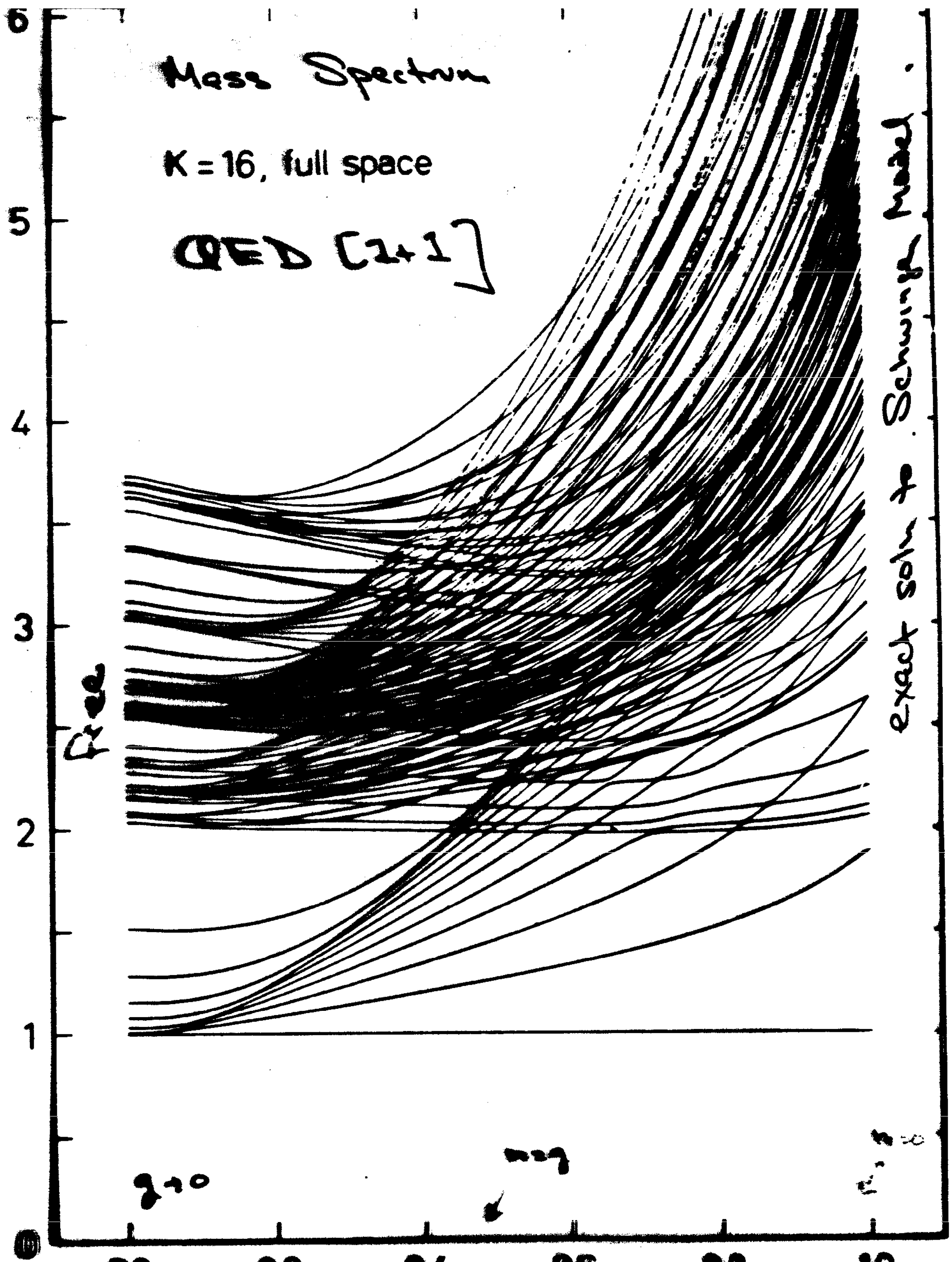
$V =$ 

$$= \frac{g^2}{\pi} \left[\frac{1}{(k^+ - \ell^+)^2} - \frac{1}{(k^+ + m^+)^2} \right]$$

From normal ordering:  $\frac{g^2}{\pi} \sum_{m=1}^{n^+} \frac{1}{m^2}$

T. Eller
H.C.P.
S.J.D

PRD 35 (87) 1493



M

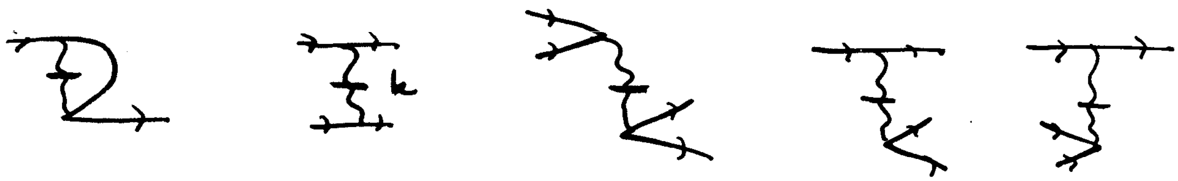
$\lambda = \frac{1}{\sqrt{1+\pi \frac{m^2}{g^2}}}$

6. The renormalized spectrum of invariant masses. — The invariant masses M_i/M_1 as calculated with the full Fock space of the massive representation for $K = 16$ is plotted versus all values of the coupling constant λ. — Note the qualitatively different parts of the spectrum. Many quasi-crossings are not resolved graphically despite the small step in the calculation, $\Delta\lambda =$ 0.001.

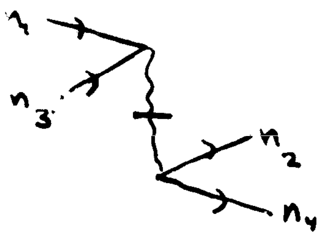
Prob ???? $\lesssim 10^{-4}$

Hornbostel, HCP, SDin
Burkardt

Interactions in QCD [1+1]



$$g^2 \, \bar{u} \gamma^+ u \, \frac{1}{(k^+)^2} \, \bar{u} \gamma^+ u$$



$$\frac{L}{2\pi} \frac{g^2}{\pi} \frac{1}{2} \left(\delta^{c_2}_{c_4} \delta^{c_3}_{c_1} - \delta^{c_3}_{c_4} \delta^{c_2}_{c_1} \right)$$

$$\sum_{n_i = \frac{1}{2}, \frac{3}{2} ..} \frac{\delta_{n_1 + n_3, \, n_2 + n_4}}{(n_1 + n_3)^2} \, b^{+c_4}_{n_4} \, b_{n_3 c_3} \, d^+_{n_2 c_2} \, d$$

No dynamical gluon in 1+1

$\frac{g}{m}$ dimensionless

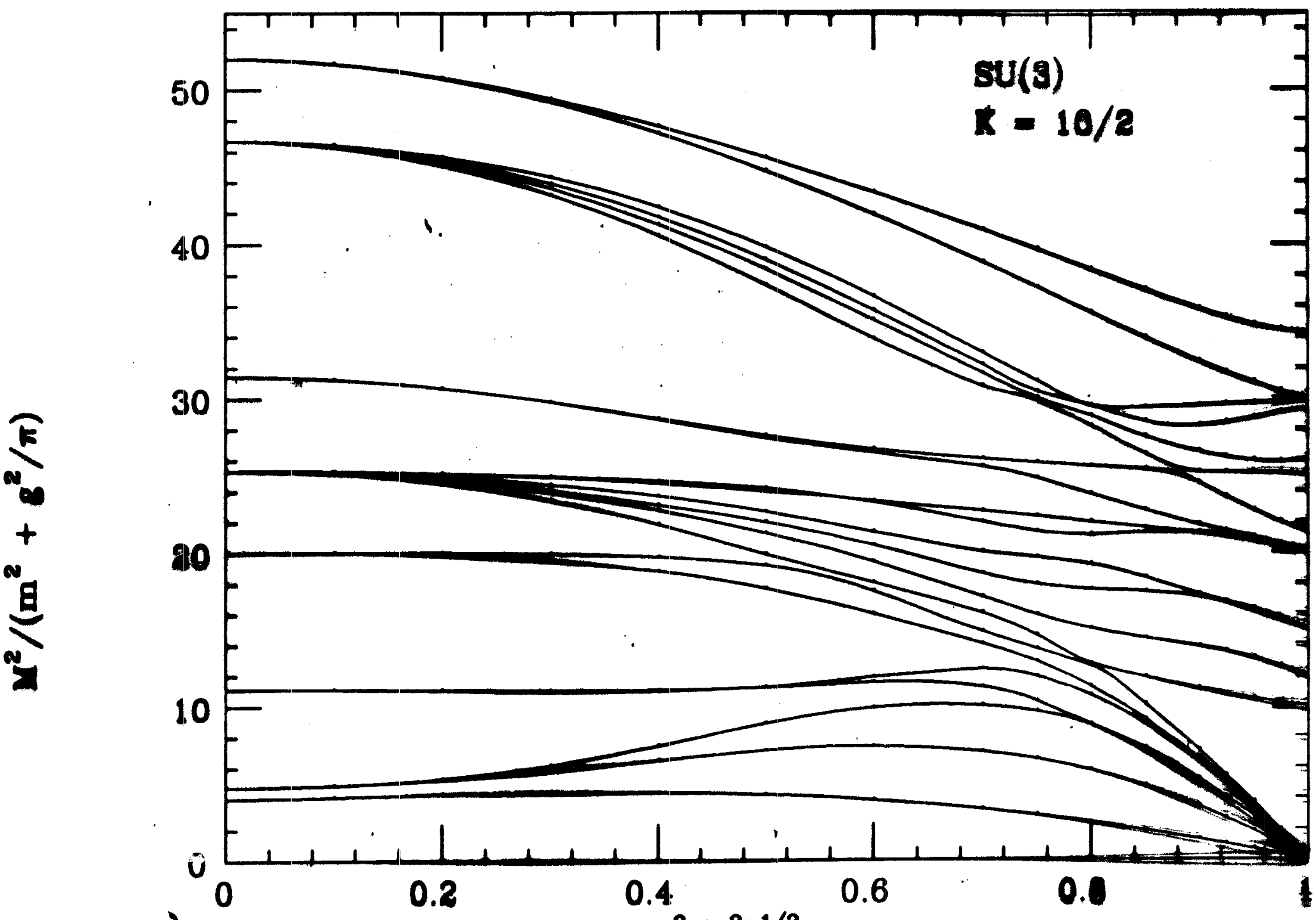
$m \to 0$
$g \to \infty$ QED[1+1] Schwinger model

$$V \Rightarrow \sum \frac{m^2}{k^+} a^+ a$$

free bosons: $m^2 = \underline{g^2}$

MESON (B=0) SPECTRUM

K. Hornbostel et al

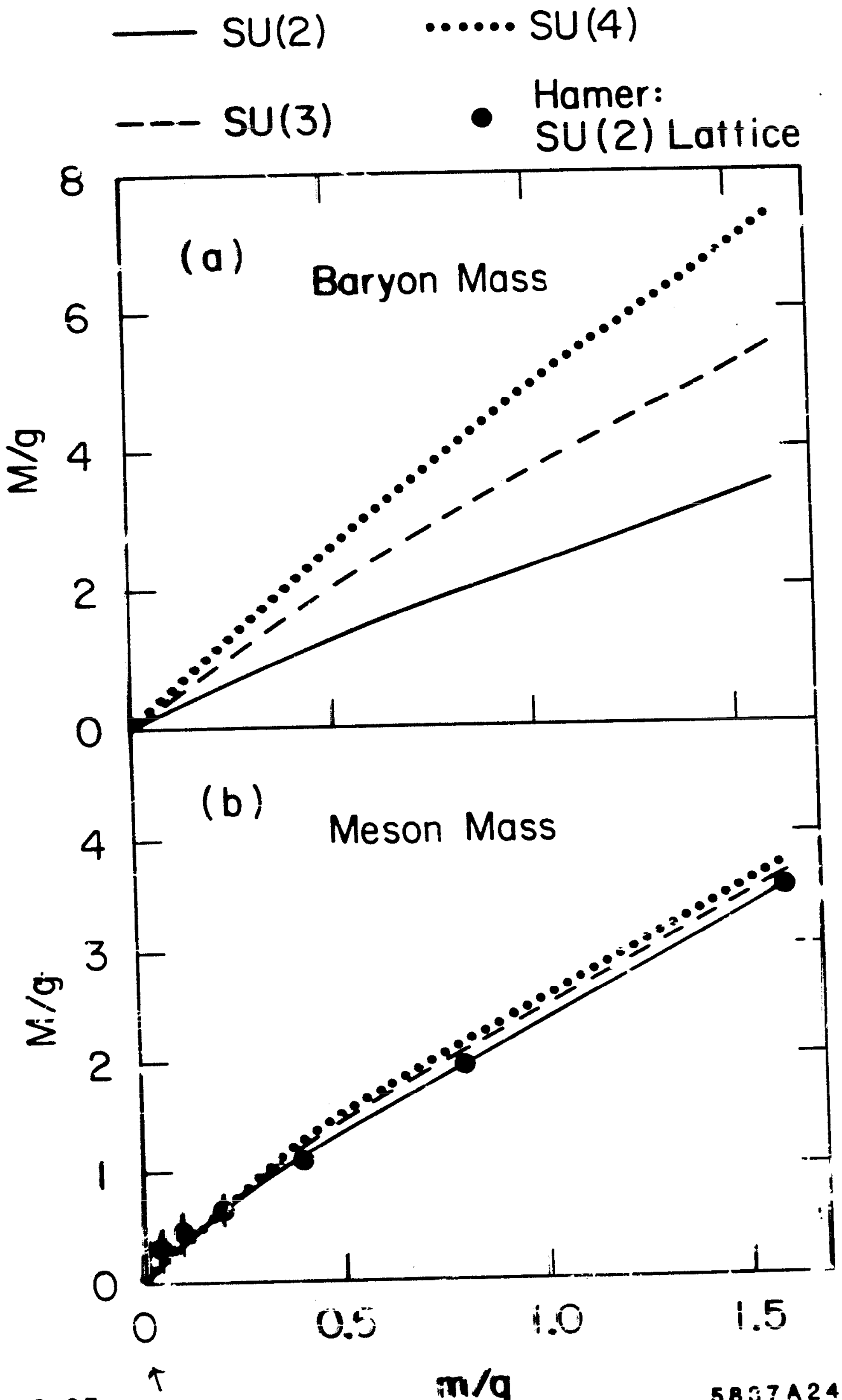


QCD [1+1]

↑ weak coupling $g = 0$

↑ strong coupling $g \to \infty$

DLCQ
Hornbostel
et al
QCD(1+1)



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↑ strong coupling limit $g \to \infty$

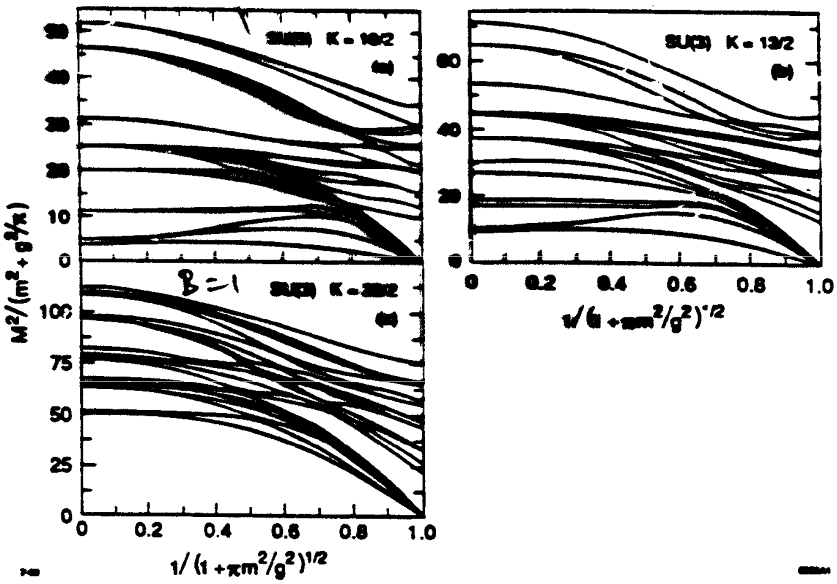


Figure 1. Spectra for $N = 3$, baryon number $B = 0, 1$ **and 2 as a function of** g/m; K fixed.

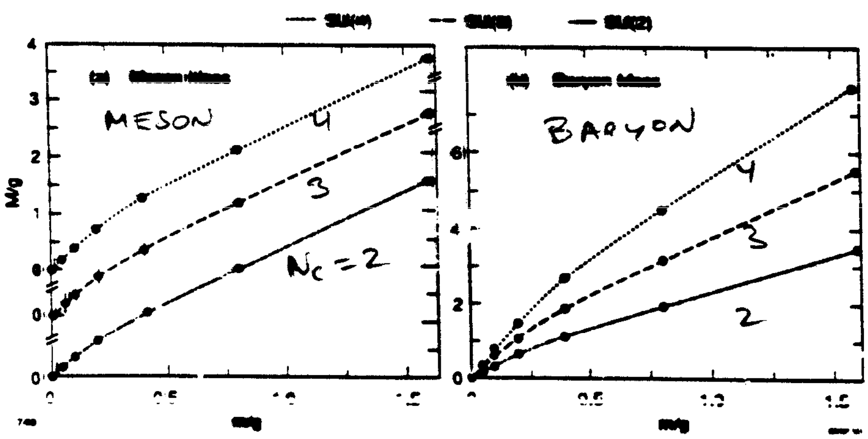


Figure 6. Extrapolated masses for $N = 2, 3$ and 4 **meson and** baryon.

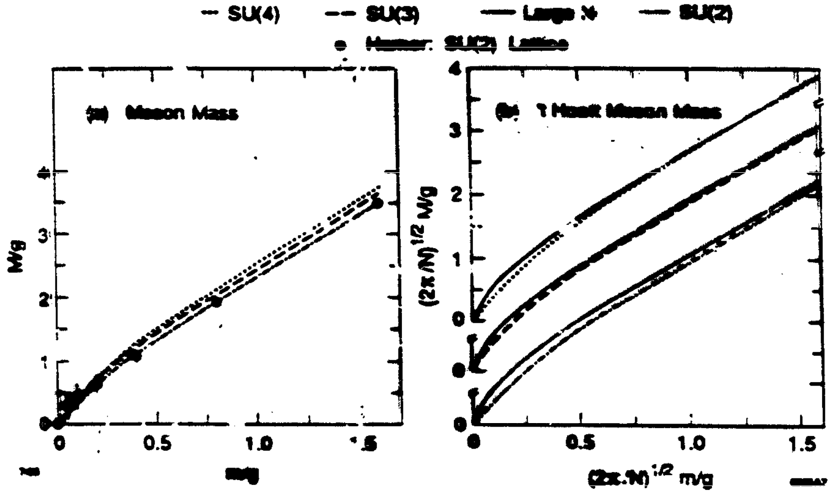


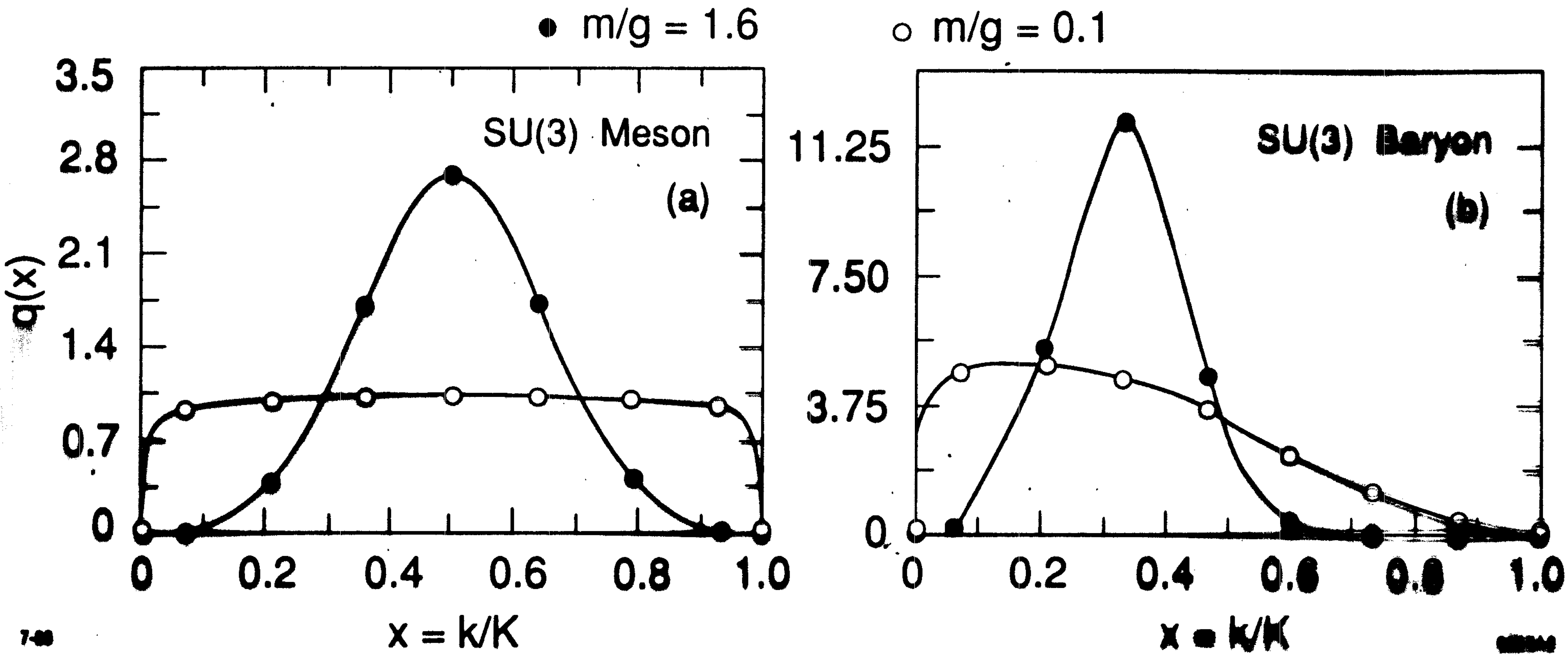
Figure 7. Comparison of $N = 2, 3$ and 4 meson masses with large-N and lattice calculations.

Spectra
QCD[1+1]

SJB
Hornbostel
Pauli

QCD(1+1)

DLCQ



● m/g = 1.6
○ m/g = 0.1
SU(3) Meson
(a)
q(x)
3.5
2.8
2.1
1.4
0.7
0
0.2
0.4
0.6
0.8
1.0
x = k/K
SU(3) Baryon
(b)
11.25
7.50
3.75
0
0.2
0.4
0.6
0.8
1.0
x = k/K

"Exact" Solution
to
QCD [1+1]

Hornbostel
Pauli, SJB

Burkhardt

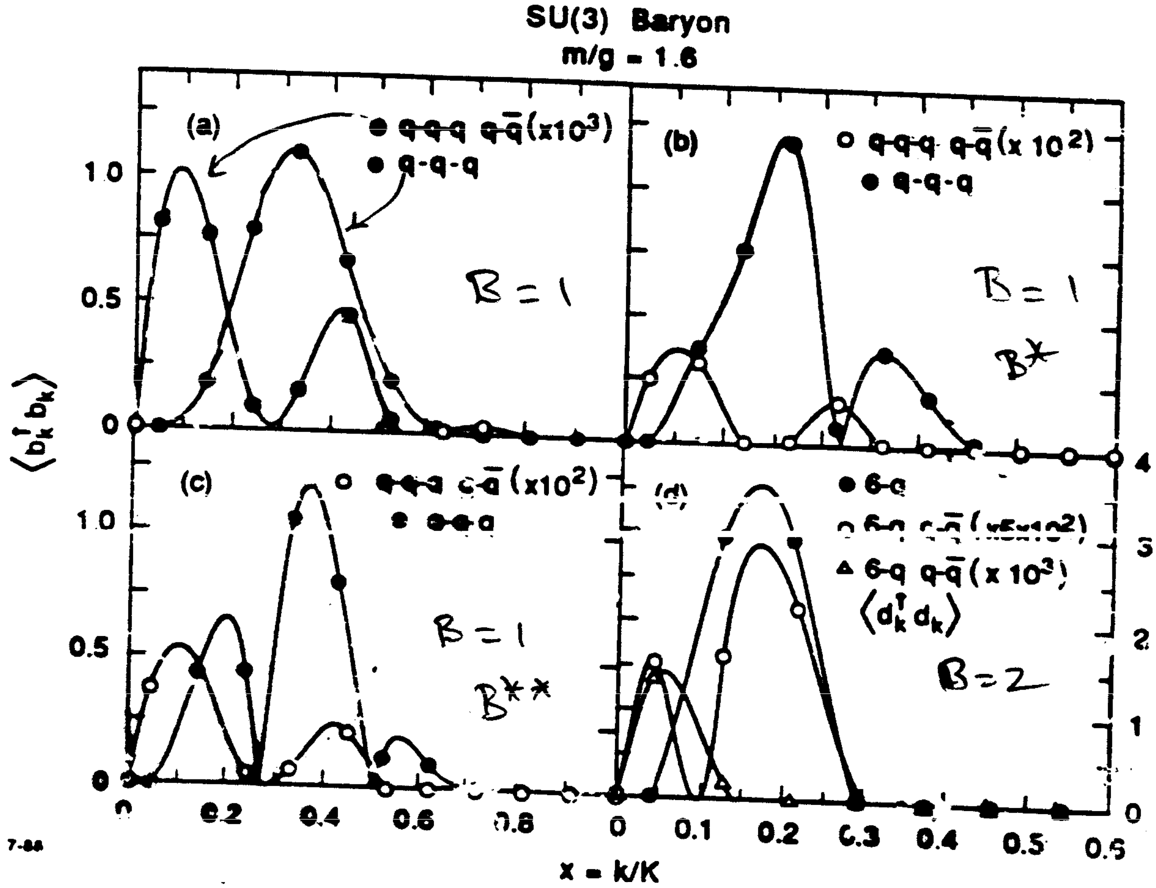


Figure 5. a-c) First three states in $N = 3$ baryon spectrum, 2K=2[illegible] d) First $B = 2$ state.