α_s FROM NON-RELATIVISTIC LATTICE QCD

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QCD PERTURBATION THEORY

CONTINUUM:

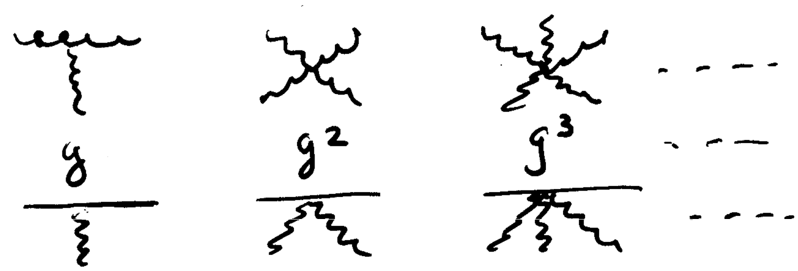
- WORKS FOR $Q >$ FEW GeV
- $\alpha_s(Q)$ INCORPORATES SCALES $Q \to \Lambda_{CUTOFF}$

LATTICE: SIMILAR, BUT

- $\Lambda_{CUTOFF} = \pi/a$ (a = SPACING)
- GLUONS: $U_\mu(x) = \mathcal{P}\, e^{-i g_{LAT} \int_x^{x+a\hat{\mu}} dx_\mu A_\mu}$

$$\frac{g_{LAT}^2}{4\pi} \equiv \alpha_{LAT}(\pi/a)$$

- VERTICES:



- FAILS FOR $Q \lesssim$ 10's – 100 GeV
  ($\neq$ NONPERT. SIMULATIONS OF
  SHORT DISTANCE QTY's)

  IF USE α_{LAT}

CULPRIT: TADPOLES

- CAN'T DISCARD (AS IN DIM. REG.)
  $\rightarrow$ IN SIMULATIONS

- WORSE THAN CONTINUUM

$\Rightarrow$ LARGE, PROCESS & SCALE INDEPENDENT CONTRIB. AT ALL ORDERS IN a

(1) RUIN CONNECTION BETWEEN

$$U_\mu \sim 1 - i g_{LAT} a A_\mu$$

SOLUTION: TADPOLE IMPROVEMENT (LEPAGE & MACKENZIE '92)

$$U_\mu \rightarrow \frac{U_\mu}{\langle U_\mu \rangle}$$

(2) RUIN CONNECTION BETWEEN α_{LAT} AND STRENGTH OF PHYSICAL GLUON:

$$\frac{\alpha_{LAT}}{q^2} + \frac{\alpha_{LAT}^2 T}{q^2} + \frac{\alpha_{LAT}^3 T^2}{q^2} + \cdots$$

$$= \frac{\alpha_{LAT}}{1 - \alpha_{LAT} T} \frac{1}{q^2} \sim 2 \alpha_{LAT} \frac{1}{q^2}$$

SOLUTION: USE PHYSICAL COUPLING

EX $$-\frac{C_f \, 4\pi \alpha_V(q)}{q^2} \equiv V(q)$$

HEAVY QK POT

EXPAND IN $\alpha_V(q)$ → RESUMS TADPOLES AUTOMATICALLY

ANYTHING REASONABLE IS OK

OUR CHOICE:

$$\frac{4\pi}{3}\alpha_P(q_{11}^*)\{1-(1.185+.07n_f)\,\alpha_P(q^*)\}$$

$$\equiv -\log W_{11}$$

$$W_{11} = \langle \tfrac{1}{3} \mathrm{Re}\,\mathrm{Tr}\, P\, e^{-i g_{LAT} \oint A\cdot dx} \rangle = \square$$

= AVE WILSON LOOP AROUND SMALLEST PLAQ.

- SMALL, EUCLIDEAN, EASY TO MEASURE
- AGREES WITH α_V TO 2ND ORDER
- KNOWN TO $\mathcal{O}(\alpha^3_{\overline{MS}})$ (ALMOST) $\left\{\begin{array}{l}\text{ALLES et al '94}\\ \text{LÜSCHER/WEISZ '95}\end{array}\right.$

⇒ USEFUL WHEN COMPUTING OTHER QTYS

SETTING THE SCALE: q^*_{11}

PRESCRIPTION: DO AS WELL AS CAN WITH AVAIL. INFO
(BRODSKY/LEPAGE/MACKENZIE '83)

$$\alpha(q^*) \int d^4q \qquad = \int d^4q \, \alpha(q)$$

$$= \int d^4q \left\{ \alpha \qquad + \alpha^2 \qquad + \cdots \right\}$$

$\equiv$ ABSORB n_f INTO $\alpha(q^*)$ $\left\{ \begin{array}{l} n_f \text{ FROM} \\ \text{VAC POL} \end{array} \right.$

$$\equiv \log q^* = \frac{\int (\log q)}{\int} = \langle \log q \rangle$$

IMPORTANT AT LOW ORDERS

HERE: $q^*_{11} = 3.40/a$

HARD PART: FIND a

SIMULATIONS:

SET $g_{LAT} \longrightarrow$ SIMULATION $\longrightarrow aM$

$$a = \frac{aM}{M_{EXPT}}$$

Υ ($b\bar{b}$) SYSTEM

- GREAT DATA
- SMALL ($\sim .15$ fm) $\rightarrow$ SMALL VOL. ERROR
- INSENSITIVE TO LT QK m's ($\langle p \rangle \sim 1.5$ GeV)
- ΔM's INSENSITIVE TO M_b
- GOOD PHENOMENOLOGY $\rightarrow$ EST ERRORS
- NON-RELATIVISTIC

$$\langle E \rangle \sim .5 \text{ GeV} \sim M_b v^2$$

$$\langle p \rangle \sim 1.5 \text{ GeV} \sim M_b v$$

$$M_b \sim 5 \text{ GeV} \quad \Rightarrow v^2 \sim .1$$

USE NON-REL. EFFECTIVE (CUT-OFF) $\mathcal{L}$ FOR b's:

$$\begin{aligned}
\mathcal{L} = {} & \psi_b^\dagger \left(\Delta_t - \frac{\mathbf{\Delta}^2}{2M_b^0} \right) \psi_b \\
& + \psi_b^\dagger \Big(-c_1 \frac{(\mathbf{\Delta}^2)^2}{8M_b^{0\,3}} + c_2 \frac{ig}{8M_b^{0\,2}} (\mathbf{\Delta}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{\Delta}) \\
& \quad - c_3 \frac{g}{8M_b^{0\,2}} \sigma\cdot(\mathbf{\Delta}\times\mathbf{E} - \mathbf{E}\times\mathbf{\Delta}) - c_4 \frac{g}{2M_b^0} \sigma\cdot\mathbf{B} \\
& \quad + c_5 \frac{a^2 \mathbf{\Delta}^4}{24 M_b^0} - c_6 \frac{a(\mathbf{\Delta}^2)^2}{16n\, M_b^{0\,2}} \Big) \psi_b
\end{aligned}$$

+ GLUE
+ LT FERMS

COMPUTING COEFFS. C_i

(1) INCLUDE ALL OPERATORS WHICH CONTRIB TO DESIRED ORDER IN a, v

(2) C_i's INCORPORATE PHYSICS FROM $M_b \to \infty$
(CUTOFF)

(3) COMPUTE SAME QTY (PERTURBATIVELY) : $\left\{ \begin{array}{l} \text{QED -} \\ \text{USE ANY GAUGE} \end{array} \right\}$
REQ: LATTICE NRQCD = CONTINUUM QCD
(TO ORDER WORKING IN a, v, α)

(4) $C_i(M_b, \alpha)$ $\longrightarrow$ SAME PARAMS AS QCD
SAME PREDICTIVE POWER

(5) USE LATTICE NRQCD FOR NON-PERT CALC

HERE WORK TO TREE LEVEL: $C_i = 1$
(BUT TADPOLE IMPROVE)

CUT OFF: $\frac{\pi}{a} < M_b$

IMPROVED: $\mathcal{O}(v^2, a^2)$

- REMOVES 1 SCALE (M_b)
  GRID $\sim \frac{1}{\langle p \rangle}$ NOT $\frac{1}{M_b}$

- SCHRÖDINGER PROPS (NOT DIRAC)

PROCEDURE:

- GENERATE (OR BORROW*) U_μ CONFIGURATIONS WEIGHTED BY $S[U, \psi_{LT}]$
- PROPAGATE MESONS IN t
- EXTRACT M's BY FITTING EXP. DECAY
- M_b^0, $m_{u,d}$ TUNED TO MATCH M_Υ, m_π

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* TYPICALLY $16^3 \times 24$
1000 PROPAGATORS
$n_f = 0$ & 2 LT QKS

THANKS TO: KILCUP et al
KOGUT et al
UKQCD COLLAB.
HEMCGC COLLAB
MILC COLLAB.

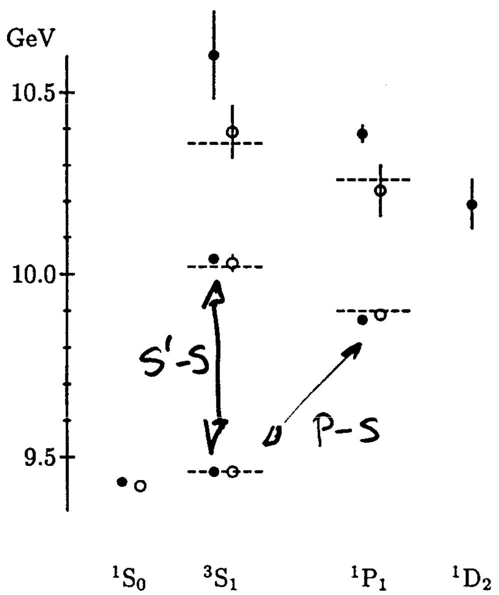
RESULTS

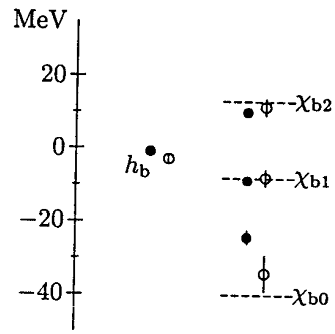
Υ SYSTEM

* only params are M_b, g

- ● $n_f = 0$
- ○ $n_f = 2$

Tadpole necessary for fine structure





FINE STRUCTURE

$^1P_1$ $\quad$ $^3P_{0,1,2}$

- E_0 SET TO $^3S_1$
- SYS ERRORS $\lesssim$ 20 MeV

- h_b SET TO SPIN AVE OF χ's
- SYS ERRORS $\lesssim$ 5 MeV

- EXTRACT

$$a_{P-S} = \frac{a\,\Delta M(\chi_b - \Upsilon)}{\Delta M(\chi_b - \Upsilon)_{EXPT}} \qquad \text{P-S SPLITTING}$$

$$a_{S'-S} = \frac{a\,\Delta M(\Upsilon' - \Upsilon)}{\Delta M(\Upsilon' - \Upsilon)_{EXPT}} \qquad \text{S'-S SPLITTING}$$

- REPEAT FOR 3 LARGER LOOPS
  - DIFF. QTY'S, DIFF EXPTS IN α_p, DIFF SCALES, DIFF NON-PERT. CONTRIBS

  $\Rightarrow$ 8 INDEP. MEASUREMENTS (+ SOME)

- EXTRAP TO $n_f = 3$

$\alpha_P^{(0)}(q_{11})$ FROM a_{P-S}

AT 5 – 15 GeV

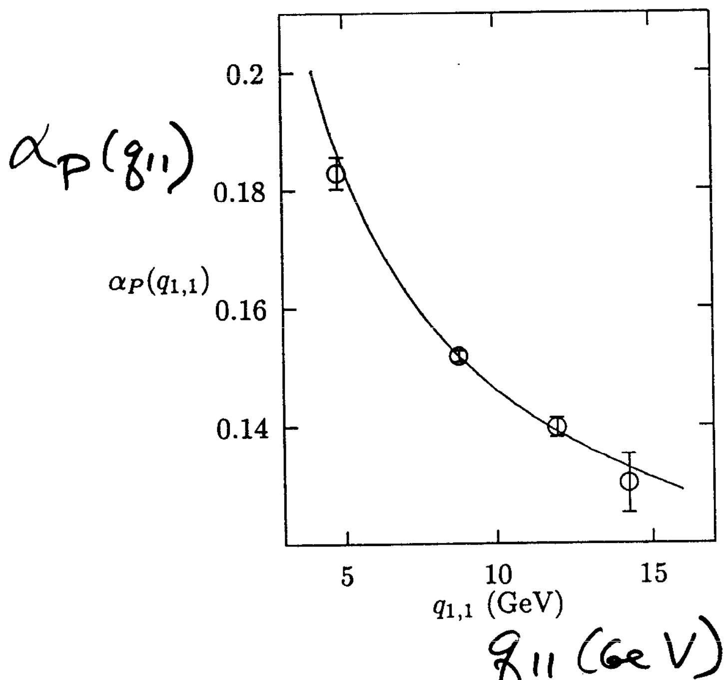


Figure 3: Values of the QCD coupling constant α_P determined from the plaquette in simulations with differing lattice spacings corresponding to $\beta = 5.7$, 6, 6.2 and 6.4, all with $n_f = 0$. The coupling constant is plotted versus the average momentum $q_{1,1}$ carried by gluons in the plaquette at the various lattice spacings, with $q_{1,1} = 3.4/a$ and a determined by $\chi_b - \Upsilon$ splittings. The line shows the coupling constant evolution predicted by third-order perturbation theory.

$\alpha_p(8.2\,\mathrm{GeV})$

$\beta = 6/g^2$	n_f	LOOP	$\alpha_p^{(n_f)}(8.2)$ FROM a_{P-S}	$a_{S'-S}$
6.0	0	1,1	.1552(10)(0)	.1525(11)(0)
		1,2	.1556(10)(6)	.1528(11)(6)
		1,3	.1560(11)(6)	.1531(11)(6)
		2,2	.1565(11)(8)	.1537(12)(7)
5.6	2	1,1	.1794(24)(0)	.1781(33)(0)
		1,2	.1777(24)(30)	.1764(32)(30)
		1,3	.1770(24)(40)	.1757(32)(40)
		2,2	.1767(23)(71)	.1754(32)(71)

$n_f = 0$: P-S , S'-S INCONSISTENT $\{\langle r\rangle_p > \langle r\rangle_S$

$n_f = 2$: BETTER

EXTRAP TO $n_f = 3$:

1,1	.1946(41)(0)	.1944(60)(0)
1,2	.1913(42)(52)	.1912(57)(53)
1,3	.1897(42)(69)	.1897(57)(70)
2,2	.1889(40)(120)	.1887(56)(123)

$n_f = 3$: PERFECT

FAVORS 3 LT FLAVORS

BEST RESULT (P-S)

$$\alpha_P^{(3)}(8.2\,\mathrm{GeV}) = .1946(41)$$

$$\Rightarrow \alpha_{\overline{MS}}^{(3)}(3.56\,\mathrm{GeV}) = .2258(93)$$

$$\Rightarrow \alpha_{\overline{MS}}^{(5)}(M_Z) = .1174(24)$$

	$\Delta\alpha_{\overline{\rm MS}}^{(5)}(M_Z)$
omit $\mathcal{O}(a^2)$ gluonic corrections	−0.6%
omit tadpole improvement of NRQCD	−0.5%
omit $\mathcal{O}(v^2, a, a^2)$ corrections in NRQCD	+0.9%
omit extrapolation (use $n_f = 2$)	−4.7%

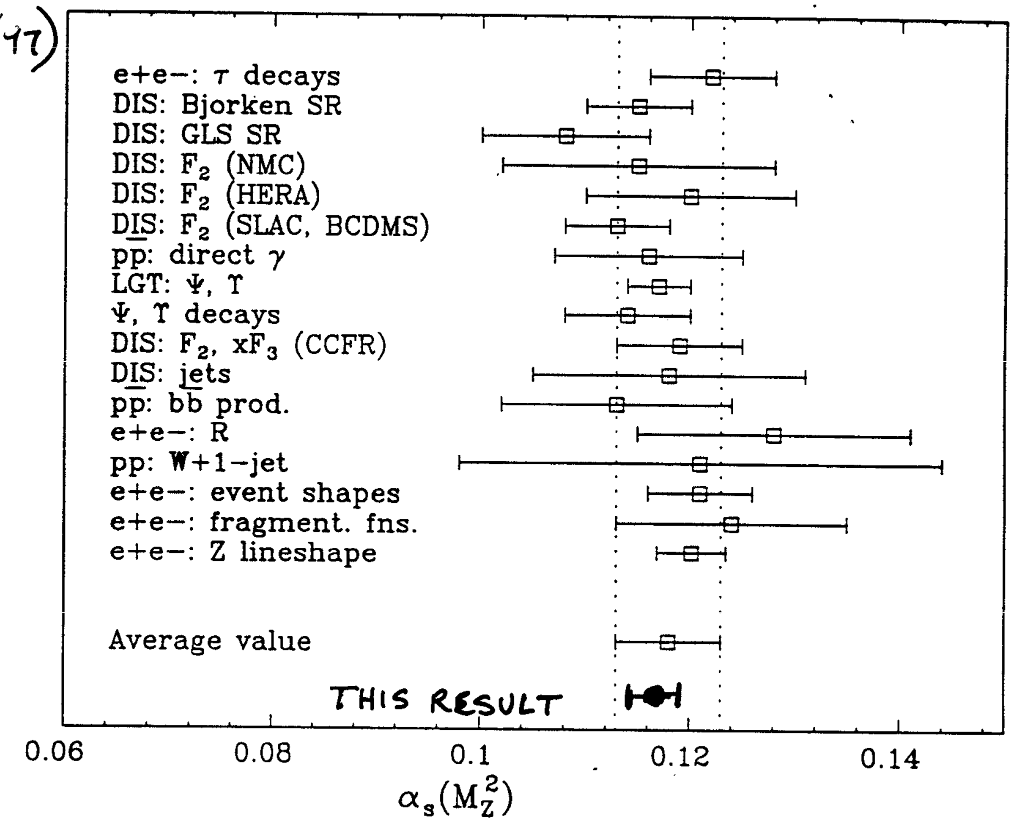
Table 8: Changes in the coupling constant at M_Z when different parts of our simulation or analysis are omitted.

ERRORS

Source	Uncertainty
Unknown n_f dependence in third-order perturbation theory	1.9%
Statistical error in determination of a^{-1}	.9%
Light-quark masses	.9%
Extrapolation in n_f	.3%
Finite a and $\mathcal{O}(v^4)$ errors	.2%
Fourth-order evolution of $\alpha_{\overline{\rm MS}}$	.01%

Table 9: Sources of error in our best determination of $\alpha_{\overline{\rm MS}}^{(5)}(M_Z)$.

(FROM BURROWS '97)



MEASURING SCALE: $\dfrac{\Delta \alpha_s}{\alpha_s} \sim \dfrac{\beta_0}{4\pi} \alpha_s \cdot \left(\dfrac{\Delta \Lambda_{QCD}}{\Lambda_{QCD}} \right)$

MODERATE ACC. IN SCALE → HIGH ACC. IN α_s (5x)

Figure 1: Average $\alpha_s(M_Z^2)$ values and errors from the 17 methods described in the text. The results are ordered vertically in Q.