Phenomenology with Improved Lattice QCD

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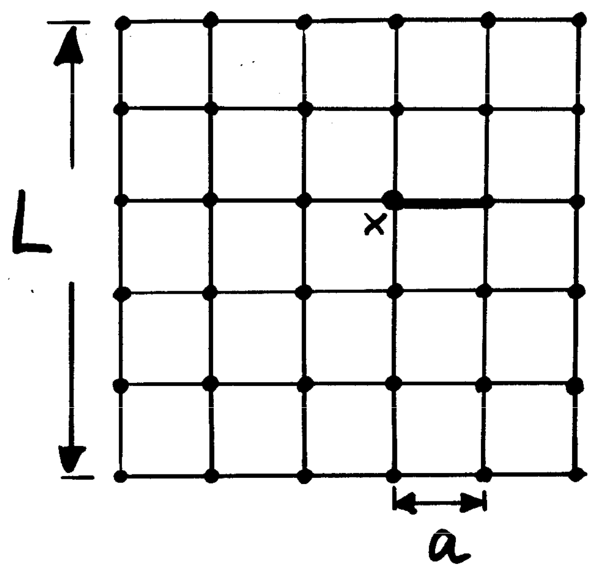
Lattice Field Theory

discretize space - time ...



$$\psi(x) = \psi_x$$

$$U_\mu(x) \approx e^{iagA_\mu(x)}$$

... discretize the QCD Lagrangian

example:

$$\partial_\mu \psi \approx \Delta_\mu \psi = \frac{1}{2a}\left(\psi_{x+\hat{\mu}} - \psi_{x-\hat{\mu}}\right)$$

better:

$$\partial_\mu \psi = \Delta_\mu \psi - \frac{1}{6} a^2 \Delta_\mu^3 \psi + \ldots$$

QCD:

$$\bar{\Psi}\gamma^\mu\partial_\mu\Psi \approx \bar{\Psi}\gamma^\mu\Delta_\mu\Psi + c_1 a^2 \bar{\Psi}\gamma^\mu\Delta_\mu^3\Psi + \dots$$

$$c_1 = c_1(\alpha_s, m_q)$$

in general:

$$S^{lat} = \sum_i c_i(\alpha_s, m_q)\cdot \mathcal{O}_i(\bar{\Psi}, \Psi, A_\mu)$$

with infinitely many terms:

$$S^{lat} = S^{cont} = \int d^4x\, \mathcal{L}^{QCD}$$

in practice: need to truncate

$$S^{lat} = S^{cont} + \mathcal{O}(a^n)$$

$\rightarrow$ introduce cut-off dependence

$$S^{lat} = \sum_n c_n \mathcal{O}_n$$

Goal: choose c_n such that

$$S^{lat} = S_{RT} \qquad (\text{while } a \neq 0)$$

Truncation criteria

- Symanzik:

  by dimension

  errors: $\sim a^n$

- "perfect action"

  locality

  errors: $\sim e^{-ar}$

Symanzik improvement

- determine the c_n:

on-shell improvement (Lüscher + Weisz):

match Green's function on the lattice with continuum equivalent:

$$\langle \cdots \rangle_{lat} = \langle \cdots \rangle_{cont} + \mathcal{O}(a^n)$$

$\rightsquigarrow$ improvement conditions on c_n

- tree-level
- 1-loop pert. thy
- non-perturbatively

Improvement

- different truncation criteria

Symanzik	Wilson "perfect action"

- need to calculate the $C_i(\alpha_s, m_q)$

Hamber + Wu	Wilson + Bell
Wetzel	Iwasaki, ...
Curci, et. al	Hasenfratz + Niedermayer
Lüscher + Weisz	de Grand, et. al.
Eguchi + Kawamoto	Bietenholz + Wiese
Sheikholeslami + Wohlert	Bietenholz, et. al.
Heatlie, et. al.	Burkhalter
Alford, et. al.	Blatter, et. al.
Jansen, et. al.	Farchioni, et. al
Kronfeld, Mackenzie, AK	Borici + de Forcrand
Lüscher, et. al.	⋮
⋮	

also: non-relativistic heavy quarks:

Lepage, et. al.

example: Wilson

- gauge action

$$S^{lat}_{gauge} \xrightarrow[a \to 0]{} \int d^4x \; \mathrm{tr}[F_{\mu\nu} F^{\mu\nu}] + O(a^2)$$

- fermion action

$$S^{lat}_{fermion} \xrightarrow[a \to 0]{} \int d^4x \; \bar{\psi} (\not{D} + m) \psi + O(a)$$

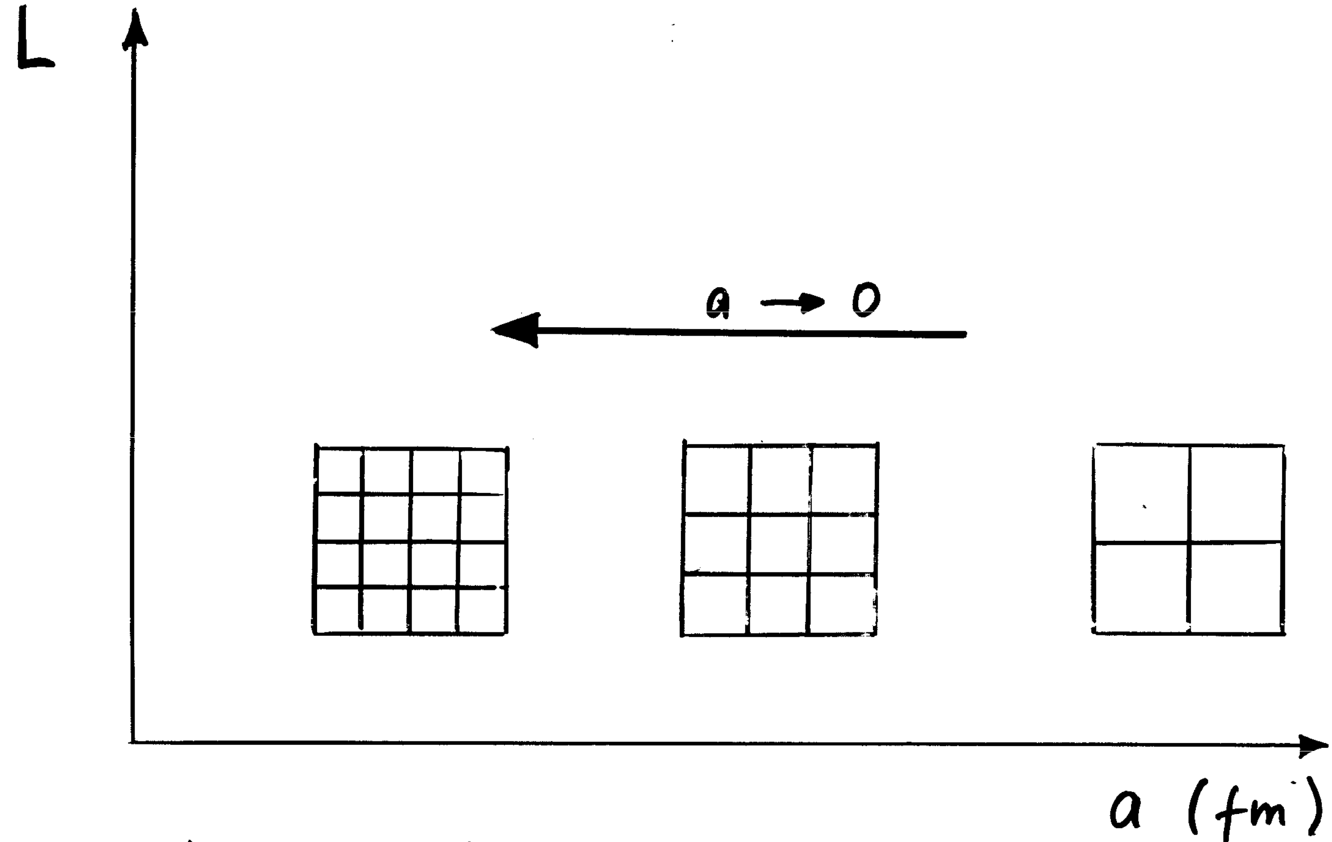
remove cut-off errors

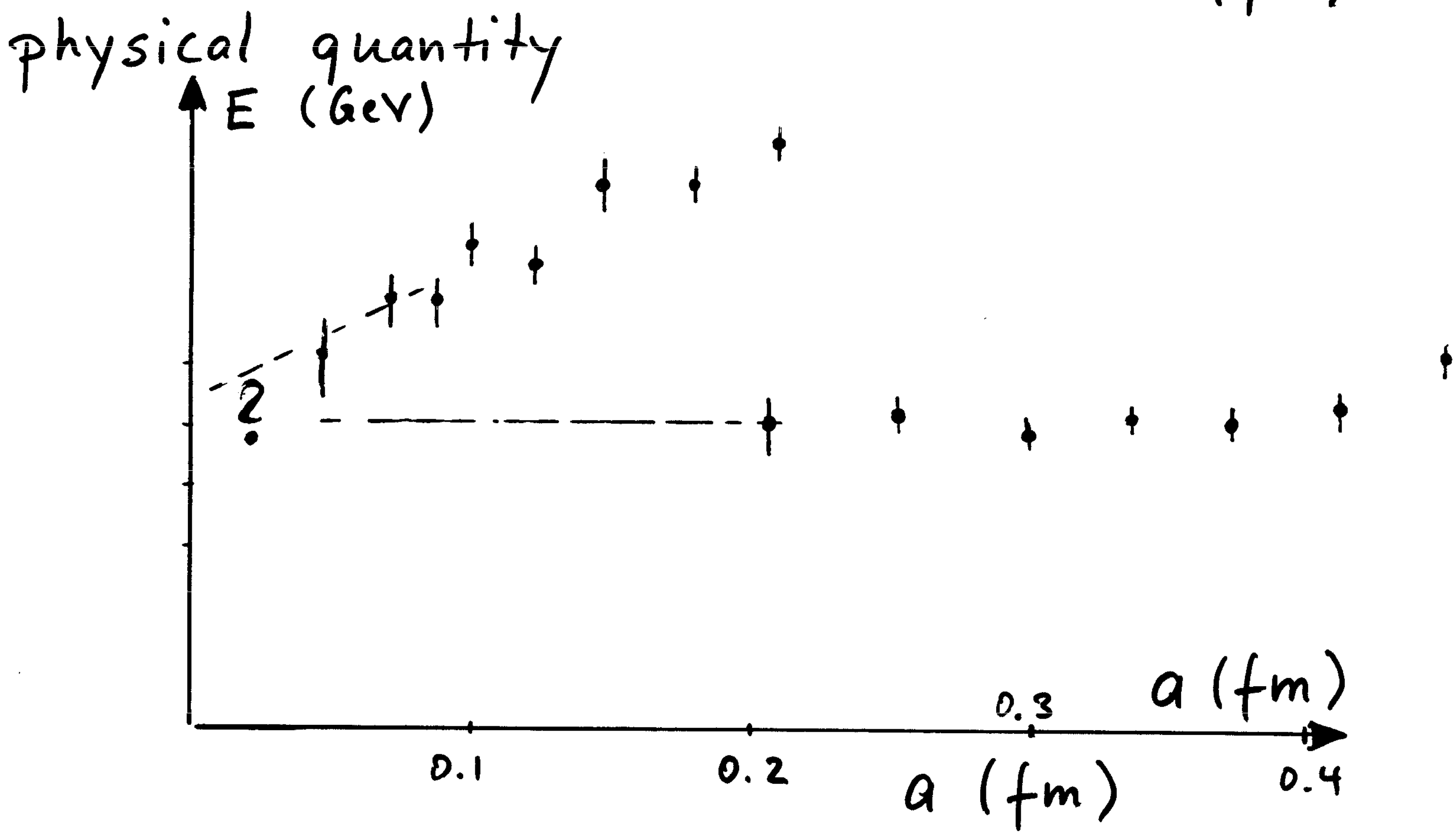
"brute force"
simple action
make a small*
$(a \to 0)$

$a \to a/2$
$N^4 \to 2^4 \cdot N^4$ points

"Improvement"
"complicated" action*
keep a "large".

* has more computational overhead ($< 10 \times$)





enormous computational savings are possible with Improvement

Heavy Quarks on the Lattice

- discretize the non-relativistic theory
  (Lepage et. al)

$$S^{lat} = \int d^4x \; \varphi^+ \left(D_t - \frac{\vec{D}^2}{2m} + \dots \right) \varphi$$

φ: 2-comp. Pauli-spinor

- discretize the relativistic theory such that the non-relativistic limit is respected. (Kronfeld, Mackenzie, AK)

$$S^{lat} = \int d^4x \; \bar{\psi} \left(\not{D} + m \right) \psi \; + \; \mathcal{O}(a^n)$$

improve action $\quad n=1 \longrightarrow n=2$

Introduction

$$S = \sum c_n \, \mathcal{O}_n$$

- Goal: choose c_n such that

$$S = S_{RT}$$

(while $a \neq 0$)

- 2 televant couplings: α_s, m

$$\rightsquigarrow \quad c_n = c_n(\alpha_s, m)$$

$a \longrightarrow 0$: $\quad ma \longrightarrow 0$

$$\alpha_s \sim \frac{1}{\beta_0 \ln(a^2\Lambda^2)} \longrightarrow 0$$

Wilson
Sheikholeslami + Wohlert
Heatlie, et. al.,
α- collaboration
⋮
expend in ma

effective S for Lattice Fermions

$$S^{lat} = \sum c_n(\alpha_s, ma) \quad O_n(\bar{\psi}, \psi, A_\mu)$$

- Wilson : $ma \ll 1$

  c_n's : expand in ma, α_s

- Eichten & Feinberg (static)

  Caswell & Lepage, Lepage, et. al (NRQCD)

  $ma > 1$

  c_n's : expand in $\frac{1}{ma}$, α_s

- Kronfeld, Mackenzie, AXK $ma \sim 1$

  c_n's : expand in α_s

  $$c_n(\alpha_s, ma) = c_n^{(0)}(ma) + \alpha_s \, c_n^{(1)}(ma) + \ldots$$

smooth interpolation between ma and $\frac{1}{ma}$ expansions

Strategy

- hadron masses, static properties

  non-perturbative QCD

  $\rightsquigarrow$ use coarse lattices $a \sim 0.2 - 0.5$ fm

  with a highly improved lattice action

  need $\sim 4^3 - 8^3$ space points

  The first tests can be done on workstations

- Standard Model Parameters

  (m_q, α_s, ...)

  perturbation theory

  $\rightsquigarrow$ use fine lattices $a \lesssim 0.2$ fm

  with an improved action (keep it simple)

  also develop non-perturbative renormalization techniques
  (Martinelli et.al, Bochicchio et.al, Alles et.al, Lüscher et.al, Jansen et.al, ...)

Tests of Improvement

- rotational invariance of $V(r)$

  → graphs

- dispersion relations, $E^2 = m^2 + p^2$
  - free fermions
  - hadrons → graphs

- cut-off (a) dependence

  note: numerical results in quenched approximation ($n_f = 0$)

  → graphs

  compare with standard results at small lattice spacings

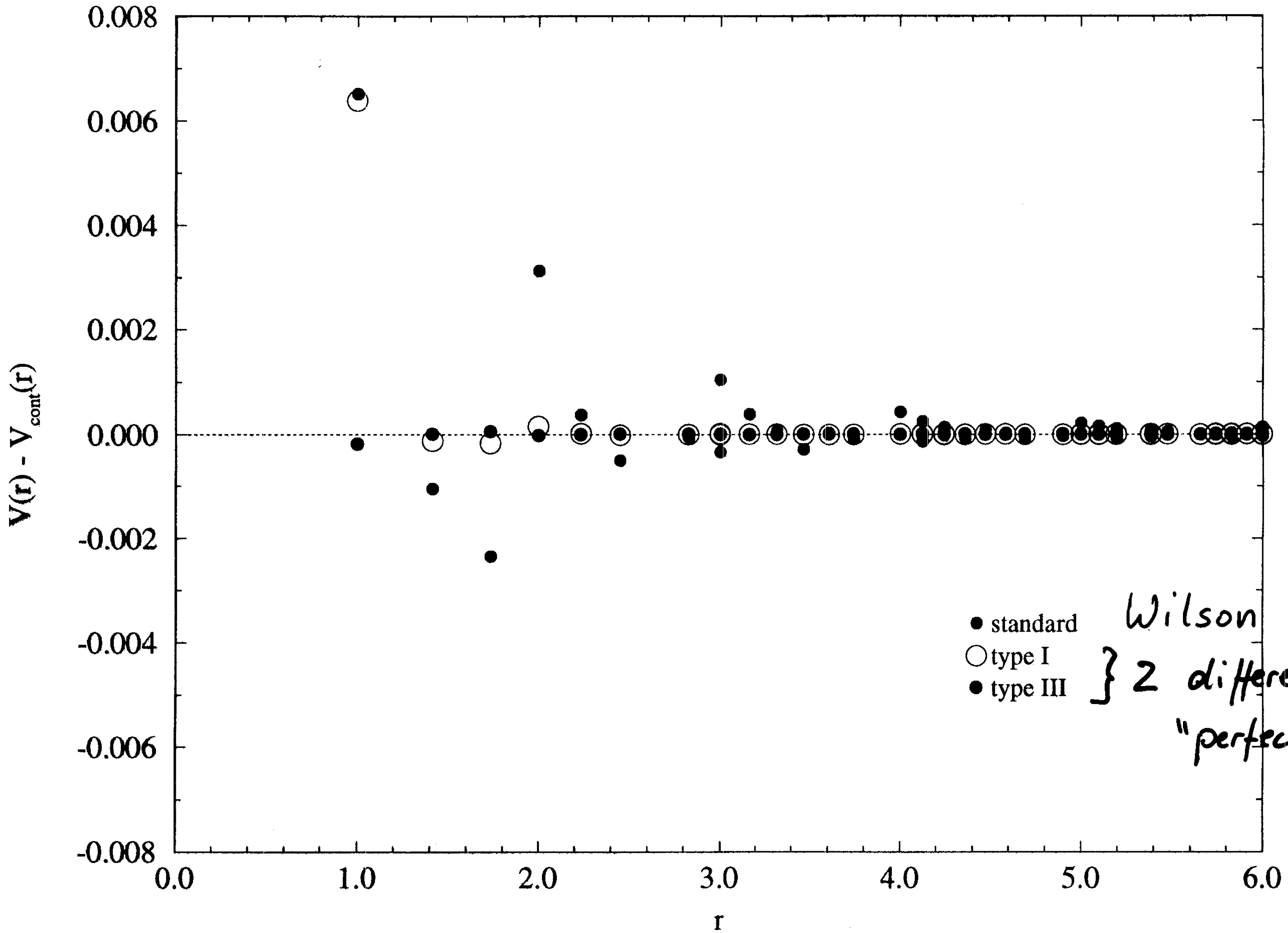
- compare to analytic results in intermediate ($L < 0.7$ fm) volumes

  need to keep $a \lesssim 0.2$ fm

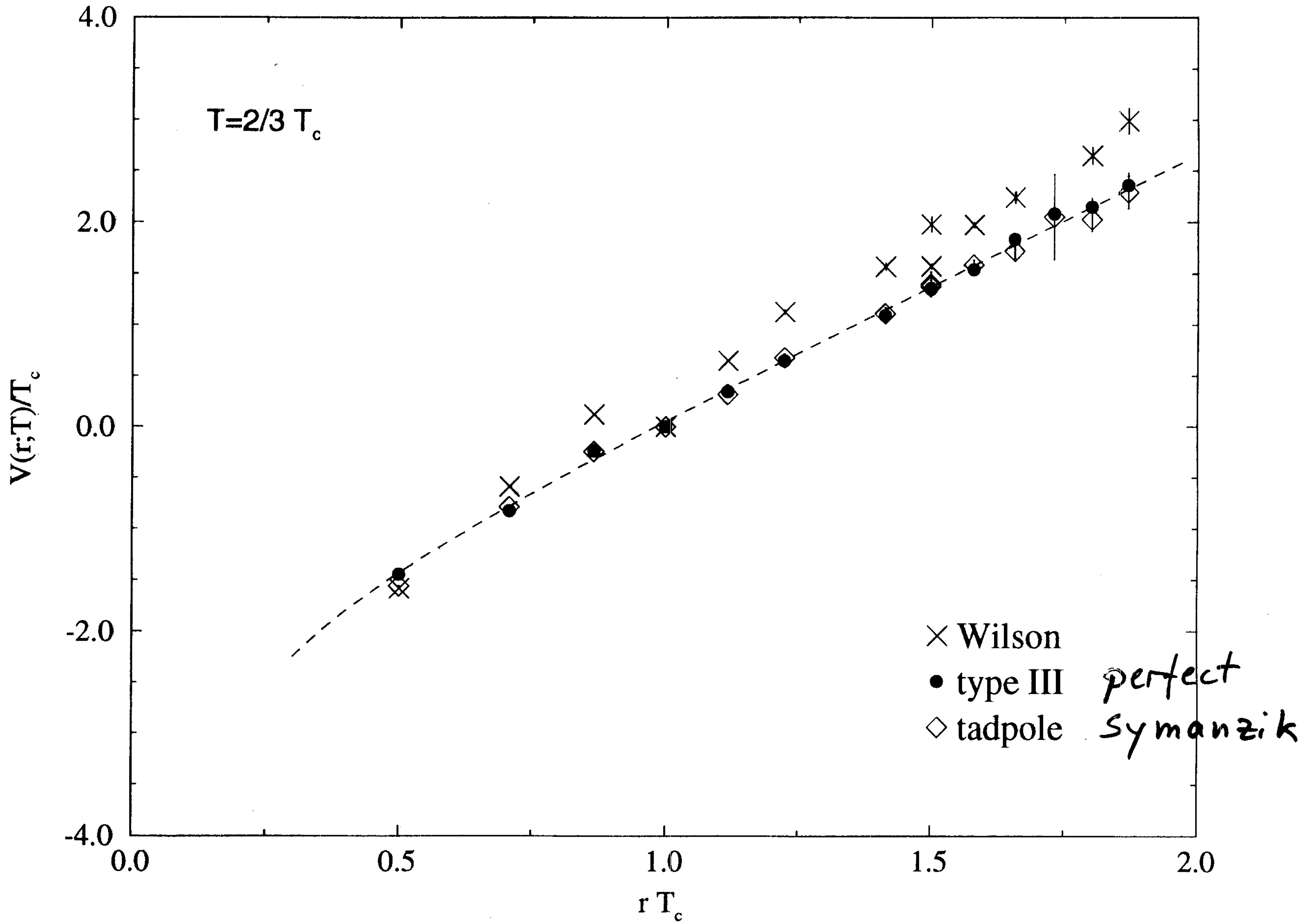
  (Garcia Perez, Snippe, van Baal)

Niedermayer @ Lattice '96

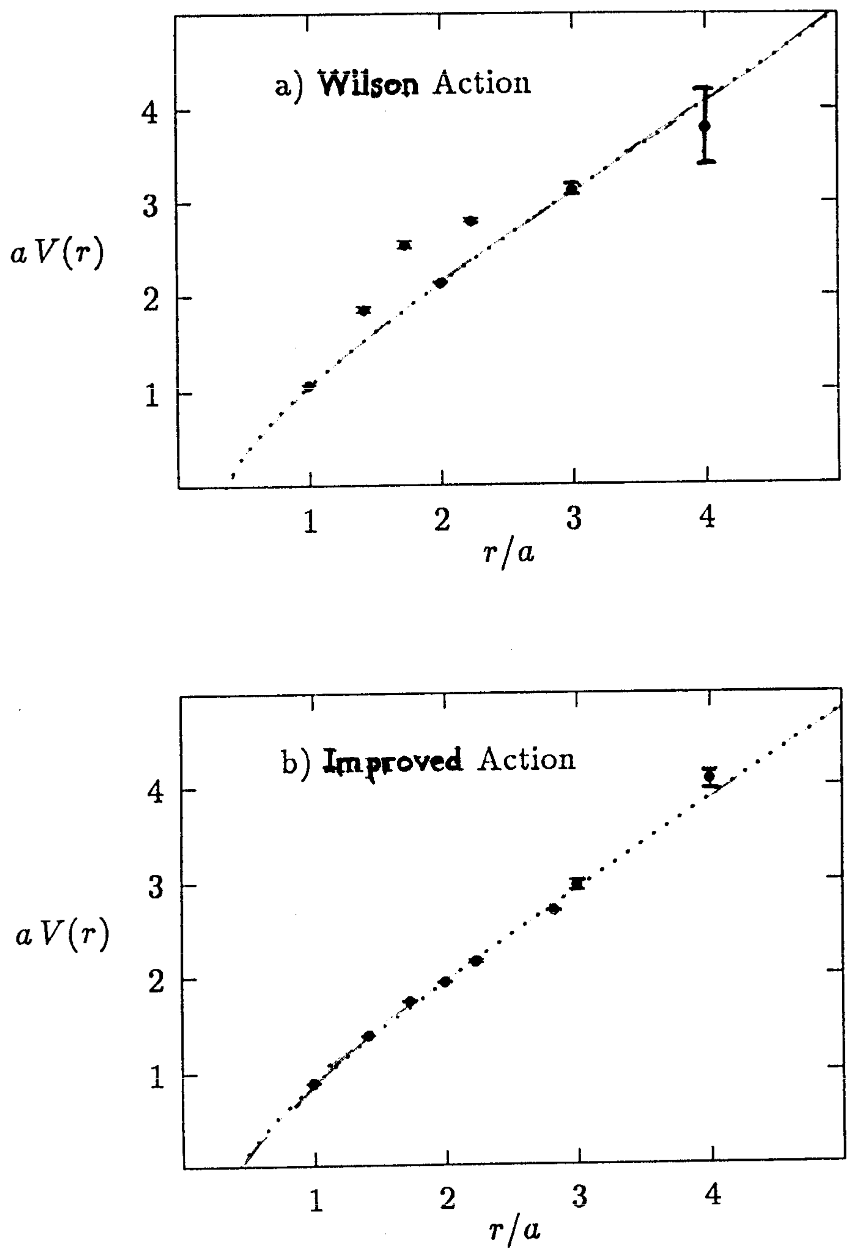
Perturbative qq-potential



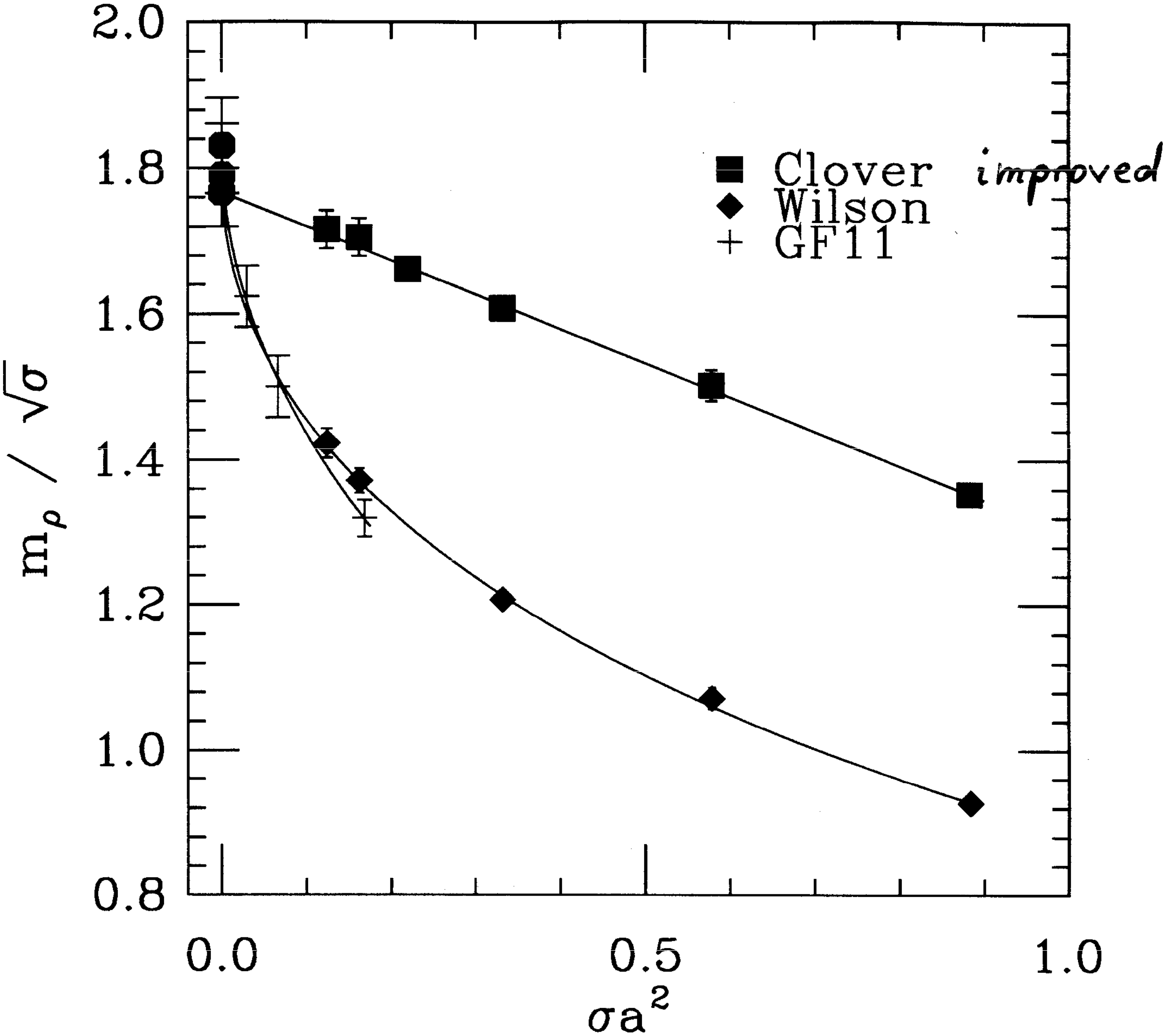
Niedermayer @ Lattice '96



Alford et. al '96



R. Edwards (SCRI) @ Lattice '96



Groups

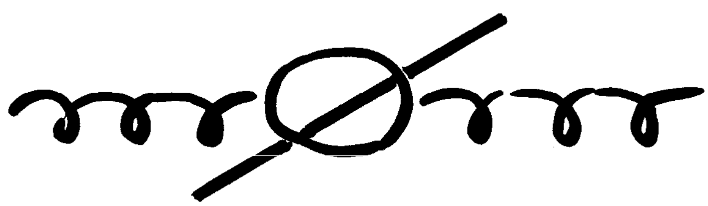
- NRQCD : Lepage, Davies, Shigemitsu, Sloan, ...
- KEK : Aoki, Fukugita, Hashimoto, Onogi, Ukawa, ...
- JLQCD : Japanese Lattice Collaboration
  Aoki, Fukugita, Ishizuka, Okawa, Onogi, Hashimoto,
- CDHW : Collins, de Grand, Heller, Wingate
- MILC : Bernard, de Grand, Heller, de Tar, Gottlieb,
  Toussaint, ...
- FNAL : Kronfeld, Mackenzie, AK, Simone, Onogi,
  Ryan , ...
- SCRI : Edwards, Heller, Collins, Sloan, ...
- SGO : Sloan, Collins, Shigemitsu, Davies, Ali-khan, ..
- UKQCD : Flynn, Sachrajda, Pendleton, Kenway,
  Richards, Davies, Michael, Catterall, Henty, ...
  Manke
- UK(NR)QCD : Catterall, Drummond, Devlin, Horgan
- ADHLM : Alford, Dimm, Hockney, Lepage, Mackenzie, ..
- APE : Allton, Martinelli, Vladikas, Rapuano, ...

- ALPHA : Lüscher, Sommer, Weisz, Wolff, de Divitiis, ...
  Wittig, Guagnelli, ...
- OSU : Kilcup, Venkataraman, Pekurovsky
- LANL : Battacharya, Gupta, Kilcup, ...
- SESAM : Bali, Schilling, Güsken, Spitz, ...
- BBS : Bernard, Blum, Soni
- Hiroshima : Hashimoto, Onogi,

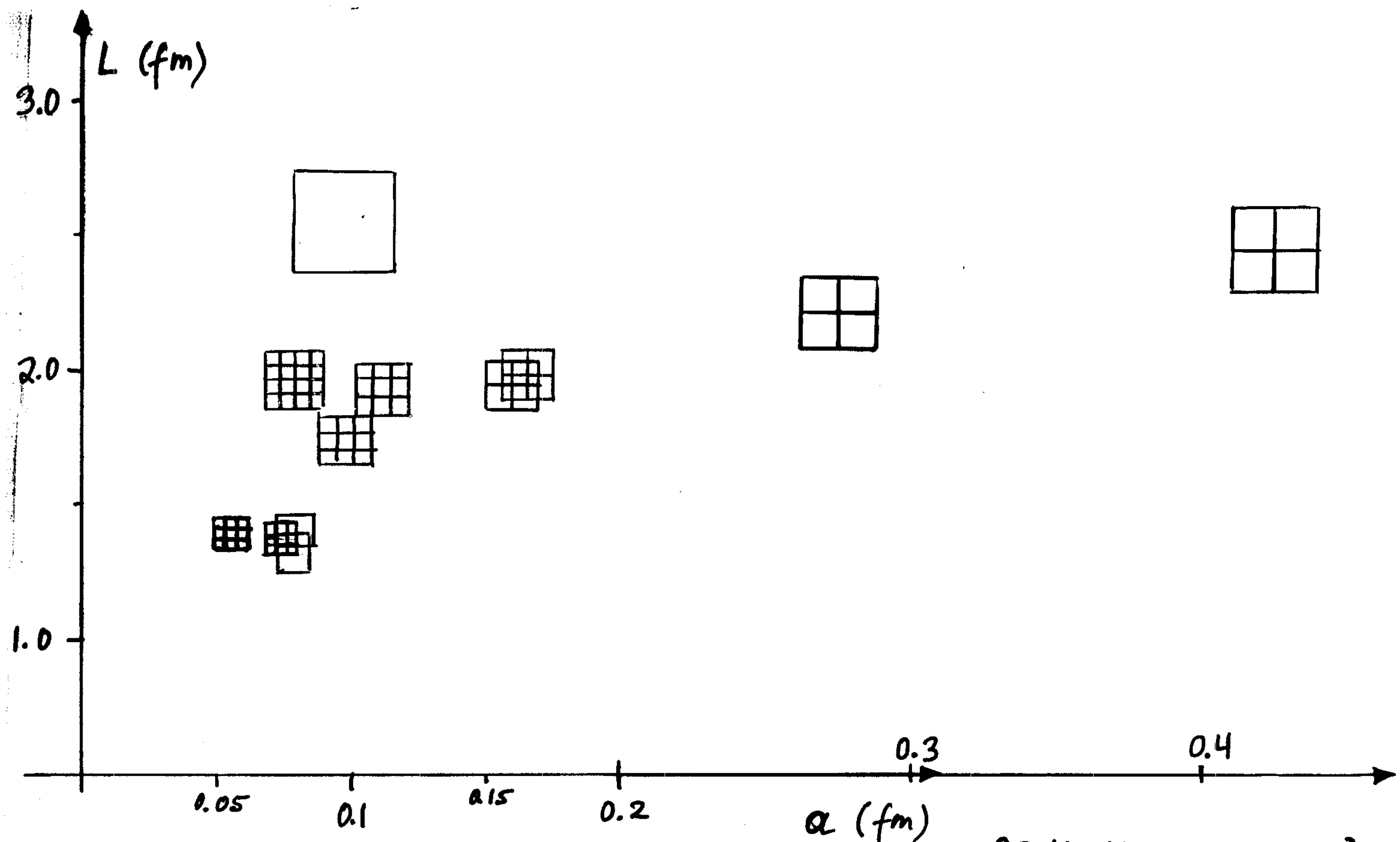
Quarkonia — $b\bar{b}$ & $c\bar{c}$ boundstates

- non-relativistic systems
  — potential model phenomenology
- use <u>relativistic</u> and <u>non-relativistic</u> Lattice actions
- Lattice calculations are easy:
  - small size compared to light hadrons
  - small widths (of states below threshold)

  ⇝ insensitive to vacuum polarization



- control over discretization (lattice spacing) errors
  - improved lattice actions
  - vary the lattice spacing



FNAL: $a \to a^2$ lattice spacing errors

NRQCD: $a, mv^4 \longrightarrow a^4, mv^6$ errors

$n_f = 0 \longrightarrow n_f = 2$ sea quarks

UK(NR)QCD: a, mv^4 errors

no spin dependent terms

KEK, CDHW: a, $n_f = 0, 2$

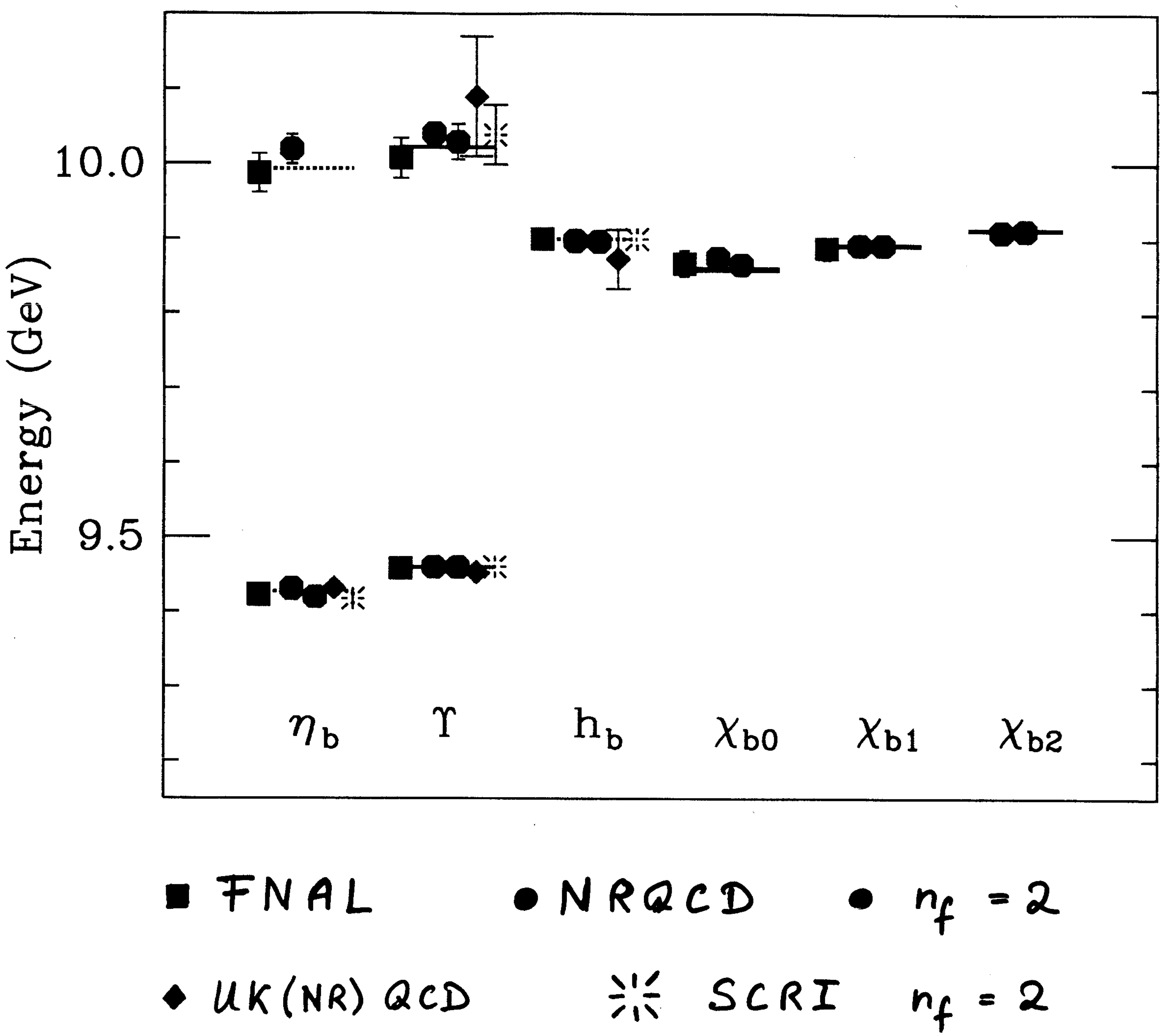
ADHLM: gauge: $a^2 \to a^4$

NRQCD: a^4, mv^6

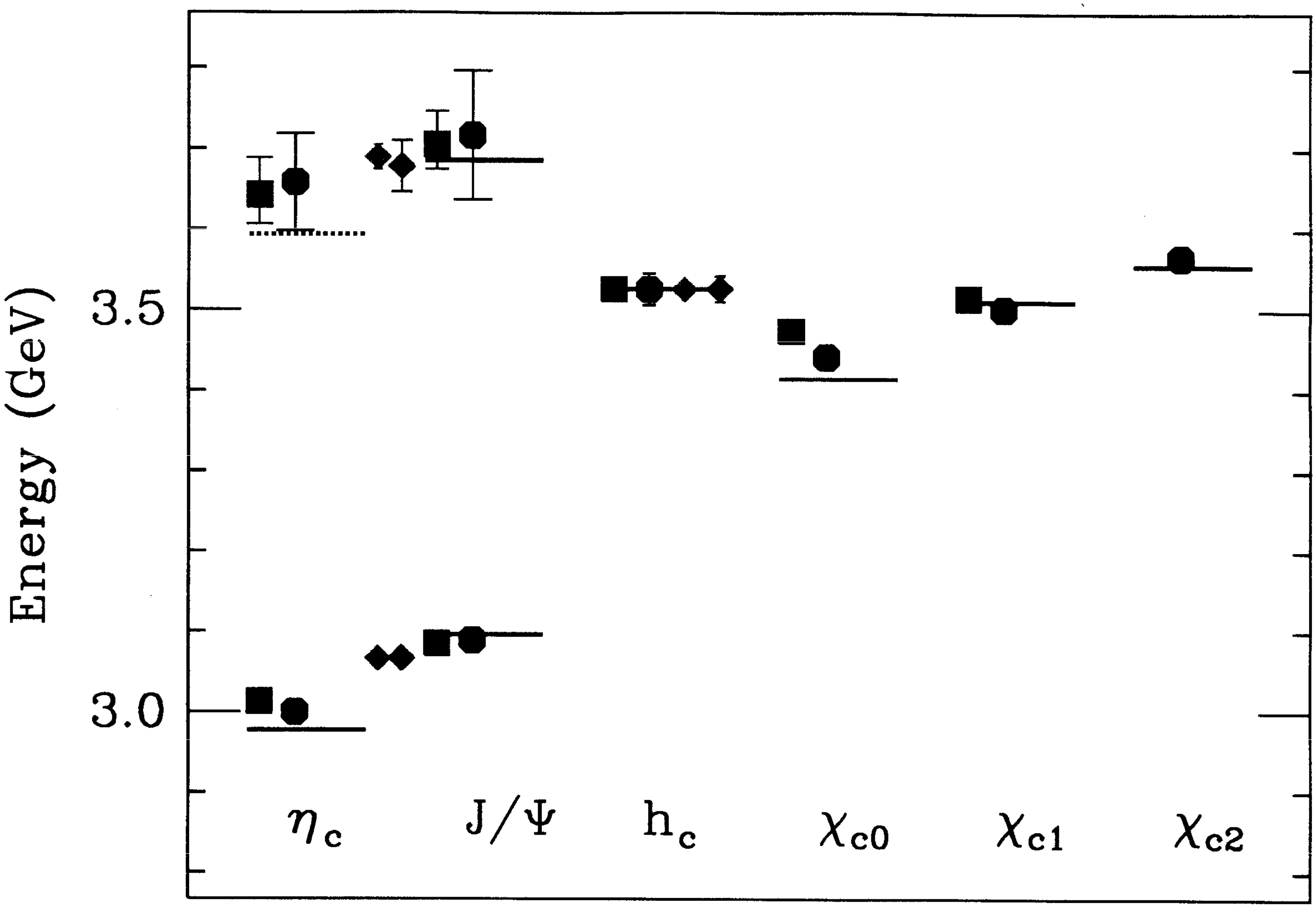
SCRI: a^2 errors

$n_f = 2$

The $b\bar{b}$ spectrum



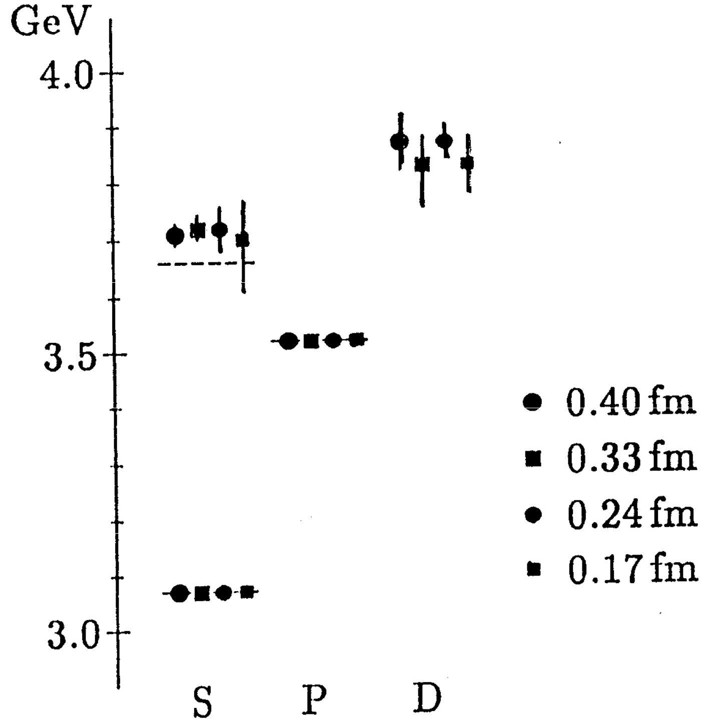
The $c\bar{c}$ spectrum



■ FNAL ● NRQCD

◆◆ ADHLM

Alford, et. al '95

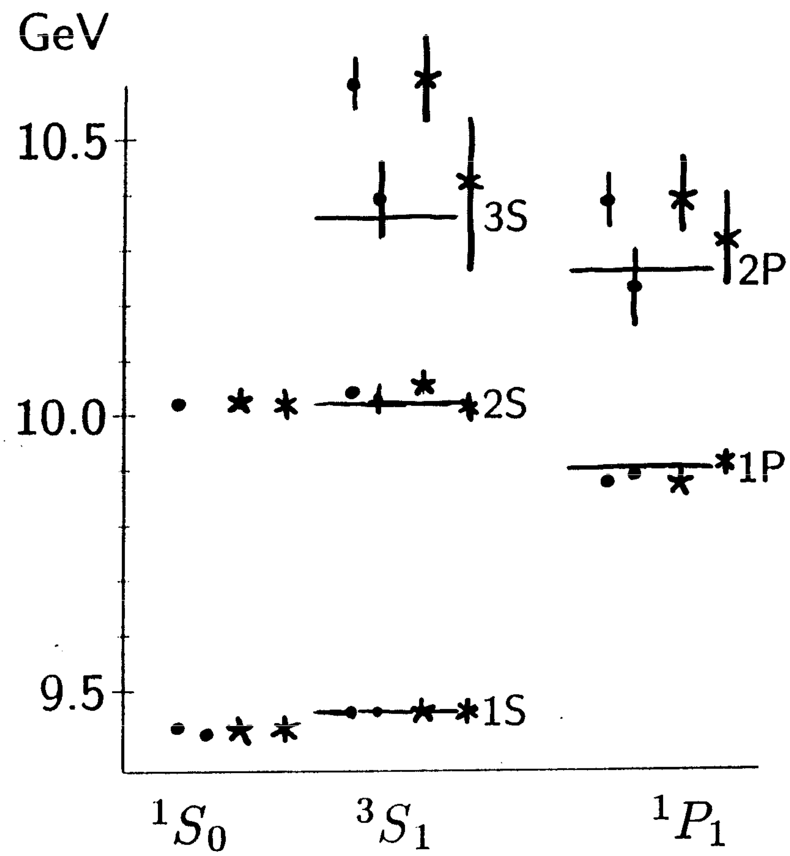


S. Güsken, A. Spitz (SESAM)

@ Lattice '97

$b\bar{b}$

Spin Independent Spectrum

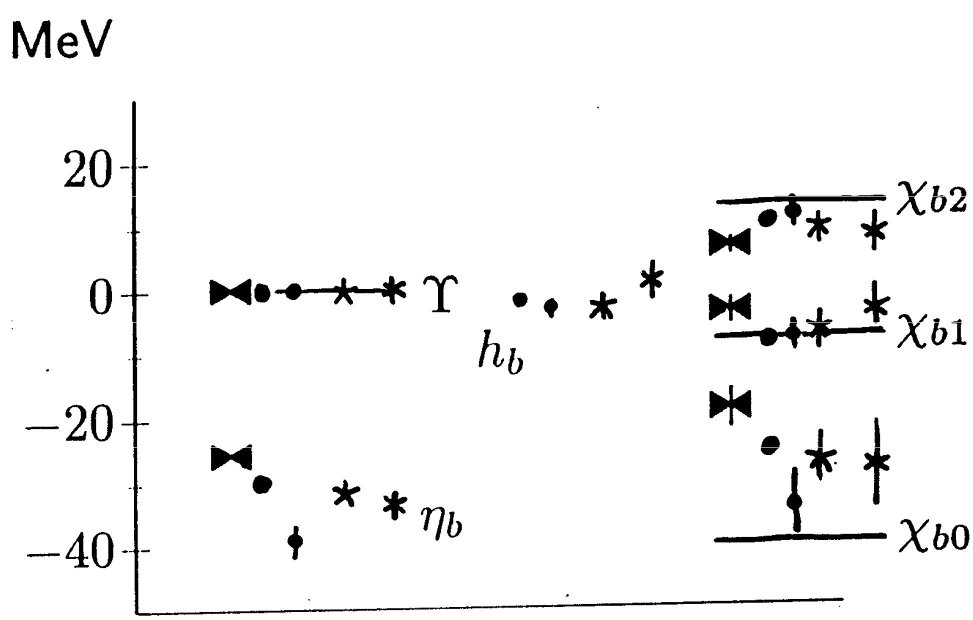


- • NRQCD $n_f = 0$ $(m v^4)$
- • NRQCD $n_f = 2$ (KS) $(m v^4)$
- ⋆ SESAM $n_f = 0$ $(m v^6)$
- ✱ SESAM $n_f = 2$ (W) $(m v^6)$

S. Güsken, A. Spitz (SESAM) @ Lattice '97

Spin Splittings

$b\bar{b}$



⧓ UKQCD $n_f = 0$ (mv^6)

• NRQCD $n_f = 0$ (mv^4)

• NRQCD $n_f = 2$ (mv^4)

* SESAM $n_f = 0$ (mv^6)

* SESAM $n_f = 2$ (W) (mv^6)

$$\boxed{\alpha_s \; , \; m_Q}$$

(Lattice) QCD action has 2 parameters

$m_Q^{(lat)}$, $\alpha_s^{(lat)}$

- spin averaged spectrum

  $a \, M(1P-1S)_{exp} = M(1P-1S)_{lat}$

  $\rightarrow$ $\underline{a \text{ in } GeV^{-1}}$ (or fm)

- $\alpha_s^{lat} \xmapsto[\text{pert. thy.}]{} \alpha_{\overline{MS}}(\pi/a)$

- tune m_Q^{lat} to M_{1S}^{exp}

  $m_Q^{lat} \xmapsto[\text{pert. thy.}]{} m_Q^{pole}$, $m_Q^{\overline{MS}}$

α_s

- choose a renormalized coupling that can be determined non-perturbatively on the Lattice from (short-distance) quantities

$\square$, V, SF, TP, P, ...

- Renormalized perturbation theory works well ($a \lesssim 0.2$ fm).

- Convert to $\overline{MS}$:

$$\alpha_{\overline{MS}} = \alpha_X + c_1 \alpha_X^2 + c_2 \alpha_X^3 + \ldots$$

For P, SF $\quad c_2 (n_f = 0)$ known
(Lüscher + Weisz, Allés, etal
Wolff + Sommer)

$\longrightarrow \Delta \alpha_{\overline{MS}} (m_z) = \pm 0.002$

lattice $\alpha \longrightarrow$ renormalized α

... perturbatively ...

$$\ln \langle \mathrm{Tr}\, U_{\Box} \rangle = -\frac{4\pi}{3} \left[\alpha_P(q^*) + c_1 \alpha_P^2(q^*) \right]$$

$$c_1 = -1.185 - \left\{ \begin{array}{ll} 0.07 & (\mathrm{KS}) \\ 0.025 & (\mathrm{W}) \end{array} \right\} n_f$$

@ 1-loop: $\alpha_P = \alpha_V$

relation to $\overline{MS}$:

$$\alpha_{\overline{MS}}(q) = \alpha_P(Q) \left[1 + \frac{2}{\pi} \alpha_P(Q) + d_2 \alpha_P^2(Q) + \ldots \right]$$

$$q = e^{-5/6} Q$$

$$d_2(n_f = 0) = 0.96$$

(Lüscher + Weisz; Alles, et al.)

including $d_2 \longrightarrow \Delta \alpha_{\overline{MS}}(m_Z) = +0.002$

need the n_f dependence @ 2-loops

- non-perturbatively

- heavy quark potential

$$V(q) = -C_F \frac{4\pi \alpha_V(q)}{q^2}$$

(T. Klassen → figure)

- Schrödinger functional (SF), twisted Polyakov Loops (TP) (Lüscher et. al, de Divitiis et. al)

$$\alpha_S(q) = \frac{g_S^2(L)}{4\pi} \qquad q = 1/L$$

compute the running using a "recursive finite size technique".

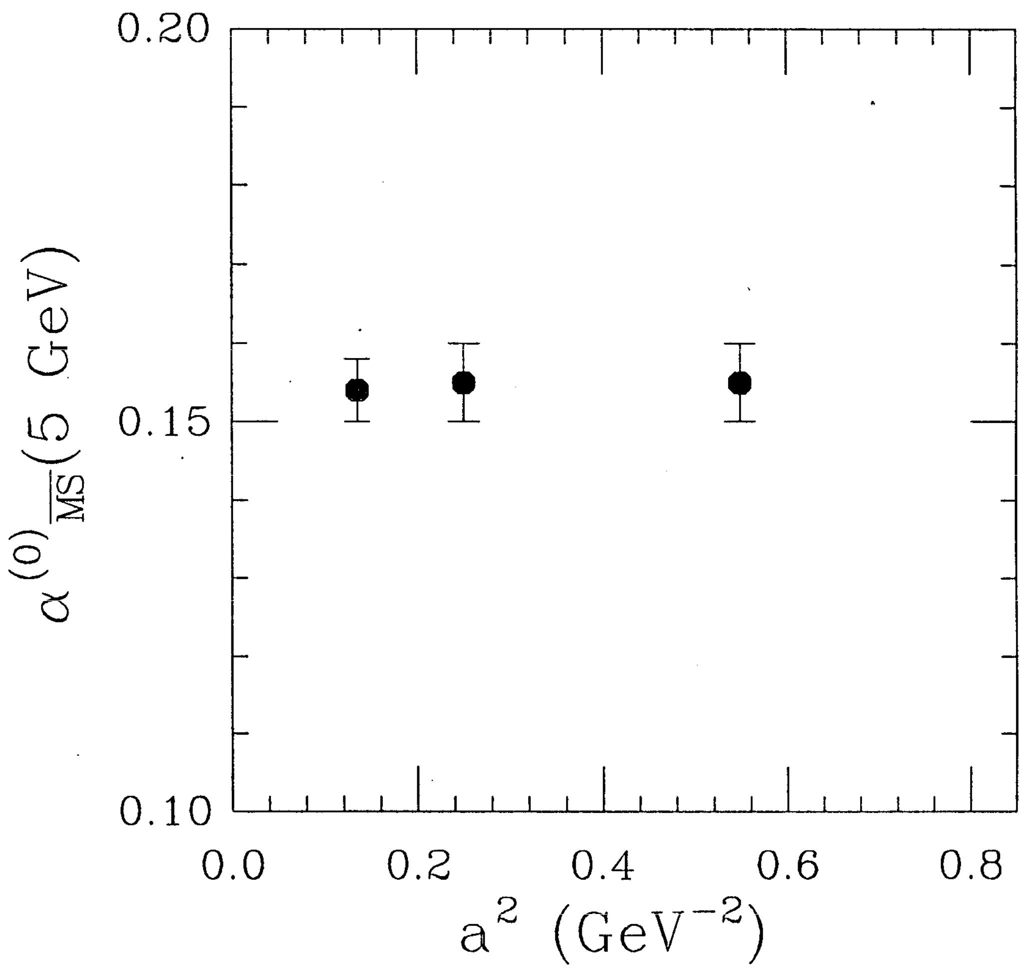
control over systematic errors

- 3-gluon vertex

(Parrinello + UKQCD)

exploratory

FNAL: • from $b\bar{b}$ 1P-1S splitting



stat. errors only

α_s cont'd

- n_f dependence

  $n_f = 0, 2 \longrightarrow n_f = 3$

- sea quark mass dependence

  Chiral perturbation thy (Rothstein + Grinstein):

$$\Delta M = \Delta M_0 \left(1 + c \sum_{i=u,d,s} m_i + \dots \right)$$

$\longrightarrow$ graph

NRQCD: extrapolate to

$m_{eff} \simeq m_s/3$ for u, d, s average

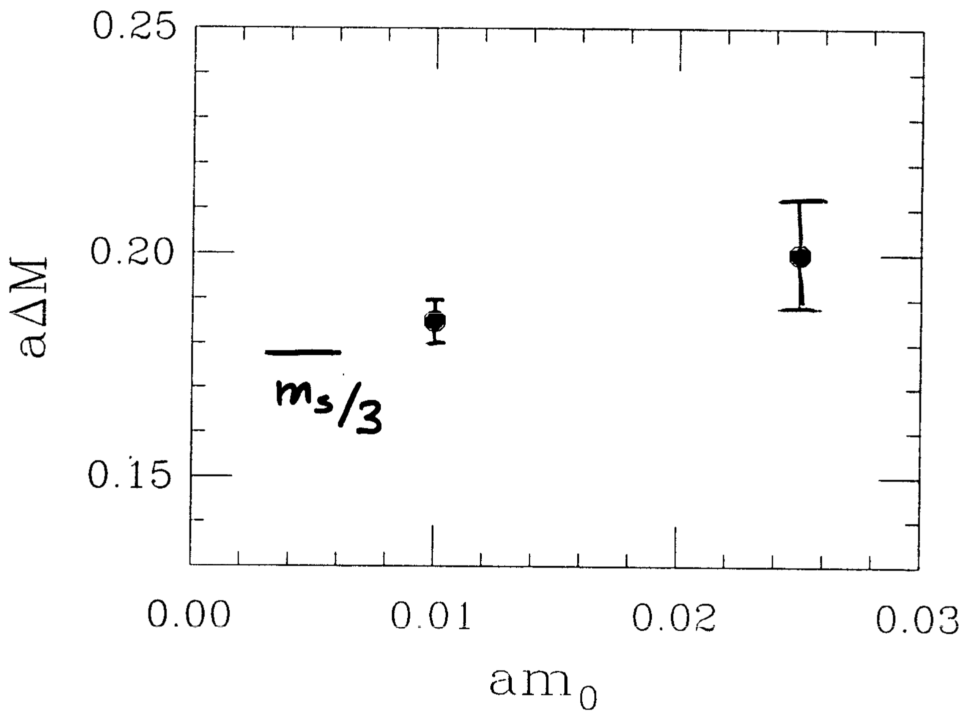
since gluon momenta $\sim 0.5 - 1$ GeV $\gg m_{u,d,s}$

$\rightarrow$ shift in ΔM is within statistical errors

- need $n_f = 3$ $m_s \neq m_{u,d}$

NRQCD '97

1P - 1S splitting vs. sea quark mass



extrapolate to $m_{eff} \sim m_s/3$ for average light quark mass

shift in ΔM is within statistical errors summary of systematic errors

on $\alpha^{(3)}_{\overline{MS}}$ (5 GeV)

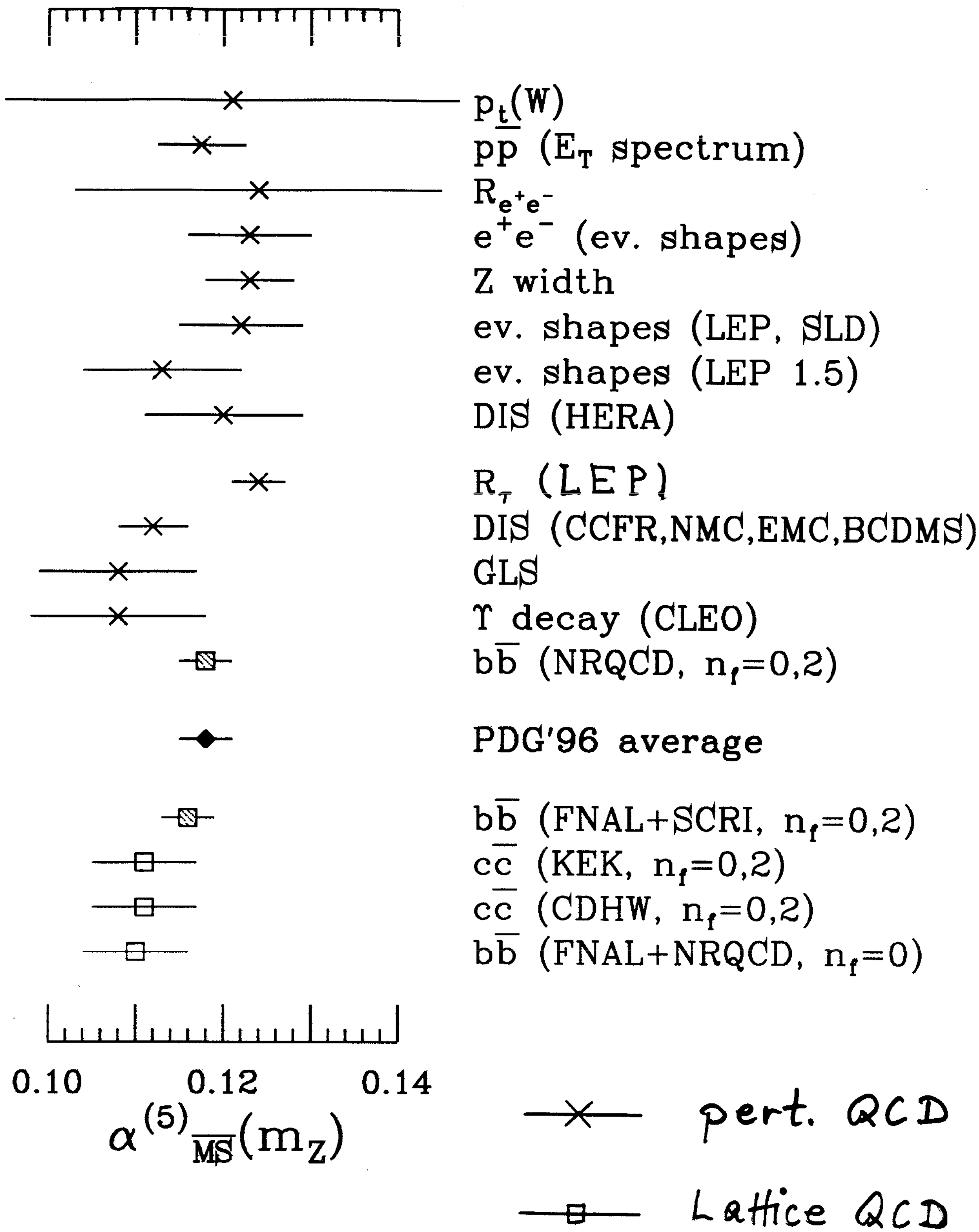
	~ 1992	now
* experimental (~ few MeV)	—	—
statistical (monte carlo)	1-3 %	—
finite lattice spacing	< 1-3 %	—
perturbative $\rightarrow \overline{MS}$	~ 5 %	$\rightarrow$ few %
sea quarks	5-8 % ($n_f=0$)	$\rightarrow$ ~ ($n_f=2$)
total	8-10 %	$\rightarrow$ few
@ m_Z :	5-7 %	$\rightarrow$ 1 %

* Note:

$b\bar{b}$: $\Delta m_{1P-1S} \simeq 8$ MeV

$\rightsquigarrow \delta \alpha_{\overline{MS}}(m_Z) \lesssim 0.5\ \%$

α_s in comparison



Light Quark Masses

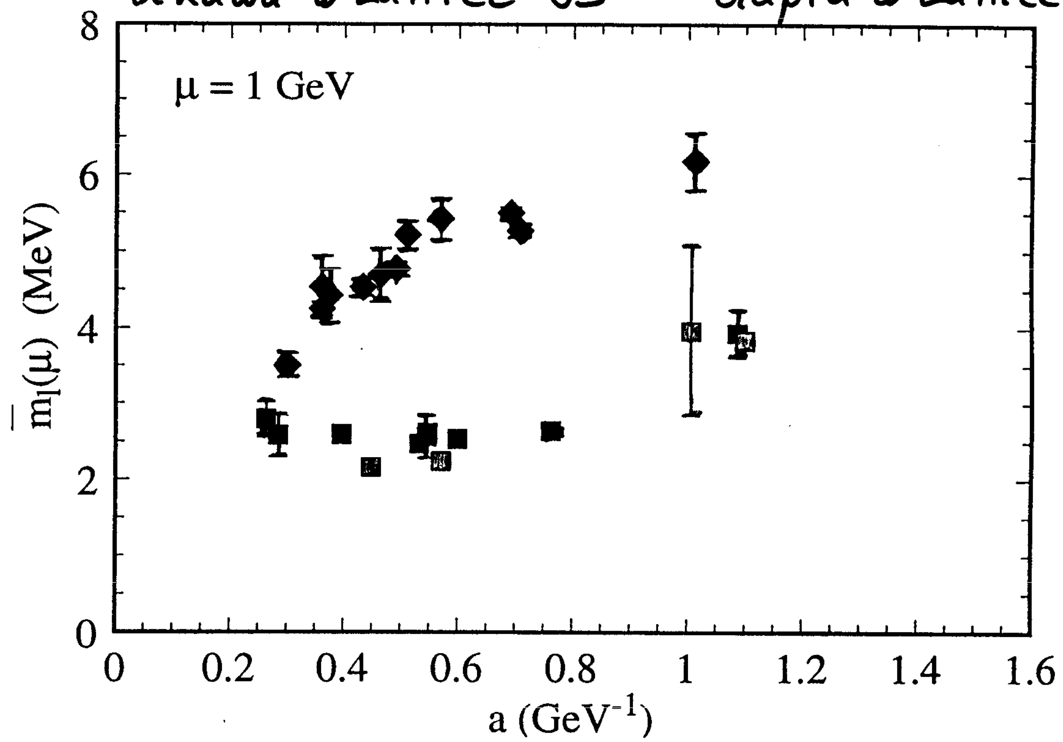
- calculate pion mass on the lattice
- adjust Lattice quark mass, m_0, so that

$$m_\pi^{lat} = m_\pi^{exp}$$

- perturbation thy

$$m_0 \longrightarrow m_{\overline{MS}} \quad (= \overline{m}) \qquad @\ q \sim \frac{\pi}{a}$$

Ukawa @ Lattice '93 Gupta @ Lattice '94



◆ Wilson fermions $n_f = 0$

■ staggered fermions $n_f = 0$

▣ " " $n_f = 2$

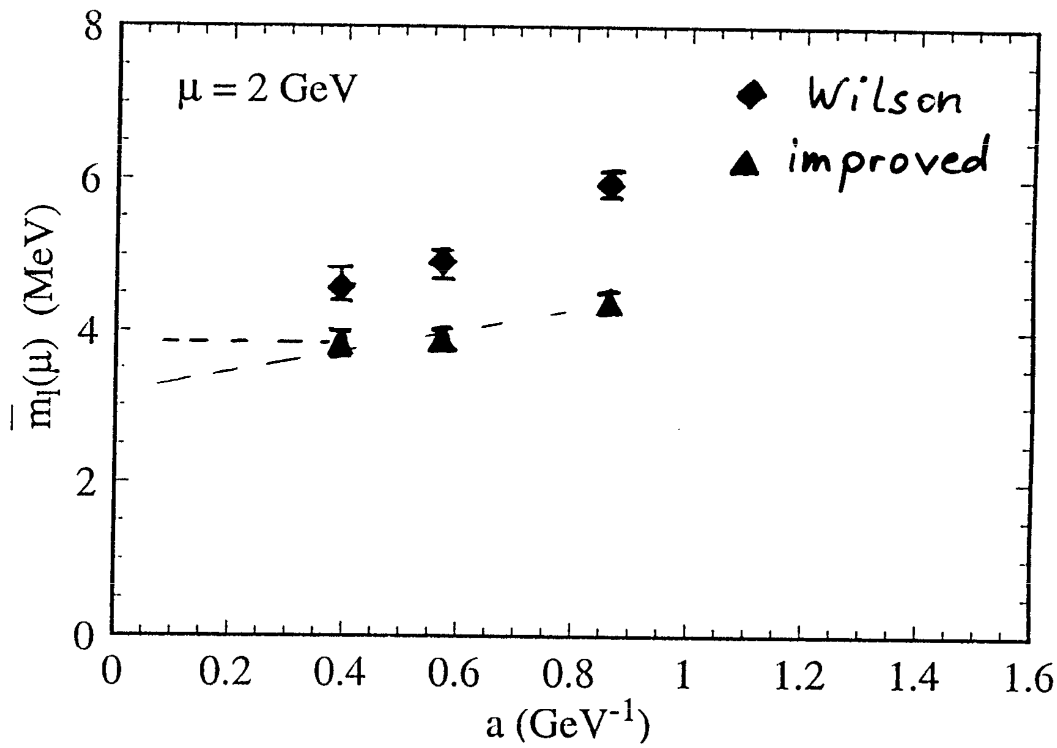
⟹ Improve the Wilson action

FNAL '96

use $\mathcal{O}(a)$ improved action

cutoff dependence from $\mathcal{O}(\alpha_s a)$, $\mathcal{O}(a^2)$, $\mathcal{O}(\alpha_s^2)$ terms

results are consistent with most other Lattice calculations



$n_f = 0$: $\overline{m}_\ell(2\ \mathrm{GeV}) = 3.6(6)\ \mathrm{MeV}$

$\overline{m}_s(2\ \mathrm{GeV}) = 95(16)\ \mathrm{MeV}$

$n_f = 3$: reduce the masses by 20-40 %

(use KEK results)

new in '97:

- CP PACS: continuum extrapolation using Wilson fermions

- new results by LANL, JLQCD, QCDSF using an improved ($\mathcal{O}(a^2)$) action consistent with FNAL results

- SESAM: Wilson fermions with $n_f = 2$

  $\rightsquigarrow$ n_f dependence consistent with previous (staggered) results.

Eichten et. al (Lattice '96)

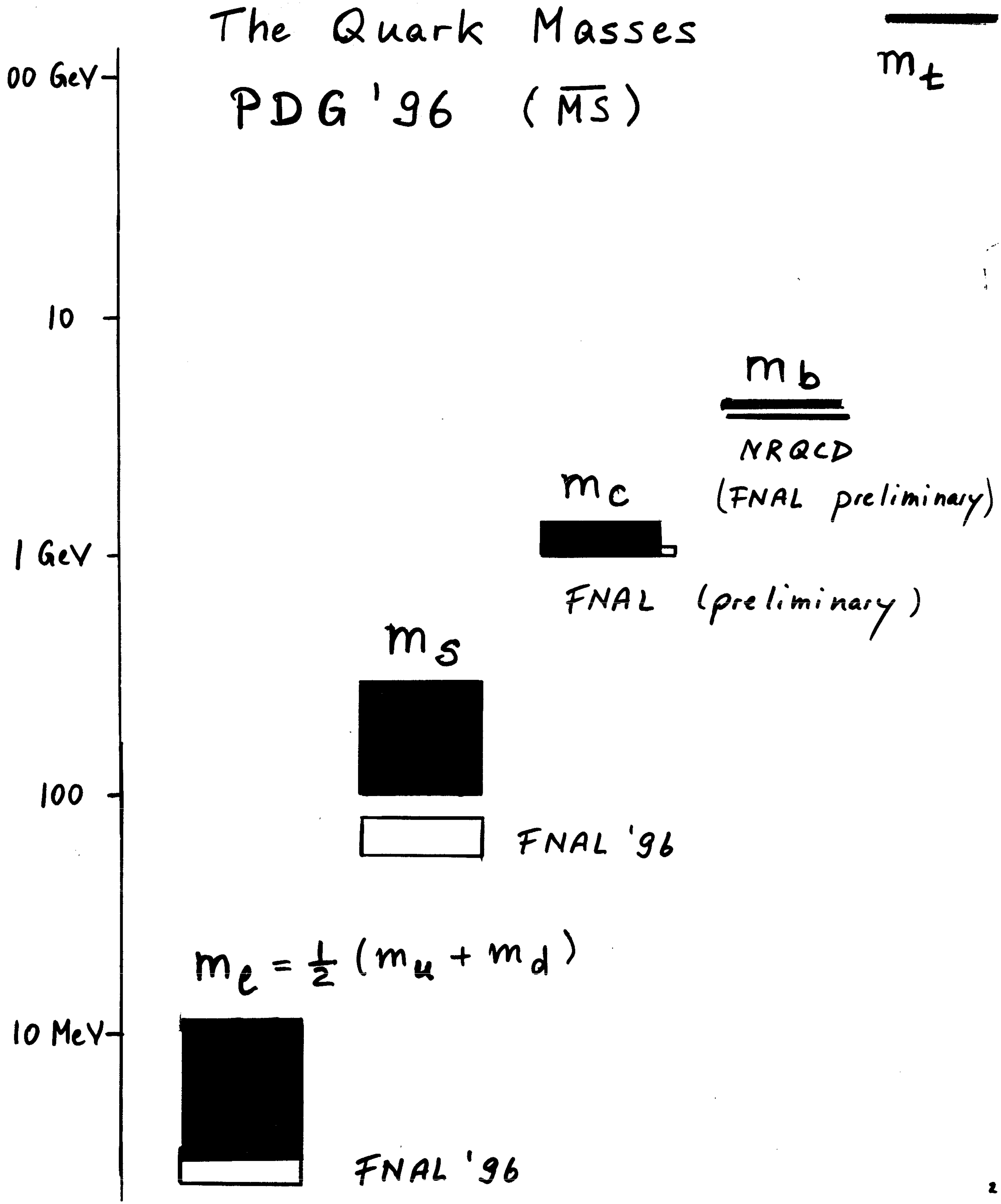
Include the electromagnetic field dynamically

to calculate charged mass splittings

$$m_{\pi^+} - m_{\pi^0} = 4.9 \pm 0.3 \text{ MeV}$$

$$\frac{m_u}{m_d} = 0.51 \pm 0.1$$

- They still need to study the systematic errors, including sea quark effects.


The Quark Masses
PDG '96 ($\overline{MS}$)
00 GeV
10
1 GeV
100
10 MeV
m_t
m_b
NRQCD
(FNAL preliminary)
m_c
FNAL (preliminary)
m_s
FNAL '96
$m_\ell = \frac{1}{2}(m_u + m_d)$
FNAL '96

$K-\bar{K}$ mixing

$$\varepsilon = (\text{known}) \cdot \mathrm{Im}\,(V_{td}^2) \cdot V_{ts}^2 \cdot f(m_t)$$

$$\cdot \langle \bar{K} | \mathcal{O}_{\Delta s=2} | K \rangle$$

$$\hookrightarrow \tfrac{8}{3} m_K^2 f_K^2 B_K$$

- systematic errors are reduced in the ratio

$$B_K = \frac{\langle \bar{K} | \mathcal{O}_{\Delta s=2} | K \rangle}{\frac{8}{3} m_K^2 f_K^2}$$

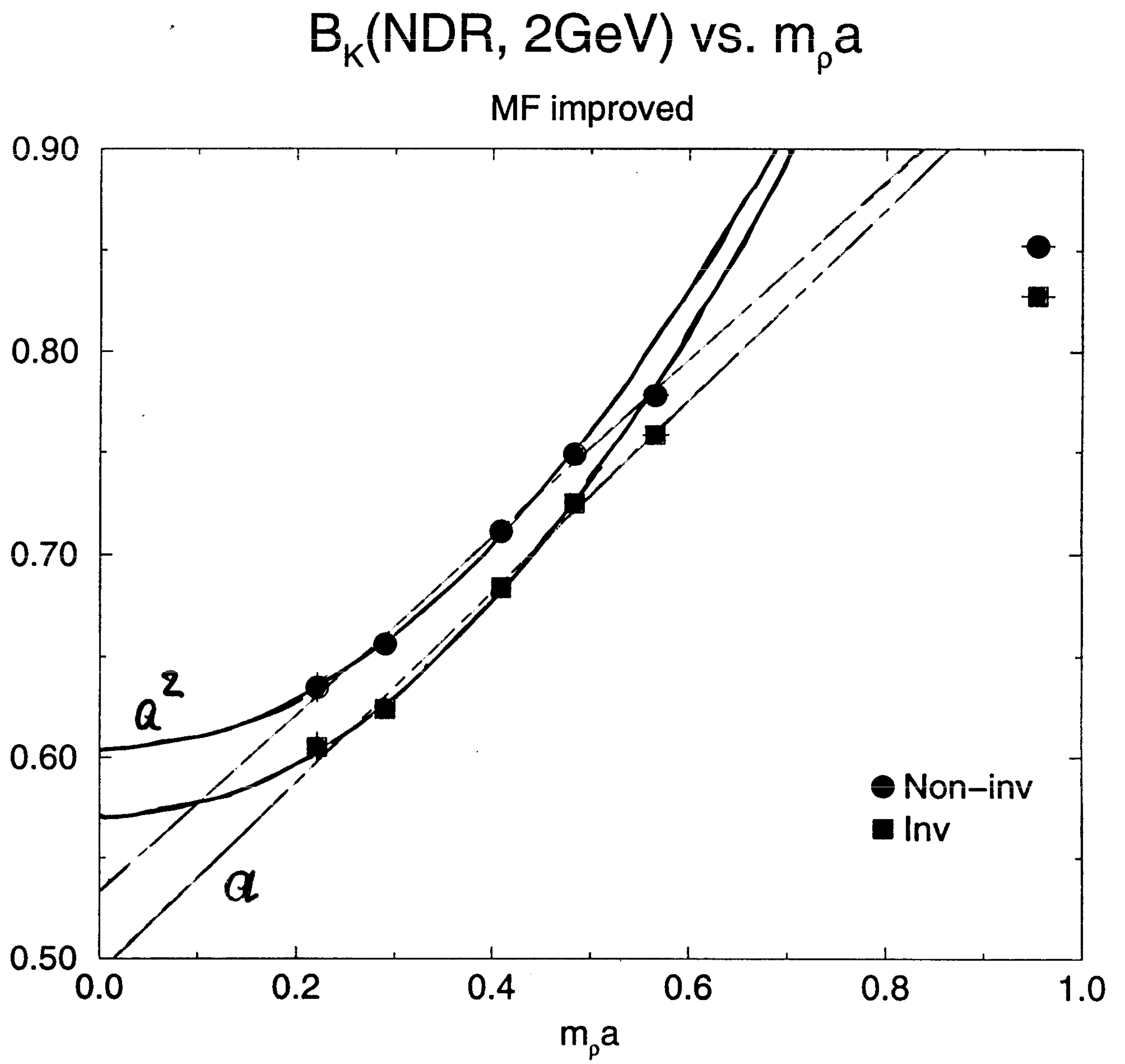
- staggered fermions :
  calculation is straightforward due to their chiral symmetry.

- cut-off (or a) dependence

  a^2 from theoretical arguments (Sharpe, JLQCD)
  confirmed by MC results (JLQCD '97)

Lattice '96

S. Aoki for JLQCD



B_K cont'd

- $n_f = 0$:

$$B_K (2 \text{ GeV}, \text{NDR}) = \begin{cases} 0.595\,(24) \text{ JLQCD '97} \\ 0.587\,(18) \text{ JLQCD} \\ 0.573\,(15) \text{ OSU} \end{cases}$$

consistent with Sharpe et. al ('93) & Ishizuka et.al. ('93)

- n_f dependence

OSU '96 $\quad n_f = 0, 2, 4 \quad m_q \sim m_s/2$

$a \simeq 0.1$ fm

$B_K(n_f = 3) = B_K(n_f = 0) \times 1.05\,(2)$

$\rightarrow$ graph

Sharpe: $\quad a \rightarrow 0 \quad (2) \longrightarrow (15)$

- $m_s \neq m_d$

$n_f = 2, 4$ results with degenerate masses only

Sharpe '96: $\quad$ expect $\times 1.05\,(5)$

using Chiral perturbation theory

Sharpe '96 :

$$B_K(2\,\text{GeV}, \text{NDR}) = 0.64\,(2)\,(10)$$

$\hookrightarrow n_f + m_q$

or $\hat{B}_K = 0.87\,(14)$ (using $\alpha_s(2\,\text{GeV}) = 0.3$)

to do :

- Lattice spacing dependence
  $\longrightarrow$ improved action (Wilson & staggered)

- n_f dependence :
  need $a \to 0$ limit

- m_q dependence
  need $n_f = 3$ $m_s \neq m_q$ $m_q \to m_{u,d}$

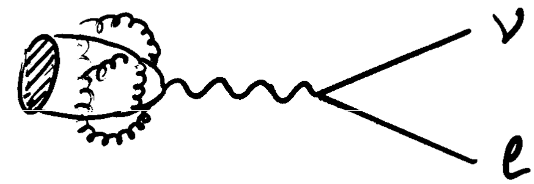
$B - \bar{B}$ mixing

$$x_d = (\text{known}) \cdot |V_{td}|^2 \cdot f(m_t) \cdot \langle \bar{B} | \mathcal{O}_{\Delta B=2} | B \rangle$$

$$\hookrightarrow \frac{8}{3} m_B^2 f_B^2 B_B$$

f_B unknown

B



HQET $\quad \phi \sim f\sqrt{m} = \text{const} + \mathcal{O}\left(\frac{1}{m}\right)$

$\mathcal{O}\left(\frac{1}{m}\right)$: sensitive to cut-off errors
use improvement

f_B

- relativistic & non-relativistic lattice actions for the heavy quark.
- new results with improved actions (JLQCD, FNAL, GLOK) preliminary
- continuum extrapolation by MILC, JLQCD, FNAL
  - results with Wilson quarks show lattice spacing dependence
  - with the improved action the a dependence is greatly reduced.
- MILC compare $n_f = 0$ & $n_f = 2$ results at $a \gtrsim 0.1$ fm

  $\rightsquigarrow$ +20% effect

  consistent with estimates from chiral pert. thy for heavy-light systems (Sharpe + Zhang '96)

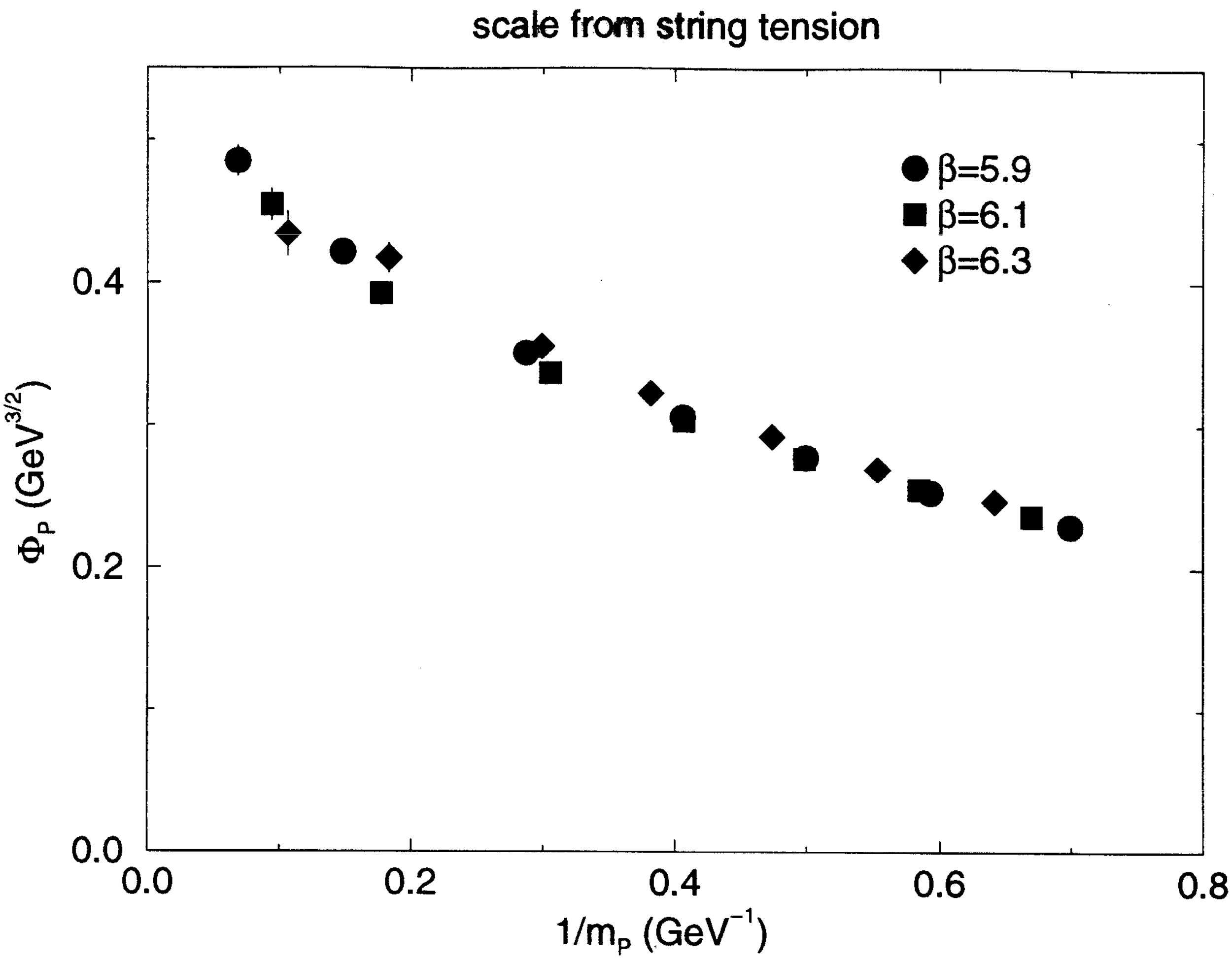
f_B & f_D cont'd

	f_B / MeV	f_{B_s} / MeV	f_D / MeV	f_{D_s} / MeV
MILC	153^{+40}_{-16}	164^{+50}_{-15}	186^{+30}_{-21}	199^{+42}_{-14}
JLQCD	163 ± 20	180 ± 24	192 ± 21	213 ± 24
FNAL	166 ± 30	184 ± 27	205 ± 28	215 ± 31
APE	180 ± 32	205 ± 35	221 ± 17	237 ± 16
UKQCD	160^{+53}_{-20}			
Hiroshima	196^{+14}_{-70}	NRQCD + Wilson (light)		
Wuppertal	230^{+20}_{-80}	NRQCD + Wilson (light)		
GLOK	148^{+24}_{-18}	(NRQCD $1/m^2$ terms)		
SGO	156^{+38}_{-12}	($n_f = 2$)		

reviews by: J. Flynn @ Lattice '96, ICHEP '96

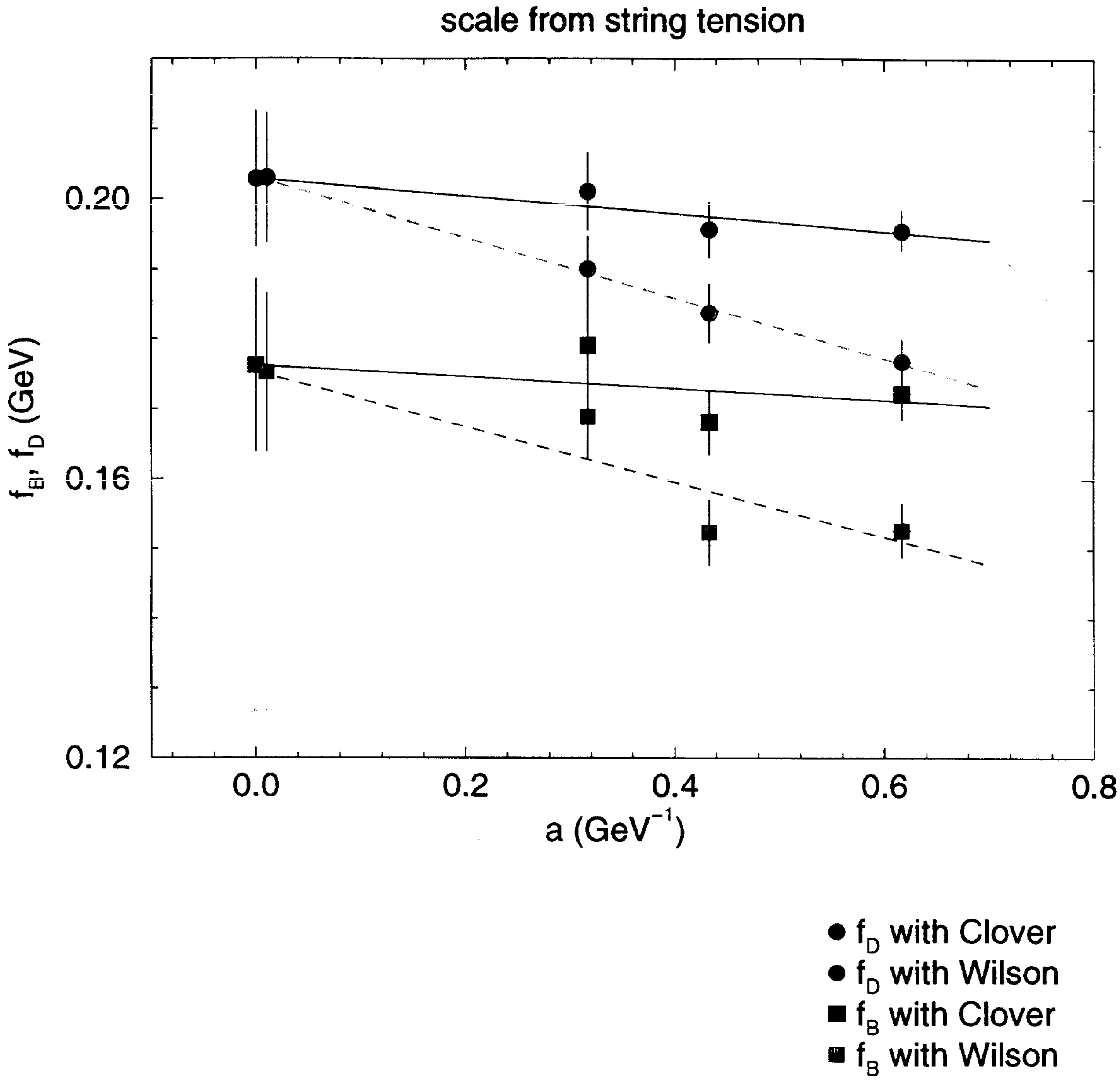
T. Onogi @ Lattice '97

S. Hashimoto (JLQCD) @ Lattice '97



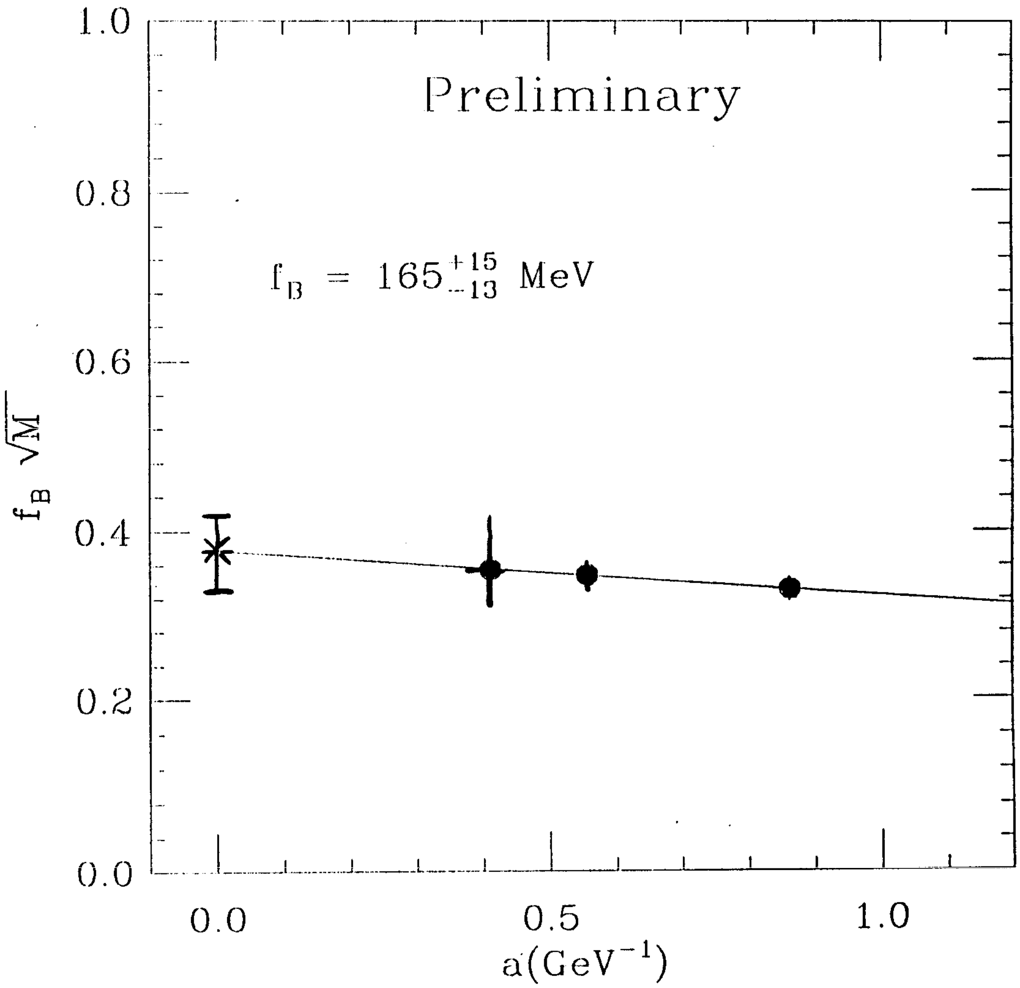
$\phi_P = f_P \sqrt{m_P}$ $\mathcal{O}(a)$ improved action

S. Hashimoto (JLQCD) @ Lattice '97

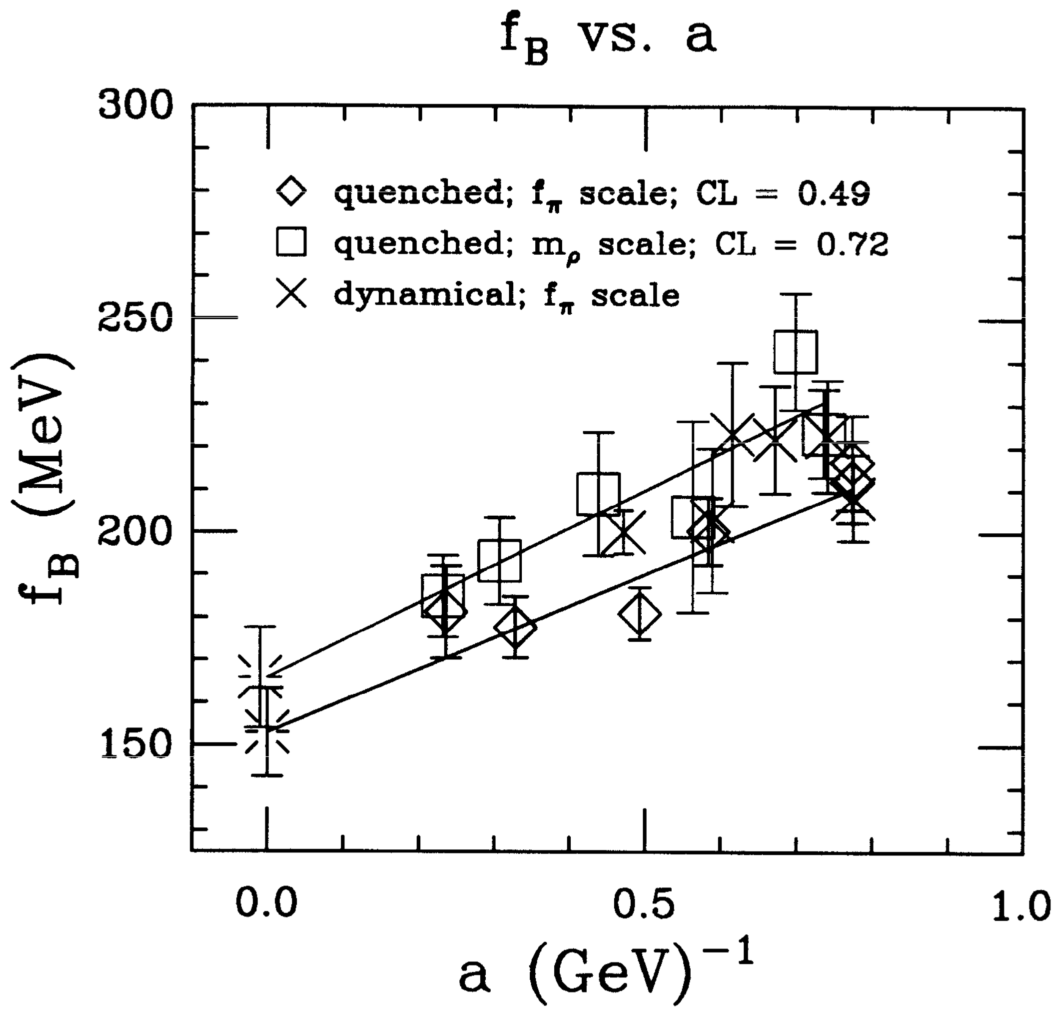


Sinead Ryan (FNAL)

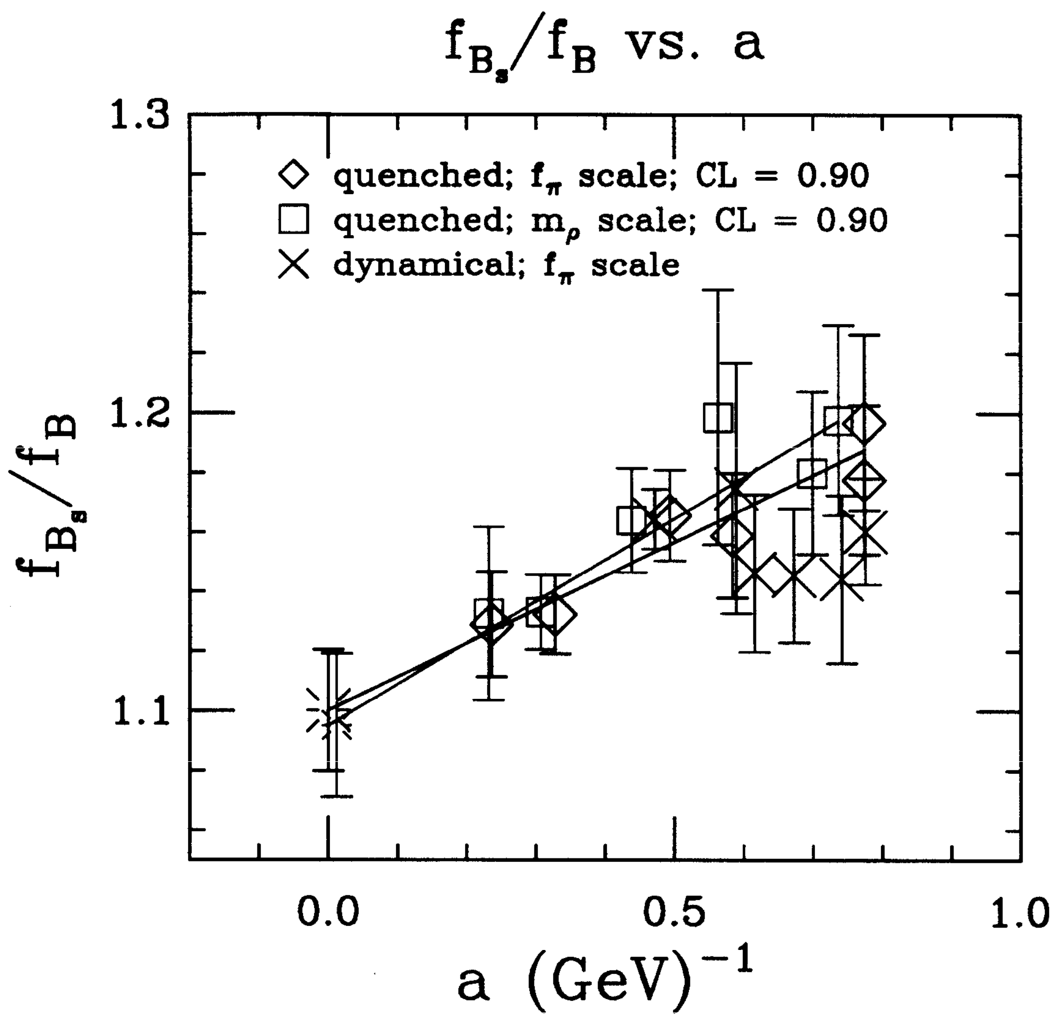
@ Lattice '97



C. Bernard (MILC) @ Lattice '97



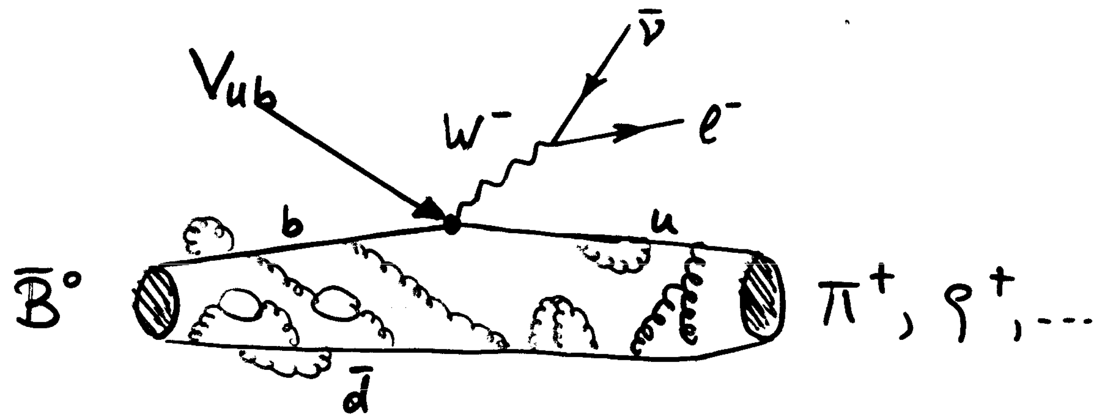
C. Bernard (MILC) @ Lattice '97



Semi-Leptonic Decays

example :

$$B \longrightarrow \pi \ell \nu \quad \rightsquigarrow V_{ub}$$



$$\frac{d\Gamma}{dq^2} \sim (\text{known}) \, |V_{ub}|^2 \cdot |f_+(q^2)|^2$$

$$\langle \pi | J_\mu | B \rangle \; : \; f_+(q^2) \;, \; f_0(q^2)$$

$$\langle \rho | J_\mu | B \rangle \; : \; V(q^2) \,, \; A_0(q^2), \; A_1(q^2)$$

$$A_2(q^2)$$

SL Decays cont'd

- q^2 dependence

need matrix elements as a function of pion momentum; but

$$\langle \pi(p) | J_\mu | B \rangle^{lat} = \langle \pi(p) | J_\mu | B \rangle^{cont} + \mathcal{O}[(ap)^n]$$

$n = 1$ for Wilson action

$n = 2$ for SW improved action

$\rightarrow$ keep $p < \frac{1}{a}$

$\rightarrow$ q^2 range is limited! $\rightarrow$ graph

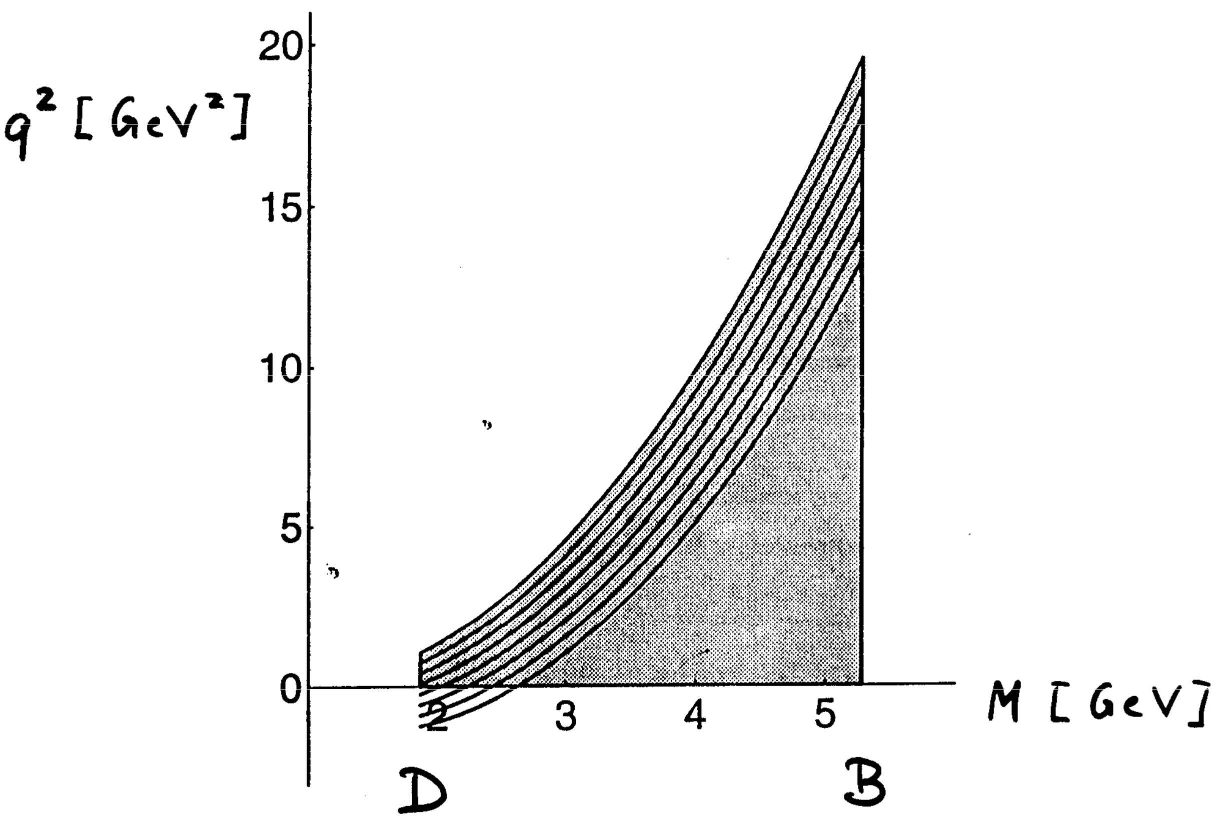
- extend it with improved actions

- Incorporate constraints from dispersion relations (UKQCD + L. Lellouch '96)

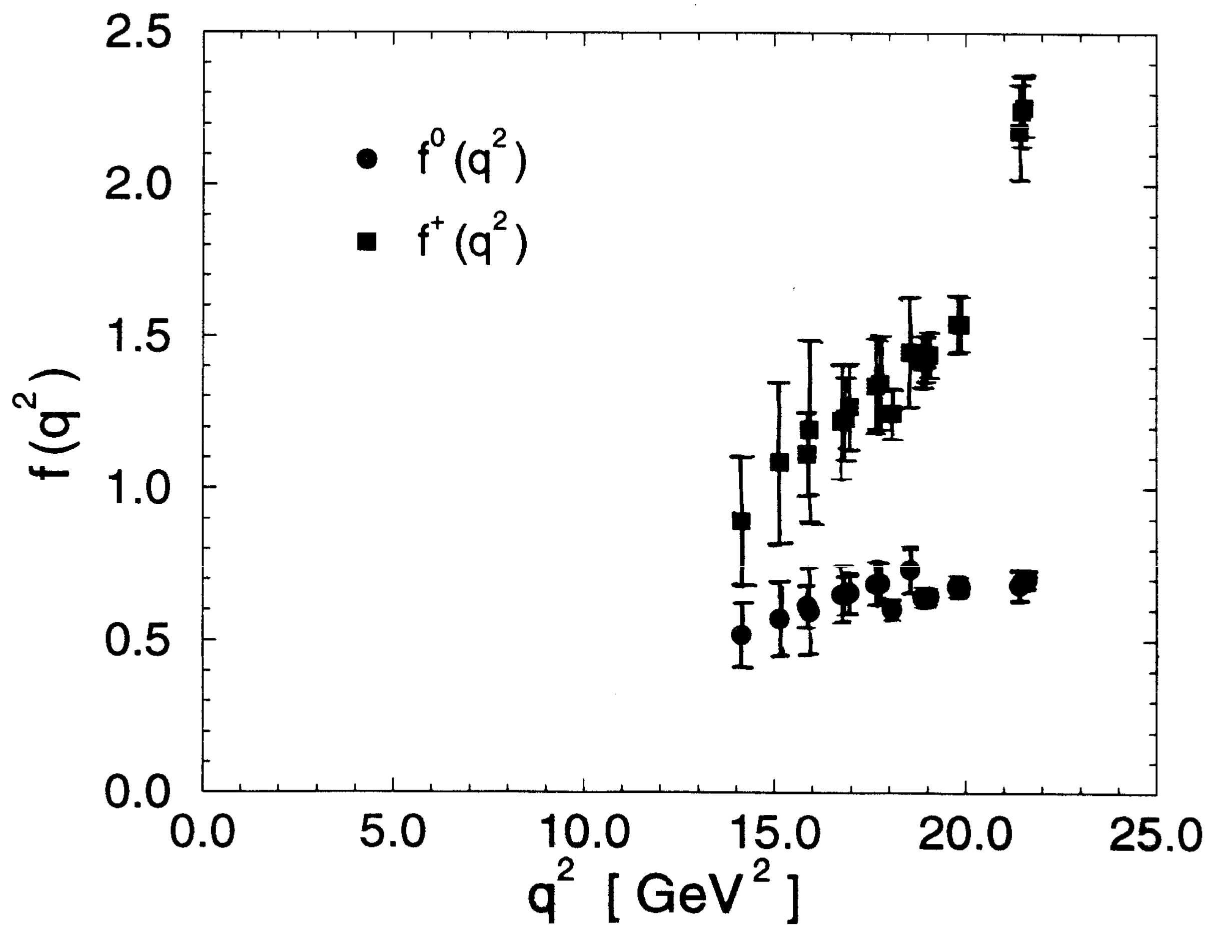
$\rightarrow$ graph

$\rightsquigarrow$ ~35% error

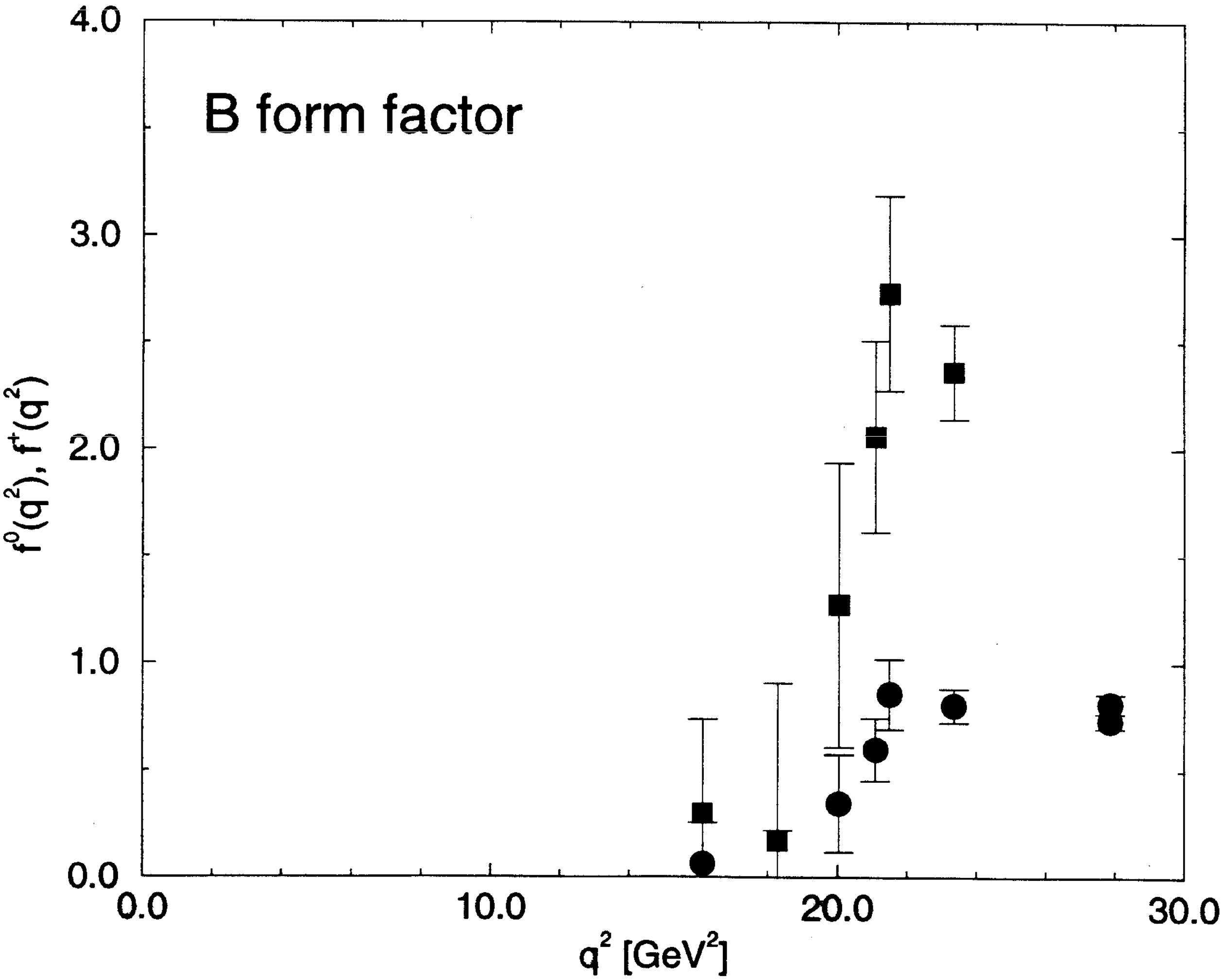
J. Flynn @ Lattice '96 & ICHEP '96



T. Onogi (Hiroshima) @ Lattice '97



T. Onogi (JLQCD) @ Lattice '97



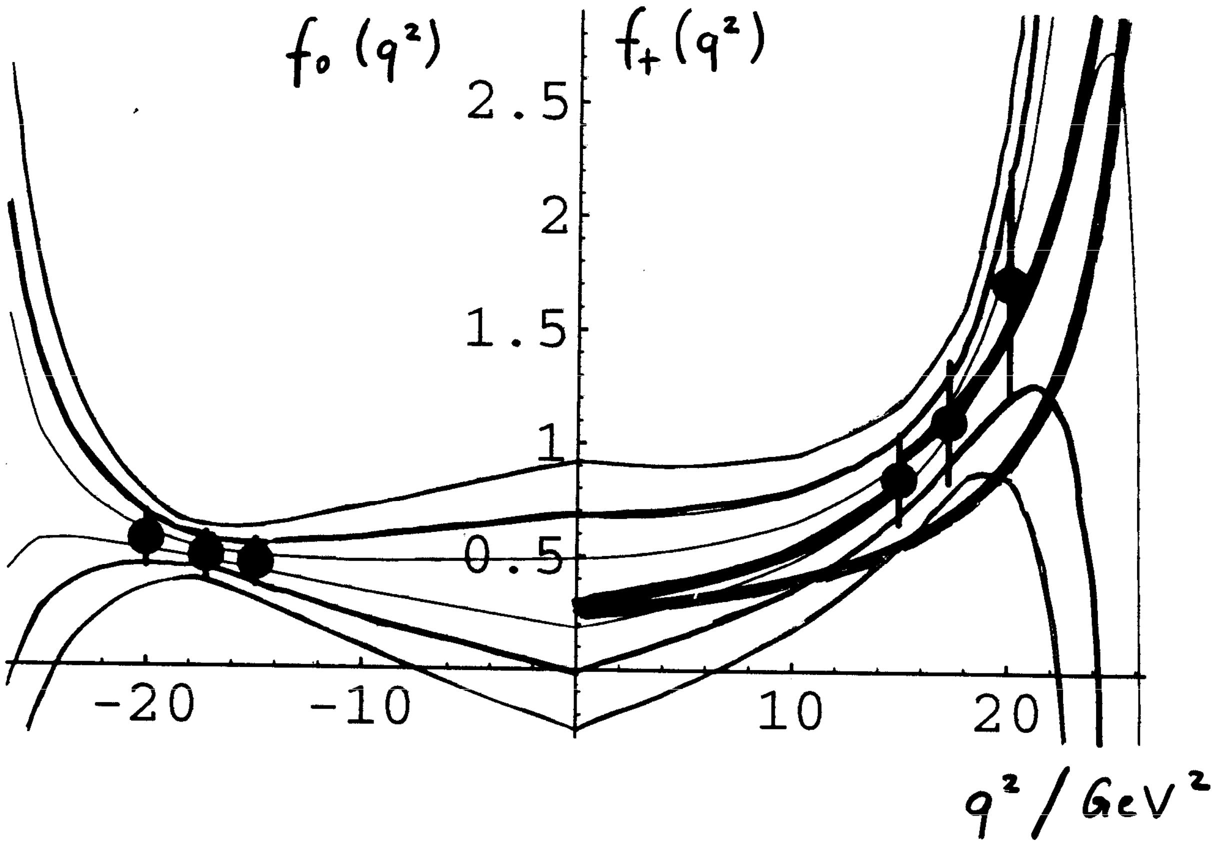
L. Lellouch '96

: UKQCD

: 30% CL

: 70% CL

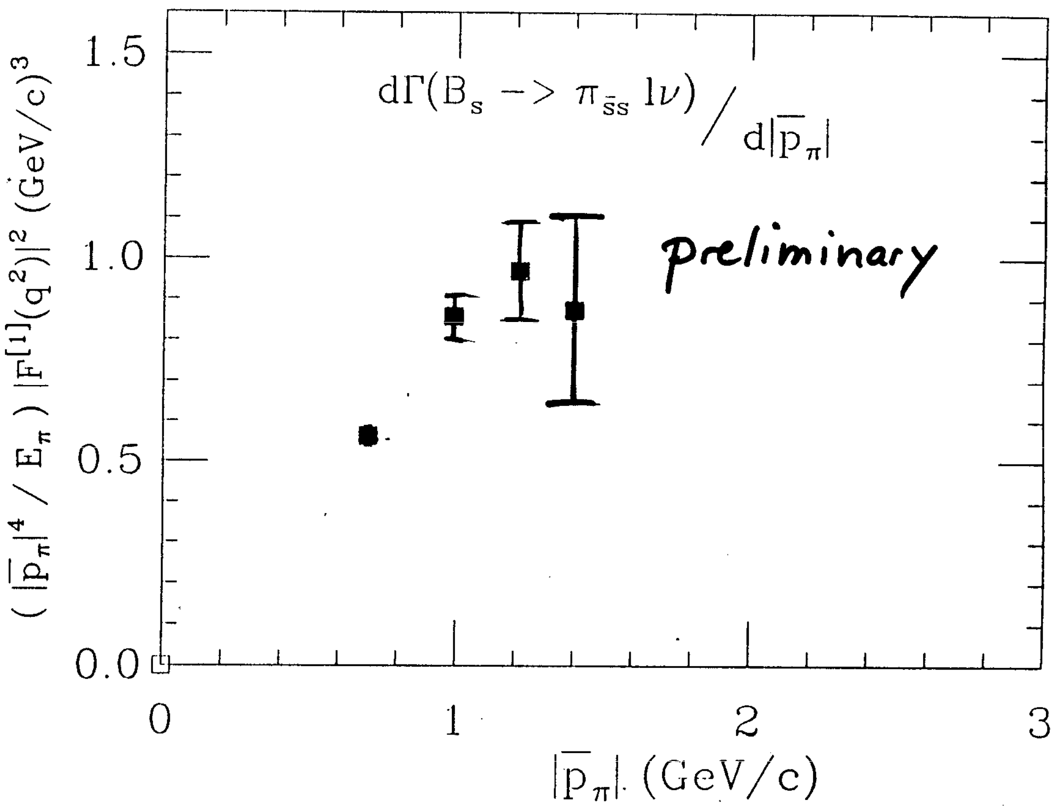
: 95% CL



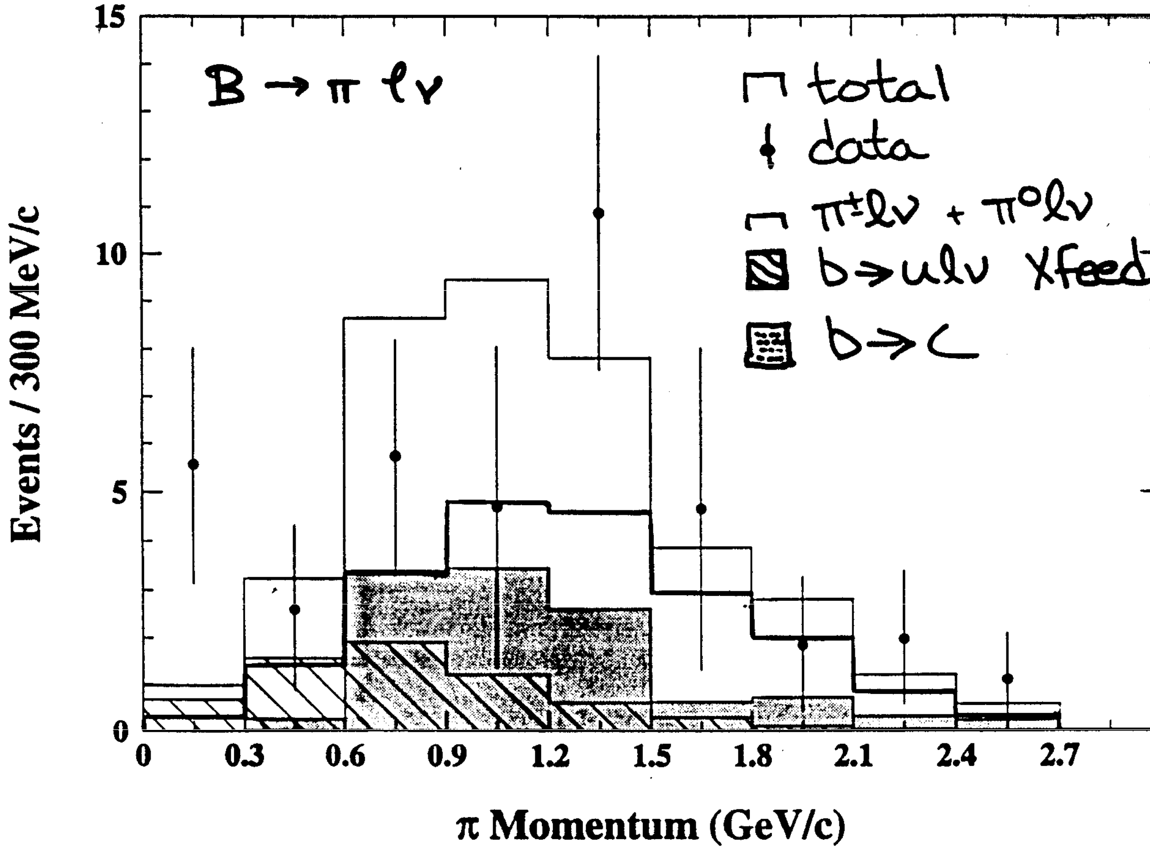
: Light-Cone Sum rule

: 3-pt sum rule

J. Simone (FNAL)



L. Gibbons CLEO



... also a lot of work on ...

- semi-leptonic B decays,
  $B \to \pi(\rho)\ \ell\nu$, $B \to D\ (D^*)\ \ell\nu$
  IW function, ...

- semi-leptonic D decays
  $D \to K^{(*)}\ \ell\nu$, $D \to \pi\ (\rho)\ \ell\nu$
  $D_s \to \phi\ \ell\nu$, ...

- rare decays, $B \to K^* \gamma$, ...

- structure functions, ...

- glueballs, hybrids, ...

- high temperature QCD, phase transition, ...

- non-leptonic weak decays,
  $\Delta I = \frac{1}{2}$ rule, ...

•
•
•

conclusions

- α_s: competitive with traditional determinations based on pert. QCD

needed: $n_f = 3$ $m_s \neq m_{u,d}$ $m_{u,d} \to 0$

$\rightsquigarrow$ $Q\bar{Q}$ spectrum from 1st principles

- Light quark masses: smaller than expected

  uncertainties still sizeable, but already smaller than standard phenomenological range.

- B_K: known to about $\sim 15\%$

- f_B: $\sim 25-30$ MeV uncertainty

- Improved actions have been and will be very important for progress