$B \to \eta'$ decays: What Do they Teach Us?

A. Zhitnitsky

University of British Columbia.

1. $B \to \eta' K$ decays. Standard mechanism
2. $B \to \eta' X$ Simple estimates.
3. Idea: $b \to c\bar{c}\,s$, $c\bar{c} \to \eta'$
4. η' as a unique meson

(In collaboration with I. Halperin)

(1) $Br\left(B \to \eta' K\right) = \left(7.8^{+2.7}_{-2.2}\right)10^{-5}$

CLEO 97

(2) $Br\left(B \to \eta' X\right) = \left(7.5 \pm 1.5 \pm 1.1\right)10^{-4}$

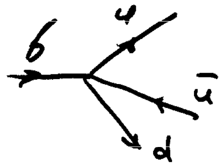
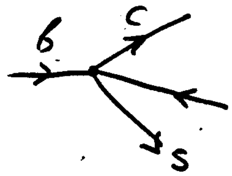
How large these numbers are?

(3) $Br\left(B \to J/\psi\, X\right) = \left(8 \pm 0.8\right)10^{-3}$

This process is due to the Cabbibo favored $b \to c\bar{c}s$ decay

Moral: (2) is only by a factor of 3 less than (3).

I. $\dfrac{\Gamma(B\to\eta' X)}{\Gamma(B\to\eta_c X)}$ [standard mechanism] $\simeq$

$=$  /  $= \left|\dfrac{V_{bu}}{V_{bc}}\right|^2 \dfrac{\langle\eta'|\bar u\gamma_\mu\gamma_5 u|0\rangle\langle X|\bar d\gamma_\mu(1+\gamma_5)b|B\rangle}{\langle\eta_c|\bar c\gamma_\mu\gamma_5 c|0\rangle\langle X|\bar s\gamma_\mu(1+\gamma_5)b|B\rangle}$

$$\sim \frac{1}{3}\left(\frac{V_{bu}}{V_{bc}}\right)^2\cdot\left(\frac{f_{\eta'}}{f_{\eta_c}}\right)^2 \sim 3\cdot 10^{-4}$$

$\langle\eta'|\bar u\gamma_\mu\gamma_5 u|0\rangle = -\frac{i}{\sqrt3} f_{\eta'} q_\mu \qquad f_{\eta'}\sim f_\pi$

$\langle\eta_c|\bar c\gamma_\mu\gamma_5 c|0\rangle = -i f_{\eta_c} q_\mu \qquad f_{\eta_c}\simeq 400$ MeV (from $\eta_c\to\gamma\gamma$)

Remarks: We estimated $\dfrac{B\to\eta' X}{B\to\eta_c X}$, not $\dfrac{B\to\eta' X}{B\to J/\psi X}$ which is measured.

However, $\dfrac{B\to\eta_c X}{B\to J/\psi X} \sim \dfrac{\langle\eta_c|\bar c\gamma_\mu\gamma_5 c|0\rangle\langle X|\bar s\gamma_\mu(1+\gamma_5)b|B\rangle}{\langle J/\psi|\bar c\gamma_\mu c|0\rangle\langle X|\bar s\gamma_\mu(1+\gamma_5)b|B\rangle}$

$$\sim\left(\frac{f_{\eta_c}}{f_\psi}\right)^2\frac{1}{1+2m_\psi^2/m_B^2}\sim 0.6.$$

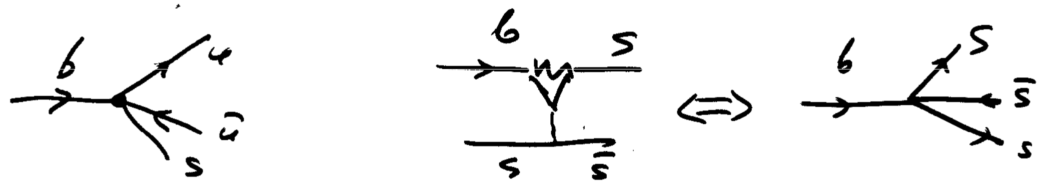
$\Rightarrow$

$$Br(B \to \eta' X) \sim 3 \cdot 10^{-4} \; Br(B \to \eta_c X) \sim$$

$$\sim 3 \cdot 10^{-4} \cdot (0.6) \cdot (8 \pm 0.8) 10^{-3} \sim 1.5 \cdot 10^{-6}.$$

(compare: $B \to \eta' X \sim (7.5 \pm 1.5) 10^{-4}$ (CLEO))

$\overline{II}$. $B \to \eta' K$ (standard mechanism).



$$\frac{B \to K \eta'}{B \to K \varphi} \simeq \frac{\langle \eta' | \bar{s} \gamma_\mu \gamma_5 s | 0 \rangle \langle K | \bar{s} \gamma_\mu b | B \rangle}{\langle \varphi | \bar{s} \gamma_\mu s | 0 \rangle \langle K | \bar{s} \gamma_\mu b | B \rangle} \sim$$

$$\sim \frac{1}{3} \left(\frac{f_{\eta'}}{f_\varphi} \right)^2 \sim 2.5 \cdot 10^{-2}.$$

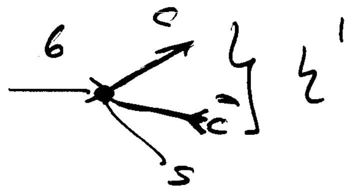
$Br(B \to K \varphi) \simeq 10^{-5}$, Therefore

$Br(B \to K \eta') \sim 2.5 \cdot 10^{-7}$

(compare: $B \to K \eta' \sim (7.8^{+2.7}_{-2.2}) 10^{-5}$, CLEO)

III. New (gluon) mechanism for $B \to \eta'$ decays

1. Idea:



$$M = \frac{G_F}{\sqrt{2}} V_{bc} V_{cs}^{*} a_1 \langle \eta' | \bar c \gamma_\mu \gamma_5 c | 0 \rangle \cdot \langle X | \bar s_L \gamma_\mu b_L | B \rangle$$

$$\langle \eta' | \bar c \gamma_\mu \gamma_5 c | 0 \rangle = -i f_{\eta'}^{(c)} q_\mu .$$ (new element of the game)

Naively: $f_{\eta'}^{(c)}$ should be very small

a) Zweig rule suppression $c\bar c \to q\bar q$

b). m_c suppression.

However: a) η' is a unique meson with 0^{+-}-quantum number, where OZI suppression does not work

b) $m_c \sim 1$ GeV is not very large parameter in hadronic scale.

2. Calculation

$$\partial_\mu \langle 0|\bar c i\gamma_\mu\gamma_5 c|\eta'\rangle = i f_{\eta'}^{(c)} q_\mu$$

$$\langle 0|2m_c \bar c i\gamma_5 c + \frac{\alpha_s}{4\pi} G^a_{\mu\nu}\tilde G^a_{\mu\nu}|\eta'\rangle = f^{(c)}_{\eta'} m^2_{\eta'}$$

$$2m_c \bar c i\gamma_5 c = -\frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde G^a_{\mu\nu} - \frac{1}{m_c^2} g^3 \frac{G^a_{\mu\nu}\tilde G^b_{\nu\lambda}G^c_{\lambda\mu} f^{abc}}{16\pi^2} + \dots O\left(\frac{1}{m_c^4}\right)$$

$$\boxed{\frac{1}{m_{\eta'}^2 m_c^2}\langle 0|g^3 \frac{G^a_{\mu\nu}\tilde G^b_{\nu\lambda} G^c_{\lambda\mu} f^{abc}}{16\pi^2}|\eta'\rangle = f^{(c)}_{\eta'}}$$

$f^{(c)}_{\eta'} \sim \frac{1}{m_c^2} \to 0 \quad (m_c \to \infty)$ as it should.

CLEO has measured this m.e.!

Theory
$\langle 0|G\tilde G G|\eta'\rangle$ – ?

Direct calculation in the Instanton Liq. Model (Shuryak + Zhitnitsky)

New experiment
$\langle p|G\tilde G G|p\rangle$

Witten Venez. Ideas, Low-Energy Th. World without light quarks.

$$f_{\eta'}^{(c)} = -\frac{1}{16\pi^2 m_c^2 m_{\eta'}^{2}} \langle 0 | g^3 G\tilde{G}G | \eta' \rangle \ = ?$$

3. $U(1)$ problem: Witten - Veneziano.

Similar matrix element $\langle 0 | G\tilde{G} | \eta' \rangle$ - ?

$$T(q^2) = i \int dx\, e^{iqx} \langle 0 | T\{ g^2 G\tilde{G}(x),\ g^2 G\tilde{G}(0) \} | 0 \rangle.$$

$N_c \to \infty$

$T(q^2) = 0$ at $q^2 = 0$ in QCD because

$G\tilde{G} \sim \partial_\mu a_\mu$, $a_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi$

$T(q^2) \neq 0$ in YM.

Q (due to Witten): How it could be ever possible in the limit of large N_c to have any influence from quarks? (They should be suppressed $1/N$).

A (Witten): η' - is unique meson.

When light quarks (u, d, s) are introduced, η' is appeared with mass $m_{\eta'}^2 \sim \frac{1}{N}$ and large mat. elements $\langle g^2 G\tilde{G} | \eta' \rangle$.

W: $\langle g^2 G\tilde{G}|\eta'\rangle \frac{1}{m_{\eta'}^2} \langle \eta'|g^2 G\tilde{G}|0\rangle =$

$$= -i\int \langle 0|T\{g^2 G\tilde{G}(x),\ g^2 G\tilde{G}(0)\}|0\rangle_{YM}$$

Our case:

a) $\langle g^3 G\tilde{G}G|\eta'\rangle \frac{1}{m_{\eta'}^2} \langle \eta'|g^2 G\tilde{G}|0\rangle =$

$$= -i\int dx\ \langle 0|T\{g^3 G\tilde{G}G(x),\ g^2 G\tilde{G}(0)\}|0\rangle_{YM}$$

It is a theorem in the limit $N_c \to \infty$.

b) Our problem is reduced to the analysis of the correlation function $\int dx \langle G\tilde{G}G,\ G\tilde{G}\rangle_{YM}$.

NSVZ: $\Rightarrow$ Correlation functions

$\int \langle G\tilde{G}G,\ G\tilde{G}\rangle_{YM} dx$ and $\int \langle GGG,\ GG\rangle_{YM} dx$ are similar $\tilde{G} \leftrightarrow G$

c) $\int i\,dx\ \langle 0|T\{G^3(x),\ \frac{\alpha_s}{4\pi} G_{\mu\nu}^2(0)\}|0\rangle_{YM} = \frac{2\cdot d}{b}\langle G^3\rangle_{YM}$

Low energy theorem.

Therefore: problem is reduced to the analysis $\langle G^3\rangle_{YM}$.

d). $$\boxed{f_{\eta'}^{(c)} = \frac{3}{4\pi^2 b}\,\frac{1}{m_c^2}\,\frac{\langle g^3 G^3\rangle_{YM}}{\langle 0|\frac{g^2}{16\pi^2} G\tilde{G}|\eta'\rangle}}$$

The problem is reduced to the

vev $\langle g^3 G^3\rangle_{YM}$.

How large $\langle g^3 G^3\rangle_{YM}$ in pure gluodynamics?

In QCD: $\langle g^3 G^3\rangle_{QCD} \simeq (0.06 \div 0.1)\ \mathrm{GeV}^6$ (AZ, many years ago)

NSVZ: $\frac{d}{dm_q}\langle g^2 G_{\mu\nu}^2\rangle = -i\int dx \langle 0|T\{g^2G^2, \bar{q}q\}|0\rangle \to$

$\to$ but this correl. function we know

exactly: $i\int dx \langle 0|T\{O, \frac{\alpha_s}{4\pi} G_{\mu\nu}^2(x)\}|0\rangle =$

$= \frac{2\cdot d}{b}\langle O\rangle$

$\langle G^2 g^2\rangle_{YM} \simeq (2\div 3)\langle g^2 G^2\rangle_{QCD}$ is

in perfect agreement with the

instanton picture: raising the m_q

diminishes the chiral suppression of

instantons and therefore increases $\langle G^2\rangle$

$$\langle g^3 G^3 \rangle_{Y.M} \simeq (2.5 \div 3)^{3/2} \langle g^3 G^3 \rangle_{QCD} \simeq$$

$$\simeq (4 \div 6) \langle g^3 G^3 \rangle_{QCD}$$

e). Instanton picture:

$$\frac{\langle g^3 G^3 \rangle}{\langle g^2 G^2 \rangle} \simeq \frac{12}{5\rho_c^2} \simeq 0.9 \text{ GeV}^2 \qquad \rho_c \simeq \frac{1}{600} \text{ MeV}^{-1}$$

f). $$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{YM} \Big|_{lattice} = \begin{matrix} 0.15 \text{ GeV}^4 & SU(2) \\ 0.1 \text{ GeV}^4 & SU(3) \end{matrix}$$

$\langle G^2 \rangle$ Condensate in the YM theory is much larger than in QCD.

$$f_{\eta'}^{(c)} \simeq (50 \div 180) \text{ MeV}$$

$$f_{\eta'}^{(c)} \simeq 140 \text{ MeV} \quad \text{(experiment)}$$

(low energy theorems + ...)

Direct calculations (E. Shuryak + A. Zhitnitsky

$$f_{\eta'}^{(c)} \simeq ?$$

4. Direct calculations

$$f_{\eta'}^{(c)} = -\frac{1}{16\pi^2 m_c^2 m_{\eta'}^2} \langle 0 | g^3 f^{abc} G_{\mu\nu}^a \tilde{G}_{\nu\lambda}^b G_{\lambda\mu}^c | \eta' \rangle$$

$$K_{22}(x) = \langle 0 | g^2 G\tilde{G}(x), \quad g^2 G\tilde{G}(0) | 0 \rangle \qquad (1)$$

$$K_{23}(x) = \langle 0 | g^2 G\tilde{G}(x), \quad g^3 G\tilde{G}G(0) | 0 \rangle \qquad (2)$$

$$K_{33}(x) = \langle 0 | g^3 G\tilde{G}G(x), \quad g^3 G\tilde{G}G(0) \rangle \qquad (3).$$

$f_{\eta'}$ we know from anomaly!

$$\langle 0 | \tfrac{1}{\sqrt{3}} (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s) | \eta' \rangle = i f_{\eta'} q_\mu .$$

$$\langle 0 | \frac{3}{\sqrt{3}} \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}_{\mu\nu} | 0 \rangle = f_{\eta'} m_{\eta'}^2$$

$$\frac{f_{\eta'}^{(c)} \sqrt{3}\, m_c^2}{f_{\eta'}} = \frac{K_{23}(x\to\infty)}{K_{22}(x\to\infty)} \sim (2.5 \div 3.5)\,\mathrm{GeV}^2$$

$$\left(\frac{f_{\eta'}^{(c)} \sqrt{3}\, m_c^2}{f_{\eta'}}\right)^2 = \frac{K_{33}(x\to\infty)}{K_{22}(x\to\infty)} \sim \left[(2.5 \div 3.5)\,\mathrm{GeV}^2\right]^2$$

$$\frac{f_{\eta'}^{(c)}}{f_{\eta'}} \simeq 1. \qquad \text{(Shuryak + Zhitnitsky)}$$

$\langle g G\tilde{G} | \eta' \rangle$ is large!

Q: What is the physics behind of this enhancement?

A: Small size instantons (or other small objects with $\rho \sim (600\,MeV)^{-1}$) give very large contribution into the high dimensional operator $G\tilde{G}$

Phenomenological Consequences for $B \to \eta'$:

1. $$\frac{B \to \eta' K}{B \to \eta' K^*} \sim \left| \frac{\langle K | \bar{s} \not{\epsilon} (1+\gamma_5) b | B \rangle}{\langle K^* | \bar{s} \not{\epsilon} (1+\gamma_5) b | B \rangle} \right|^2 \sim \frac{B \to \psi + K}{B \to \psi + K^*} \sim 0.5$$

2. 0^{+-} is special channel. We do not expect other hadrons: $f_2, f_4, \ldots$ in $B \to$ decays

3. A spectrum is unique: in the $m_b \to \infty$ limit, it is free quark decay $b \to \eta' s$

5. $\langle 0 | G\tilde{G} | \eta' \rangle$

Direct calculations

Low-energy theorems

$\boxed{\text{New experimental results.}}$

Proton spin problem:

$$\frac{1}{2}\langle N | \frac{4}{9}\bar{u}\gamma_\mu\gamma_5 u + \frac{1}{9}\bar{d}\gamma_\mu\gamma_5 d + \frac{1}{9}\bar{s}\gamma_\mu\gamma_5 s + \ldots + \frac{4}{9}\bar{c}\gamma_\mu\gamma_5 c | N \rangle$$

?

$$\langle N | \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s + \ldots | N \rangle = g_A \bar{N}\gamma_\mu\gamma_5 N$$

$$\langle N | \frac{3\alpha_s}{4\pi} G_{\mu\nu}\tilde{G}_{\mu\nu} | N \rangle = 2m_N g_A \bar{N} i\gamma_5 N$$

$$g_A = 0.31 \pm 0.07.$$

(in quark model $g_A \sim 1$). So-called "spin-crisis"

Exact Kühn-Zakharov theorem:

$$\langle N | \frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}_{\mu\nu} | N \rangle = -\frac{2}{3b} M_p \bar{p} i\gamma_5 p$$

$$g_A \simeq -\frac{2}{3b} \qquad b = \frac{11}{3}N_c - \frac{2}{3}N_f$$

(similar results were tested in SQCD)

6. Intrinsic charm contribution to $g_A^{(c)}$

$$\langle N | \bar{c}\gamma_\mu \gamma_5 c | N \rangle = g_A^{(c)} \bar{N}\gamma_\mu \gamma_5 N$$

$$\langle N | 2m_c \bar{c} i\gamma_5 c + \frac{\alpha_s}{4\pi} G_{\mu\nu}\tilde{G}_{\mu\nu} | N \rangle = 2m_N g_A^{(c)} \bar{N} i\gamma_5 N$$

$$\to -\frac{\alpha_s}{4\pi} G_{\mu\nu}\tilde{G}_{\mu\nu} - \frac{1}{16\pi^2 m_c^2} g^3 G^a_{\mu\nu} \tilde{G}^b_{\nu\lambda} G^c_{\lambda\mu} f^{abc} + \ldots$$

$$\langle N | -\frac{1}{16\pi^2 m_c^2} g^3 G^a_{\mu\nu} \tilde{G}^b_{\nu\lambda} G^c_{\lambda\mu} f^{abc} | N \rangle =$$

$$= 2m_N g_A^{(c)} \bar{N} i\gamma_5 N$$

η' dominance:

$$g_A^{(c)} = \frac{1}{2m_N} g_{\eta' NN} \langle 0 | -\frac{1}{16\pi^2 m_c^2 m_{\eta'}^2} g^2 G\tilde{G}G | \eta' \rangle$$

$$g_A^{(c)} \text{ (theory) } \simeq (0.2 \div 0.5)$$

$$g_A^{(c)} \text{ (exp. DIS) } \simeq 0.3.$$

The most important factor: $f_{\eta'}^{(c)}$ is large,

$\Rightarrow g_A^{(c)}$ is also very large number!

Conclusion

1. η' is unique meson.

   It has large gluon m. elements

   $$\langle G^4/\eta' \rangle \to (1\,\mathrm{GeV})^4$$

2. This scale (1 GeV) penetrates to all hadrons: $\eta' NN$, ...

   Therefore, this new scale should be seen everywhere in hadronic physics.

3. This scale originates from small-size strong fluctuations (instantons?)

4. The situation reminds me the J/ψ discovery in 1974 when "hidden charm" was observed simultaneously in $\begin{cases} e^+e^- & \text{SLAC} \\ p & \text{Brookhaven} \end{cases}$

We believe we are facing a similar case when two groups $\left\{ \begin{array}{l} \text{CLEO} \quad B \to \eta' \\ \text{EMC, DIS} \to \langle p | \bar{c}\gamma_\mu\gamma_5 c | p \rangle \end{array} \right\}$

see "intrinsic charm" in the axial channel