POWER COUNTING IN NON-RELATIVISTIC EFFECTIVE FIELD THEORIES

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OUTLINE:

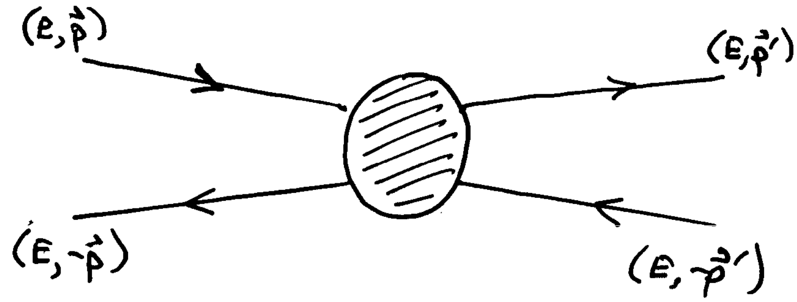
① INTRODUCTION: EFT & Power Counting

② Manifest v counting in NRQCD → potential & radiation gluons and the multipole expansion

③ EFT for NN scattering? Where life gets harder...

④ Conclusions

Want to Study: near threshold scattering of heavy particles (bound state regime) in an effective field theory



$E \sim m + \frac{\vec{p}^{\,2}}{2m}$ (+small)

Why?

→ $\underbrace{\text{N}}_{}$RQCD (quarkonium production / decay) [Bodwin, Braaten, Lepage, ...]

non-relativistic

→ NRQED (precision calculations made easy! ex: relativistic corrections to positronium) [Labelle, Hoang, ...]

→ $e^+e^- \Rightarrow$ hadrons near threshold (NR sum rules for α_s, m_b [Voloshin], $t\bar{t}$ production [Fadin & Khoze, Strassler & Peskin, ...])

→ NN scattering in chiral perturbation theory [Weinberg, Van Kolck, Kaplan Savage & Wise ...]

deuteron: can we get it from a Chiral $\mathcal{L}$? $\Lambda(1405)$? (weakly bound states)

large scattering length is $^1S_0$ channel

Effective Field Theory Propaganda

→ near threshold, nonrelativistic effects (eg. $\bar{Q}Q$ pair production) are suppressed → simplest to work in an EFT where such effects are integrated out

⇒ calculational simplicity
⇒ forces you to concentrate on relevant degrees of freedom
⇒ symmetries / other simplifications of leading order theory are manifest
⇒ systematic expansion

→ need a POWER COUNTING scheme to determine importance of ∞ number of operators (based on small parameter... $\frac{\Lambda_{QCD}}{M_Q}$, $\frac{p}{\Lambda_{\chi SB}}$, $\frac{p}{M_W}$, ...)

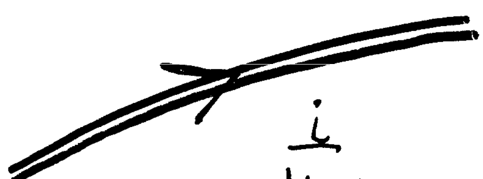
→ power counting not always straight forward...

→ once power counting is established, corrections to leading results straightforward to compute (turn the crank...)

Qu: what is the correct EFT for systems of 2 heavy particles near threshold?

<u>Single</u> heavy particle near mass-shell $\Rightarrow$ HQET

$$\mathcal{L}_{HQET} = \bar{\psi}_h (iD\cdot v)\, \psi_h + (\text{higher dimension operators})$$



$\dfrac{i}{k\cdot v}$

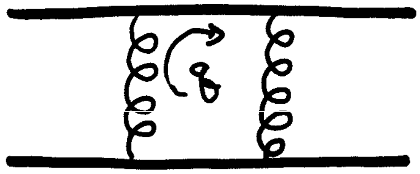
$$p^\mu = m_Q v^\mu + \underbrace{k^\mu}_{\substack{\text{small} \\ (O(\Lambda_{QCD}))}}$$

$\Rightarrow$ power counting is simple ... Λ_{QCD} is only scale $< m_Q$

$\therefore$ higher dim'n operators are suppressed by $\left(\dfrac{k}{m_Q}\right)^N \sim \left(\dfrac{\Lambda_{QCD}}{m_Q}\right)^n$ $\Rightarrow$ power counting is just dimensional analysis

But... HQET with 2 heavy particles w/same velocity is sick in the IR...

(take $v^\mu = (1, \vec{0})$ from now on)



$$\sim \int \frac{d^4 q}{(2\pi)^4} \frac{i}{q_0 + i\varepsilon} \frac{i}{-q_0 + i\varepsilon} \times [\text{gluon prop's}]$$

$\Rightarrow$ pole at $q_0 = 0$ has pinch singularity $\Rightarrow$ divergence (particularly nasty... can't regulate with m_g, DR,...)

WHY? - $Q\bar{Q}$ bound states ... as $M_Q \to \infty$, states fall into infinitely deep potential well ... the kinetic energy operator $\left(P^2/2M_Q\right)$ prevents this

$\therefore$ $P^2/2M$ must be leading order $\Rightarrow$ power counting not as simple as $1/M_Q$

Scales size^{-1} of bound states $\sim m\alpha_s \sim mv$

energy " " " $\sim m\alpha_s^2 \sim mv^2$

(NB $\alpha_s \gg \Lambda/m_Q$ as $m_Q \to \infty$)

(HQET: all components of 4-momentum offshell by $\sim \Lambda_{QCD}$)

$$\mathcal{L}_{NRQCD} = \bar{\psi}\left(iD_0 - \frac{D^2}{2M}\right)\psi + \cdots$$

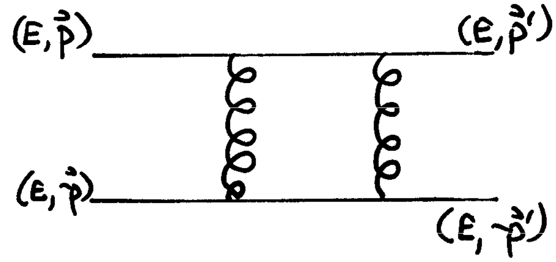
NRQCD Power Counting:

→ relevant expansion parameter is v

→ space & time derivatives have different v scaling

$$\left.\begin{aligned} &\bar\psi\,(i\partial_0)\,\psi \sim v^2 \\ &\bar\psi\,\frac{(i\vec\partial)^2}{2M}\,\psi \sim v^2 \end{aligned}\right\}$$ both operators are leading order → cures IR sickness

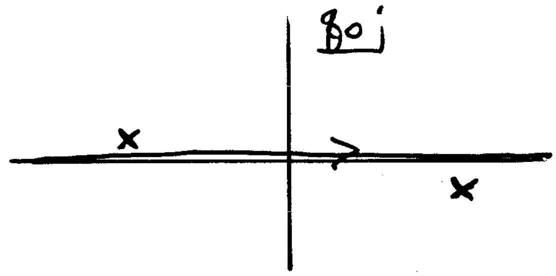
→ v scaling of higher dim'n operators worked out by Lepage et al. (1992)



$$\sim \int \frac{d^4q}{(2\pi)^4}\,\frac{i}{E+q_0-\frac{(\vec p+\vec q)^2}{2M}+i\varepsilon}\,\frac{i}{E-q_0-\frac{(\vec p+\vec q)^2}{2M}+i\varepsilon}$$

$$\times\,\frac{i}{(p-q)^0-(\vec p-\vec q)^2}\,\frac{i}{(q-p')^0-(\vec q-\vec p\,')^2}$$

q_0 integral ... poles due to quark propagators are enhanced (both quarks are almost on shell at same time)

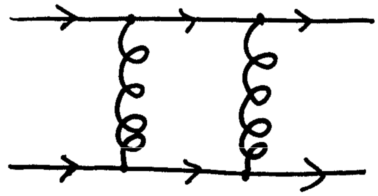


NB: poles are on opposite sides of axis due to $+i\varepsilon$'s

$$\therefore\ \text{graph} \sim -iMQ\int\frac{d^3\vec q}{(2\pi)^3}\,\frac{1}{q^2-ME}\,\frac{1}{\mathcal{O}\!\left(\frac{1}{MQ^2}\right)-(\vec p-\vec q)^2}\,\frac{1}{\mathcal{O}\!\left(\frac{1}{M^2}\right)-(\vec q-\vec p\,')^2}$$

MQ enhancement (this is why HQET power counting failed)

small (neglect) (NB: can't necessarily do this in a loop integral!)



$$\sim -iM_Q \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{\vec{q}^2 - ME} \frac{1}{(\vec{p}-\vec{q})^2} \frac{1}{(\vec{q}-\vec{p}')^2}$$

Comments:

(1) graph enhanced by $\frac{M_Q}{|\vec{p}|} \sim \frac{1}{v}$ relative to nonladder graphs

(2) poles in q_0 plane from gluon propagators don't get enhancement & neglect + component of propagator (subleading)

$\to$ effective gluon propagator $\sim \frac{1}{\vec{k}^2} \xrightarrow{F.T.} \frac{1}{r}\delta(t)$

(Instantaneous potential)

$\leadsto$ this is, of course, well known.
$\bar{Q}Q$ bound states exist for arbitrary weak coupling $\Rightarrow$ Pert. theory must break down for arb. weak coupling

n loop ladder graph $\sim \left(\frac{\alpha}{v}\right)^n$ ... for $v \lesssim \alpha$ (i.e. near threshold) must sum all ladder graphs
$\Leftrightarrow$ solve Schrödinger eqn in Coulomb potential

$\boxed{\text{Manifest } v \text{ Scaling in NRQCD}}$

(ML & A.V. Manohar, B. Grinstein & I. Rothstein, ML & M. J. Savage)

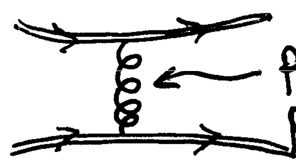
GOAL: ① powers of v manifest in operators
② loop integrals don't mix powers of v ($\therefore$ need mass independent regulator $\Rightarrow$ DIM. REG.)
↳ not essential for EFT, but convenient

<u>Relevant Scales</u>:

Non relativistic quarks: $E \sim mv^2$
$p \sim mv$

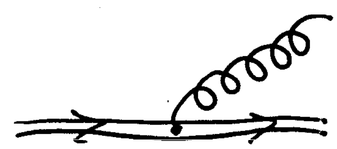
Gluons? depends on what you look at...

① (potential scattering)



$\left.\begin{array}{l} p^0 \sim mv^2 \\ p^i \sim mv \end{array}\right\}$ far off shell, ~instantaneous

② (bremsstrahlung)



$\left.\begin{array}{l} p^0 \sim mv^2 \\ p^i \sim mv^2 \end{array}\right\}$ on shell

$\Rightarrow$ both characteristic scales of gluon are required to reproduce the IR physics of QCD ("potential" & "radiation" gluons)

$\Rightarrow$ powers of v for quarks & potential gluons can be made manifest by introducing new coordinates

$$\left.\begin{aligned} \vec{X} &\equiv m_Q v\, \vec{x} \\ T &\equiv m_Q v^2 t \end{aligned}\right\} \quad \begin{aligned} \vec{P} &= \vec{p}/m_Q v \sim 1 \\ \mathcal{E} &= E/m_Q v^2 \sim 1 \end{aligned}$$

$\therefore$ ∂'s in the rescaled theory are all $\mathcal{O}(1)$ & v's are explicit in couplings

treat subleading terms as <u>insertions</u> $\Rightarrow$ loop integrals don't mix powers of v

$\Rightarrow$ rescaling of ψ & A^μ dictated by normalization of kinetic terms

$$\mathcal{L} = \psi^\dagger (i\partial_0 - g/\sqrt{v}\, A_0)\psi$$
$$- \tfrac{1}{2}\psi^\dagger (i\vec{\nabla} - g\sqrt{v}\,\vec{A})^2 \psi$$
$$-\tfrac{1}{4}(\partial_i A_j - \partial_j A_i - g\sqrt{v}\, f_{abc} A_i^b A_j^c)^2$$
$$+\tfrac{1}{2}(\partial_i A_0 - v\,\partial_0 A_i - g\sqrt{v}\, f_{abc} A_i^b A_0^c)^2$$
$$+ \cdots$$

$\boxed{\text{COMMENTS}}$

1) ∂_0 & $\frac{\partial^2}{\partial M}$ same size

2) effective A_0 coupling is $g/\sqrt{v}$ $\Rightarrow$ for $v \lesssim \alpha$ (near threshold) theory is strongly coupled $\Rightarrow$ must sum A_0 exchange to all orders

3) $\partial_0 A_i$ (& $\partial_0 A_0$ from gauge fixing term) are subleading in v

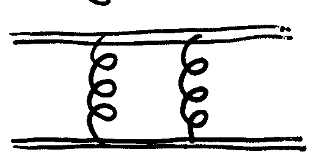
$$D_{00} \sim \frac{i}{\vec{k}^2}$$

$$D_{ij} \sim \frac{i}{\vec{k}^2}\left[\delta_{ij} - (1-\xi)\frac{\vec{k}_i \vec{k}_j}{\vec{k}^2}\right]$$

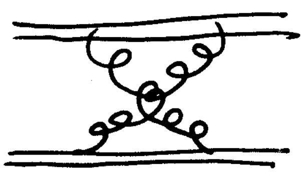
(not quite Coulomb gauge ...)

$\Rightarrow$ instantaneous potential manifest in any gauge

4) only ladder graphs survive (to <u>all</u> orders in v)



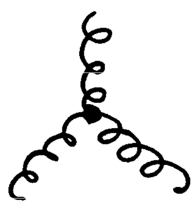
$$\sim \int d^3\vec{q} \times [\text{gluon prop's}] \int dq_0 \underbrace{\frac{1}{q_0 + \ldots + i\varepsilon} \; \frac{1}{-q_0 + \ldots + i\varepsilon}}_{\text{poles on opposite sides of axis}}$$



$\Rightarrow$ q_0 poles on same side of axis
$\therefore$ integral vanishes

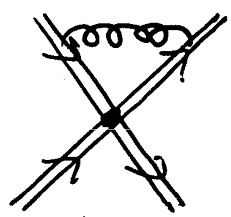
(2) + (3) + (4) $\Rightarrow$ for $v \lesssim \alpha$, sum all ladder graphs w/ instantaneous gluon exchange
$\Leftrightarrow$ solve Schrödinger equation in Coulomb potential (now exact)

$\Rightarrow$ subleading corrections correspond to insertions of higher order operators in ladder sum (simple to organize) -- in particular,



triple gauge vertex is $\mathcal{O}(g\sqrt{v})$

$\leadsto$ so much for potential gluons. What about radiation gluons? $\Rightarrow$ absent from EFT! (no pole in gluon propagator at $(k^0)^2 = \vec{k}^2$)



- vanishes to all orders in v! (not a ladder graph)
- in full QCD, $\sim \ln m_g$

$\therefore$ EFT misses IR physics of full QCD (unless v counting violated $\leadsto$ unsatisfactory --)

Radiation Gluons

- since $E, p \sim mv^2$ for radiated gluons, appropriate rescaling is

$$T \equiv m_Q v^2 t \quad \text{(as before)}$$
$$\vec{Y} \equiv m_Q v^2 \vec{x} \quad \text{(LONG wavelength compared with quarks, potential gluons)}$$

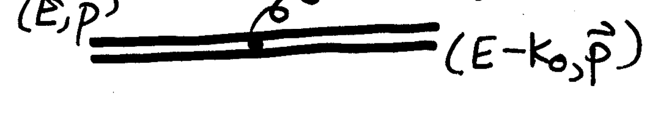
→ just an overall rescaling ∴ no effect on purely gluonic piece of $\mathcal{L}$ (so triple gauge vertex not suppressed —)

COUPLING: $\psi^\dagger(T,\vec{X})\,\psi(T,\vec{X})\,A_0(T,\vec{Y}=v\vec{X})$

$$= \psi^\dagger(T,\vec{X})\,\psi(T,\vec{X})\,\underbrace{\left[A_0(T,0) + v\,\vec{x}\cdot\vec{\nabla}A_0(T,0) + \cdots\right]}_{}$$

multipole expansion required for power counting [Grinstein & Rothstein, Labelle]

NB: momentum is NOT conserved at $A\psi^\dagger\psi$ vertex (as expected, since $p_{gluon} \sim mv^2 \ll p_{quark} \sim mv$)



∴ radiation gluons do not contribute to potential scattering (can't transfer momentum) to ALL orders in v

2 different sets of gluon modes w/different v scaling $\Rightarrow$ need 2 separate fields (or radiation gluon + nonlocal interactions [Grinstein & Rothstein])

going back to unrescaled coordinates (simpler to work with)

$$\mathcal{L}_{NRQCD} = \psi_h^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} - gA_p^0 - gA_R(0,t) \right) \psi_h$$

$$- \frac{1}{4}\left(\nabla^i A_p^j - \nabla^j A_p^i\right)^2 + \frac{1}{2}\left(\nabla A_p^0\right)^2 - \frac{1}{4} G_{\mu\nu R} G^{\mu\nu}_R$$

$$- \frac{1}{2\alpha}\left(\nabla \cdot A_p\right)^2 - \frac{1}{2\alpha}\left(\partial_\mu A^\mu_R\right)^2 + O(v, g\sqrt{v})$$

SIMPLE EXAMPLE – Non-Relativistic Yukawa Theory (NRY) with massless scalar

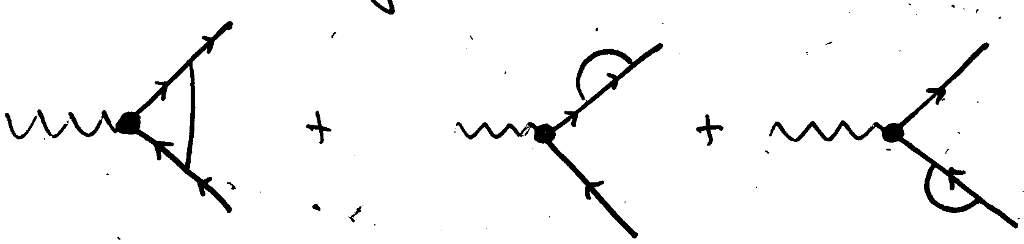
(no complications due to gauge invariance)

$$\mathcal{L}_{FULL} = \bar{\psi}(i\not{\partial}-m)\psi + \tfrac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - g\bar{\psi}\psi\varphi$$

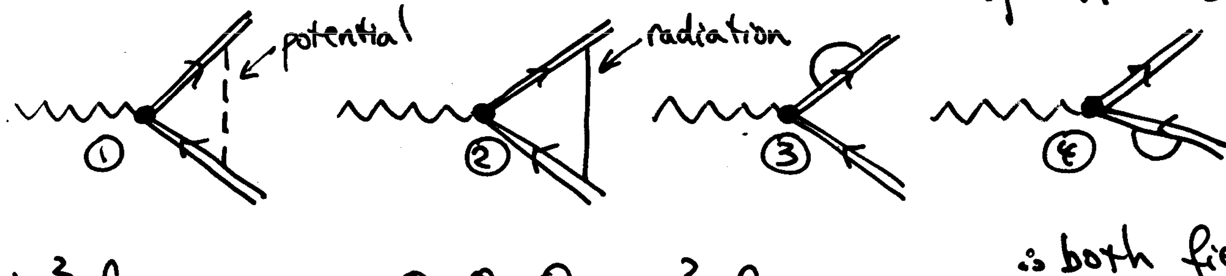
$$\mathcal{L}_{NR} = \psi_h^{+}\left(i\partial_0 + \frac{\nabla^2}{2m}\right)\psi_h + \tfrac{1}{2}\partial_\mu\varphi_R\,\partial^\mu\varphi_R - \tfrac{1}{2}(\nabla\varphi_p)^2$$
$$- g\,\psi_h^{+}\psi_h\varphi_p - g\,\psi_h^{+}\psi_h\,\varphi_R(0,x) + \cdots$$

→ EFT has both potential & radiation scalars
→ regulate with scalar mass to show explicitly IR divergences

One-loop Matching Conditions for External Current:



$$iA_{full} = \left[\frac{-ig^2}{8\pi\beta} + \frac{g^3}{2\pi^2} + O(\beta)\right]\ln m_\varphi + \text{finite}$$
$$\left(\beta \equiv \sqrt{1-4m^2/s}\right)$$

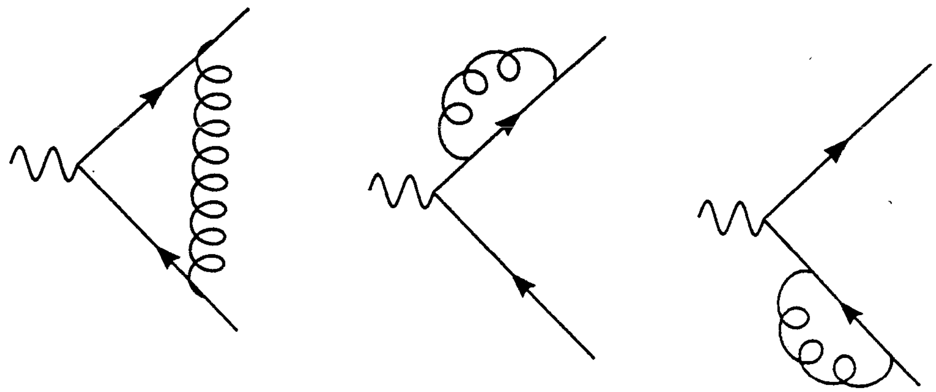


①: $\frac{-ig^2}{8\pi\beta}\ln m_\varphi + \cdots$ (Coulomb phase)

② + ③ + ④ : $\frac{g^2}{2\pi^2}\ln m_\varphi + \cdots$ (KLN divergence)

∴ both fields required to reproduce IR behaviour of full theory

QCD Amplitude



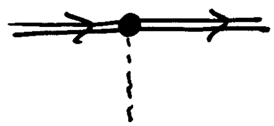
$$
\begin{aligned}
i\mathcal{A}_{\rm QCD} = {} & u_h^\dagger \boldsymbol{\sigma}^i v_h \left(1 - \frac{2g^2}{3\pi^2}\right) - \frac{1}{2m^2} u_h^\dagger \mathbf{p}\cdot\boldsymbol{\sigma}\mathbf{p}^i v_h \left(1 - \frac{g^2}{3\pi^2}\right) \\
& + \frac{g^2}{12\pi^2} u_h^\dagger \boldsymbol{\sigma}^i v_h \left[\frac{m}{|\mathbf{p}|}\left(\pi^2 + i\pi\left(\gamma_E + \frac{2}{d-4} + \ln\frac{\mathbf{p}^2}{\pi\mu^2}\right)\right)\right. \\
& \qquad + \frac{3|\mathbf{p}|}{2m}\left(\pi^2 + i\pi\left(\gamma_E - 2 + \frac{2}{d-4} + \ln\frac{\mathbf{p}^2}{\pi\mu^2}\right)\right) \\
& \qquad \left. + \frac{\mathbf{p}^2}{3m^2}\left(\frac{2}{3} - 8\gamma_E - \frac{16}{d-4} - 8\ln\frac{m^2}{4\pi\mu^2}\right)\right] \\
& + \frac{g^2}{12\pi^2}\frac{u_h^\dagger \mathbf{p}\cdot\boldsymbol{\sigma}\mathbf{p}^i v_h}{2m^2}\left[\frac{m}{|\mathbf{p}|}\left(-\pi^2 - i\pi\left(\gamma_E - 2 + \frac{2}{d-4} + \ln\frac{\mathbf{p}^2}{\pi\mu^2}\right)\right)\right] \\
& + O(v^3)
\end{aligned}
$$

Higher order terms contributing to $e^+e^- \to$ hadrons at $\mathcal{O}(g^2 v)$:

DARWIN:

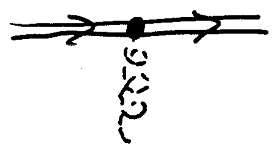
$$\mathcal{L}_D = \frac{g}{8m^2}\left(\psi_h^\dagger T^a \psi_h\right)\nabla^2 A_p^{0a}$$
$$\mathcal{O}(v^{3/2})$$

SPIN-ORBIT:



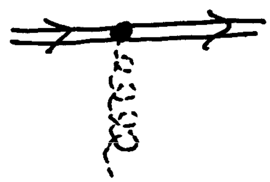
$$\mathcal{L}_{SO} = \frac{ig}{4m^2}\varepsilon^{ijk}\left(\psi_h^\dagger T^a \sigma^i \nabla^j \psi_h\right) \times \nabla^k A_p^{0a}$$
$$\mathcal{O}(v^{3/2})$$

p · A:



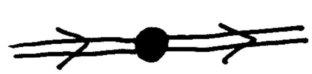
$$\mathcal{L}_{pA} = \frac{g}{2m}\psi_h^\dagger\left(\vec{A}_p\cdot\vec{\nabla} + \vec{\nabla}\cdot\vec{A}_p\right)\psi_h$$
$$\mathcal{O}(v^{1/2})$$

Fermi: (Chromomagnetic Moment)



$$\mathcal{L}_F = -\frac{g}{2m}\psi_h^\dagger \sigma\cdot(\nabla\times A_p)\psi_h$$
$$\mathcal{O}(v^{1/2})$$

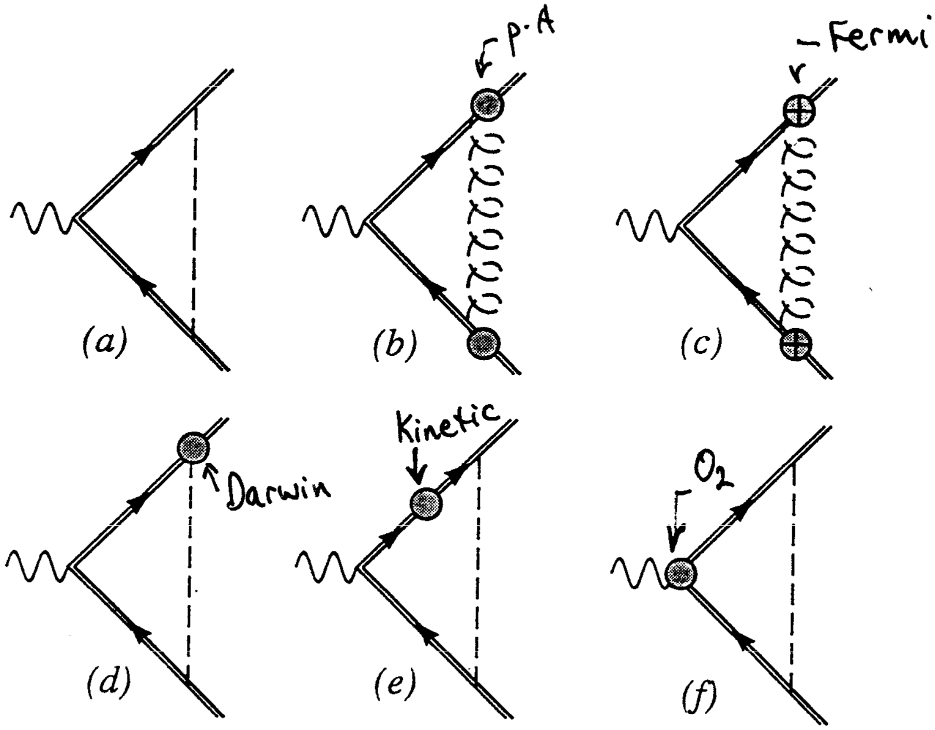
Relativistic Kinetic Correction



$$\mathcal{L}_K = \frac{1}{8m^3}\psi_h^\dagger \nabla^4 \psi_h$$
$$\mathcal{O}(v^2)$$

(+ antiquark terms)

NRQCD Amplitude



$$(a) = c_1 \frac{g^2}{12\pi^2} u_h^\dagger \boldsymbol{\sigma}^i v_h \left[\frac{m}{|\mathbf{p}|} \left(\pi^2 + i\pi \left(\gamma_E + \frac{2}{d-4} + \ln \frac{\mathbf{p}^2}{\pi\mu^2} \right) \right) - \frac{i\pi}{4} \frac{|\mathbf{p}|}{m} \right]$$

$$(b) = c_1 \frac{g^2}{12\pi^2} u_h^\dagger \boldsymbol{\sigma}^i v_h \frac{|\mathbf{p}|}{m} \left(\pi^2 + i\pi \left(\gamma_E - 1 + \frac{2}{d-4} + \ln \frac{\mathbf{p}^2}{\pi\mu^2} \right) \right)$$

$$(c) = c_1 \frac{g^2}{12\pi^2} \left(u_h^\dagger \boldsymbol{\sigma}^i v_h \frac{|\mathbf{p}|}{m} + \frac{m}{|\mathbf{p}|} \frac{u_h^\dagger \mathbf{p}\cdot\boldsymbol{\sigma}\mathbf{p}^i v_h}{m^2} \right) \left(-\frac{i\pi}{2} \right)$$

$$(d) = c_1 \frac{g^2}{12\pi^2} u_h^\dagger \boldsymbol{\sigma}^i v_h \frac{|\mathbf{p}|}{m} (-i\pi)$$

$$(e) = c_1 \frac{g^2}{12\pi^2} u_h^\dagger \boldsymbol{\sigma}^i v_h \frac{|\mathbf{p}|}{2m} \left[\pi^2 + i\pi \left(\gamma_E + \frac{1}{2} + \frac{2}{d-4} + \ln \frac{\mathbf{p}^2}{\pi\mu^2} \right) \right]$$

$$(f) = -c_2 \frac{g^2}{12\pi^2} \frac{u_h^\dagger \mathbf{p}\cdot\boldsymbol{\sigma}\mathbf{p}^i v_h}{m^2} \frac{m}{2|\mathbf{p}|} \left(\pi^2 + i\pi \left(\gamma_E - 3 + \frac{2}{d-4} + \ln \frac{\mathbf{p}^2}{\pi\mu^2} \right) \right)$$
$$- c_2 \frac{g^2}{12\pi^2} u_h^\dagger \boldsymbol{\sigma}^i v_h \frac{|\mathbf{p}|}{m} \left(\frac{i\pi}{2} \right)$$

$$
\begin{aligned}
i\mathcal{A}_{\text{NRQCD}} &= c_1 u_h^\dagger \boldsymbol{\sigma}^i v_h - \frac{c_2}{2m^2} u_h^\dagger \mathbf{p}\cdot\boldsymbol{\sigma}\mathbf{p}^i v_h \\
&+ \frac{g^2}{12\pi^2} u_h^\dagger \boldsymbol{\sigma}^i v_h \left[c_1 \frac{m}{|\mathbf{p}|}\left(\pi^2 + i\pi\left(\gamma_E + \frac{2}{d-4} + \ln\frac{\mathbf{p}^2}{\pi\mu^2}\right)\right)\right. \\
&\qquad \left. + \frac{3|\mathbf{p}|}{2m}\left(c_1\left(\pi^2 + i\pi\left(\gamma_E - \frac{5}{3} + \frac{2}{d-4} + \ln\frac{\mathbf{p}^2}{\pi\mu^2}\right)\right) - \frac{i\pi}{3}c_2\right)\right] \\
&+ \frac{g^2}{12\pi^2}\frac{u_h^\dagger \mathbf{p}\cdot\boldsymbol{\sigma}\mathbf{p}^i v_h}{2m^2}\left[\frac{m}{|\mathbf{p}|}\left(-c_2\left(\pi^2 + i\pi\left(\gamma_E - 3 + \frac{2}{d-4} + \ln\frac{\mathbf{p}^2}{\pi\mu^2}\right.\right.\right.\right.
\end{aligned}
$$

Matching Conditions

$$
J_\mu \bar\psi\gamma^\mu\psi \to c_1\mathbf{O}_1 + c_2\mathbf{O}_2 + c_3\mathbf{O}_3 + \ldots + O(v^4)
$$

where

$$
\begin{aligned}
\mathbf{O}_1 &= J_i \psi_h^\dagger \boldsymbol{\sigma}^i \chi_h \\
\mathbf{O}_2 &= \frac{1}{4m^2} J_i \left[\psi_h^\dagger \left(\overrightarrow{\boldsymbol{\nabla}}\cdot\boldsymbol{\sigma}\overrightarrow{\boldsymbol{\nabla}}^i + \overleftarrow{\boldsymbol{\nabla}}\cdot\boldsymbol{\sigma}\overleftarrow{\boldsymbol{\nabla}}^i\right)\chi_h\right] \\
\mathbf{O}_3 &= \frac{1}{2m^2} J_i \left[\psi_h^\dagger \boldsymbol{\sigma}^i \left(\overrightarrow{\boldsymbol{\nabla}}^2 + \overleftarrow{\boldsymbol{\nabla}}^2\right)\chi_h\right]
\end{aligned}
$$

Karplus & Klein (1952)

$$
\begin{aligned}
c_1 &= 1 + \frac{8\alpha_s}{3\pi} + O(\alpha_s^2) \\
c_2 &= 1 - \frac{4\alpha_s}{3\pi} + O(\alpha_s^2) \\
c_3 &= -\frac{\alpha_s}{9\pi}\left(\frac{2}{3} - 8\ln\frac{m^2}{\mu^2}\right) + O(\alpha_s^2).
\end{aligned}
$$

EFT for Nucleon-Nucleon Scattering?

[Weinberg, Van Kolck, Kaplan Savage & Wise, ...]

- potentially useful for NN scattering (elastic & inelastic) ...
  deuteron? $\Lambda(1405)$? ... bulk nuclear matter ... Kaon condensation ...

$\Rightarrow$ $N\pi$ interactions are described for $p \ll \Lambda_{\chi SB} \sim 1\,\text{GeV}$ by chiral perturbation theory ... can this be used to describe NN scattering in the same regime?

($\sim$ HQET $\Rightarrow$ NRQCD)

OBSERVATION - perturbation theory alone isn't promising

$^1S_0$ channel: scattering length $\sim [8.5\,\text{MeV}]^{-1} \gg \Lambda_{\chi SB}^{-1}$

$\Downarrow$ corresponds to coefficient of $N^\dagger N N^\dagger N$ operator $\Rightarrow$ huge!

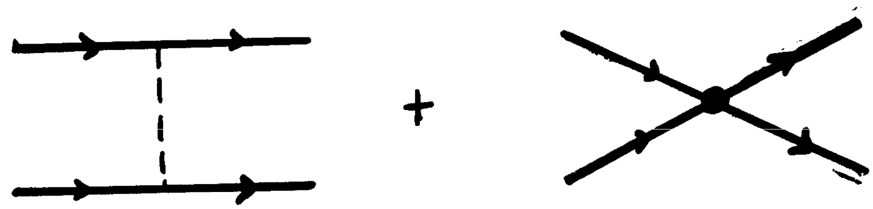
$^3S_1$ channel: light bound state (deuteron)

$\therefore$ if we just look at perturbation theory, EFT breaks down at $p \sim$ few MeV $\ll \Lambda_{\chi SB}$

beyond perturbation theory ... Schrödinger equation in potential given by EFT ...

$$\mathcal{L}_{INT} = \frac{g_A}{f_\pi} N^\dagger \sigma N \cdot \partial\pi - \frac{1}{2} C_S (N^\dagger N)^2 - \frac{1}{2} C_T (N^\dagger \sigma N)^2$$

$+ \cdots$ (higher order in chiral expansion)

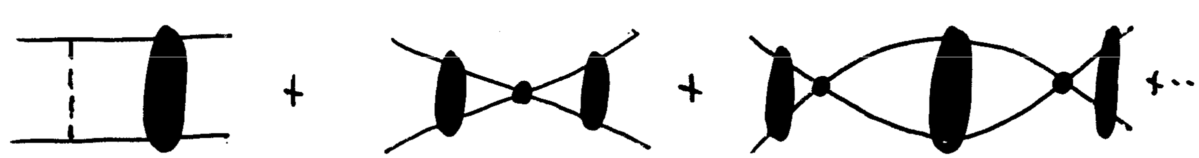


$$V(\vec{q}) = C - \frac{4\pi\alpha_\pi}{\vec{q}^{\,2} + m_\pi^2} + \cdots$$

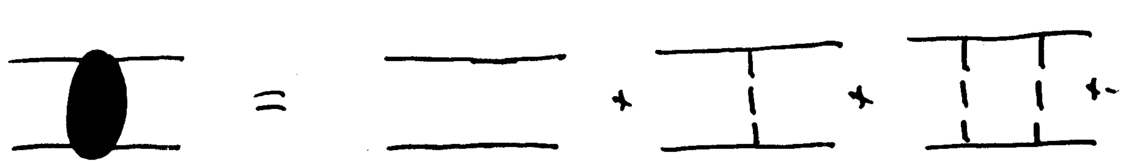
δ fn potential

Yukawa potential

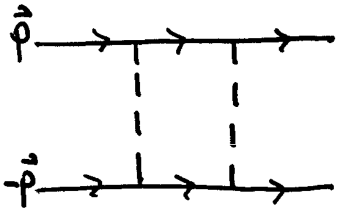
$$\alpha_\pi = \frac{g_A^2\, m_\pi^2}{8\pi^2 f_\pi^2}$$

$iA =$ 

where



Power Counting?



$\sim M_N p$ (n loop ladder graph $\sim (M_N p)^n$)

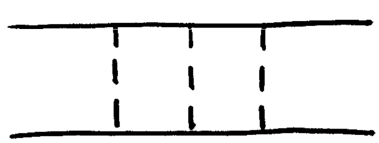
$\Rightarrow$ vanishes at threshold (unlike Coulomb pot'l ... Yukawa pot'l has no bound state for arbitrarily weak coupling)

$\Rightarrow$ need a different power counting to justify retaining only bubble sum

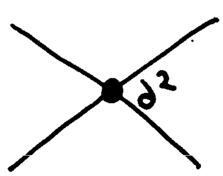
Weinberg: at leading order, sum all terms $\sim (p M_N)^\ell$

n^{th} subleading $\sim \underbrace{p^n}_{\text{higher order operator}} \underbrace{(p M_N)^\ell}_{\text{bubble sum with leading potential}}$

Qu: is this consistent with power counting of chiral perturbation theory?

a) 

$\sim M_N p^2 \Rightarrow$ analytic in external momentum

b) 

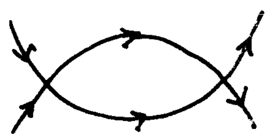
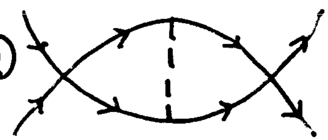
- higher dimension operator $\sim p^2$ ... can we consistently neglect (b) and keep (a)?

Power Counting of Local Operators - 2 approaches

I: Consistency within EFT (Kaplan, Savage & Wise)

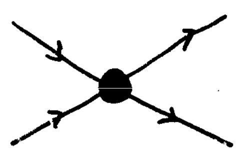
$\Rightarrow$ divergent graphs require counterterms of same order

$^1S_0$ channel: only divergences in leading order sum are

1  2 

1) linear divergence - infinite part vanishes in DR

2) $\sim M_N \alpha_\pi \ln \mu \sim M_N^2 m_\pi^2 \ln \mu$

$\Rightarrow$ renormalizes  $\sim N^\dagger M_q N N^\dagger N$

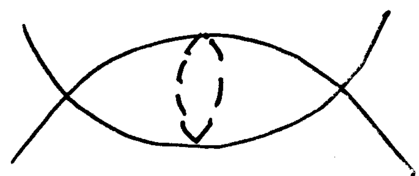
quark mass matrix

SUBLEADING IN CHIRAL EXPANSION!

$\therefore$ all orders of m_π^2 in higher dim'n ops $\Rightarrow \mathcal{O}(1)$ (enhanced by powers of M_N) $\Rightarrow$ chiral power counting is not consistent w/ bubble sum

$\therefore$ no SU(3) symmetry for scattering amplitudes

$\Rightarrow$ fortunately, no $M_N^2 p^2$ divergence (would then require complete series of local operators $\sim (M_N p)^N$ i.e. complete form factor --- no predictive power!) ... leading divergence of this form is

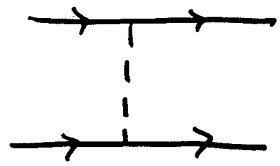


$\sim M_N^2 p^4 \Rightarrow$ subleading (but still enhanced over naive estimate)

NB... this is <u>not</u> just an artifact of dim'l reg'n... same problem also arises w/cutoff

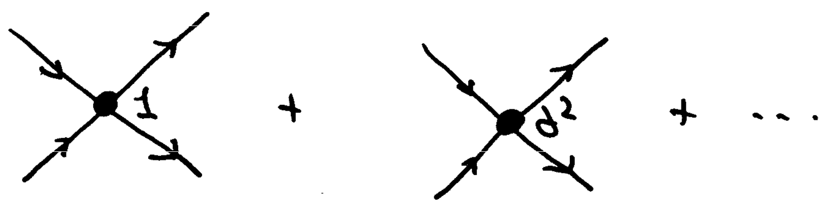
$^3S_1$ channel - worse! $\Rightarrow$ no number of divergent ladder graphs at leading order...

II: Toy Model (ML & A.V. Manohar)

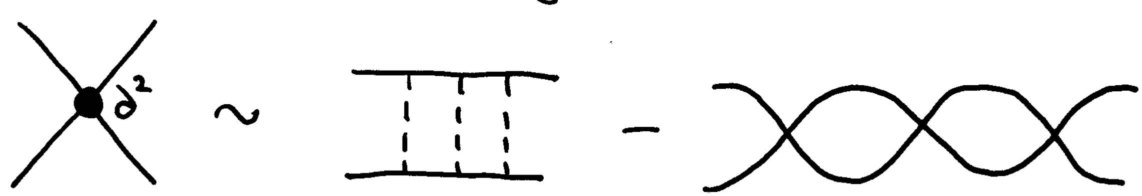


Yukawa theory, integrate out exchanged scalar

$\Longrightarrow$



$\rightarrow$ higher dimension operators are generically same size as loop graphs



$\sim M_N^2 p^2$ $\therefore$ M_N power counting of loop graphs only doesn't appear consistent in this theory ...

Conclusion? - not yet understood how to obtain a consistent power counting for NN scattering ...