"Small x behaviour of the chirally-odd parton distribution $h_1(x, Q^2)$"

R. Kirschner, (Leipzig)
L. Mankiewicz, (München/Warsaw)
A. Schäfer (Regensburg)
L. Sz. (Leipzig/Warsaw)

Zeit. f. Ph. C74(97)501

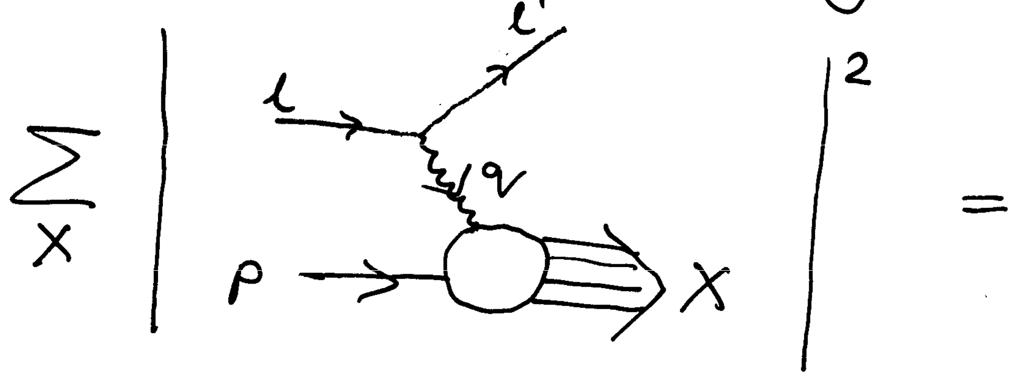
Plan:

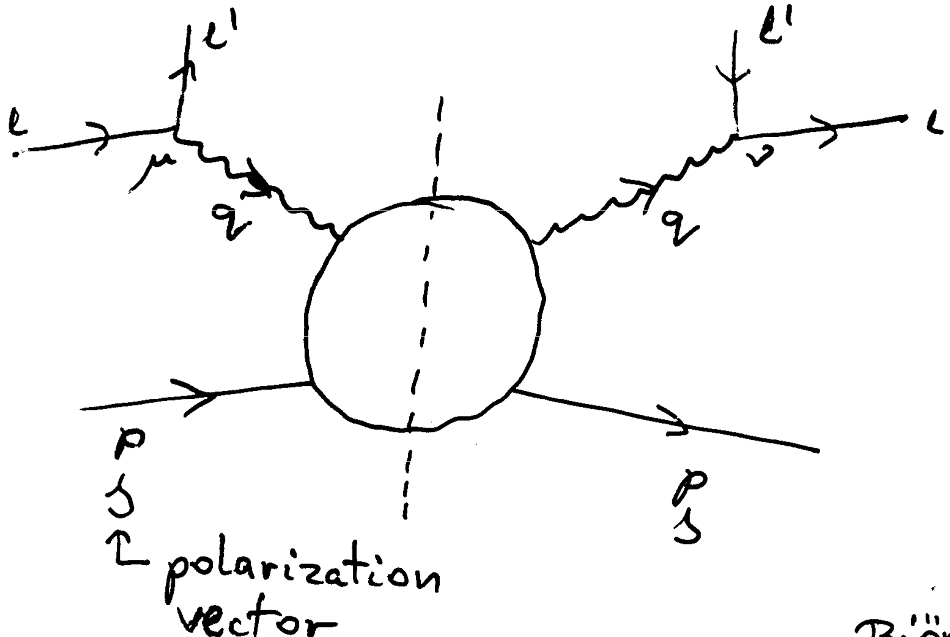
- motivation

  g_1, g_2 versus h_1, h_2

- small x behaviour of h_1
- small x beh. versus DGLAP evolution

Deep Inelastic Scattering (DIS)





virtual Compton scatterin

Björken var.

$$x = -\frac{q^2}{2pq}$$

$$W_{\mu\nu}(p, q, s) \equiv$$

$$\equiv \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, q^2)$$

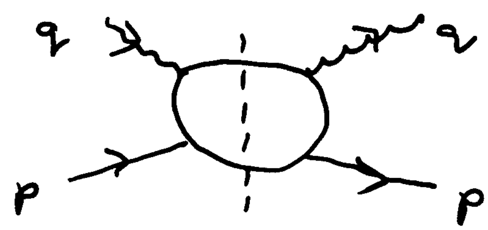
unpolarized structure f

$$+ \left(p_\mu - q_\mu \frac{pq}{q^2}\right)\left(p_\nu - q_\nu \frac{pq}{q^2}\right) \frac{1}{pq} F_2(x, q^2)$$

$$+ i\, \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha s^\beta \frac{m}{pq}\, g_1(x, q^2) \quad \parallel$$

polarized str. f.

$$+ i\, \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha \left(s^\beta - p^\beta \frac{sq}{pq}\right) g_2(x, q^2) \quad \perp$$



high-energy scattering

$$s = (p+q)^2 = m^2 + \underbrace{q^2}_{-Q^2} + 2pq \longrightarrow \infty$$

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{s + Q^2 - m^2}$$

Björken limit of DIS:

$$s \gtrsim Q^2 \gg \Lambda^2$$

$$x \sim \mathcal{O}(1)$$

perturbation theory o.k.
$\Lambda \sim 200\,\text{MeV}$

evolution in Q^2: $\left(\ln \frac{Q^2}{\Lambda^2}\right)^n$ ONE SCALE!
GLAP

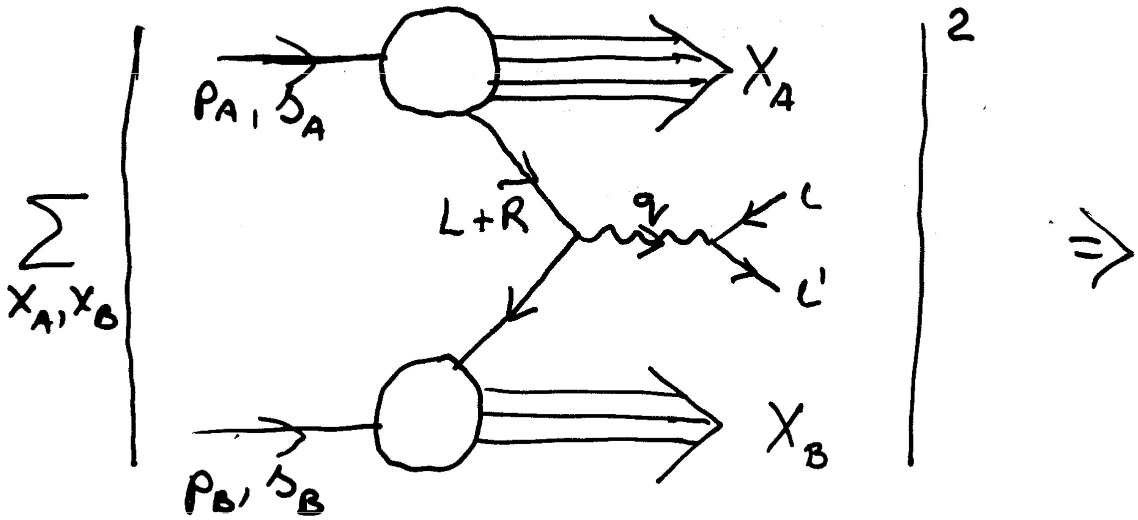
Small-x limit of DIS:

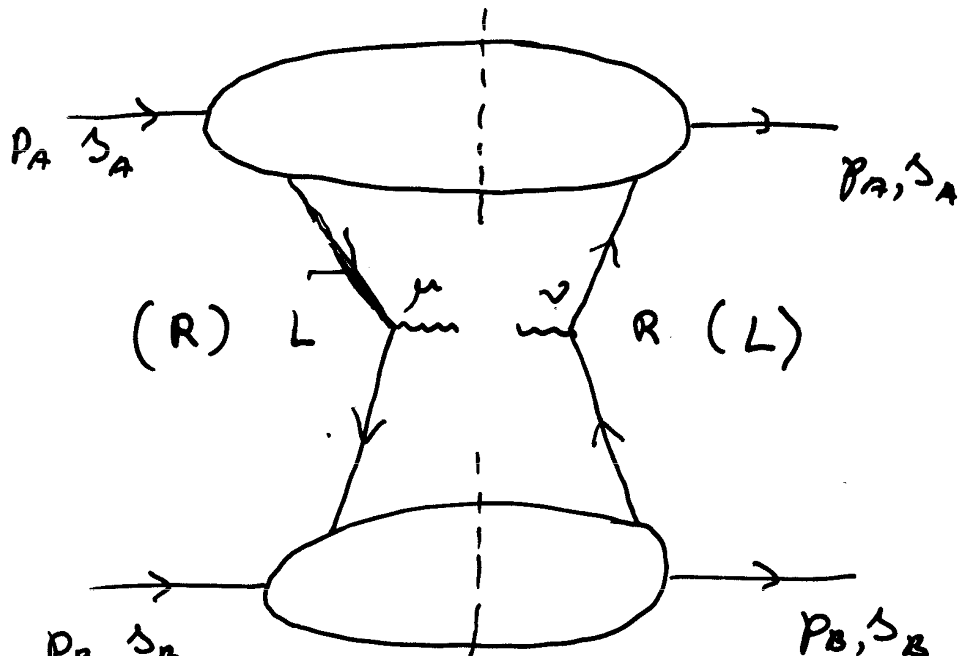
$$s \gg Q^2 \gg \Lambda^2$$

$$x \sim \frac{Q^2}{s} \longrightarrow 0$$

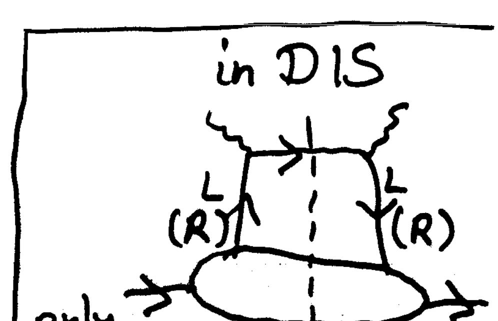
evolution in $\frac{1}{x}$: $\left(\ln \frac{1}{x}\right)^n$ BFKL   TWO SCALES!
and in Q^2: $\left(\ln \frac{Q^2}{\Lambda^2}\right)^n$

Drell - Yan process (DY)





two quark lines can have opposite chiralities (the same helicities) $\Longrightarrow$ chirally-odd structure f
$h_1(x, q^2)$, $h_L(x, q^2)$



$$W^{\mu\nu} \Rightarrow$$

$$(s_{A\perp}\cdot s_{B\perp})(p_A^\mu p_B^\nu + p_A^\nu p_B^\mu - g^{\mu\nu} p_A p_B) +$$

$$+\, p_A\cdot p_B\,(s_{A\perp}^\mu s_{B\perp}^\nu + s_{A\perp}^\nu s_{B\perp}^\mu)\,\frac{1}{M^2}\sum_a e_a^2\, h_1^a(x)\, h_1^{\bar a}(y)$$

$$-\sum_a e_a^2\,\frac{p_A\cdot s_B}{p_A\cdot p_B}\,(p_A^\mu s_{A\perp}^\nu + p_A^\nu s_{A\perp}^\mu)\; h_1^a(x)\, h_L^{\bar a}(y)$$

$$-\sum_a e_a^2\,\frac{p_B\cdot s_A}{p_A\cdot p_B}\,(p_B^\mu s_{B\perp}^\nu + p_B^\nu s_{B\perp}^\mu)\; h_L^a(x)\, h_1^{\bar a}(y)$$

$$xy = \frac{Q^2}{s}$$

$h_1(x)$ transverse str. f.

$h_L(x)$ Longitudinal str. f.

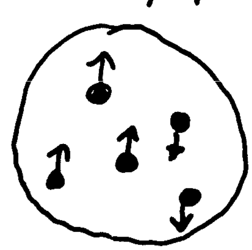
DIS:

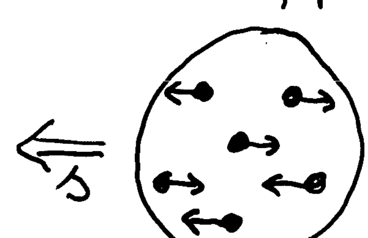
$g_1(x)$ $\parallel$

$g_2(x)$ $\perp$

$g_1(x)$ versus $h_1(x)$:

$s \Uparrow \uparrow p$ $\qquad\qquad$ $\uparrow p$





$g_1(x) \sim \#\uparrow - \#\downarrow$

helicity asymmetry

$h_1(x) \sim \#\leftarrow - \#\rightarrow$

transversity asymmetry

Conclusion :

LONGITUDINAL AND TRANSVERSE SPIN EFFECTS APPEAR IN QCD ON EQUAL FOOTING.

	L	T
Twist - 2	$g_1(x, Q^2)$	$h_1(x, Q^2)$
Twist - 3	$h_L(x, Q^2)$	$g_T(x, Q^2)$

$$\boxed{\left(\frac{M}{\sqrt{Q^2}}\right)^{t-2}}$$

Experiment and $h_1(x,q^2)$, $h_L(x,q^2)$:

no experimental data ?

Proposed experiments:

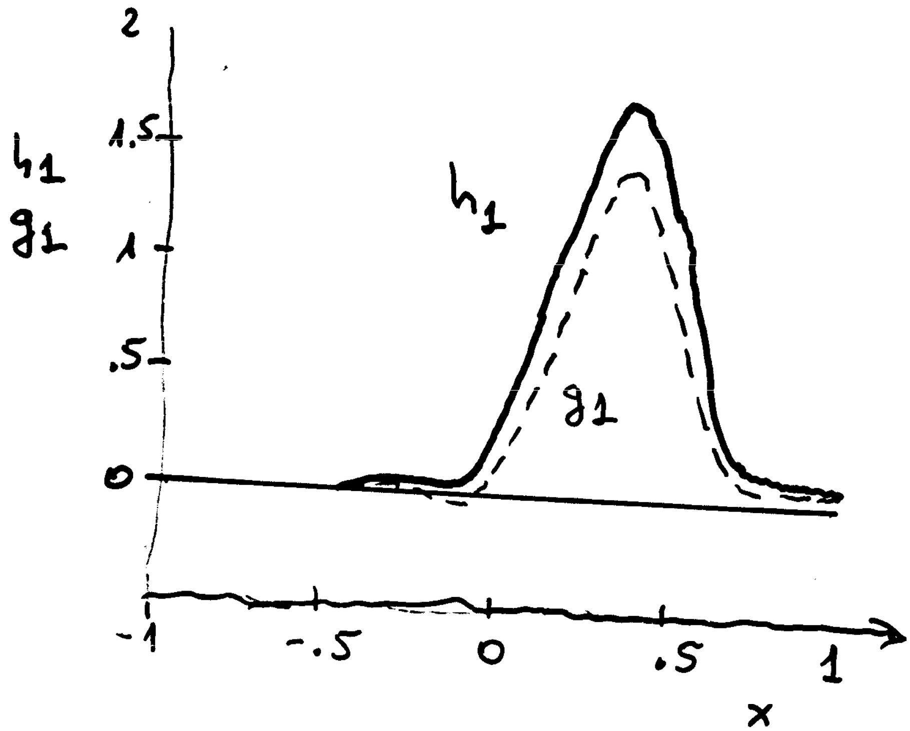
- RHIC spin collaboration at BNL (USA)
- COMPASS collaboration at CERN

Theoretical status of g_1 versus h_1

- For non-relativistic limit

$g_1 = h_1$ i.e. $g_1 - h_1$ = relativ. effects

- Bag model calculations Jaffe, Ji '93



- QCD sum rules Ioffe Khodjamirian '95

h_1 is of the same order as g_1

- inequalities

$|h_1| \leq F_1(x)$ $|g_1| \leq F_1$

$F_1 + g_1 \geqslant 2|h_1|$ Soffer '95

- leading order Q^2 evolution

  GLAP: Artru, Mekhfi '90

  but anomalous dim's:

  Bukhvostov etal '85

- next-to-leading order Q^2 evolution:

  Kumano and Miyama ph/970642

  Vogelsang ph/970651

  Hayashigaki et al ph/970720

Our aim: small x behaviour of h_1

g_1 h_1

Bartels, Ermolaev, Ryskin

NS : Zeit. f. Ph. C70(96)273

S : Zeit. f. Ph. C72(96)627

Operator definition of $h_1(x, Q^2)$

- on the light-cone   Jaffe + Ji '91, '93

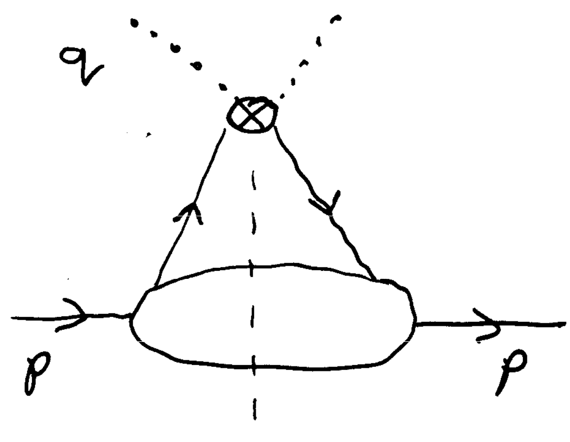
$$\frac{i}{2}\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}\, e^{i\lambda x}\, \langle p, s_\perp | \bar{\Psi}(0)\, \sigma_{\mu\nu}\, n^\nu \gamma_5\, \Psi(\lambda n) | p, s_\perp\rangle$$

Björken var. $x = \frac{Q^2}{2pq}$

light-cone vector

$n^\mu = q^\mu + x p^\mu$

$n^2 = 0$



$q^2 = -Q^2$

$$= h_1(x, Q^2)\, s_{\perp\mu}\, np$$

Remark:

definition consistent with the factorization thm up to twist-2 terms.

- Ioffe + Khodjamirian '95

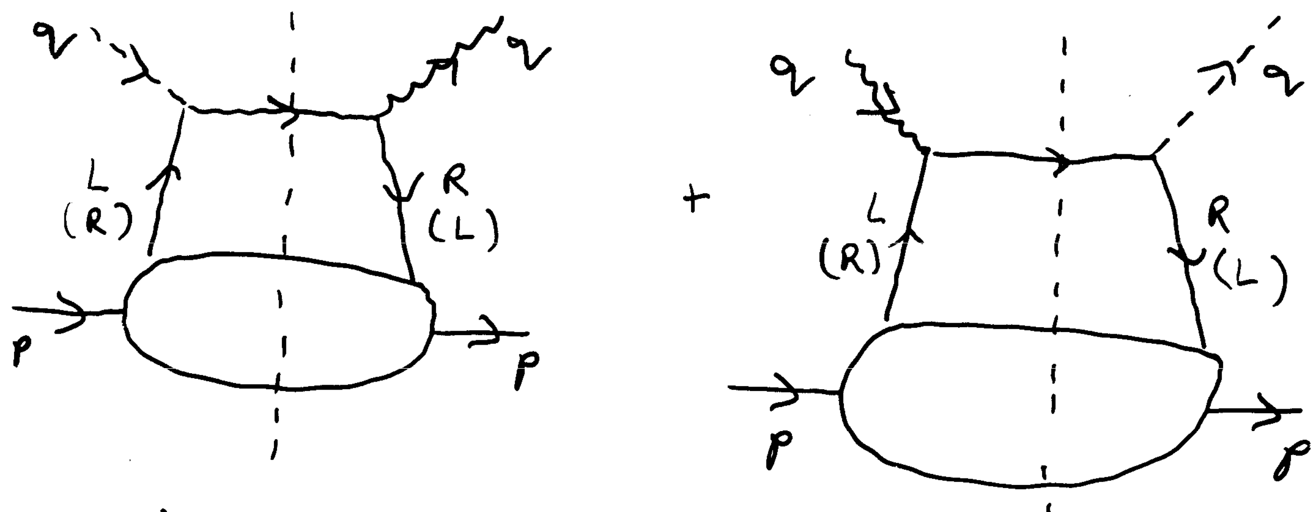
$$T_\mu(p,q,s_\perp) =$$

$$= \frac{i}{2} \int d^4x \, e^{iqx} \langle p, s_\perp | T\left(j_{5\mu}(x) j(0) + j(x) j_{5\mu}(0) \right) | p, s_\perp \rangle$$

$$= \tilde{h}_1(x,Q^2) \, s_{\perp\mu}$$

$$j_{5\mu}(x) = \bar{\psi}(x) \gamma_5 \gamma_\mu \psi(x) \qquad j(x) = \bar{\psi}(x)\psi(x)$$

$$h_1(x,Q^2) = -\frac{1}{\pi} \, \mathrm{Im} \, \tilde{h}_1(x,Q^2)$$



Remark:

(i) h_1 above has POSITIVE SIGNATURE: $h_1(x,Q^2) = h_1(-x,Q$

(ii) if $T(j_\mu(x) j_5(0) - j_5(x) j_\mu(0)) \Rightarrow$ NEGATIVE SIGNATURE

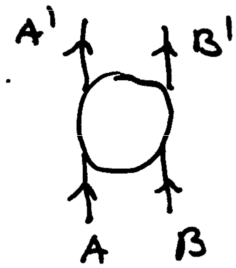
Small x asymptotics $\equiv$

Regge asymptotics

$$s \gg Q^2 \gg \mu^2$$

$$x \sim \frac{Q^2}{s} \longrightarrow 0$$

Conventionally:

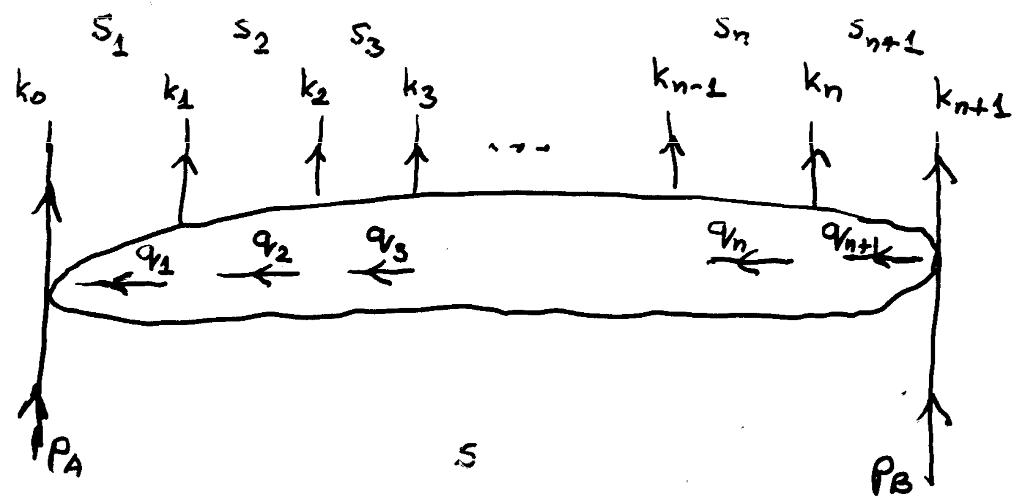


$$s = (p_A + p_B)^2 \longrightarrow \infty$$

$$t = (p_A - p_{A'})^2 = \text{const}$$

$$p_A^2,\ p_{A'}^2,\ p_B^2,\ p_{B'}^2 \sim \text{const}$$

Multi-Regge Kinematics:



$$s = (p_A + p_B)^2 \to \infty$$
$$s_1 = (k_0 + k_1)^2 \to \infty$$
$$s_2 = (k_1 + k_2)^2 \to \infty$$
$$\vdots$$
$$s_{n+1} = (k_n + k_{n+1})^2 \to \infty$$

$$q_i^2 \approx q_{\perp i}^2 \approx \text{const.}$$

$$k_{0-} \gg k_{1-} \gg k_{2-} \cdots \gg k_{n+1\,-}$$

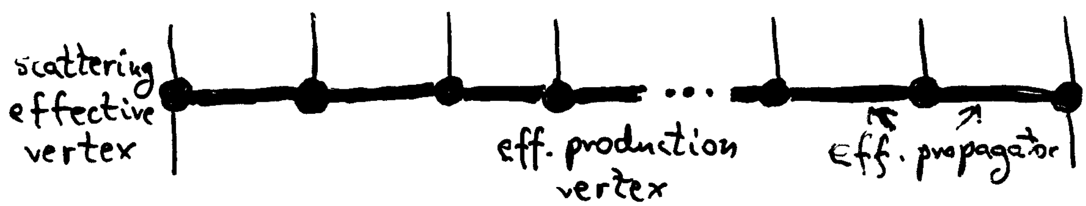
$$k_{0+} \ll k_{1+} \ll \cdots \ll k_{n+1\,+}$$

$$s_1 \cdot s_2 \cdot \cdots \cdot s_{n+1} = s\,(-k_{\perp 1}^2)\cdot(-k_{\perp 2}^2)\cdot \cdots \cdot(-k_{\perp n}^2)$$

Gribov et al '68

Cheng + Wu '68

Fadin, Kuraev, Lipatov '76



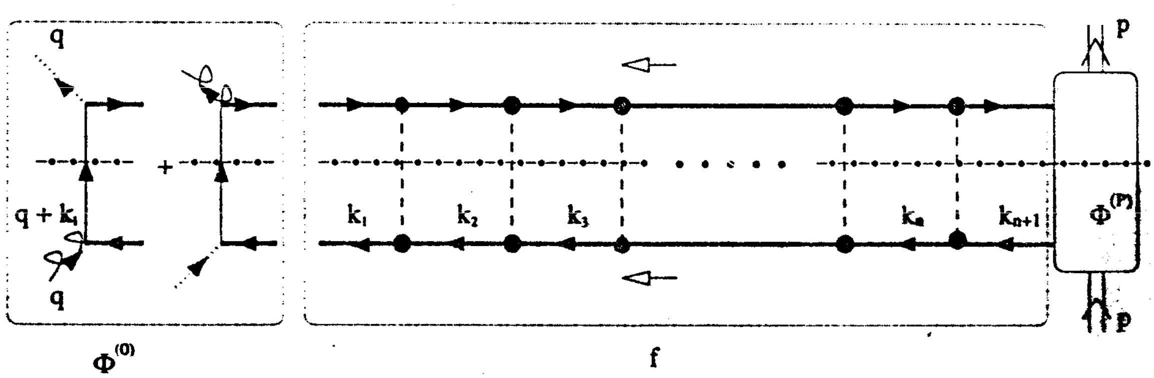


Fig. 2

- **L**eading **L**ogarithm **A**pproximation
- the impact representation:

$$A(s,t=0) \sim \int \frac{d^2 k_1}{(|k_1|^2)^2} \, \Phi^{(0)}(k_1) \cdot f(s, k_1, k_{n+1}) \cdot$$

$$\cdot \, \Phi^{(P)}(k_{n+1}) \, \frac{d^2 k_{n+1}}{(|k_{n+1}|^2)^2}$$

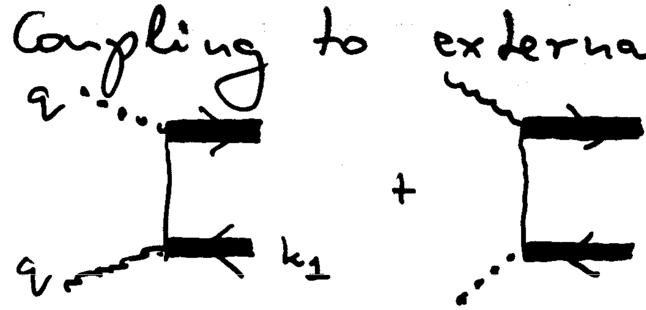
impact factor

- partial wave:

$$f(\omega, k, k') = \int_1^\infty d\left(\frac{s}{\mu^2}\right) \left(\frac{s}{\mu^2}\right)^{-\omega-1} f(s, k, k')$$

$$f(s, k, k') = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{\mu^2}\right)^{\omega} f(\omega, k, k')$$

Coupling to external currents:



$$\gamma_5 \gamma_\mu (\hat{k}_1 + \hat{q}) + (\hat{k}_1 + \hat{q}) \gamma_5 \gamma_\mu \Rightarrow$$

$\Rightarrow$ in MRK: k_1 negligible in comp. to q

Remark:

Complex # notation:

$$k^\mu = \frac{2}{\sqrt{s}} \left(q_1^\mu k_- + p^\mu k_+ \right) + k_\perp^\mu$$

$q_1 = q + xp$ $\qquad \kappa = k_\perp^1 + i k_\perp^2$ $\qquad \gamma = \gamma^1 + i \gamma^2$

$$T_\mu \Rightarrow T^* = -(T_1 - i T_2) = S^* \tilde{h}_1(x, Q^2)$$

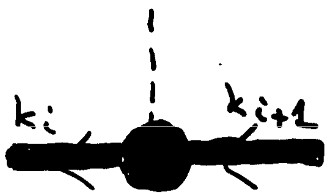
$$\Downarrow$$

$$\gamma_5 \gamma^* (\hat{k}_1 + \hat{q}) + (\hat{k}_1 + \hat{q}) \gamma_5 \gamma^* \xrightarrow{MRK}$$

$$\Rightarrow \underbrace{q_-}_{\sim \sqrt{s} \to \infty} \gamma^* \gamma_+ + \underbrace{q_+}_{\sim x \to 0} \gamma_- \gamma^*$$

Reggeon interaction :

Fadin Sherman '77

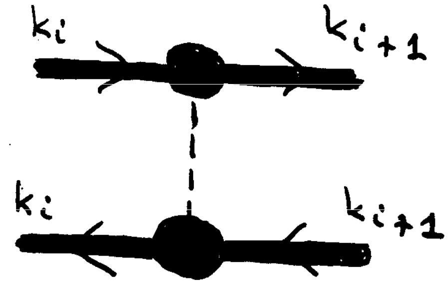


$$\hat{V}^\mu = \gamma^\mu_\perp - \frac{\hat{k}_{i\perp}}{k_i p} p^\mu - \frac{\hat{k}_{i+1\perp}}{k_{i+1} q_1} q_1^\mu$$

direct production — bremsstrahlung

$\hat{V}^\mu k_\mu = 0$
gauge inv.

Interaction kernel :



$$V^\mu \otimes V^\nu \, g_{\mu\nu} \quad \Rightarrow \text{MRK}$$

gluon propag.

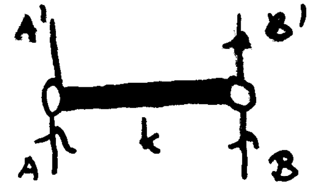
$$-\frac{1}{2}\left(\gamma \otimes \gamma^* + \gamma^* \otimes \gamma\right) \frac{|k_i|^2 + |k_{i+1}|^2}{|k_i - k_{i+1}|^2}$$

$\uparrow\downarrow$ chiral even F_1^{NS}

$$-\frac{1}{|k_i - k_{i+1}|^2}\left[\gamma^* \otimes \gamma^* \, k_{i+1} k_i + \gamma \otimes \gamma \, k^*_{i+1} k^*_i\right]$$

$\uparrow\uparrow$ chiral odd h_1

Reggeon propagator :



$$\underset{\text{particle}}{\frac{1}{\hat{k}_\perp}} \longrightarrow \frac{1}{\hat{k}_\perp}\, \underset{\text{reggeon}}{s^{\alpha(t)}} =$$

$$= \frac{1}{2} \left\{ \frac{1}{\kappa^*}\, \gamma^*\, s^{\alpha_F^*(\kappa)} + \frac{1}{\kappa}\, \gamma\, s^{\alpha_F(\kappa)} \right\}$$

$$\alpha_F(\kappa) = \frac{g^2 C_F}{(2\pi)^3}\, \bar{\alpha}_F(\kappa)$$

$$\llcorner \int \frac{d^2 q}{|q-\kappa|^2} \cdot \frac{\kappa}{q}$$

Partial wave :

$$A(s) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{\mu^2}\right)^{\omega} \mathcal{A}(\omega)$$

$$\mathcal{A}(\omega) = \int_1^{\infty} d\left(\frac{s}{\mu^2}\right) \left(\frac{s}{\mu^2}\right)^{-\omega-1} A(s)$$

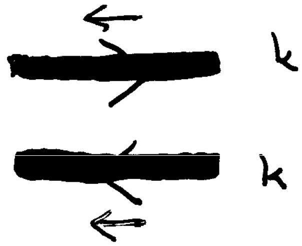
Reggeon field t
Gribov '6.

1-reggeon :

 $$\Rightarrow \frac{1}{\omega - \alpha_F} \cdot \frac{\gamma}{\kappa} + \frac{1}{\omega - \alpha_F^*} \cdot \frac{\gamma^*}{\kappa^*}$$

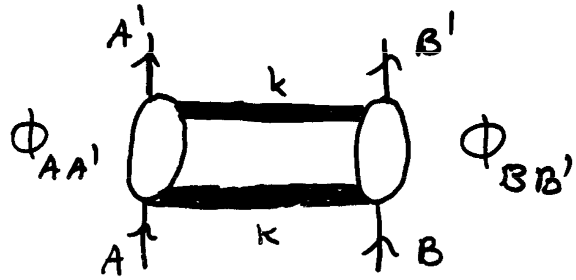
$$\mathcal{A}(\omega) = \Phi_{AA'}\, \frac{1}{\omega - \alpha_F} \cdot \frac{1}{\kappa}\, \Phi_{BB'}(\kappa)$$

2 - reggeons t-channel state :



$\Rightarrow$ $t = 0$

$$\frac{1}{\omega - 2\alpha_F(k)} \cdot \frac{\gamma}{k} \otimes \frac{\gamma}{k} + \frac{1}{\omega - 2\alpha_F^*(k)} \cdot \frac{\gamma^*}{k^*} \otimes \frac{\gamma^*}{k^*}$$

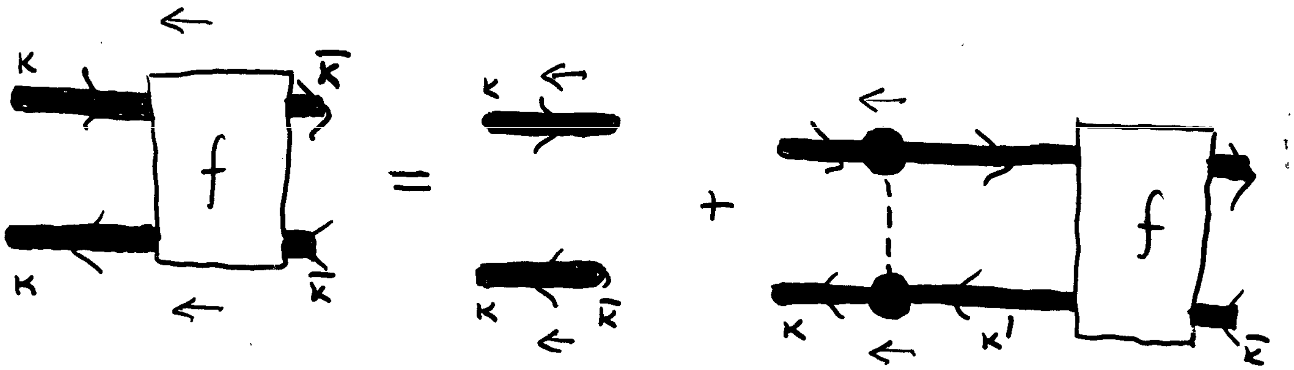


$$A(\omega) = \int d^2k \; \Phi_{AA'}(k) \frac{1}{\omega - 2\alpha_F(k)} \cdot \frac{1}{k^2} \; \Phi_{BB'}(k)$$

In our case the coupling to external currents is

$$q_- \gamma^* \gamma_+ \underset{\displaystyle \overset{\parallel}{1}}{\Phi^{(0)}}$$

Equation for f: Analogon of BFKL for $q\bar{q}$ ↑↑



$$f(\omega, \kappa, \bar{\kappa}) = \frac{\delta^2(\kappa - \bar{\kappa})}{\omega - 2\alpha_F(\kappa)} +$$

$$+ \frac{1}{\omega - 2\alpha_F(\kappa)} \cdot \frac{g^2 C_F}{(2\pi)^3} \cdot \int d^2\kappa' \cdot \frac{2}{|\kappa - \kappa'|^2} \cdot \frac{\kappa'}{\kappa} \cdot f(\omega, \kappa', \bar{\kappa})$$

$$\alpha_F(\kappa) = \frac{g^2 C_F}{(2\pi)^3} \cdot \int \frac{d^2 q}{|q - \kappa|^2} \cdot \frac{\kappa}{q}$$

quark Regge trajectory

Eigenfunctions :

$$\phi_{n,\nu}(\kappa) \equiv |\kappa|^{2i\nu}\left(\frac{\kappa}{|\kappa|}\right)^{n+1} \qquad \begin{array}{l}\nu \in [-\infty, +\infty] \\ n \in \mathbb{Z}\end{array}$$

Eigenvalues :

$$\omega(n,\nu) = \frac{g^2 C_F}{4\pi^2}\left\{-\psi\left(\tfrac{1}{2}+i\nu+\tfrac{|n|}{2}\right)-\psi\left(\tfrac{1}{2}-i\nu+\tfrac{|n|}{2}\right)+ + 2\psi(1)\right\}$$

the same as for $\mathbb{P}$

Solution :

$$f(\omega, k_\perp, \bar{k}_\perp) = = \frac{1}{2\pi^2 \kappa^2}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} d\nu \, \frac{|\kappa|^{2i\nu}\left(\frac{\kappa}{|\kappa|}\right)^{n+1} |\bar{\kappa}|^{-2i\nu}\left(\frac{\bar{\kappa}^*}{|\bar{\kappa}|}\right)^{n+1}\left(\frac{\bar{\kappa}}{\bar{\kappa}^*}\right)}{\omega - \omega(n,\nu)}$$

Decoupling of leading singularity $\qquad \Phi^{(0)} = 1!$

leading sing. for $\omega(0,0) = \frac{g^2 C_F}{\pi^2}\ln 2$

$$\int \frac{d^2 k_\perp}{\kappa^2}\kappa = 0 \qquad \Rightarrow \left(\frac{1}{x}\right)^{\omega(0,0)}$$

since $\int_0^{2\pi} d\theta \, e^{i\theta} = 0$

Next singularity

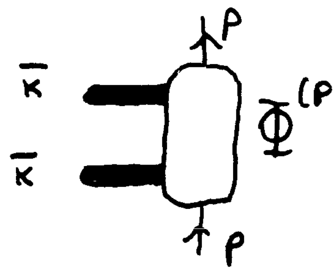
$$\omega(1,0) = 0 \quad \Rightarrow \quad \left(\tfrac{1}{x}\right)^0$$

$$f^{(n=1)}(\omega, k, \bar{k}) =$$

$$= \frac{1}{2\pi^2} \frac{1}{|k|^2} \int_{-\infty}^{\infty} d\nu \, \frac{\left(\frac{|k|}{|\bar{k}|}\right)^{2i\nu}}{\omega - \omega(1,\nu)}$$

<u>Coupling to the proton:</u> Ansatz

$$\Phi^{(p)}(\bar{k}) = \frac{\hat{p}}{\sqrt{s}} (S_\perp \bar{k}_\perp) \hat{k}_\perp \gamma_5 \tilde{F}^{(p)}(|\bar{k}|^2)$$

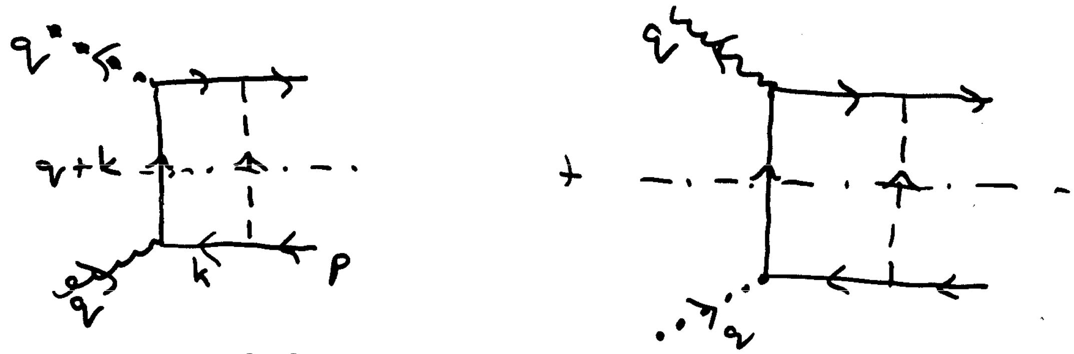


$$h_1(x, Q^2)\Big|_{\text{Regge}} =$$

$$= \frac{1}{2} \int d^2\bar{k} \left(1 + \Phi\left(\frac{\ln \frac{Q^2}{|\bar{k}|^2}}{\sqrt{4\Omega_0 \ln(\frac{1}{x})}} \right) \right) \tilde{F}^{(p)}(|\bar{k}|^2)$$

$$\Omega_0 = \frac{g^2 C_F}{2\pi^2} \zeta(3)$$

GLAP evolution of $h_1(x, Q^2)$



$$\mathrm{Disc}\, T_\mu^{(1)} =$$

$$= \frac{i}{2} g^2 C_F \int \frac{d^4k}{(2\pi)^4} \frac{N_\mu^{\rho\sigma} d_{\rho\sigma}}{(k^2+i\varepsilon)^2} (-2\pi i)^2 \delta_+((p-k)^2)\, \delta_+((q+k)^2)$$

axial gauge:

$q_1 A = 0 \qquad d_{\rho\sigma}(k) = g_{\rho\sigma} - \frac{q_{1\rho} k_\sigma + q_{1\sigma} k_\rho}{q_1 k}$

only this term contributes

$$N_\mu^{\rho\sigma} d_{\rho\sigma} = -\frac{4 s k^2}{q_1(p-k)} \left[2 S_{\perp\mu}(q_1 k) + k^2 S_{\perp\mu} - k_\mu (S_\perp k) \right]$$

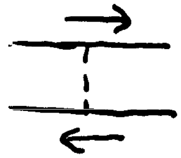
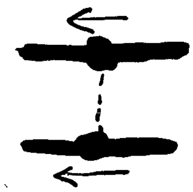
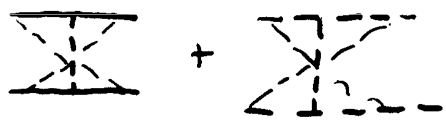
$$\Rightarrow S_{\perp\mu} \frac{(-i) g^2 C_F}{2\pi} \frac{x}{1-x} \int^{Q^2} \frac{d|k|^2}{|k|^2}$$

$$\Rightarrow \qquad P^{(0)}(x) = 2 C_F \frac{x}{1-x}$$

Remark:

in Regge limit $x \to 0$

$h_1\big|_{GLAP} \longrightarrow x \ln Q^2$ i.e. $\frac{1}{s}$ behavior non-leading!

	$g_1(x,Q^2)$ B+E+R	$h_1(x,Q^2)$ K+M+S+Sz
NS	 $\left(\frac{1}{x}\right)^{\sqrt{\frac{8}{3\pi}\alpha_s}}$	 $\left(\frac{1}{x}\right)^{0}$
S	 $\left(\frac{1}{x}\right)^{\omega_s}$ $\omega_s \approx 1{,}01$ $(\omega_{\mathbb{P}} \sim 1{,}5)$	No
Kind of asymptotics	double-log $\left(g^2 \ln^2 \frac{1}{x}\right)^n$	single-log $\left(g^2 \ln \frac{1}{x}\right)^n$
Regge beh. versus $GLAP\|_{x\to 0}$	smooth transition	$h_1\|_{GLAP} \sim x$ non-leading i.e. withing LLA no connection