Effective Action Approach to Semi-hard Processes

L. Szymanowski
(Leipzig Univ. & SINS Warsaw)

- Motivation
- Sketch of the construction of the Eff. Action

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Semi-hard processes $\equiv$

processes with two large momentum scales

$\Rightarrow$ related to the Regge asymptotics:

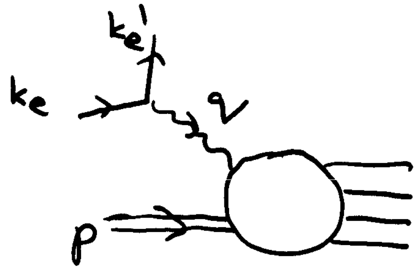
- DIS at small x $\quad s \gg Q^2 \gg m_p^2$
- quasi-elastic scattering at relatively large t $\quad s \gg |t| \gg m$
- inclusive mini-jet production with large E_T $\quad s \gg E_T^2 \gg m$

$\Rightarrow$ not related to Regge asymptotics

- fragmentation of jets

$$E_J \gg x_E E_J \gg m$$

DIS:



$$s = (p+q)^2 = m_p^2 - Q^2\left(1-\frac{1}{x}\right)$$

$$Q^2 = -q^2 = -(k_e - k_e')^2$$

$$\nu m_p = p(k_e - k_e')$$

$$x = \frac{Q^2}{2\nu m_p} \qquad y = \frac{p(k_e-k_e')}{pk_e}$$

unpolarized DIS:

$$\frac{d\sigma}{dQ^2\,dx}(x,Q^2,y) =$$

$$= \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y-\frac{m_p x y}{2pk}\right)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

F's carry information about $\gamma^* p$ inter,

Two asymptotics:

1° $\quad s \gtrsim Q^2 \gg m_p^2$

2° $\quad s \gg Q^2 \gg m_p^2$

- $s \gtrsim Q^2 \gg m_p^2$

one scale

one large log : $\ln \frac{Q^2}{m_p^2}$

$$x = \frac{Q^2}{s + Q^2 - m_p^2} \sim \mathcal{O}(1)$$

$$F(x,Q^2) = \sum_i \int_x^1 dz \; \underbrace{C_i\left(\alpha_s(Q^2), \tfrac{x}{z}, Q^2\right)}_{\text{hard subprocess cross-section}} \; \underbrace{f_i(z,Q^2)}_{\text{distribution function}}$$

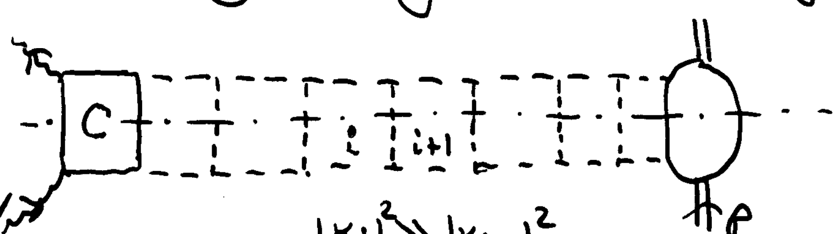
collinear factorization

Evolution in Q^2 : LLA

DGLAP
Dokhshitzer
Gribov
Lipatov
Altarelli
Parisi

$$\frac{d}{d(\ln \frac{Q^2}{Q_0^2})} f_i(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \underbrace{P_{ij}\left(\tfrac{x}{y}, Q^2\right)}_{\text{splitting f.}} f_j(y,Q^2)$$

In the physical gauge (axial g.):



$|k_i|^2 \gg |k_{i+1}|^2$

transverse mom. ordering
(transverse integr. easy)

$q' = q - xp$

$k = \sqrt{\frac{2}{s}}\left(k_- q' + k_+ p\right) + k$

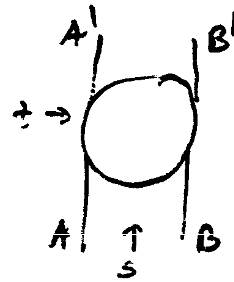
$k_\perp^\mu \to k$

- $s \gg Q^2 \gg m_p^2$ two scales

two large log's : $x = \frac{Q^2}{s} \to 0$

$$\ln\left(\frac{1}{x}\right) \quad \text{and} \quad \ln\left(\frac{Q^2}{m_p^2}\right)$$

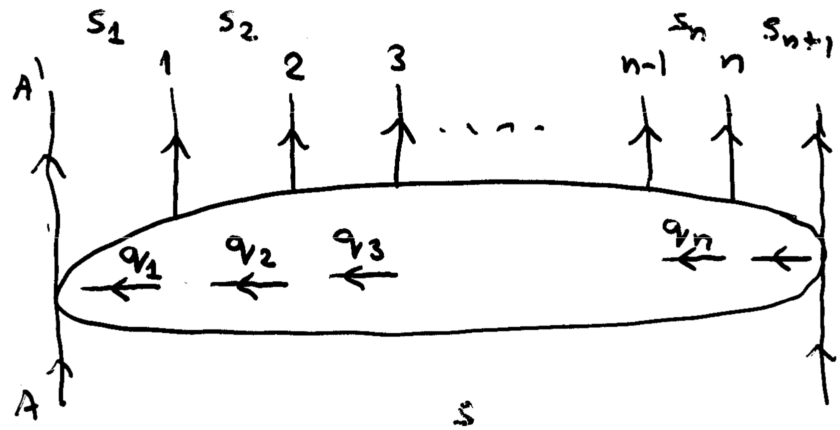
$\Rightarrow$ Regge region



$$\infty \leftarrow s \gg p_{A}^2, p_{B}^2, \; p_{A'}^2, p_{B'}^2, \; t \; (=0)$$

summation of $\left(\alpha_s \ln\frac{1}{x}\right)^n \overset{LLA}{\Rightarrow}$ BFKL

Multi-Regge Kinematics (MRX)



$$k^\mu = \frac{2}{\sqrt{s}}\left(k_- p_A^\mu + k_+ p_B^\mu\right) + \underbrace{k_\perp^\mu}_{\hookrightarrow \, \mathrm{k}}$$

$$q_{-1} \gg q_{-2} \gg \cdots \gg q_{-n} \gg q_{-n+1}$$

$$q_{+n+1} \gg q_{+n} \gg \cdots \gg q_{+2} \gg q_{+1}$$

no ordering in $|q_i|^2$!!

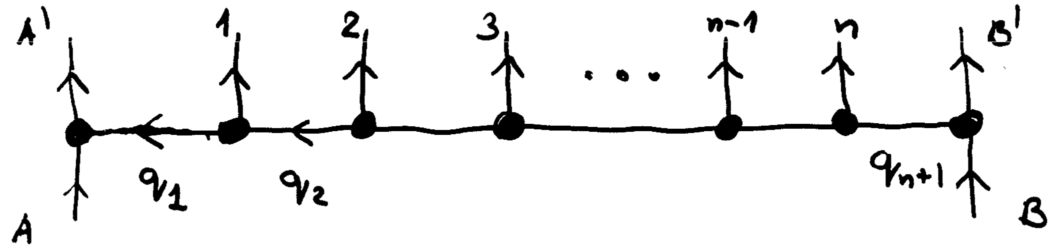
$$q_{+l}\, q_{-l} \ll |q_{\perp l}^2| \quad \text{finite}$$

$$s_i \approx q_{-i-1}\, q_{+i+1} \to \infty \qquad s \to \infty$$

$$\prod_{i=1}^{n+1} s_i = s \prod_{i=1}^{n} |q_i - q_{i+1}|^2$$

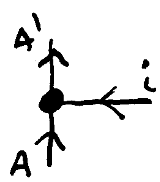
• QCD in M.R.K : Fadin Kuraev Lipatov '76

sum of F.D. for $2 \to 2+n$



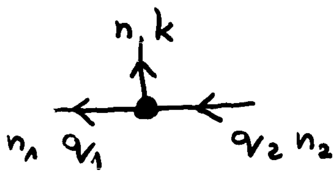
tree F.D

$$\frac{1}{q_{\perp i}^2}$$



$$\sim \quad g\,(T^i)_{A'A}\,\delta_{\lambda_A \lambda_{A'}}$$

helicity conserv.

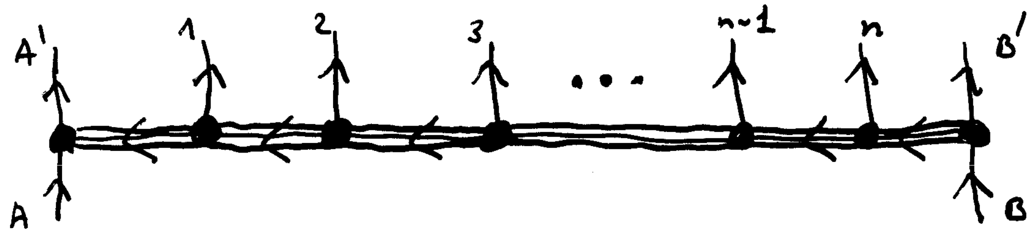


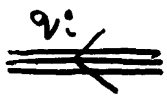
$\sim$

$$g\,f^{n n_1 n_2}\left[-(q_1-q_2)_\perp^\mu + p_A^\mu\left(\frac{p_B k}{p_A p_B} + \frac{q_1^2}{p_A k}\right) - p_B^\mu\left(\frac{p_A k}{p_A p_B} + \frac{q_2^2}{p_B k}\right)\right]\varepsilon_\mu(k)$$

gauge inv. $C_\mu k^\mu = 0$ bremsstrahlung

Loop corrections in LLA:





$$\frac{(s_i)^{\alpha_G(q_i)}}{q_{\perp i}^2} \leftarrow \text{Regge traj.}$$

$$\alpha_G(q) = \frac{N}{2} \cdot \frac{g^2}{(2\pi)^3} \cdot |q|^2 \int \frac{d^2k}{|k|^2\,|q-k|^2}$$

- the same effective vertices as in the tree approx.

Properties and consequences of BFKL:

- BFKL $\leftrightarrow$ separation of the $\parallel$ and $\perp$ degrees of freedom in LLA pQCD

- 2-dim conformal symmetry in the impact space $\quad k \xrightarrow{\text{Fourier t.}} z$
  $\Rightarrow$ solution for $t \neq 0$ $\quad z \to \frac{az+b}{cz+d}$
  no "hard" scale $\quad ad - bc \neq 0$

- diffusion in $\ln \frac{|k|^2}{|k_0|^2}$
  random walk in $\ln \frac{|k|^2}{|k_0|^2}$ space
  $\Rightarrow$ $|k|^2$ can enter non-pert. region
  $\Rightarrow$

- necessity of unitarization
  - possible violation of Froissart bound
    $\Rightarrow$ $\sigma \sim s^{\omega_0}$ $\quad \sigma \sim \ln^2 s$
  - screening corrections

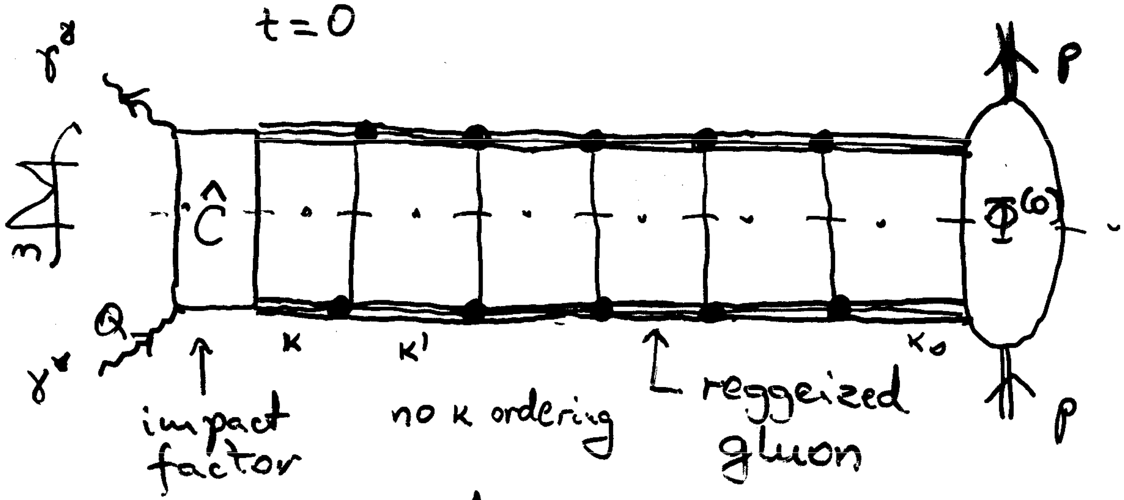
$$\left(\frac{1}{x}\right)^{\omega_0} \sim x G(x) = \frac{d}{d(\ln \frac{1}{x})} n_G(x, Q^2)$$

naive parton model not valid

BFKL equation

Balitski
Fadin
Kuraev
Lipatov '78

LLA



$$F(x,Q^2) = \int d^2\kappa \int_x^1 \frac{dz}{z}\ \hat{C}(\alpha_s, z, \kappa, Q^2)\, \tilde{f}(z,\kappa)$$

k_T - factorization

unintegrated gluon density

$$\frac{d}{d(\ln\frac{1}{x})}\tilde{f}(x,\kappa) = \frac{g^2 N}{(2\pi)^3}\int d^2\kappa' \left[\frac{2}{|\kappa-\kappa'|^2}\tilde{f}(x,\kappa') - \frac{|\kappa|^2}{|\kappa'|^2|\kappa-\kappa'|^2}\tilde{f}(x,\kappa)\right]$$

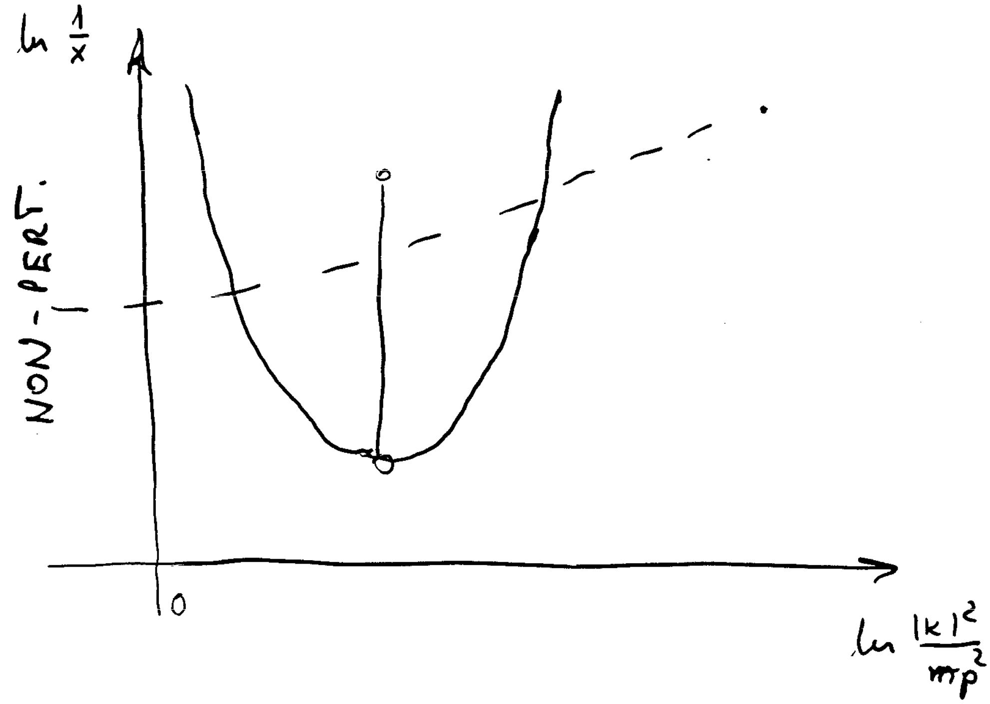
evolution in $\ln\frac{1}{x}$

reggeization

Solution for $\ln\frac{1}{x}\to\infty$

$$\tilde{f}(y,\kappa,\kappa_0) \sim \left(\frac{1}{x}\right)^{\omega_0} \frac{\exp\left(-\dfrac{\ln\frac{|\kappa|^2}{|\kappa_0|^2}}{2D^2\ln\frac{1}{x}}\right)}{\sqrt{2\pi D^2 \ln\frac{1}{x}}}$$

$$\omega_0 = \frac{g^2N}{2\pi^2}\, 2\ln 2 \qquad D^2 = \frac{g^2N}{\pi^2}\, 7\zeta(3)$$



ln 1/x
NON-PERT.
0
ln |k|²/m_p²

Different approaches to BFKL and unitarization
(Belief: BFKL is more fundamental than LLA pQCD)

- t- and s-channel unitarity — V. Fadin, L. Lipatov
- reggeon field theory — J. Bartels, A. White
- new degrees of freedom - colour dipoles — A. H. Mueller
- high-energy OPE — I. Balitsky
- renorm. group approach to longitudinal degrees of freedom — L. McLerran & collab.
- new efficient methods for calculating higher-loop diagrams:
  * cutting techniques — Z. Bern
  * string inspired methods — L. Magnea
- multi-Regge effective action (EA) — L. Lipatov

Unitarity corrections:

- restoring unitarity conditions in ALL s-subchannels

  ⇒ s-channel multi-particle interm. states which obey MRK

  Generalized LLA — J. Bartels, T.T. Wu

  R. trajectories and vertices like in LLA

- related to region beyond MRK — V. Fadin, L. Lipatov, R. Fiore, M. Kotsky

  corrections to R. trajectories and vertices

Motivation for EA:

summarizes LLA results in a simple form
$\Rightarrow$ starting point for a systematic improvements

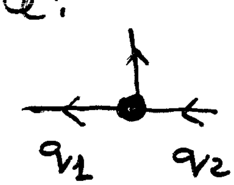
- EA involves scattering fields (s-channel) and exchanged fields (t-channel) interacting by means of scattering and production vertices

  example: deriv. of BFKL

- particularly simple form of effective vertices in the helicity basis and in complex notation for $\perp$ vectors

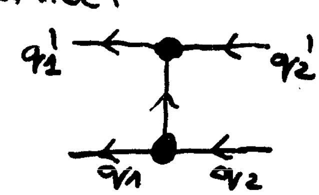
  - the LLA effective vertices can be read off from multi-Regge tree amplitudes

example:



$$\begin{cases} \dfrac{q_1 q_2^*}{q_1 - q_2} & - \\ \\ \text{c.c.} & + \end{cases}$$

BFKL kernel:



$$= \frac{q_1 q_2^* \, q_1'^* q_2' + \text{c.c.}}{|q_1 - q_2|^2}$$

- derivation of EA from the original QCD action

  - QCD (gluons + quarks)
    leading power of s $\Rightarrow$ BFKL
    $j=1$ Kirschner, Lipatov, S.

  - quantum gravity Kirschner, Sz.

  - non-leading reggeons
    $\Rightarrow$ polarized structure f.
    $j=0$ like quarks g_1 F_3^{γ} Kirschner, Sz.

Yang-Mills action in the light-cone axial gauge $A_- = 0$

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)}$$

where

$$\begin{aligned}
\mathcal{L}^{(2)} &= -2A^{a*}(\partial_+\partial_- - \partial\partial^*)A^a \\
\mathcal{L}^{(3)} &= -\frac{g}{2}J^a_- A^a_+ - \frac{g}{2}j^a A'^a \\
\mathcal{L}^{(4)} &= \frac{g^2}{8}J^a_- \partial_-^{-2} J^a_- - \frac{g^2}{8}j^a j^a
\end{aligned}$$

and

$$A^a_+ = \partial_-^{-1}(\partial A^a + \partial^* A^{a*}), \qquad \sim \partial_\varsigma A^\varsigma$$

$$A'^a = i(\partial A^a - \partial^* A^{a*}) \qquad \sim \epsilon_{\varsigma\sigma}\partial^\varsigma A^\sigma$$

$$J^a_- = i(A^* T^a \overleftrightarrow{\partial}_- A)\,,$$

Later:
$j^a \to \partial^*$
$j^{*a} \to \partial$

$$j^a = (A^* T^a A)$$

Conventions: $\partial_+ x_- = \partial_- x_+ = \partial x = \partial^* x^* = 1 \qquad (AT^a B) = -if^{abc}A^b \overline{B}^c$

Separation of modes:

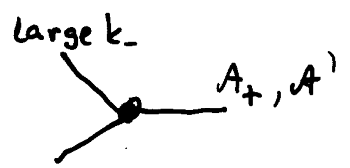
$$A \to A_t + A_s + A_1$$

$$\begin{aligned} A_t &: |k_+ k_-| \ll |\kappa|^2 \\ A_s &: |k_+ k_- - |\kappa|^2| \ll |\kappa|^2 \\ A_1 &: |k_+ k_-| \gg |\kappa|^2 \end{aligned}$$

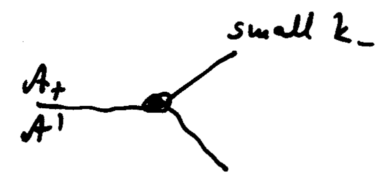
$$\mathcal{L}_{kin.} = -\frac{1}{2} A_s^a \Box A_s^{a*} - \frac{1}{2} A_1^a \Box A_1^{a*} - \frac{1}{2} A_t^a \Box A_t^{a*}$$

$$\Box = 4(\partial_+ \partial_- - \partial \partial^*)$$

$\mathcal{L}_1^{(3)} =$



or



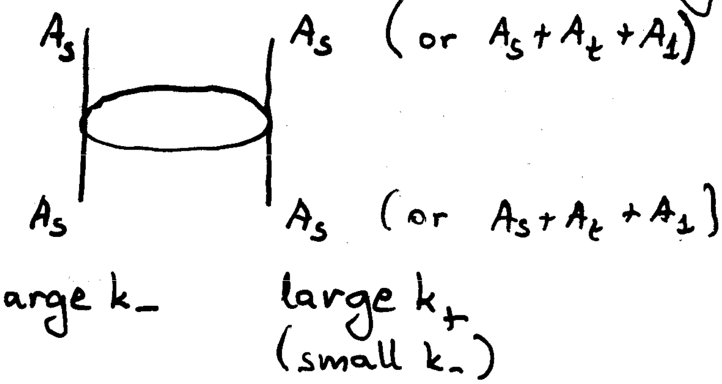
$$-\frac{g}{2}[J_-^a - \frac{1}{2} \cdot \frac{\partial_-}{\partial \partial^*}(\partial J^a + \partial^* J^{a*}) - i\frac{\partial_-^2}{\partial^*}(\frac{1}{\partial_-}(\partial A + \partial^* A^*)T^a A)$$

$$- i\frac{\partial_-^2}{\partial}(\frac{1}{\partial_-}(\partial A + \partial^* A^*)T^a A^*)]\mathcal{A}_+^a$$

$$- g[j^a + \frac{i}{4}\frac{1}{\partial \partial^*}(\partial J^a - \partial^* J^{a*}) - \frac{1}{2}\frac{\partial_-}{\partial^*}(\frac{1}{\partial_-}(\partial A + \partial^* A^*)T^a A)$$

$$+ \frac{1}{2}\frac{\partial_-}{\partial}(\frac{1}{\partial_-}(\partial A + \partial^* A^*)T^a A^*)]\mathcal{A}'^a$$

Quasi elastic scattering :



Consider as an example

$$\mathcal{L}^{(4)} = \frac{g^2}{8} J_-^a \frac{1}{\partial_-^2} J_-^a - \frac{g^2}{8} j^a \cdot j^a$$

$$\Downarrow \quad A = A_{\text{large } k_-} + A_{\text{small } k_-}$$

large k_-

$$\mathcal{L}^{(4)}_{scatt} = \frac{g^2}{4} J_-^a \frac{1}{\partial_-^2} J_-^a - \frac{g^2}{2} j^a\, j^a - \frac{g^2}{4} j_D^r\, j_D^r$$

large k_+

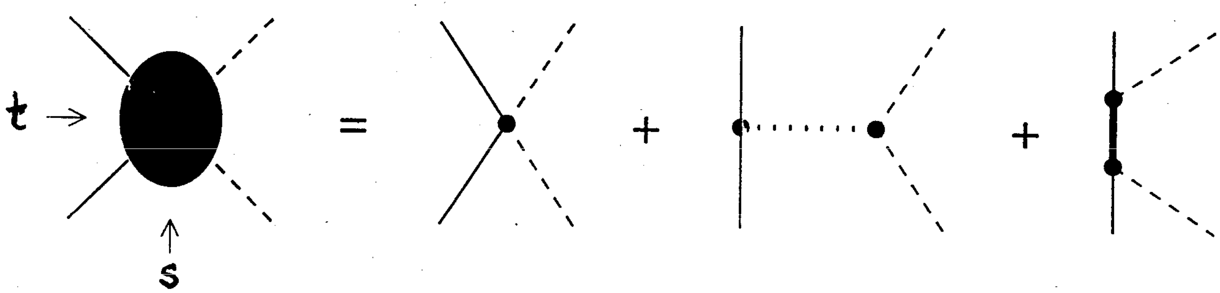
where

$$j_D^r = (A^* D^r A) \ ,$$
$$(T^e)_{ab}(T^e)_{cd} + (T^e)_{ac}(T^e)_{bd} = (D^r)_{ad}(D^r)_{cb}$$

current in symmetric representation
(t-channel exchange)

Complete quartic terms:

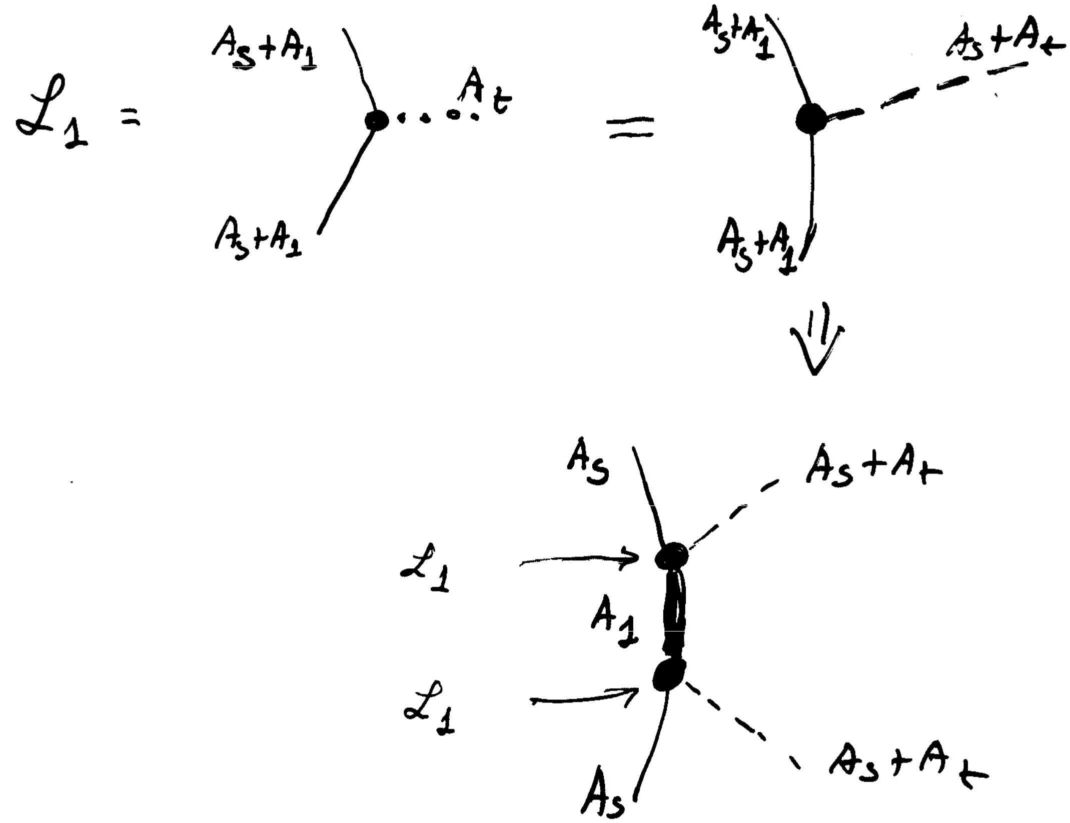
$$\mathcal{L}^{(4)}_{tot} = \mathcal{L}^{(4)}_{scatt} + < \mathcal{L}^{(3)}_{1}\ \mathcal{L}^{(3)}_{1} >_{A_t} + < \mathcal{L}^{(3)}_{1}\ \mathcal{L}^{(3)}_{1} >_{A_1}$$



Different line forms represent different modes: full line - scattered modes, dotted line - exchange modes, bold line - heavy modes, dashed line - the sum of all modes.

Integration over "heavy modes" A_1:

- approximate, with use of equations of motion



$\mathcal{O}(s^1)$ $\quad\quad$ $\mathcal{O}(s^0)$

$$\mathcal{L}^{(4)}_{tot,scatt} = \frac{g^2}{8} J^a_- \, \frac{1}{\partial\partial^*} \, J^a_{+R} - \frac{g^2}{16}(\partial_+ J^a_-) \, \frac{1}{\partial\partial^*} \, (\partial_- J^a_{+R})$$

"A_+ chann

$\mathcal{O}(s^0)$ $\quad\quad$ factorization

$$\underbrace{+g^2 j^a_s \; j^a_{sR} - \frac{g^2}{2} j^a \; j^a_R}_{\text{"}A'\text{ channel"}} - \frac{g^2}{4} j^r_D \; j^r_{DR} + \mathrm{O}(s^{-1})$$

symm. repr.

Currents with "R":

currents without R plus

$$A^a \to A^a_R = -\tfrac{\partial^*}{\partial} A^{a*}$$

gauge tr.

and

$$j^a_s = j^a + \frac{i}{4} \frac{1}{\partial\partial^*} (\partial J^a - \partial^* J^{a*})$$

$$\mathcal{L}_{eff,scatt} = \mathcal{L}_{kin.} + \mathcal{L}_{s-} + \mathcal{L}_{s+}$$

leading reggeon (BFKL)

$$\mathcal{L}_{kin.} = -2A_s^{a*}(\partial_-\partial_+ - \partial\partial^*)A_s^a \overbrace{- 2\mathcal{A}_+^a\partial\partial^*\mathcal{A}_- + \mathcal{A}^a_{(+)}\mathcal{A}^a_{(-)}}^{\text{positive parity}}$$

$$\underbrace{-\mathcal{A}_s'^{a(+)}\mathcal{A}_s'^{a(-)} + 2\mathcal{A}_2'^{a(+)}\mathcal{A}_2'^{a(-)}}_{\text{negative parity}} + B^{r(+)}B^{r(-)}$$

$$\mathcal{L}_{s-} = -\frac{g}{2}J_-^a\mathcal{A}_+^a - \frac{g}{4}\left(\frac{\partial_+}{\partial\partial^*}J_-^a\right)\mathcal{A}^a_{(+)} - gj_s^a\mathcal{A}_s'^{a(+)} - gj^a\mathcal{A}_2'^{a(+)} - \frac{g}{2}j_D^r B^{r(+)}$$

$$\mathcal{L}_{s+} = \mathcal{L}_{s-}\left(+ \leftrightarrow -,\ J_-^a \to J_{+R}^a,\ j_s^a \to j_{sR}^a,\ j^a \to j_R^a,\ j_D^r \to j_{DR}^r \right)$$

Including fermions:

- only as scattered fields
- light-cone decomposition:

$$\Psi = \Psi_+ + \Psi_- \qquad\qquad \frac{\gamma_+\gamma_-}{2} \qquad \frac{\gamma_-\gamma_+}{2}$$

$$\gamma_- \Psi_+ = 0 = \gamma_+ \Psi_-$$

Ψ_+ is eliminated

$$\Psi_- = f u_{--} + \bar{f} u_{-+} \qquad \gamma u_{-+} = 0 = \gamma^* u_{+-}$$

two chiralities

- SUSY transformations:

$$\delta A = 2 \bar{\alpha} \gamma \Psi_-$$

$$\delta \Psi_- = i \partial_- (\gamma A^* + \gamma^* A) \gamma_+ \alpha$$

$$\alpha = \alpha_+ u_{+-} + \alpha_+^* u_{++}$$

A_- unchang. algebra closes on ∂_-

for one chirality: $\alpha_+ = \alpha_r + \alpha_i$

$$\delta_i A = f \qquad \delta_i A^* = f^* \qquad \delta_i f = 2i\partial_- A \qquad \delta_i f^* = 2i\partial_- A^*$$

$$\delta_r A = f \qquad \delta_r A^* = -f^* \qquad \delta_r f = -2i\partial_- A \qquad \delta_r f^* = 2i\partial_- A$$

Currents:

$$J_-^a = i(A^* T^a \overleftrightarrow{\partial}_- A) + f^* T^a f$$

$$J^a = i(A^* T^a \overleftrightarrow{\partial}^* A) - \frac{1}{2}\left(\frac{1}{\partial_-} f^* T^a \partial^* f\right)$$

$$j^a = A^* T^a A \qquad \mathcal{K}^a = A^* T^a A - \frac{i}{4}\left(f^* T^a \overleftrightarrow{\partial_-^{-1}} f\right)$$

SUSY MULTIPLETS:

$$\begin{pmatrix} \mathcal{A}_+ \\ u \\ \bar{u} \\ \mathcal{A}' \end{pmatrix} \quad \begin{pmatrix} \mathcal{J}_- \\ I \\ \vdots \\ j \end{pmatrix} \quad \begin{pmatrix} \mathcal{J} \\ \chi \end{pmatrix} \quad \begin{pmatrix} \mathcal{J}^a \\ \chi^a \end{pmatrix} \quad \begin{pmatrix} \mathcal{K} \\ \vdots \end{pmatrix}$$

in odd parity channel:

$$\dot{j}^a \dot{j}^a_R \longrightarrow \frac{1}{2} \dot{j}^a \dot{j}^a_R + \frac{1}{2} \mathcal{K}^a \mathcal{K}^a_R$$

$$+ \quad \dot{j}_s \dot{j}_s$$

$$\Downarrow$$

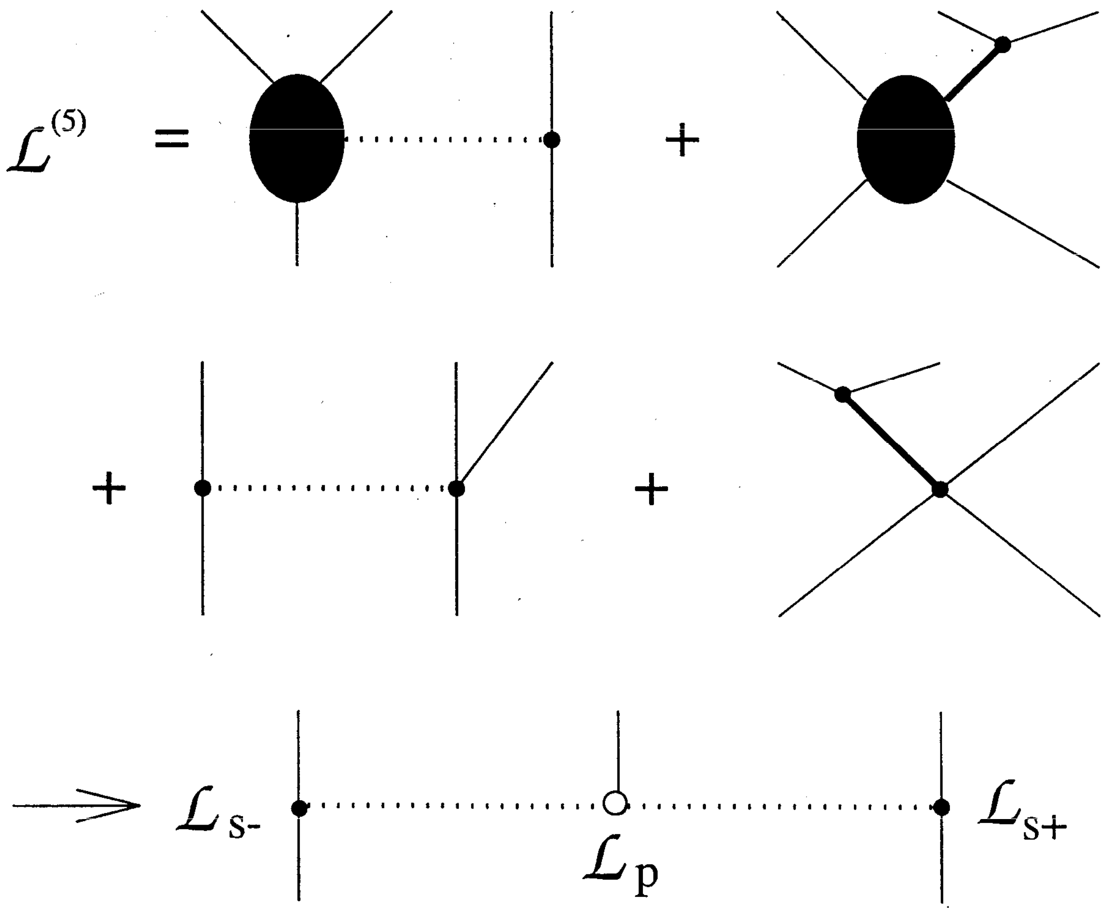
3 reggeons near $j=0$!

without fermions: 2 reggeons ?

$$\mathcal{L}^{(5)} = < \mathcal{L}^{(4)}_{tot}\ \mathcal{L}^{(3)}_1 >_{A_t} + < \mathcal{L}^{(4)}_{tot}\ \mathcal{L}^{(3)}_1 >_{A_1}$$

$$+ < \mathcal{L}^{(3)}_1\ \mathcal{L}^{(4)}_{scatt} >_{A_t} + < \mathcal{L}^{(3)}_1\ \mathcal{L}^{(4)}_{scatt} >_{A_1}$$

$$= < \mathcal{L}_{s-}\ \mathcal{L}_p\ \mathcal{L}_{s+} >_{A_t}$$



$$\mathcal{L}_p\Big|_{A_\pm\, A_{(+)\,(-)}} =$$

leading v. (BFKL)

$$A_- - A^* - A_+$$

$$= \frac{ig}{4}\Big\{\quad 2\,\partial^* A_-\, T^a\, \partial A_+\ \ \frac{1}{\partial} A^{*a}$$

$$A_{(-)} - A^* - A_{(+)}$$

$$+\Big[-\frac{3}{2} A_{(-)} T^a A_{(+)} + \frac{1}{2}\frac{\partial^*}{\partial} A_{(-)} T^a \frac{\partial}{\partial^*} A_{(+)}$$

$$+ \frac{1}{\partial} A_{(-)} T^a \partial A_{(+)} + \partial^* A_{(-)} T^a \frac{1}{\partial^*} A_{(+)}\Big]\frac{1}{\partial} A^{*a}$$

$$A_- - A^* - A_{(+)}$$

$$+\Big[\partial^* A_- T^a \frac{1}{\partial^*} A_{(+)} - \frac{1}{\partial\partial^*}\Big(\partial\partial^* A_- T^a A_{(+)} + \partial^* A_- T^a \partial A_{(+)}\Big)\Big]\frac{\partial_+}{\partial} A^{*a}$$

$$A_{(-)} - A^* - A_+$$

$$+\Big[\frac{1}{\partial} A_{(-)} T^a \partial A_+ - \frac{1}{\partial\partial^*}\Big(A_{(-)} T^a \partial\partial^* A_+ + \partial^* A_{(-)} T^a \partial A_+\Big)\Big]\frac{\partial_-}{\partial} A^{*a}\Big\}$$

$$+\ c.c.$$

$$\mathcal{L}_p|_{\mathcal{A}'_s\mathcal{A}'_2} =$$

$\mathcal{A}_s - A^{\gamma} - \mathcal{A}_s$

$$-\frac{ig}{4}\{ \quad 2(\frac{1}{\partial}\mathcal{A}_s^{\prime(-)}T^a\partial\mathcal{A}_s^{\prime(+)}) + 2(\partial^*\mathcal{A}_s^{\prime(-)}T^a\frac{1}{\partial^*}\mathcal{A}_s^{\prime(+)})$$

$$+ \quad (\frac{\partial^*}{\partial}\mathcal{A}_s^{\prime(-)}T^a\frac{\partial}{\partial^*}\mathcal{A}_s^{\prime(+)}) + (\mathcal{A}_s^{\prime(-)}T^a\mathcal{A}_s^{\prime(+)})$$

$$+ \quad 4(\mathcal{A}_2^{\prime(-)}T^a\mathcal{A}_2^{\prime(+)})$$

$\mathcal{A}_2 - A^{\gamma} - \mathcal{A}_2$

$$+ \quad 2(\frac{1}{\partial}\mathcal{A}_s^{\prime(-)}T^a\partial\mathcal{A}_2^{\prime(+)}) + 2(\mathcal{A}_s^{\prime(-)}T^a\mathcal{A}_2^{\prime(+)})$$

$\mathcal{A}_s - A^{\gamma} - \mathcal{A}_2$

$\mathcal{A}_2 - A^{\gamma} - \mathcal{A}_s$

$$+ \quad 2(\partial^*\mathcal{A}_2^{\prime(-)}T^a\frac{1}{\partial^*}\mathcal{A}_s^{\prime(+)}) + 2(\mathcal{A}_2^{\prime(-)}T^a\mathcal{A}_s^{\prime(+)})\}\frac{1}{\partial^*}A^{a*} + c.c.$$

$$\mathcal{L}_p\Big|_{A' A_{+ (+)}} =$$

$$A_S^{(-)} - A^* - A_+$$

$$= \frac{g}{2}\Big\{ \Big[\frac{1}{\partial} A_S^{(-)} T^a \partial A_+ + \frac{1}{\partial}\left(A_S^{(-)} T^a \partial A_+\right)\Big] \frac{\partial_-}{\partial} A^{*a}$$

$$- \frac{1}{\partial}\left(A_2^{(-)} T^a \partial A_+\right) \frac{\partial_-}{\partial} A^{*a} \qquad A_2^{(-)} - A^* - A_+$$

$$+ \Big[\frac{3}{2} A_S^{(-)} T^a A_{(+)} + \frac{1}{\partial} A_S^{(-)} T^a \partial A_{(+)} \qquad A_S^{(-)} - A^* - A_{(+}$$

$$+ \partial^* A_S^{(-)} T^a \frac{1}{\partial^*} A_{(+)} + \frac{1}{2}\frac{\partial^*}{\partial} A_S^{(-)} T^a \frac{\partial}{\partial^*} A_{(+)}\Big] \frac{1}{\partial} A^{*a}$$

$$A_2^{(-)} - A^* - A_{(+}$$

$$- \frac{1}{2}\partial^*\left(A_2^{(-)} T^a \frac{1}{\partial^*} A_{(+)}\right) \frac{1}{\partial} A^{*a} \Big\} + \text{c.c.}$$

Summary:

- in pert. Regge region gluon interactions contribute not only to leading asympt. ($\Rightarrow$ BFKL, $j=1$) but also to subleading asy.

- generalization of EA to terms $\mathcal{O}(s^0)$
  $\Rightarrow$ identification of non-leading gluonic reggeons near $j=0$

  (i) partner of A_+ : $A_{(+)} \sim \partial A_t + \partial^+ A_t^*$
  $\delta_{\lambda_1\lambda_2}$ physical meaning?

  (ii) A' channel: A_S and A_2
  $\lambda_1\delta_{\lambda_1\lambda_2}$ contribute to g_1^S

  (reggeiz. of scalar?) physical meaning and splitting?

  (i) and (ii) contribute to spin-flip photon str. f.

  (iii) symmetric represent. $\mathcal{B}^{r(+)}$
  physical meaning?

  $8 \otimes 8 = 1 + \cdots$

- prod. vertices $\Rightarrow$ BFKL type kernels
  - small x beh. of structure f
  - symmetries of kernels

- relation to Gell-Mann et al '60 method
  Grisaru Schnitzer