Light-Cone QCD Sum Rules & Exclusive Hadronic Amplitudes

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$\boxed{\text{LC SR}}$ = $\boxed{\begin{array}{c}\text{LC expansion}\\ \langle h| j_1(x)\, j_2(0)|0\rangle \\ x^2 \simeq 0 \quad h = \pi, \gamma, \rho, \ldots\end{array}}$ $\oplus$ $\boxed{\begin{array}{l}\text{Dispersion relations}\\ \text{Quark-hadron duality}\\ \text{Borel transform}\end{array}}$

hard exclusive processes in QCD
[Brodsky, Lepage
Chernyak, Zhitnisky
Efremov, Radyushkin]

QCD Sum rules
[Shifman, Vainshtein
Zakharov]

First applications:

$\Sigma \to p\gamma$ [Balitsky, Braun, Ko[illegible]

$\left.\begin{array}{l}\pi NN\\ NN\gamma\\ \rho\omega\pi\end{array}\right\}$ [Braun, Filyanov]

$f^+_{B\to\pi}(0)$ [Chernyak, I. Zhitnisky]

Recent applications:

$f^+_{B\to\pi}(p^2)$ [Belyaev, A.K., R.Rückl]

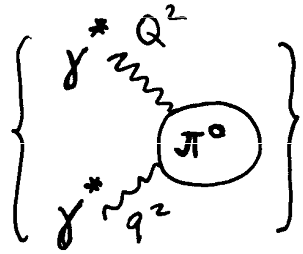
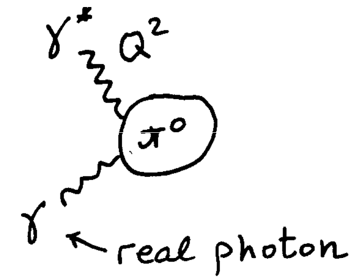
$B^*B\pi$ [Braun, Belyaev, A.K., R.Rück

$F_\pi(Q^2)$ [Braun, Halperin]

. . . .

Using duality & LC SR to estimate preasymptotic behaviour of $F_{\gamma^*\gamma\to\pi^0}(Q^2)$

recent CLEO data at $1 \leq Q^2 \lesssim 10\,\mathrm{GeV}^2$

$\lim_{q^2\to 0}$  $= F_{\gamma^*\gamma\to\pi^0}(Q^2) \equiv$ 

Main points:

1. $F_{\gamma^*\gamma^*\pi^0}(Q^2, q^2)$ is calculable using LC OPE at $Q^2, q^2 >> \Lambda^2_{QCD}$

2. Use dispersion relation in q^2-channel

3. Employ duality in ρ-channel

4. Calculate $\gamma^*\rho\pi$ form factor from LC SR

Result:

$$F_{\gamma^*\gamma\to\pi^0}(Q^2) = \lim_{q^2\to 0} F_{\gamma^*\gamma^*\to\pi^0}(Q^2, q^2)$$

* limit well defined
* correct asymptotic behaviour ($Q^2\to\infty$)
* preasymptotics in terms of LC OPE
* prediction for $\gamma^*\rho\to\pi^0$

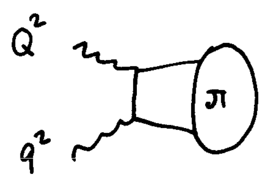
(1) $$F_{\gamma^*\gamma^*\pi^0}(Q^2, q^2)\,\varepsilon_{\mu\nu\alpha\beta}\,q_1^\alpha q_2^\beta =$$

$$= i\int d^4x\, e^{-iq_1x}\langle \pi^0(q_1+q_2)\,|\,T\{J_\mu(x), J_\nu(0)\}\,|0\rangle$$

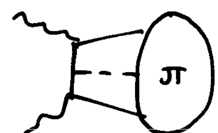
$$J_\mu = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d \qquad q_1^2 = -Q^2, \quad q_2^2 = -q^2$$

LC OPE:

$F_{\gamma^*\gamma^*\pi^0}$ =

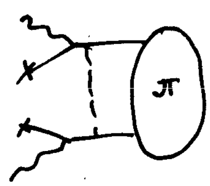


twist 2, 4

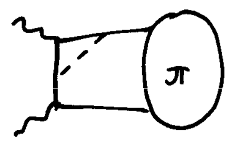


twist 4

$q^2 \to 0$ is impossible!



$\alpha_s\langle\bar{q}q\rangle$ tw 3



$O(\alpha_s)$ tw 2

$$\langle \pi(p)\,|\,\bar{u}(x)\gamma_\mu\gamma_5 u(0)\,|0\rangle$$

$$= -\frac{i p_\mu f_\pi}{\sqrt{2}}\int_0^1 du\, e^{iupx}\left[\varphi_\pi(u) + x^2 g_1(u)\right] +$$

$$+ \frac{f_\pi}{\sqrt{2}}\left(x_\mu - \frac{x^2 p_\mu}{(px)}\right)\int_0^1 du\, e^{iupx} g_2(u) + O(x^4)$$

tw2 $\quad \varphi_\pi(u) = 6u(1-u)\left[1 + \sum_{i=2,4,..} a_i C_i^{3/2}(2u-1)\right]$

tw4 $\quad g_1(u) = \frac{5}{2}\delta^2(1-u)^2u^2 + \dots \qquad g_2(u) = \frac{10}{3}(1-u)u(2u-1)$

①′ The explicit answer: [Brodsky-Lepage]

$$F^{OPE}_{\gamma^*\gamma^*\pi}(Q^2,q^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1 du\,\frac{\varphi_\pi(u)}{q^2u+Q^2(1-u)}$$

$$+\frac{4f_\pi}{\sqrt{2}}\int_0^1 du\,\frac{\{g_1(u)+G_2(u)\}}{[q^2u+Q^2(1-u)]^2} + O(G^{\mu\nu}) + O(\alpha_s)\ \ldots$$

[Braaten]

$$G_2(u) = -\int_0^u g_2(v)\,dv$$

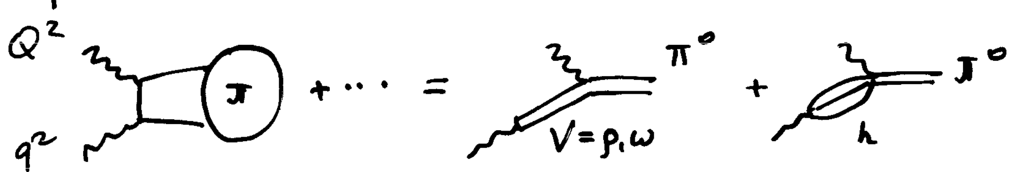
in progress

$$+\frac{\alpha_s\langle\bar q q\rangle}{q^2Q^2}\int\frac{\varphi^{tw3}(u)}{q^2u+Q^2(1-u)}$$

[Gorsky, Braun, Filyanov]

valid at $Q^2, q^2 \gg \Lambda^2_{QCD}$, no $q^2\to 0$ limit

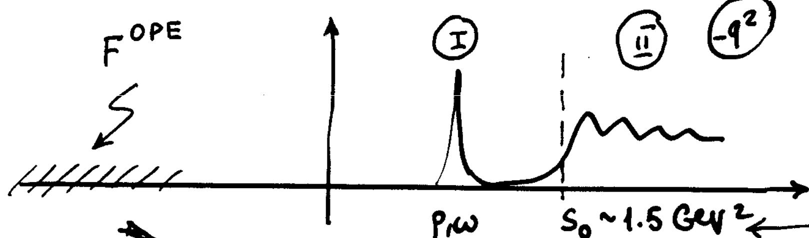
② Dispersion relation: $(-q^2)$-plane Q^2 fixed



$$F_{\gamma^*\gamma^*\pi}(Q^2,q^2) = \sum_{V=\rho,\omega}\underbrace{\frac{f_V f_{\gamma^*V\pi}(Q^2)}{m_\rho^2+q^2}}_{\mathrm{I}} + \underbrace{\int_{s_0}^{\infty}\frac{ds\cdot\rho^h(Q^2,s)}{s+q^2}}_{\mathrm{II}}$$

valid at $q^2\to 0$

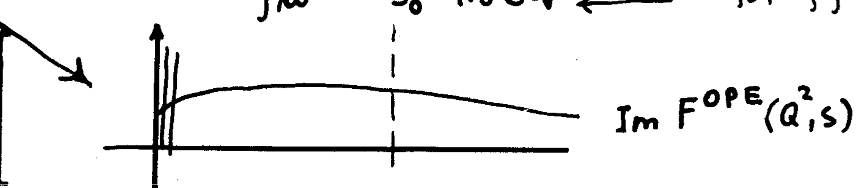
③ Use of quark-hadron duality in ρ-channel



SVZ SR for f_ρ

II

$$\rho^h(Q^2,s) = \frac{1}{\pi}\mathrm{Im}\,F^{OPE}(Q^2,s)\cdot\theta(s-s_0)$$



④ The LC Sum rule for $\gamma^* \rho \pi$ form factor

$-q^2$ spacelike

$$F_{\gamma^*\gamma^*\pi}(Q^2,q^2) \simeq F^{OPE}_{\gamma^*\gamma^*\pi}(Q^2,q^2)$$

$$\sum_V \frac{f_V f_{\gamma^* V \pi}(Q^2)}{m_\rho^2+q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\mathrm{Im}\, F^{OPE}(Q^2,s)\,ds}{s+q^2} = \frac{1}{\pi}\int_0^{\infty} \frac{\mathrm{Im}\, F^{OPE}(Q^2,s)\,ds}{s+q^2}$$

After subtraction of higher states and Borel:

$$\text{Ⓘ} \Rightarrow \boxed{\sum_V f_V f_{\gamma^* V \pi}(Q^2) = \frac{1}{\pi}\int_0^{s_0} ds\, \mathrm{Im}\, F^{OPE}(Q^2,s)\, \cancel{M^2}\, e^{-\frac{s}{M^2}+\frac{m_\rho^2}{M^2}}}$$

twist 2: $$\frac{1}{\pi}\mathrm{Im}\, F^{OPE}(Q^2,s) = \frac{\sqrt{2}}{3} f_\pi \frac{\varphi_\pi(u(s))}{s+Q^2}$$

$$u = Q^2/s+Q^2$$

Substitute Ⓘ & Ⓘ︎Ⓘ in the disp. relation

$$F_{\gamma^*\gamma^*\pi}(Q^2,q^2) = \frac{\frac{1}{\pi}\int_0^{s_0} ds\, \mathrm{Im}\, F^{OPE}(Q^2,s)\, e^{-\frac{s}{M^2}+\frac{m_\rho^2}{M^2}}}{m_\rho^2+q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} \frac{ds \cdot \mathrm{Im}\, F^{OPE}(Q^2,s)}{s+q^2}$$

put $q^2 \to 0$!

$$\boxed{\begin{aligned} Q^2 F(Q^2) &= \frac{\sqrt{2} f_\pi}{3} \left\{ I_1(Q^2) + I_2(Q^2) \right\} + \cdots \\ I_1(Q^2) &= \int_0^{Q^2/s_0+Q^2} du \left\{ \frac{\varphi_\pi(u)}{1-u} - 50 \frac{\delta^2}{Q^2} u^2 (2u-1) \right\} \\ I_2(Q^2) &= \frac{Q^2}{m_\rho^2} \int_{Q^2/s_0+Q^2}^{1} du \left\{ \frac{\varphi_\pi(u)}{u} - 50 \frac{\delta^2}{Q^2} (1-u) u (2u-1) \right\} * \\ &\quad * \exp\left\{ -\frac{(1-u)Q^2}{u M^2} + \frac{m_\rho^2}{M^2} \right\} \end{aligned}}$$

* parameters: s_0, M^2 fixed from QCD sum rule
for f_ρ : $s_0 = 1.5\ \mathrm{GeV}^2$ $M^2 = 0.6 \div 0.8\ \mathrm{GeV}^2$

$\langle \pi | \bar{q} G q | 0 \rangle \to \delta^2 = 0.2\ \mathrm{GeV}^2$ [Novikov, Shifman, Vainshtein, Zakharov]

** $O(\alpha_s)$ corrections are very important, demand separate analysis (Im-part, ln's, scale choice etc.)

$\boxed{\text{lower limit}}$

*** higher twist corrections $\sim -40\%$ at $Q^2 = 1\ \mathrm{GeV}^2$
-5% at $Q^2 = 10\ \mathrm{GeV}^2$

$O(G_{\mu\nu})$ and $O(\alpha_s \langle \bar\psi \psi \rangle)$ are presumably small and have diff. signs

	$Q^2 = 10$	$1\ \mathrm{GeV}^2$
**** I_1 dominates at large Q^2	80%	40%
(2) (small)	20%	60%

***** I_2 can be measured in $\gamma^* \pi \to \rho$
not valid at $Q^2 \to \infty$? (Feynman mechanism)

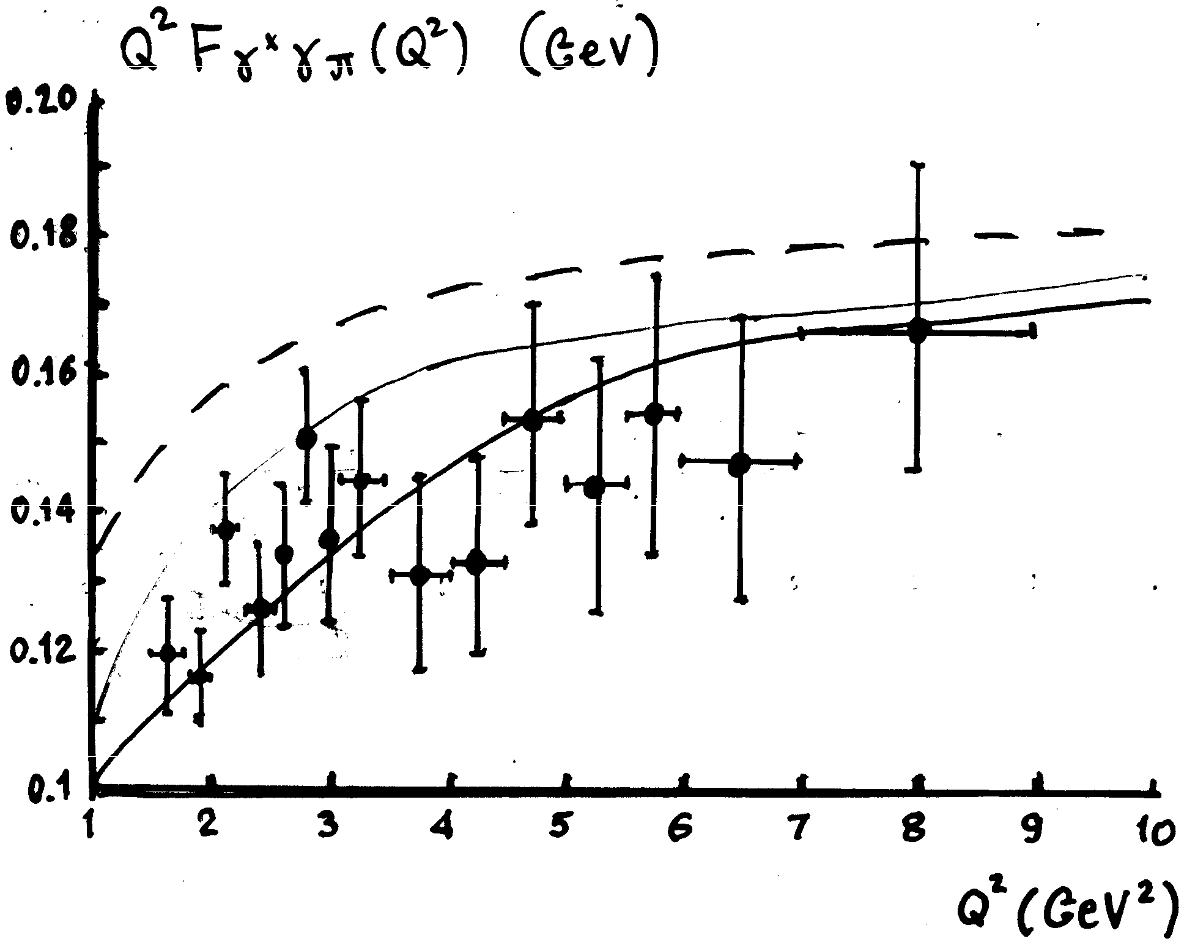
$f_\pi = 132$ MeV

$$F_{\gamma^*\gamma\pi}(Q^2) = \frac{\sqrt{2}}{4\pi^2 f_\pi}\,\frac{1}{\left(1 + Q^2/4\pi^2 f_\pi\right)}$$

[CLEO data] [Brodsky-Lepage interpolation]

twist 2

twist 2 $\oplus$ twist 4 $\Big\}$ $\varphi_\pi^{as} = 6u(1-u)$



Sensitivity to $\varphi_\pi(u)$:

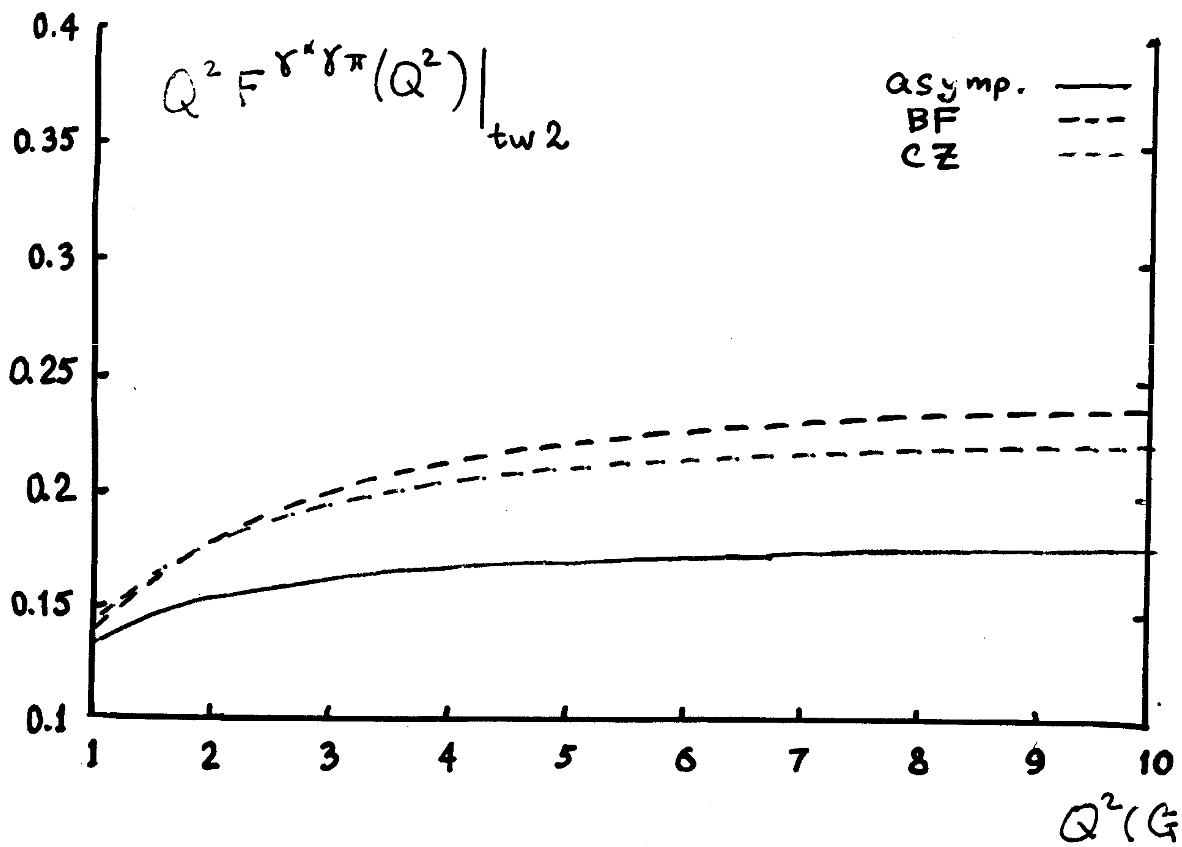
asympt. $\varphi_\pi(u) = 6u(1-u)$

CZ $\begin{cases} \varphi_\pi(u) = 6u(1-u)\left[1 + a_2 C_2^{3/2}(2u-1)\right] \\ a_2 = 2/3 \quad (\mu = 0.5\,\mathrm{GeV}) \end{cases}$

LO evolution of a_2, a_4
$\mu = \sqrt{Q^2}$

BF $\varphi_\pi(u) = 6u(1-u)\left[1 + a_2 C_2^{3/2}(2u-1) + a_4 C_4^{3/2}(2u-1)\right]$

$a_2 = 2/3\,,\ a_4 = 0.43 \quad (\mu = 0.5\,\mathrm{GeV})$



Conclusions

* Use of duality & LC QCD Sum rules allows to estimate $F_{\gamma^*\gamma\pi}(Q^2)$ in "preasymptotic" region $1\ \mathrm{GeV}^2 \lesssim Q^2 \lesssim 10\ \mathrm{GeV}^2$ entirely in terms of pion wave function on the light-cone. [Radyushkin Ruskov 3-point]

* To improve the estimate quantitatively
  (a) calculate $O(G_{\mu\nu})$ $O(\alpha_s\langle\bar\psi\psi\rangle)$
  (b) include $O(\alpha_s)$

* CLEO data $\Rightarrow$ φ_π^{asympt} preferred

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