Parton-hadron duality in QCD sum rules (1)
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(1) Introduction

QCD sum rules - known from 1979 (SVZ), since then - thousand papers.

<u>First</u> - 2point sum rules. Basic idea-

$$\Pi(Q^2) = \int \frac{Im\,\Pi(s)}{s+Q^2} = \frac{\beta^2}{Q^2+m_n^2} + \int_{s_0}^{\infty} \frac{Im\,\Pi^{pert}(s)}{s+Q^2}$$

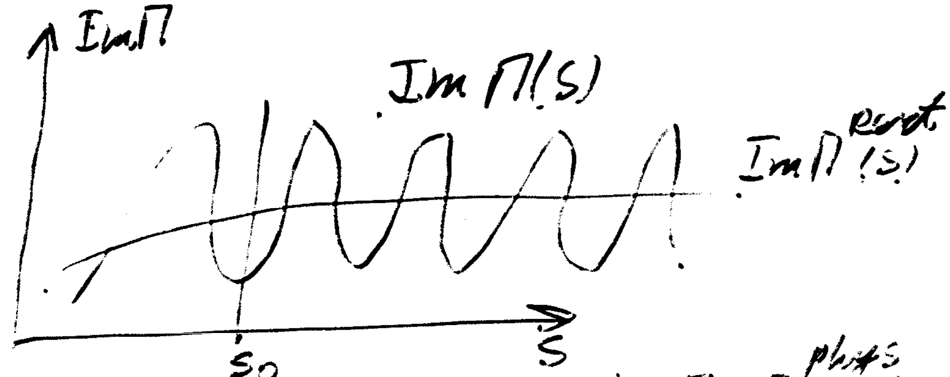
$$\parallel$$

$$\sum \frac{a_n}{Q^{2n}}$$

or

$$\sum \frac{a_n}{M^{2n}} = \beta^2 e^{-\frac{m_n^2}{M^2}} + \int_{s_0}^{\infty} Im\,\Pi^{pert} e^{-\frac{s}{M^2}}$$

The approach is based on <u>duality</u>:



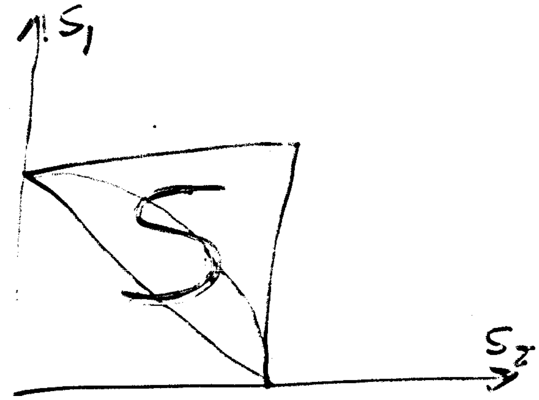
in eq. above there must $Im\,\Pi^{phys}$ instead of "pert", which is, of course, unknown.

i.e. we assume: average resonance curve is approximated by smooth O.P.E. curve, calculated in Euclid (even more suprising).

<u>Later</u> – 3-point sum rules.

$$\Pi(Q^2) = \int \mathrm{Im}\,\Pi(s_1, s_2)\, e^{-\left(\frac{s_1}{M_1^2}+\frac{s_2}{M_2^2}\right)} =$$

$$= f_1^2 g\, e^{-\frac{m_1^2}{M_1^2}+\frac{m_2^2}{M_2^2}} + \int_S \mathrm{Im}\,\Pi^{pert}(s_1, s_2)\, e^{-\left(\frac{s_1}{M_1^2}+\frac{s_2}{M_2^2}\right)}$$



(Here for simplicity we assume diagonal transitions, $f = i$)

g – unknown coupling constant.

Once again basic idea – <u>duality</u>

$\mathrm{Im}\,\Pi^{pert}$ approximates $\mathrm{Im}\,\Pi^{exact}$

smooth
Euclidean

oscillating
Minkowski

In this case, even if local duality fails one can consider generalised duality – integrate in direction orthogonal to diagonal.

aim of the talk: study the assumption of duality in 3-point sum rules. ③

Unfortunately, it is yet not possible ye to study duality in a model independent way in QCD (see, however, Blok, Shifman, Zhang; Blok hep-ph 9707287), in preparation

So our goal will be to study duality in quantum mechanical analogues of the 2- and 3-point QCD sum rules.

We shall consider 3 cases:

harmonic oscillator $V = \frac{\omega r^2}{2}$

linear oscillator $V = \alpha r$

linear + coulomb $V = \alpha r - \beta/r$

We shall build 2-point sum rules (NSVZ w DK)

3-point sum rules for operators

$O_1 = r^2$, $O_2 = \partial^2$, $O_3 = r$

(for harmonic oscillator sum examples were considered by Bigi, Shifman, Uraltsev hep-ph/970324)

We shall see: in 2-point functions duality always work

in 3-point - it may fail 100%, and it is hard to control.
(small continuum, good plato, wrong answer)

The reason for this investigation - a stream of publications on QCD sum rules in HQET.

Example - recent controversy over λ_1

$$\lambda_1 \equiv \langle B | \frac{-\vec{D}^2}{2m_B} | B \rangle$$

Ball, Braun $\quad \lambda_1 = 0.5 \pm 0.1 \ \text{GeV}^2$

Neubert $\quad \lambda_1 \sim 0.1 \ \text{GeV}^2$

M. Wise, Gremm, Kapustin, Ligetti ; $\lambda_1 \sim 0.2 \div 0.3 \ \text{GeV}^2$
V. Chernyak
(from experimental CLEO data)

② 2-point sum rules

QM analogue of polarisation operator is a propagator

$$S(0,0;0,T) \equiv S(0,T) = \sum |\psi_n(0)|^2 e^{-E_n T}$$

$\|$

asymptotics for small T

$$S^{har}(0,T) = \left(\frac{1}{4\pi T}\right)^{3/2}\left(1 - \frac{1}{4}T^2 + \dots\right) \qquad (\omega \equiv 1)$$

$$S^{L}(0,T) = \frac{1}{(4\pi T)^{3/2}}\left(1 - \frac{\sqrt{\pi}}{2}T^{3/2} + \frac{5}{12}T^3 + \dots\right) \qquad (\lambda = 1)$$

$$S^{qc} = \frac{1}{(4\pi T)^{3/2}}\Big(1 + \varepsilon_0\sqrt{\pi}\,T^{3/2} + \varepsilon_0^2\frac{\pi^2}{5}T - \frac{\varepsilon_0 3}{2}T^2$$
$$- \frac{\pi}{2}^2 T^{3/2} + 5/12\, T^3 + \dots\Big)$$

$$\varepsilon(x) = \frac{8\pi(4m^2/\lambda)^{1/3}}{27\ln(5 + 8(2m\lambda)^{1/3}x)}$$

$\varepsilon_0 \equiv \varepsilon(x_0)$, $x_0 = 1.188$, $\alpha_s \sim 0.3$

$$\alpha_s(r) = \frac{2\pi}{9\ln(5 + 8/r)}, \quad x = (2m\lambda)^{1/3} r \qquad V = \lambda r - \frac{4}{3}\frac{\alpha_s}{r}$$

(to maximally immitate NRQM)

Results: 1) $E^{har} = 3/2$, $E^{s.r.} \sim 3/2$, $E_c \approx 2.6$

2) $E^{L} = 2.338$, $E^{s.r.} = 2.35 \pm 0.05$ $\quad E_c \sim 3.1$

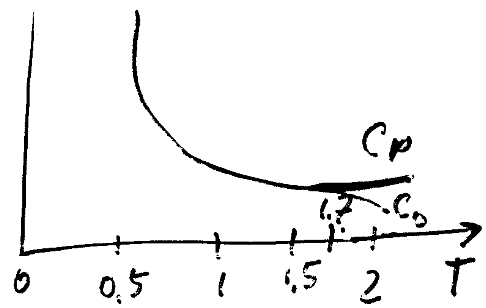
3) $E^{qc} = 1.83$, $E^{s.r.} = 1.9 \pm 0.05$ $\quad E_c \sim 3$

In addition one may consider curves

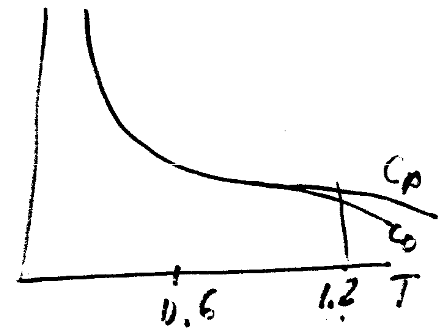
$$C_0(T) \equiv \frac{S^{ex}(0,T) - |\psi_0(0)|^2 e^{-E_0 T}}{|\psi_0(0)|^2 e^{-E_0 T}}$$

$$C_p(T) \quad \frac{\int_{E_c}^{\infty} dE\, \sigma_p(E)\, e^{-ET}}{|\psi_0(0)|^2 e^{-E_0 T}}$$

In the case of duality these curves must well match each other.



Harmonic oscillator



Linear oscillator

conclusion: duality always works.

③ 3-point sum rules

analogue (Blok, Shifman)

$$S(\tau_1, \tau_2) = \int d^3r\, K(0, \tau_1+\tau_2, \vec r, \tau_1)\,\hat O_i(\vec r)\cdot$$
$$\cdot K(\vec r, \tau_1; 0, 0) =$$
$$= \sum_{l,n} \psi_l(0)\psi_n^{\circ}(0)\,\langle l | O_i | n\rangle \cdot$$
$$\cdot e^{-E_n\tau_1} e^{-E_l\tau_2}$$

The sum rule is:

$$S(\tau_1, \tau_2) = |\psi_0(0)|^2 \langle 0|\hat O|0\rangle\, e^{-E_0(\tau_1+\tau_2)} +$$
$$+ \int_{s_0}^{\infty} ds_1\, ds_2\, \sigma_h(s_1, s_2)\, e^{-s_1\tau_1 - s_2\tau_2}$$

$$\updownarrow$$

$$\sigma_p(s_1, s_2)$$

We take the integrated sum rules over $\frac{s_1 - s_2}{2} = A$
consider symmetric case $\tau_1 = \tau_2 = T/2$

we get

$$S(T/2, T/2) = |\psi_0(0)|^2 \langle 0|O|0\rangle\, e^{-E_0 T} + \int_{s_0}^{\infty} \tilde\sigma(s)\, e^{-sT}$$

$S(T/2, T/2)$ is calculated for short T using explicit equations for K for free particle:

$$G_0(\vec r, t, \vec r', t') = \frac{1}{(4\pi(T-t'))^{3/2}} \exp\left(-\frac{(\vec r - \vec r')^2}{4(T-t')}\right)$$

consider now different examples.

A) Harmonic oscillator (see also Bigi, Uraltsev, Shifman)

$O_1 = r^2/6 \qquad O_2 = -\partial^2$

$$S_1(T/2, T/2) = \frac{1}{32\pi^{3/2}} \frac{1}{T^{1/2}} \left(1 - \frac{1}{3}T^2 + \frac{11}{540}T^4 \dots\right.$$

$$S_2(T/2, T/2) = \frac{3/2}{(4\pi)^{3/2}} \frac{1}{T^{5/2}} \left(1 - \frac{1}{6}T^2 + \frac{5}{288}T^4 + \dots\right.$$

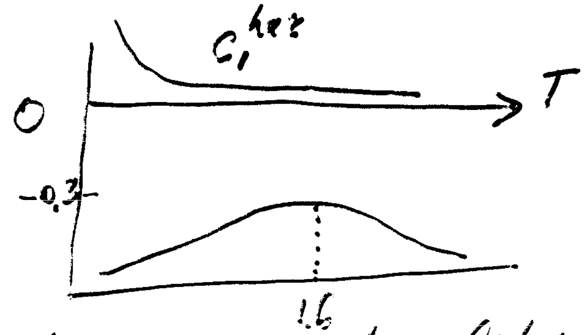
We first check duality by introducing new functions

$$C_i(T) = S_i(T/2, T/2) - |\psi_0(0)|^2 e^{-E_0 T} \langle 0|O_i|0\rangle_{exact}$$

$$C_{pi}(T) = \int_{E_c}^{\infty} dE\, \sigma_p(E)\, e^{-ET}$$

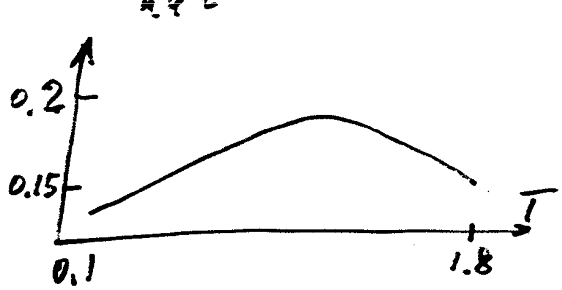
partonic spectral density (smooth, $\sim \sqrt{E}$ or $E^{3/2}$)

Then for duality to hold, $C_i(T) \sim C_{pi}(T)$



$E_c = 2.5$

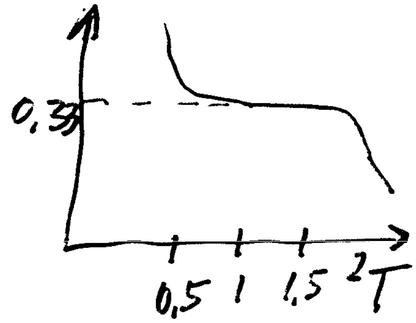
We see - no duality window. But if one builds s.r.: $\langle O_1\rangle_{har}$



$\langle O_1\rangle_{exact} = 1/3$

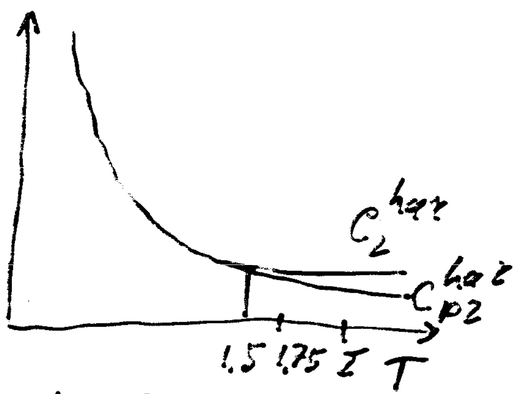
$E_c = 2.5$

s.r. "works" with plato & small continuum.



sum rule for $N=3$
- with 3r. taken explicitly
- right answer.

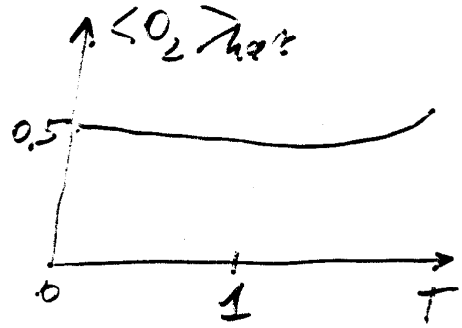
O_2 - Duality works



$O_2 = -\partial^2$

$E_c = 2$

- duality works



$\langle O_2 \rangle_{exact} = \frac{1}{2}$

sum rules work.
(but - big continuum, more than 50%.)

note: $-\partial^2$ and r^2 are partners
by means of the virial theorem
but sum rule and duality work for $-\partial^2$
(analogue of Ball, Braun) and do not for r^2
(Analogue of Neubert)

Although for O_2 - big continuum,
for O_1 - small.
note also: suppression is by $e^{-(E_0+E_1)T/2}$ $5^{3/2}$ $\sim 5/2$

$\Rightarrow E_c \sim \frac{E_0+E_1}{2}$ 2

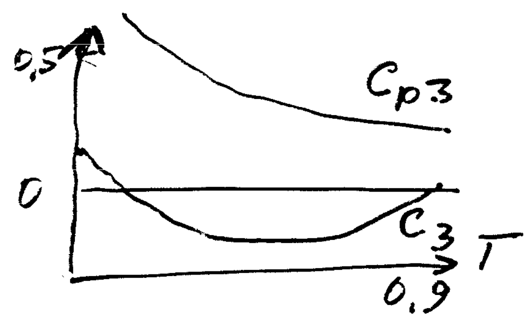
$E_{c_{2,5}}^3 < E_{c_2} \sim 2$

B) consider now
linear oscillator.
O_2, O_3 - virial partners

$$S_1 = \frac{1}{32\pi^{3/2}} \frac{1}{T^{1/2}} \left(1 - \left(\sqrt{\pi} - \frac{4}{3\sqrt{\pi}}\right) T^{3/2} + \left(\frac{277}{128} - \frac{16\sqrt{2}}{15}\right) T^3 + \ldots\right)$$

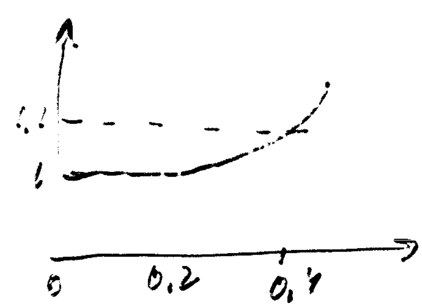
$$S_2 = \frac{3}{16\pi^{3/2}} \frac{1}{T^{5/2}} \left(1 - \frac{4}{3\sqrt{\pi}} T^{3/2} + \left(\frac{52\sqrt{2}}{45} - \text{[illegible]}\right) T^3 + \ldots\right)$$

$$S_3 = \frac{1}{4\pi^2} \frac{1}{T} \left(1 - \left(\sqrt{2\pi} - \frac{7\sqrt{\pi}}{8}\right) T^{3/2} + \left(\frac{35}{48} - \frac{5\pi}{64}\right) T^3 + \ldots\right)$$



$E_c = 3.3$

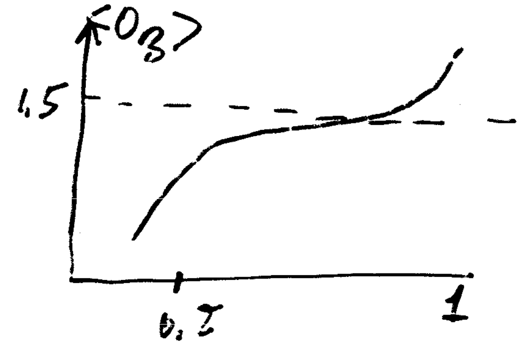
duality picture -
- no way to have duality



$\langle O_3 \rangle_{exact} = 1.559$

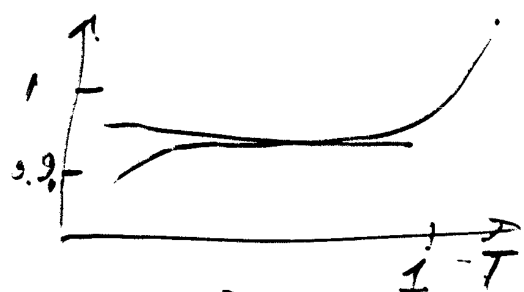
sum rule

for $N=11$



physical reason the same: nondiagonal transitions are big and <u>sign alternating</u>.

On the other hand, for O_2 they work well (but big continuum)



$\langle O_2 \rangle \sim 1 \pm 0.05$

$E_c = 2.8$

$\langle O_2 \rangle_{exact} = 0.972$

$(E_0^1 \approx 2.3 \quad E_c^2 = 3.4$

$E_1^1 \approx 4.1$

O_1 – like O_3.

c) linear + coulomb
most involved calculations
but physical situation the same

4) Implications for HQET (possible) ⑫

$\langle -\partial^2 \rangle$ - analogue for λ_1

$\langle r \rangle$ (or $\langle r^2 \rangle$) - analogues of Neubert virial theorem

we see : 1) for virial partners sum rules do not work - duality breaking, answers underestimated.

2) for ∂^2 big continuum (like in Ball, Braun) + $\omega_c^3 < \omega_c^2$

redoing Ball - Braun calculations

with $\omega_c^3 = 0.7 - 0.9$ GeV (instead of $\omega_c^3 \sim 1.2$ GeV $= \omega_c^2$)

one get

$$\lambda_1 = 0.3 \pm 0.15 \text{ GeV}^2$$

- in agreement with Wise et al and Chernyak

⑥ Conclusions

1) We see that duality in 3-point sum rules may fail completely due to sign alternatig transitions (with small continuum, plato quite good)
   in fact, very small continuum dependence always seems to be suspicious

2) w_c^3 may be different (smaller) than w_c^2 if duality works

3) sum rules for virial partners may work quite differently

4) the situation seems to be worse for s/rules with spectral density near diagonal

5) Implications for QCD may be better
   a) in QCD situation may be suppressed n/d transitions may be suppressed stronger
   b) but one must be very careful first to check duality if one is interested in quantative answers
   c) our results support
   $$\lambda_1 \sim 0.3 \pm 0.5 \text{ GeV}^2.$$

6) future work must be done, in particular, on relativistic models and on real QCD.