# DEEP INELASTIC SCATTERING

# AND

# THE TWIST - 2

# PION WAVE FUNCTION

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# OUTLINE

I. DEEP INELASTIC SCATTERING AND THE PION WAVE FUNCTION IN THE LIGHT-CONE QCD SUM RULE APPROACH.
V. M. BELYAEV, M.B.J. PHYS. REV. D 52 (1997) 1481

II. CONSTRAINED ANALYSIS OF PION WAVE FUNCTION

- CHERNYAK - ZHITNITSKY (1984): MOMENT ANALYSIS $M_2$
- RADYUSHKIN - RUSTOV (1996): $\gamma\gamma^* \to \pi$
- BRAUN - FILYANOV (1989): $g_{\pi NN}$
- BELYAEV - MBJ (1997): DEEP-INELASTIC SCATTERING

V. M. BELYAEV, M.B.J "TWIST-2 PION LIGHT-CONE WAVE FUNCTION", LOS ALAMOS PREPRINT LA-UR-97-1119; hep-ph/9702207

III. COMPARISON TO LIGHT-FRONT QUARK MODEL
V. M. BELYAEV, M.B.J "PION LIGHT-CONE WAVE FUNCTIONS AND LIGHT-FRONT QUARK MODEL" LOS ALAMOS PREPRINT LA-UR-97-2874; hep-ph/

# CONVENTIONAL QCD SUM RULES

SHIFMAN, VAINSHTEIN, ZAKAROV (1979)

CORRELATOR: $\Pi(p^2) = \int e^{ip\cdot x} d^4x \langle 0 | T\{J_1(x) J_2(0)\} | 0 \rangle$

1. PHENOMENOLOGICAL EVALUATION (RHS):

$$\Pi(p^2) = \Pi^h(p^2) = \frac{1}{\pi} \int \frac{\rho^h(s)\, ds}{p^2 + s} + \text{SUBTRACTIONS}$$

$$\rho^h(s) = f\, \delta(p^2 - M_R^2) + \theta(s - s_0)\, \mathrm{Im}\, \Pi^{CONT}(s)$$

2. THEORETICAL EVALUATION (LHS):

$$\Pi(p^2) = \Pi^{QCD}(p^2) = A_1 p^2 \log p^2 + A_2 \log p^2 + \frac{A_3}{p^2} + \cdots$$

WILSON OPE, LOCAL CONDENSATES

3. BOREL TRANSFORM:

$$\mathcal{B}\{\Pi^h(p^2)\} = \mathcal{B}\{\Pi^{QCD}(p^2)\}$$

OR

$$\boxed{\frac{1}{\pi} \int \rho^h(s)\, e^{-s/M^2} = A_1 M^4 + A_2 M^2 + A_3 + \cdots}$$

QCD SUM RULE

SUCCESSFUL QUANTITATIVE EXPLANATION OF HADRON PROPERTIES (MASSES, COUPLINGS) IN QCD.

# DEEP INELASTIC SCATTERING IN LIGHT-CONE SUM RULE APPROACH

## CONVENTIONAL SUM RULE APPROACH:

- WILSON O.P.E. OF T.O.P. OF CURRENTS AT SMALL DISTANC
- LOCAL CONDENSATES
- ORDERED WITH INCREASING DIMENSION

  SHIFMAN, VAINSHTEIN, ZAKAROV (1979)

  APPLICATION TO $q(x)$: IOFFE (1985, 1986), BELYAEV, IOFFE (198

## LIGHT-CONE QCD SUM RULES

FOLLOWS CONVENTIONAL APPROACH EXCEPT

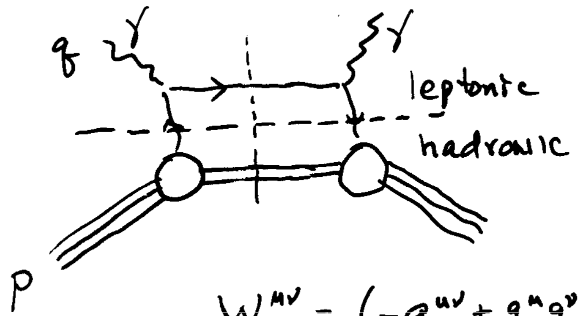
- EXPRESSED IN TERMS OF NONLOCAL CONDENSATES NEAR LIGHT CONE $k^2=0$
- MATRIX ELEMENTS OF NONLOCAL CONDENSATES DEFINE HADRON WAVE FUNCTIONS
- ORDERED WITH INCREASING TWIST $\sim\left(\frac{\Lambda_{QCD}}{M^2}\right)^{t-2}$

  BALITSKY, BRAUN, KOLESNICHENKO (1986, 1989)

# DEEP INELASTIC SCATTERING

RELATIONSHIP OF QUARK DISTRIBUTION FUNCTION $q(x)$ TO DEEP INELASTIC SCATTERING

HADRONIC TENSOR $W^{\mu\nu} = \sum_X \langle ps|J^\mu|X\rangle (2\pi)^4 \delta^{(4)}(p+q-p_X)\langle X|J^\nu|ps\rangle$




$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1 + \left(p^\mu - \frac{\nu}{q^2}q^\mu\right)\left(p^\nu - \frac{\nu}{q^2}q^\nu\right)\frac{M_T}{\nu}F_2$$

Bjorken limit $-q^2 = Q^2 \to \infty$, $\nu \to \infty$, $x = \frac{Q^2}{2M_T\nu}$ = fixed

Parton model $F_2(x) = x\,q(x)$

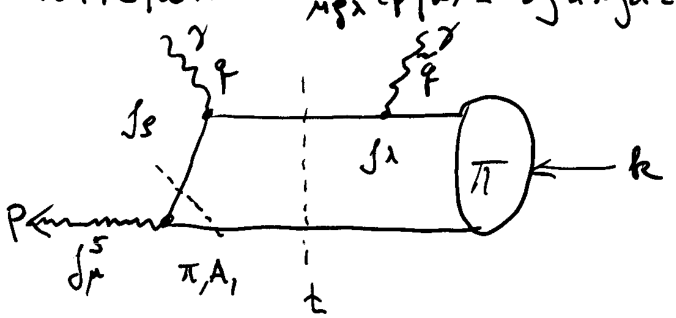
SUM RULE EXPLOITS CONNECTION TO VIRTUAL COMPTON AMPLITUDE $T^{\mu\nu}$

$$2\pi W^{\mu\nu} = \mathrm{Im}\, T^{\mu\nu}$$

$$T^{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle ps|T\{J^\mu(x)J^\nu(0)\}|ps\rangle$$

# QCD LIGHT-CONE SUM RULE

Correlator: $T_{\mu\rho\lambda}(pqk) = -i\int d^4x \int d^4z\, e^{ip\cdot x} e^{iq\cdot z} \langle 0| T\{ j^5_\mu(x)\, j^{d}_\rho(z)\, j^{d}_\lambda(0)\}|\pi(k)\rangle$




$$\left\{\begin{array}{l} k^2=0\,,\quad q^2=\tilde{q}^2=-Q^2\,,\quad t=(p-k)^2=0 \\ s=(p+q)^2\,,\quad (2k,\,p+q) = s+Q^2 \\ 2p\cdot k = p^2 \qquad \boxed{t_{SING} = -\dfrac{p^2 x}{1-x}} \end{array}\right.$$

RHS: RETAIN PHYSICAL $\pi$ INTERMEDIATE STATE

$$\mathrm{Im}\, T_{\mu\rho\lambda} = \frac{1}{2i}\left[T_{\mu\rho\lambda}(p^2, q^2, s+i\varepsilon) - T_{\mu\rho\lambda}(p^2, q^2, s-i\varepsilon)\right]$$

$$\to \frac{p_\mu f_\pi}{p^2}\, \underbrace{\mathrm{Im}\left\{ i\int d^4z \langle \pi(p)| T\{ j_\rho(z)\, j_\lambda(0)\}|\pi(k)\rangle e^{iq\cdot z}\right.}_{2\pi W_{\rho\lambda}}$$

$\Rightarrow \quad \mathrm{Im}\, T_{\mu\rho\lambda}: \quad p_\mu p_\rho p_\lambda \cdot \dfrac{4\pi x^2}{q^2}\, t(p^2, x)$

BOREL TRANSFORM $\Rightarrow$ $\boxed{t_{RHS}(M^2, x) = -g(x) + \int ds\, \rho(s,x)\, e^{-s/M^2}}$

LHS: RETAIN TWIST-2: PION $(q\bar q)$ WAVE FUNCTION $f_2$

TWIST-4: 2 PARTICLE $(q\bar q)$ WF $\quad f_4$

3 PARTICLE $(q\bar q g)$ WF $\quad g_4$

BOREL TRANSFORM $\Longrightarrow$ $\boxed{t_{LHS}(M^2, x) = -f_2(x) - \frac{1}{M^2}\left(f_4(x) + g_4(x)\right)}$

# TWIST-2 CONTRIBUTION

$$\langle 0 | \bar{u}(x) \gamma_\mu \gamma_5 d(0) | \pi(k) \rangle \cong i k_\mu f_\pi \int_0^1 dv \, e^{-ik v \cdot x} \varphi_\pi(v)$$

$\varphi_\pi(v) \equiv$ TWIST-2 PION WAVE FUNCTION

FIND, AFTER STRAIGHTFORWARD CALCULATION

$$f_2(x) = \varphi_\pi(x)$$

SUM RULE $\Rightarrow$ $\boxed{q(x) = \varphi_\pi(x) + \text{TWIST-4}}$.

## REMARKS

1. $q(x) = \varphi_\pi(x) =$ AMPLITUDE IN QCD L-C SUM RULE
2. $q(x) = \varphi_\pi^2(x) =$ PROBABILITY IN QUARK-PARTON MODEL
3. (1.) + (2.) $\Rightarrow$ METHOD WORKS WHERE $\varphi_\pi(x) \simeq \varphi_\pi^2(x)$, $\boxed{\varphi_\pi(x) \simeq 1}$
4. MUST CALCULATE HIGHER TWIST CORRECTIONS.

   SHOW HIGHER TWIST $\simeq 0$ WHERE $q(x) \simeq 1$.

# ILLUSTRIATIVE CALCULATIONS

* WAVE FUNCTIONS WE USE

• CHOOSE ASYMPTOTIC PARAMETRIZATION

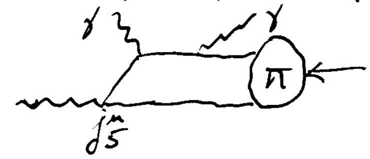
$a_2 = a_4 = 0$ TWIST-2

$\varepsilon = 0$ TWIST-4

* RESULTS

# TWO-PARTICLE LIGHT-CONE WAVE FUNCTIONS

e.g. BRAUN - FILYANOV (1990)




$$< 0|\bar{u}(x)\gamma_\mu\gamma_5 d(0)|\pi(k) >= ik_\mu f_\pi \int_0^1 dv e^{-i(kx)v}(\varphi_\pi(v) + x^2 g_1(v) + O(x^4))$$
$$+f_\pi\left(x_\mu - \frac{x^2}{kx}k_\mu\right)\int_0^1 du e^{-i(kx)v}(g_2(v) + O(x^2)), (13)$$

## TWIST - 2

$$\varphi_\pi(u) = 6u(1-u)\{1 + a_2 C_2^{3/2}(2u-1) + a_4 C_4^{3/2}(2u-1)$$
$$+a_6 C_6^{3/2}(2u-1) + \ldots\}.$$

Here $C_n^{3/2}$ are the Gegenbauer polynomials;

$$C_2^{3/2}(x) = \tfrac{3}{2}(5x^2-1),\ C_4^{3/2}(x) = \ 15/8(21x^4 - 14x^2 + 1).$$

## TWIST - 4

$$g_1(u) = \frac{5}{2}\delta^2\bar{u}^2u^2 + \frac{1}{2}\epsilon\delta^2[\bar{u}u(2+13\bar{u}u) + 10u^3(2-3u+\frac{6}{5}u^2)ln(u)$$
$$+10\bar{u}^3(2-3\bar{u}+\frac{6}{5}\bar{u}^2)ln(\bar{u})],$$
$$g_2(u) = \frac{10}{3}\delta^2\bar{u}u(u-\bar{u}),$$
$$G_2(u) = \frac{5}{3}\delta^2\bar{u}^2u^2,$$

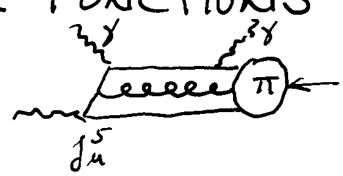
$\bar{u} = 1-u$ , $G_2(u) = -\int_0^u du' g_2(u')$

$< \pi|g_s\bar{d}\tilde{G}_{\alpha\mu}\gamma_\alpha u|0 >= i\delta^2 f_\pi q_\mu$ $\quad \delta^2 = 0.2\ GeV^2$ @ $\mu = 1\ GeV$

| |
|---|
| $\epsilon$, $a_2$, $a_4$, and $a_6$ QUANTIFY DEVIATION FROM ASYMPTOTIC FORM |

# THREE-PARTICLE LIGHT-CONE WAVE FUNCTIONS (TWIST-4)

eg. Braun Filyanov (1990)




$$
\begin{aligned}
&\langle 0|\bar{u}(x) g_s G_{\alpha\beta}\gamma_\gamma\gamma_5(z) d(0)|\pi(k)\rangle \\
&= f_\pi (k_\alpha g_{\beta\gamma} - k_\beta g_{\alpha\gamma}) \int \mathcal{D}\alpha_i e^{-ikx\alpha_1 - ikz\alpha_3} f(\alpha_i) \\
&+ \frac{f_\pi}{(kx)} k_\gamma (x_\alpha k_\beta - x_\beta k_\alpha) \int \mathcal{D}\alpha_i e^{-ikx\alpha_1 - ikz\alpha_3} f_x(\alpha_i) \\
&+ \frac{f_\pi}{(kz)} k_\gamma (z_\alpha k_\beta - z_\beta k_\alpha) \int \mathcal{D}\alpha_i e^{-ikx\alpha_1 - ikz\alpha_3} f_z(\alpha_i),
\end{aligned}
$$

$$
\begin{aligned}
&i\langle 0|\bar{u}(x) g_s \tilde{G}_{\alpha\beta}(z)\gamma_\gamma d(0)|\pi(k)\rangle \\
&= -f_\pi (k_\alpha g_{\beta\gamma} - k_\beta g_{\alpha\gamma}) \int \mathcal{D}\alpha_i e^{-ikx\alpha_1 - ikz\alpha_3} \tilde{f}(\alpha_i) \\
&- \frac{f_\pi}{(kx)} k_\gamma (x_\alpha k_\beta - x_\beta k_\alpha) \int \mathcal{D}\alpha_i e^{-ikx\alpha_1 - ikz\alpha_3} \tilde{f}_x(\alpha_i) \\
&- \frac{f_\pi}{(kz)} k_\gamma (z_\alpha k_\beta - z_\beta k_\alpha) \int \mathcal{D}\alpha_i e^{-ikx\alpha_1 - ikz\alpha_3} \tilde{f}_z(\alpha_i).
\end{aligned}
$$

$$\int \mathcal{D}\alpha_i \equiv \int_0^1 d\alpha_1 \cdots d\alpha_3\, \delta(1-\alpha_1-\alpha_2-\alpha_3)$$

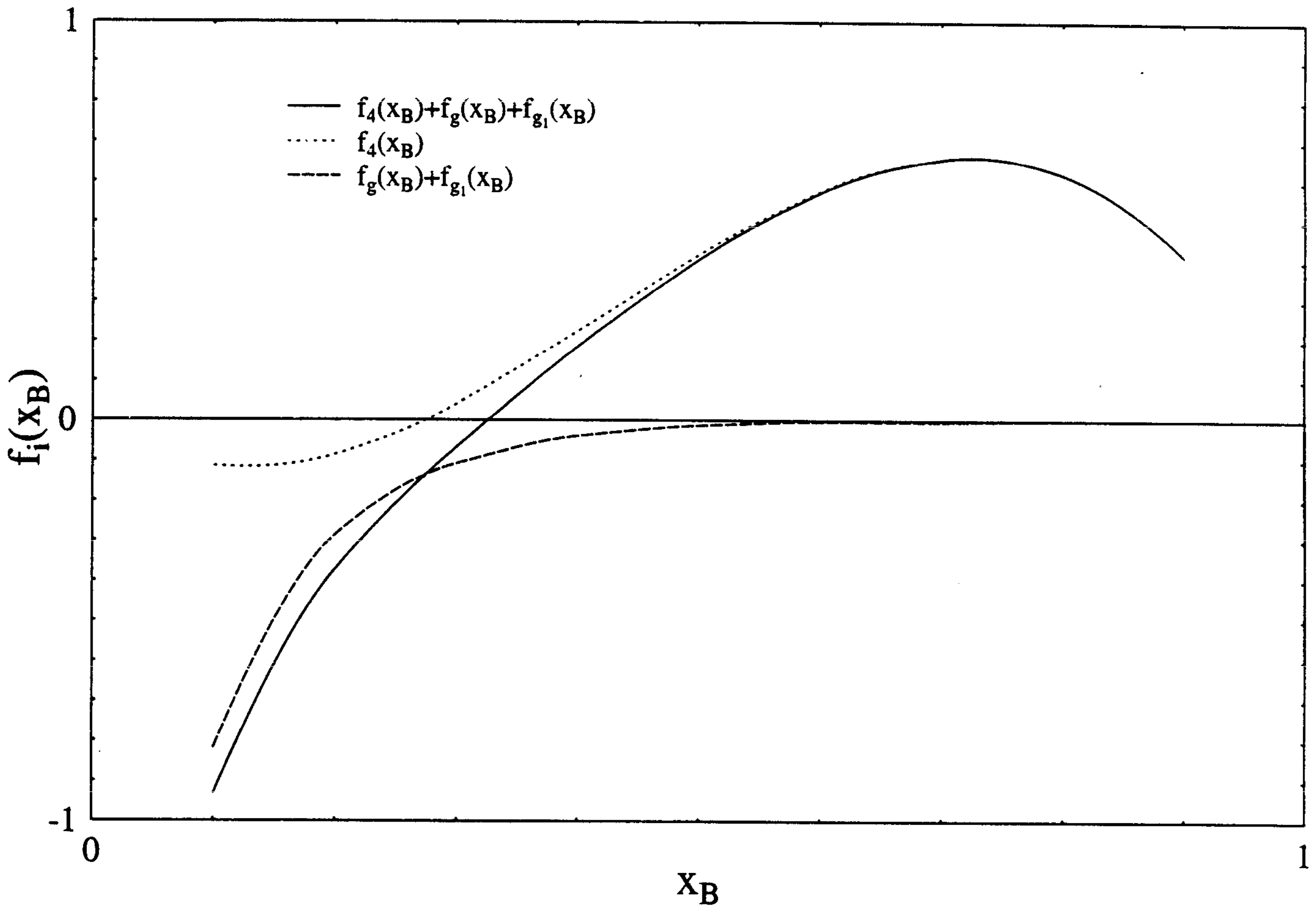
$$
\begin{aligned}
f(\alpha_i) &= \varphi_\perp(\alpha_i), \\
f_x(\alpha_i) + f_z(\alpha_i) &= \varphi_\parallel(\alpha_i) + \varphi_\perp(\alpha_i), \\
\tilde{f}(\alpha_i) &= \tilde{\varphi}_\perp(\alpha_i), \\
\tilde{f}_x(\alpha_i) + \tilde{f}_z(\alpha_i) &= \tilde{\varphi}_\parallel(\alpha_i) + \tilde{\varphi}_\perp(\alpha_i).
\end{aligned}
$$

$$
\begin{aligned}
\varphi_\perp(\alpha_i) &= 30\delta^2(\alpha_1-\alpha_2)\alpha_3^2[\tfrac{1}{3} + 2\epsilon(1-2\alpha_3)], \\
\varphi_\parallel(\alpha_i) &= 120\delta^2\epsilon(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3, \\
\tilde{\varphi}_\perp(\alpha_i) &= 30\delta^2\alpha_3^2(1-\alpha_3)[\tfrac{1}{3} + 2\epsilon(1-2\alpha_3)], \\
\tilde{\varphi}_\parallel(\alpha_i) &= -120\delta^2\alpha_1\alpha_2\alpha_3[\tfrac{1}{3} + \epsilon(1-3\alpha_3)].
\end{aligned}
$$

$$\langle\pi| g_s \bar{d}\, \tilde{G}_{\alpha\mu}\gamma_\alpha u|0\rangle = i\,\delta^2 f_\pi q_\mu$$

$\delta^2 = 0.2\ \mathrm{GeV}^2$ at $\mu = 1\ \mathrm{GeV}$

$\epsilon$: DEVIATION FROM ASYMPTOTIC FORM

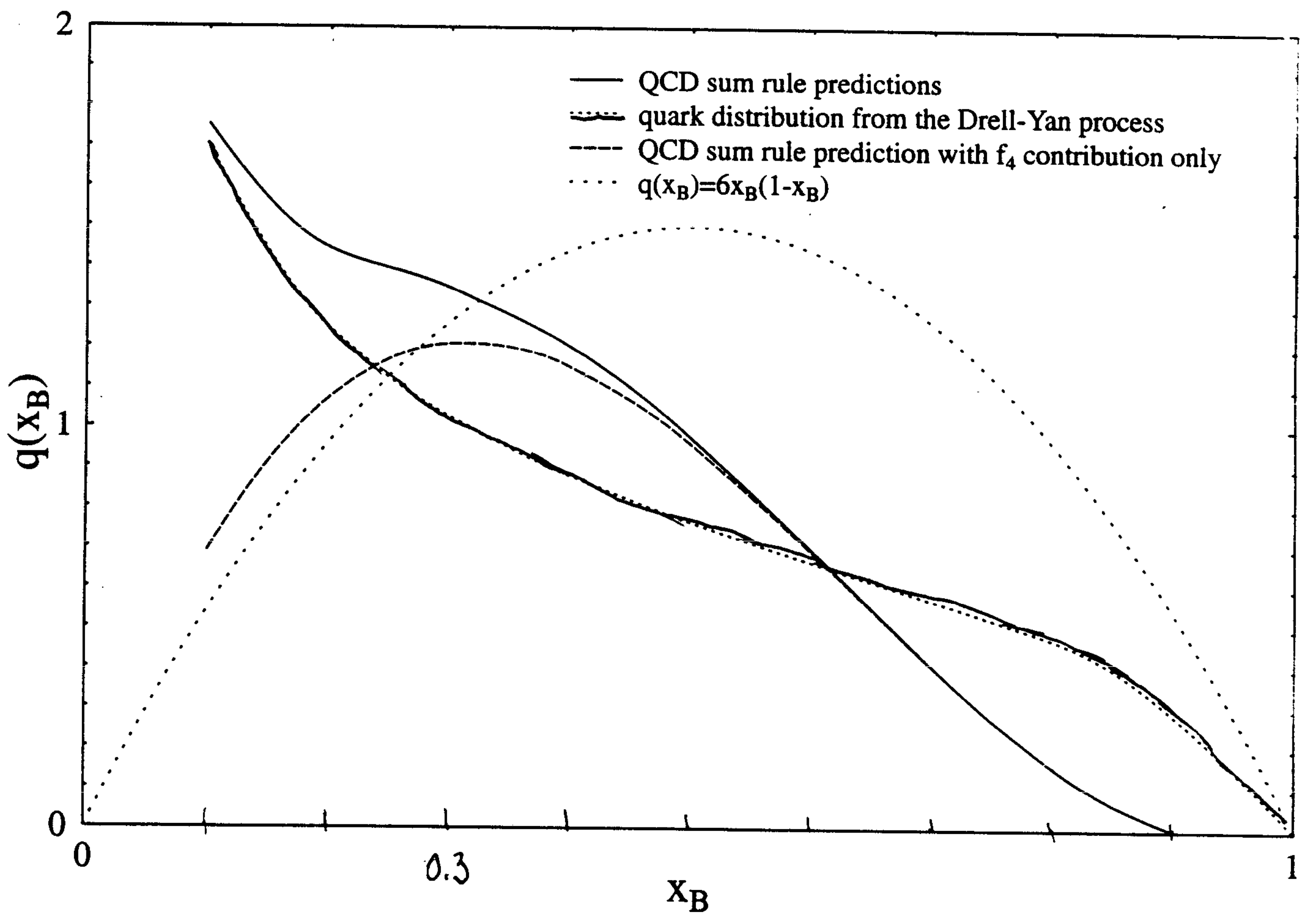


$f_4(x_B)+f_g(x_B)+f_{g_1}(x_B)$
$f_4(x_B)$
$f_g(x_B)+f_{g_1}(x_B)$
1
0
-1
0
1
$f_i(x_B)$
$x_B$

"EXPERIMENTAL DATA" $\mu^2 = 1\ GeV^2$ ⟶

Glück, Reya, Vogt (1995)

Drell Yan



# CONCLUSIONS

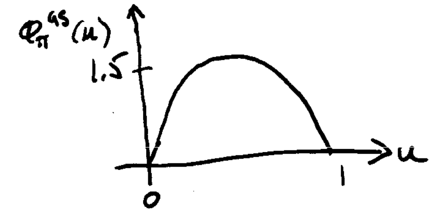
$\varphi_\pi(u) = 1. \pm 0.2$ for $u \simeq 0.3$

TWIST-4 CORRECTIONS ARE SMALL HERE ($u \simeq 0.3$)

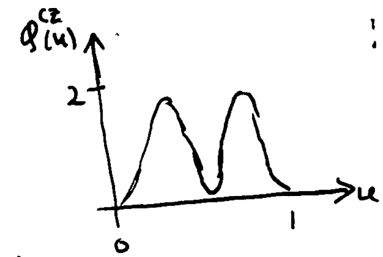
N.B. SUPPORT FOR ASYMPTOTIC TWIST-4 WAVE FUNCTIONS WILL BE GIVEN IN OUR ANALYSIS OF THE LFQM (BELOW).

# CONSTRAINED ANALYSIS OF $\varphi_\pi(u)$

## RELEVANT MODELS

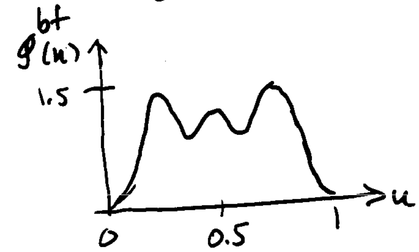



1. Asymptotic: $\varphi_\pi^{as}(u) = 6u(1-u)$




2. Chernyak-Zhitnitsky: $\varphi_\pi^{cz}(u) = 30u(1-u)(1-2u)^2$

   $\pi$ form factor $5 \leq Q^2 \leq 10\,GeV^2$ in perturbative QCD




3. Braun-Filyanov: $\varphi_\pi^{bf}(u=0.5) = 1.2$

   $\pi NN$ Coupling constant

4. Constraint of Radyushkin-Rustov $\gamma\gamma^* \to \pi$

$$I = \int_0^1 \frac{\varphi_\pi(u)}{u}\, du = 2.4 \pm ?$$

$$\left.\begin{array}{l} I^{as} = 3 \\ I^{cz} = 5 \\ I^{bf} = 5.07 \end{array}\right\} \; I = 2.4 \text{ suggests asymptotic w.f.}$$

5. Belyaev-MBJ $\varphi_\pi(0.3) = 1.0 \pm 0.2$

$\varphi_\pi^{as}(0.3) = 1.26$

$\varphi_\pi^{cz}(0.3) = 1.02$

$\varphi_\pi^{bf}(0.3) = 0.72$

# APPLICATION OF CONSTRAINTS

Recall: $\phi_\pi(u) = 6u(1-u)\left\{1 + a_2 C_2^{3/2}(2u-1) + a_4 C_4^{3/2}(2u-1) + \cdots\right\}$

Chernyak-Zhitnitsky

$$m_2 = \int_0^1 u^2 \phi_\pi(u)\, du = \frac{3}{70}(7 + 2a_2) = 0.35 \pm 0.05$$

$$\Rightarrow \quad 0 < a_2 < 1.2$$

Braun-Filyanov

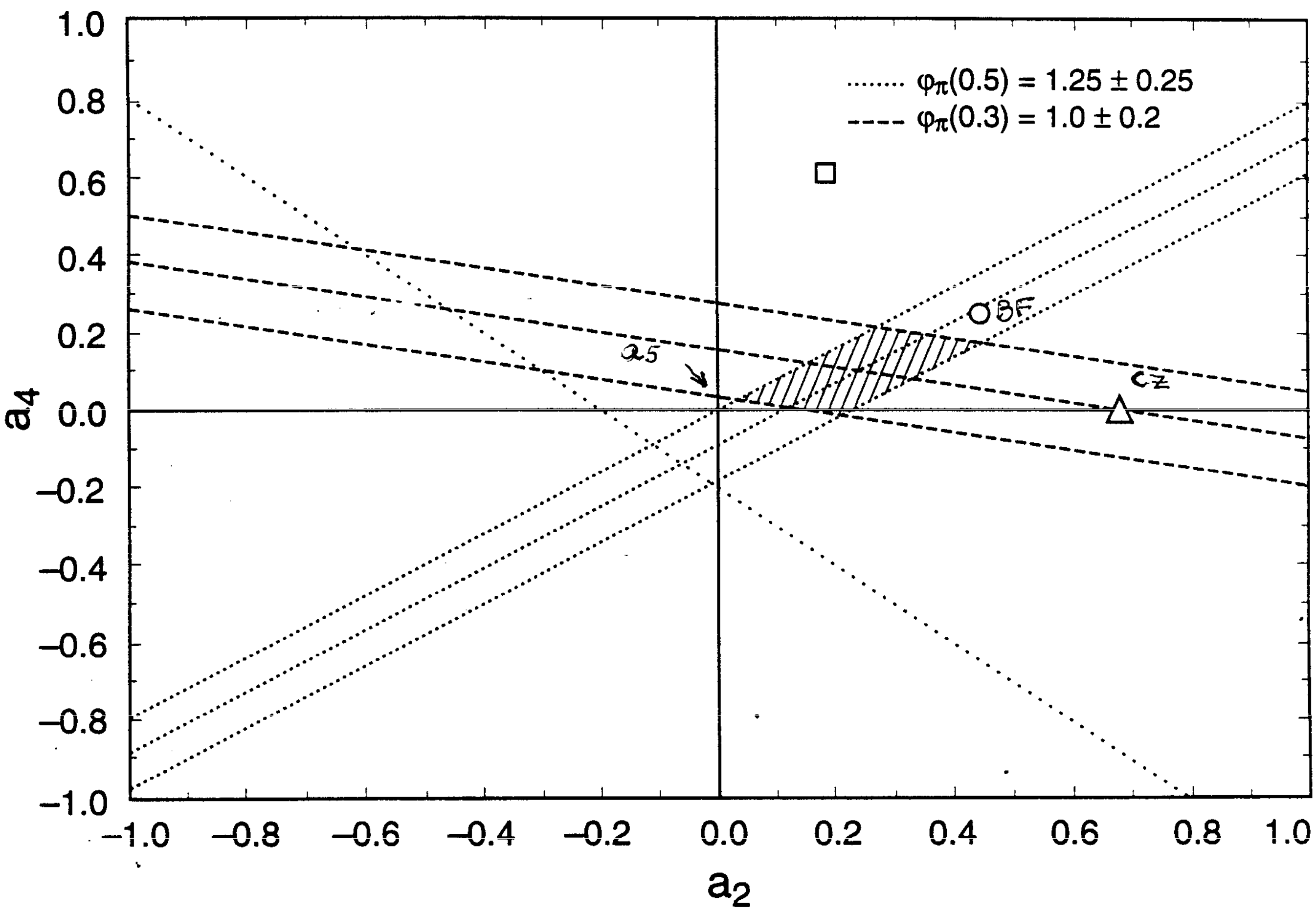
$$\phi_\pi(0.5) = \frac{3}{2}\left(1 - \frac{3}{2}a_2 + \frac{15}{8}a_4\right) = 1.25 \pm 0.25$$

Belyaev-MBJ

$$\phi_\pi(0.3) = 1.26(1 - 0.3a_2 - 1.317a_4) = 1 \pm 0.2$$

Radyushkin-Rustov

$$I = \int_0^1 \frac{\phi_\pi(u)}{u}\, du = 3(1 + a_2 + a_4) = 2.4 \pm ?$$



φπ(0.5) = 1.25 ± 0.25
φπ(0.3) = 1.0 ± 0.2
BF
a5
CZ
a4
a2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0

# CONCLUSION

Best Values:

$$\left.\begin{aligned} a_2 &= 0.25 \pm 0.25 \\ a_4 &= 0.1 \pm 0.12 \end{aligned}\right\} \varphi_\pi(u) \text{ is nearly asymptotic}$$

Prediction:

$$I \equiv \int_0^1 du \, \frac{\varphi_\pi(u)}{u} = 4 \pm 1$$

# COMPARISON TO LIGHT-FRONT QUARK MODEL 

LFQM: $\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 u(0)|\pi^+(p)\rangle = i f_\pi P^+$

$\pi$ $$\phi(p) = \sqrt{\frac{(2\pi)^3}{N_c}}\ \frac{1}{\pi^{3/4}\beta^{3/2}}\ e^{-p^2/2\beta^2}$$

$$\Rightarrow f_\pi = \frac{\sqrt{3}\,m}{\pi^{5/4}\beta^{3/2}}\int_0^\infty \frac{p^2 e^{-p^2/2\beta^2}}{(p^2+m^2)^{3/4}} = 0.130\ \text{GeV}$$

$\left.\begin{array}{l}\beta = 0.3194\ \text{GeV}\\ m = 0.25\ \text{GeV}\end{array}\right\}$ JAUS(1991)

$\Rightarrow$ HADRON PROPERTIES, FORM FACTORS $Q^2 \sim 1\,\text{GeV}^2$

TRANSLATE ORIGIN TO FIND

$$\langle 0|\bar{d}(0)\gamma_+\gamma_5 u(x)|\pi(P)\rangle = \frac{iP_+\sqrt{3}\,m}{\pi^{5/4}\beta^{3/2}}\int_0^1 \frac{e^{-i\xi P^+x^-}\,d\xi}{[4\xi(1-\xi)]^{3/4}}\int d^2p_\perp\ \frac{e^{-\frac{1}{2\beta^2}\left[\frac{p_\perp^2+m^2}{4\xi(1-\xi)}-m^2\right]}}{(p_\perp^2+m^2)^{1/4}}\ e^{ip_\perp\cdot x_\perp}$$

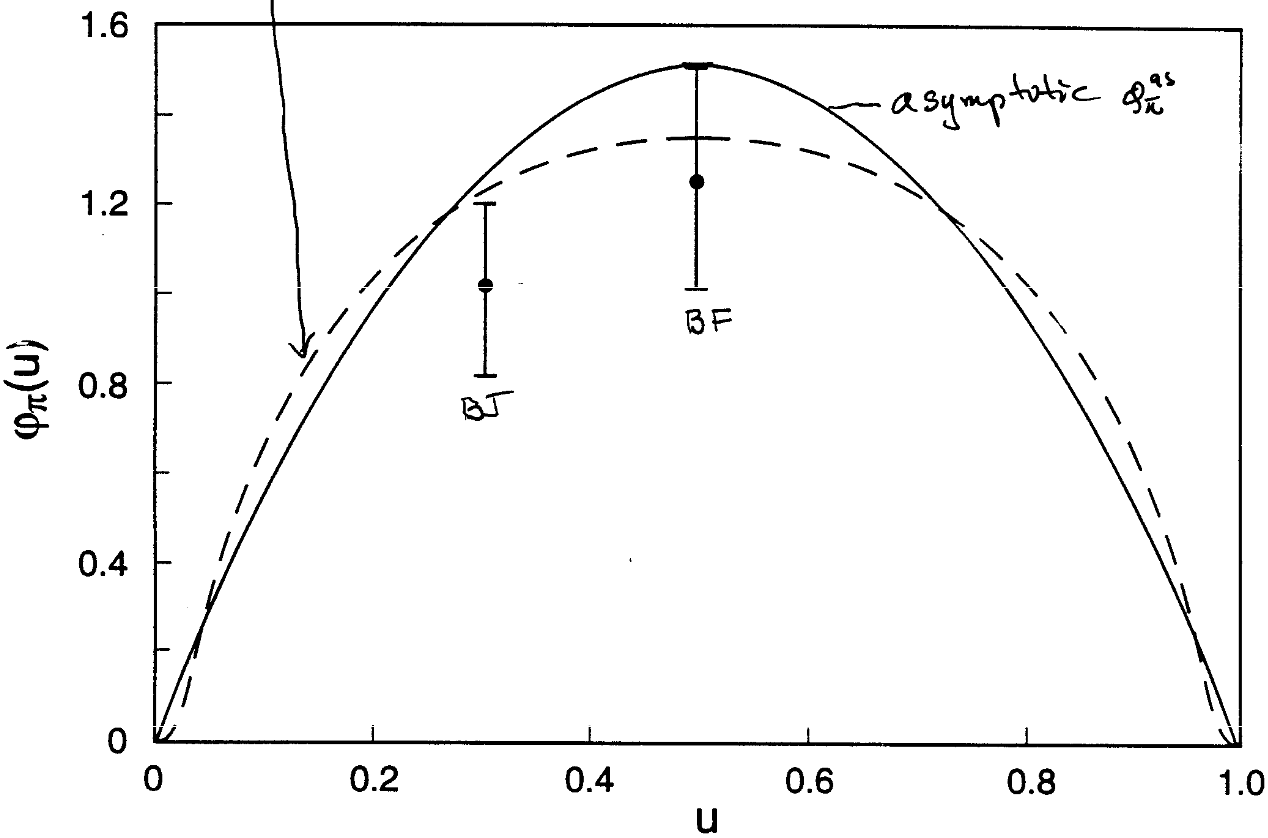
LIGHT-CONE PION WAVE FUNCTION

$$\langle 0|\bar{d}(0)\gamma_+\gamma_5 u(x)|\pi(P)\rangle = if_\pi P_+\int_0^1 e^{-iuP^+x^-}\left[\varphi_\pi(u) - x_\perp^2 g_1(u) - x_+^2 G_2(u)\right]du$$

$$\Rightarrow\quad f_\pi\varphi_\pi(u) = \frac{2^{1/4}\sqrt{6}\,m}{\pi^{5/4}}\,e^{\frac{m^2}{2\beta^2}}\,\Gamma(3/4, z_0)\ ,\quad z_0 = \frac{m^2}{8\beta^2 u(1-u)}$$

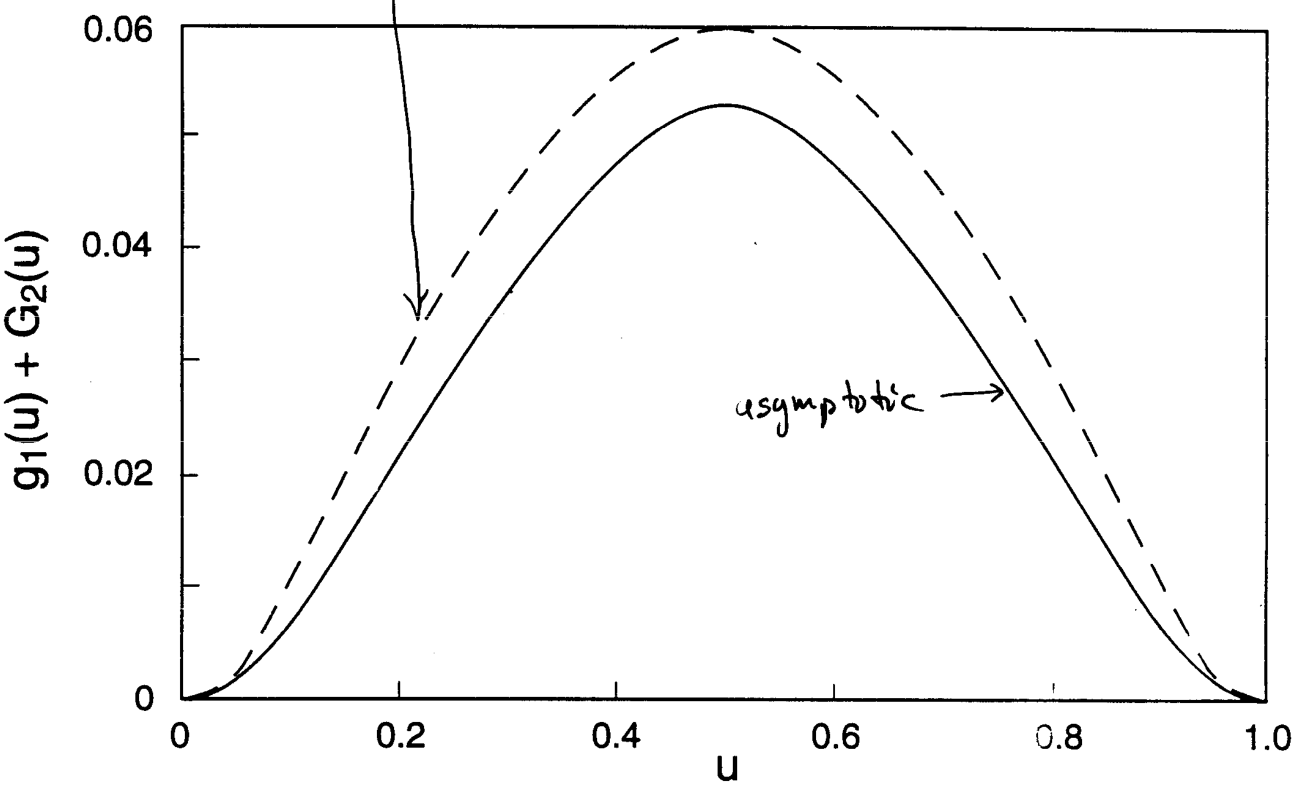
$$f_\pi[g_1(u)+G_2(u)] = \frac{2^{1/4}\sqrt{6}\,e^{\frac{m^2}{2\beta^2}}}{4\pi^{5/4}}\,m^3\left[\frac{1}{z_0}\Gamma(\tfrac{7}{2}, z_0) - \Gamma(\tfrac{3}{4}, z_0)\right]$$

$$f_\pi \varphi_\pi(u) = \frac{2^{1/4}\sqrt{6}\, m}{\pi^{5/4}}\, e^{\frac{m^2}{2\beta^2}}\, \Gamma(3/4, z_0)$$




TWIST-2

$$f_\pi \left[g_1(u) + G_2(u)\right] = \frac{2^{1/4}\sqrt{6}\, e^{\frac{m^2}{2\beta^2}}}{4\pi^{5/4}} m^3 \left[\frac{1}{z_0}\Gamma\left(\tfrac{7}{2}, z_0\right) - \Gamma\left(\tfrac{3}{4}, z_0\right)\right]$$




TWIST - 4

# CONCLUSIONS, LFQM

1. $g_1(u)$, $G_2(u)$ are nearly asymtotic

   JUSTIFIES CHOICE OF TWIST-4 WF USED TO ESTIMATE HIGHER TWIST CONTRIBUTIONS TO $g(x)$.

2. $\varphi_\pi(u)$ is nearly asymptotic; but, it is even more consistent with constraints of Braun-Filyanov and Belyaev-MBJ.

## OVERALL RESULTS

1. $\varphi_\pi(0.3) = 1 \pm 0.2$,

WHICH ARISES FROM AN ANALYSIS OF DEEP INELASTIC SCATTERING IN THE LIGHT-CONE QCD SUM RULE APPROACH, PROVIDES A NEW AND USEFUL CONSTRINT FOR DETERMINING THE TWIST-2 PION WAVE FUNCTION $\varphi_\pi(u)$.

2. $\varphi_\pi(x)$ : $a_2 = 0.25 \pm 0.25$, $a_4 = 0.1 \pm 0.12$

WHICH RESULTS FROM A NEW CONSTRAINED ANALYSIS, INCLUDING 1. ABOVE