Light Cone Wave Functions

and

Vacuum Structure of QCD: Lessons, Experience, Problems.

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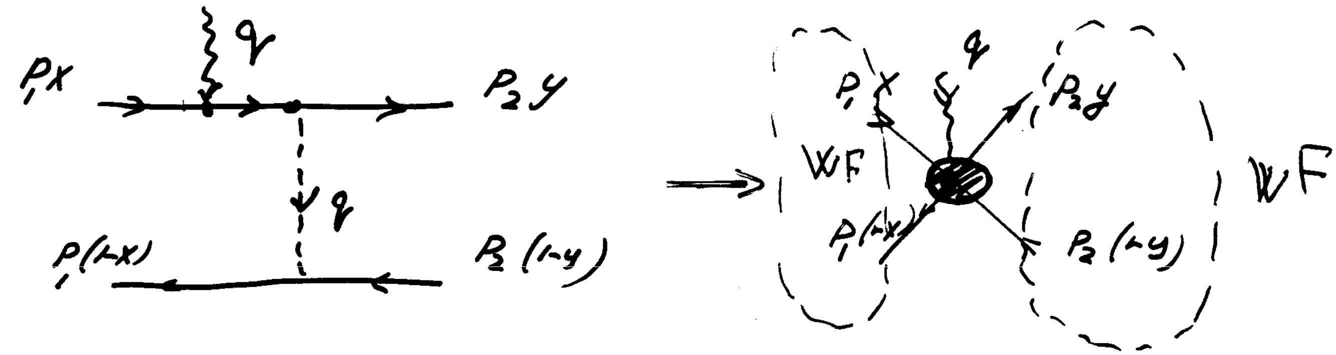
I. Factorization theorem. Idea. (OPE)



$z \sim Q^{-1} \to 0 \qquad zQ \simeq 1$

1. We calculate small distance contribution explicitly (asymptotic freedom).

2. All nontrivial, large distance physics is hidden into the nonperturbative WF

3. WF can not be found by perturbative technique, but should be extracted from elsewhere.

4. $z^2 \simeq 0$. Therefore, we have only one variable: $\Psi(zQ)$

5. Dimensional counting rules are reproduced within QCD

$\underline{II}$ Remark on a Definition of WF.

1.

$$\langle \bar{\Psi} i\gamma_5 \Psi | \pi \rangle \simeq \frac{-2\langle \bar{\Psi}\Psi \rangle}{f_\pi} \sim \frac{2\,(250\,MeV)^3}{133\,MeV} \sim f_\pi \cdot 2\,GeV$$

$$\langle \bar{\Psi} i\gamma_5 (\sigma_{\mu\nu} G_{\mu\nu}) \Psi | \pi \rangle \sim \frac{\langle \bar{\Psi} i g \sigma G \rangle}{f_\pi} \sim f_\pi \cdot GeV^3$$

$$\langle \bar{\Psi} i\gamma_5 (\sigma G g)^n \Psi | \pi \rangle \sim f_\pi \cdot (GeV)^{2n+1}.$$

Arbitrary number of gluons in the matrix elements are not suppressed!

2. However at large Q^2 there is a unique classification. (Only valence quarks contribute).

3. When Q^2 is getting smaller and smaller, the question 1. is becoming relevant! (the possible answer what happens at small energies will be given at the end).

III WF. Definition.

1. $$\langle 0|\bar d(z)\gamma_\mu\gamma_5\, e^{ig\int_{-z}^{z} A_\mu dx_\mu}\, u(-z)|\pi(q)\rangle =$$

$$= i f_\pi q_\mu \Psi(qz) = i f_\pi q_\mu \int_{-1}^{1} \varphi(\xi) e^{i\xi zq} d\xi$$

$$= \sum \frac{i^n}{n!} \langle 0|\bar d(0)\gamma_\mu\gamma_5 (i z_\mu \overleftrightarrow{D}_\mu)^n u(0)|\pi(q)\rangle$$

$$i\vec{D}_\mu = i\vec{\partial}_\mu + g A_\mu^a \frac{T^a}{2} \qquad \Psi(0) = \int_{-1}^{1} \varphi(\xi) d\xi = 1$$

2. $$\langle 0|\bar d\gamma_\mu\gamma_5 (i\overleftrightarrow{D}_\nu z_\nu)^n u|\pi(q)\rangle = i f_\pi q_\mu (zq)^n \langle \xi^n\rangle =$$

$$= i f_\pi q_\mu (zq)^n \int_{-1}^{1} \varphi(\xi)\xi^n d\xi \qquad \int \Psi_{BL}(\vec{k}_\perp, \xi) d\vec{k}_\perp^2 = \varphi(\xi$$

Remarks:

a) DA (distribution amplitude) $\varphi(\xi)$ describes the distribution of the total longitudinal momentum between the quark and antiquark carrying the momenta xq_z and $(1-x)q_z$.

$$\xi = x - (1-x) = 2x-1$$

b) If we knew all matrix elements we could restore the whole $\varphi(\xi)$.

3. Problems:

a) What is the shape $\varphi(\xi)$ - ?

b). What is the behavior $\varphi(\xi \to \pm 1)$?

c). How one can find the moments $\langle \xi^2 \rangle$ - ? $\langle \xi^4 \rangle$ - ? ...

4. The way to answer:

a) Quark - hadron duality

b). Dispersion Relations

c) QCD sum rules

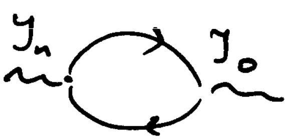
IV. Duality and Dispersion Relations

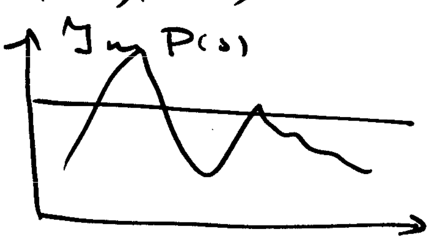
1. $J_n = \bar{d}(zD)^n \gamma_\mu u$

$$P_n(q^2) = i\int dx\, e^{iqx} \langle 0| J_n(x)\, J_0(0)|0\rangle$$

$$P_n(q^2) = \frac{1}{\pi}\int_0^\infty ds\, \frac{\mathrm{Im}\, P_n(s)}{s-q^2}$$ (dispersion relation)

$$\mathrm{Im}\, P_n^{pert}(s) = \frac{3}{4\pi(n+1)(n+3)}$$





perturbative

$$\int_0^{s_\pi} \mathrm{Im}\, P_n^{pert.}(s)\, ds = \int_0^{\infty} \mathrm{Im}\, P_n^{\pi}(s)\, ds$$ quark-hadron duality

$$\langle 0| J_n |\pi\rangle \sim \langle \bar q D^n q|\pi\rangle \sim f_\pi \langle \xi^n\rangle \sim \frac{1}{n^2} \quad n \to \infty$$

$$\int \varphi(\xi)\,\xi^n \sim \frac{1}{n^2} \Rightarrow \boxed{\varphi(\xi) \sim (1-\xi^2), \quad \xi \to \pm 1}$$

Strict result!

2. How to calculate the numbers $\langle \xi^2 \rangle$, $\langle \xi^4 \rangle$ - ?

One needs to calculate power corrections to be sensetive to the given resonance (model dependent results)

V Phenomenology (modern problems)

Why do we need to calculate the power corrections?

Brodsky and Co $\downarrow$ Perturbative QCD works!

$Q^2 F_\pi(Q^2) \simeq \text{const}$

$Q^4 F_N(Q^2) \sim \text{const}$

$s^7 \frac{d\sigma}{dt}(\gamma p \to \pi^+ n) \sim f(t/s)$.

(quark counting rules)

Isgur and Co $\downarrow$ Perturbative QCD gives much smaller contribution than "soft" terms

If the leading contribution can not provide the observable absolute values (I.), than, how can one explain the good functional agreement with experiment (B).

Scientific Q!

What the matrix elements are?

$$\langle 0|\bar{d}\,\gamma_{\lambda 5}(D_\mu t_\mu)^{2n} u|\pi\rangle \sim \langle k_\perp^{2n}\rangle (t^2)^n, \quad t_\mu = (0,\vec{t},0), \ t q.$$

VI. Definition.

$$\langle 0|\, \bar d(x)\gamma_\mu\gamma_5\, e^{ig\int A_\mu dx_\mu}\, u(0)\, |\pi(q)\rangle =$$

$$= i f_\pi q_\mu\ \Psi(zq, z^2)$$

a) The moments in longitudinal direction are defined as follow

$$\langle 0|\, \bar d\gamma_\mu\gamma_5\, (i z_\mu \overleftrightarrow{D}_\mu)^n u(0)\, |\pi\rangle = i f_\pi q_\mu (qz)^n \int \varphi(\xi)\xi^n d\xi =$$

$\underline{z^2 = 0 \text{ is a projector}}$ $\quad \hookrightarrow = \langle \xi^n\rangle\, i f_\pi q_\mu (zq)^n$

b) The moments in transverse direction are defined in a similar way:

$$\langle 0|\, \bar d\gamma_\mu\gamma_5\, (i\vec{D}_\mu t_\mu)^{2n} u\, |\pi(q)\rangle_{\vdash} = i f_\pi q_\mu\, (-t^2)^n \frac{(2n-1)!!}{(2n)!!} \cdot \langle \vec{k}_\perp^{2n}\rangle$$

transverse vector $t_\mu = (0\ \ \vec{t}\ \ 0)$; $t_\mu q_\mu = 0$.

$$\frac{(2n-1)!!}{(2n)!!} \longleftrightarrow \int (\cos\varphi)^{2n} d\varphi \Big/ \int d\varphi$$

c) This definition is different from a gauge-variant definition $\langle 0 | \bar{d} \gamma_\nu \gamma_5 \partial_\perp^2 u | \pi \rangle$ because the physical transverse gluons are participants of this definition.

d) Infinite number of gluons are present (automatically) in such a definition.

But this is the only gauge invariant way to define WF.

e) For any reasonable WF an arbitrary high moment $\langle 0 | \bar{u} \partial_\perp^{2n} \gamma_\mu \gamma_5 d | \pi \rangle \neq 0$.

Therefore, an arbitrary moment $\langle 0 | \bar{u} \gamma_\mu \gamma_5 D_\perp^{2n} d | \pi \rangle \neq 0$. (finite)

VII Strict constraints on the nonperturbative w.f.

1. The Idea

Calculation of the specific correlation function in QCD $\longrightarrow$

$\longrightarrow$ Dispersion relations convert this information into $\rightarrow$

$\rightarrow$ knowledge about hadronic matrix elem.

$$\int e^{iqx} dx \, \langle 0 | T \{ \bar\psi \gamma_\mu \gamma_5 \psi(x) \, , \, \bar\psi \gamma_\nu \gamma_5 (\vec{D} \cdot z)^n \psi \} | 0 \rangle$$

$Q^2 \to \infty$ this correlator can be calculated.

We assume that π-meson contribution to the dispersion integral is not zero. $\rightarrow$ $\frac{1}{\pi}\int_0^{S_0} \mathrm{Im}\, P(s)^{pert}\, ds = \frac{1}{\pi}\int_0^{\infty} \mathrm{Im}\, P^{\pi}$

$$\langle \bar\psi (Dz)^n \gamma_\mu \gamma_5 \psi | \pi \rangle \underset{n\to\infty}{\rightarrow} \frac{1}{n^2} \sim \int \Psi(\vec{K}_\perp^2, x)(x_1 - x_2)^n dx \, d\vec{K}_\perp^2$$

● 1 $\quad \varphi(\xi) \equiv \int \Psi(\vec{K}_\perp^2, \xi)\, d\vec{K}_\perp^2 \sim 1 - \xi^2 \, , \quad \xi \to 1$

$$\langle \xi^n \rangle = \int \varphi(\xi)\, \xi^n d\xi \sim \frac{1}{n^2}$$

• 2 $$\int \Psi(k_\perp^2, \xi)(\vec{k}_\perp^2)^\ell d\vec{k}_\perp^2 \sim (1-\xi^2)^{\ell+1}$$

$\rightarrow \dfrac{\vec{k}_\perp^2}{1-\xi^2}$ - the only allowed combination.

$$\rightarrow \Psi(k_\perp^2, \xi) \neq \Psi(\vec{k}_\perp^2) \otimes \varphi(\xi)$$

Factorization does contradict to QCD

• 3 $$\langle \vec{k}_\perp^{2n} \rangle \sim \int \Psi(\vec{k}_\perp^2, \xi)(\vec{k}_\perp^2)^n d\vec{k}_\perp^2 d\xi \sim n!$$

$$\Psi(\vec{k}_\perp, \xi) \sim e^{-\frac{\vec{k}_\perp^2}{1-\xi^2}} \qquad 1-\xi^2 \sim x(1-x)$$

$$\int \Psi(\vec{k}_\perp^2, \xi)(\vec{k}_\perp^2)^n \sim n!$$

good news

bad. news

$$\varphi(\xi) = \int \Psi(\vec{k}_\perp^2, \xi) d\vec{k}_\perp \sim 1-\xi^2$$ distribution amplitude

Function $\Psi(\vec{k}_\perp^2 \xi)$ has been derived from QCD. It satisfies all constraints on matrix elements. Does it accidentally coincide with quark model?

X First applications

Drell - Yan Formula for π- meson form - factor

$$F_\pi(Q^2) = \int \frac{dx\, d^2k_\perp}{16\pi^3} \Psi^+(x_1, \vec{k}_\perp + x_2 \vec{q}_\perp) \Psi(x_1, \vec{k}_\perp)$$

1. Ψ_{CQM}
2. Ψ_{QCD}
3. Ψ_{QCD+} $\qquad R = \frac{\langle k_\perp^4 \rangle}{\langle k_\perp^2 \rangle^2} \simeq 4.$

The main results:

a) "Soft" mechanism might be responsible for success of dimensional counting rules

b) This contribution (formally, higher twist) can Mimic (imitate) the leading twist behavior

XI. Model WF.

1) $$\Psi(\vec{k}_\perp^{\,2}, x)_{CQM} = A\, e^{-\frac{\vec{k}_\perp^{\,2}+m^2}{8\beta^2 x\bar{x}}}$$

2) $$\Psi(\vec{k}_\perp^{\,2}, x)_{QCD} = A\, e^{-\frac{\vec{k}_\perp^{\,2}}{8\beta x\bar{x}}} \cdot \left(1 + g\left(\xi^2 - \frac{1}{5}\right)\right]$$

$g = 0 \qquad \varphi(\xi) = \int \Psi \, d\vec{k}_\perp^{\,2} \sim x\bar{x} \rightarrow$ asymp.

$g = 5 \qquad \varphi(\xi) = \int \Psi \, d\vec{k}_\perp^{\,2} \sim \bar{x} x \, (x - \bar{x})^2 \rightarrow CZ$

3. $$\Psi(\vec{k}_\perp^{\,2}\, x)_{QCD+} =$$

$$= A \left\{ e^{-\frac{\vec{k}_\perp^{\,2}}{8\beta^2 x\bar{x}}} + c\left[e^{-\left(\frac{k_\perp^2}{8\beta\bar{x}x} - \ell\right)^2} \right] \right\} \left(1 + g\left(\xi^2 - \frac{1}{5}\right)\right)$$

$c \simeq 0{,}15$

For $u \simeq 1/2$ the second pick is located at $\vec{k}_\perp^{\,2} \sim 1\ \mathrm{GeV}^2$ and 10 times smaller.

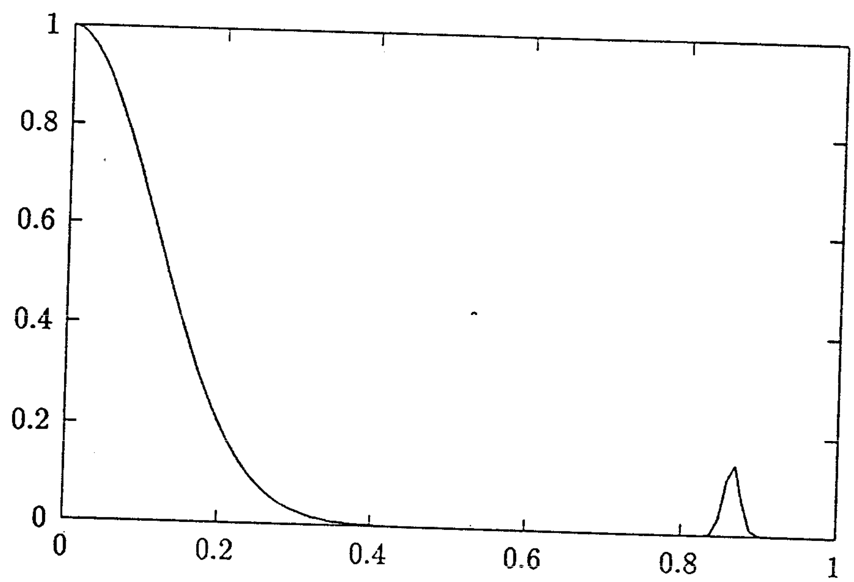
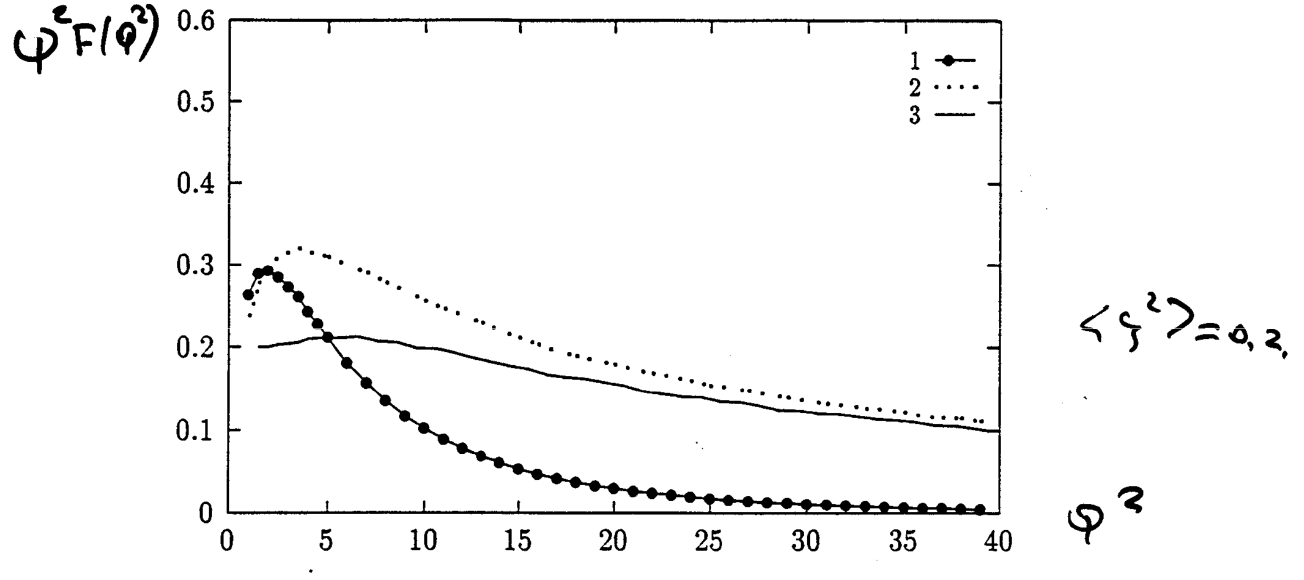
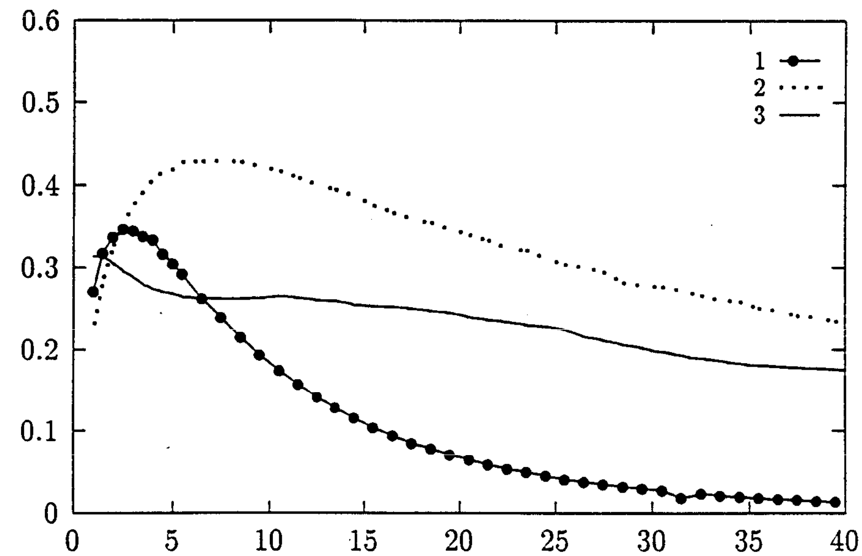


Fig.1



$\varphi^2 F(\varphi^2)$
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
40
1
2
3
$\langle \xi^2 \rangle = 0.2$
φ^2

Fig.2



0.6
0.5
0.4
0.3
0.2
0.1
0
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5
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35
40
1
2
3

Fig.3

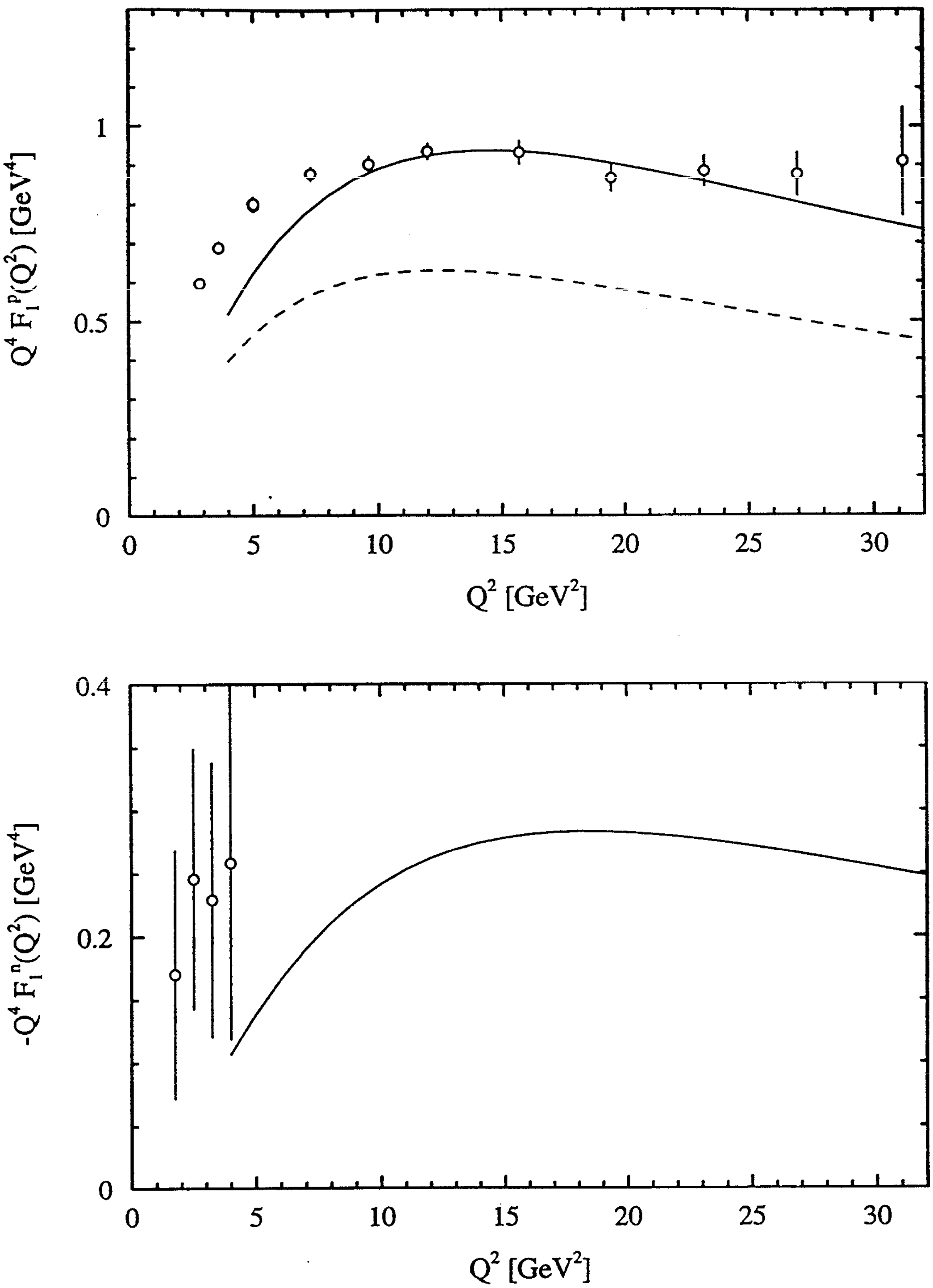
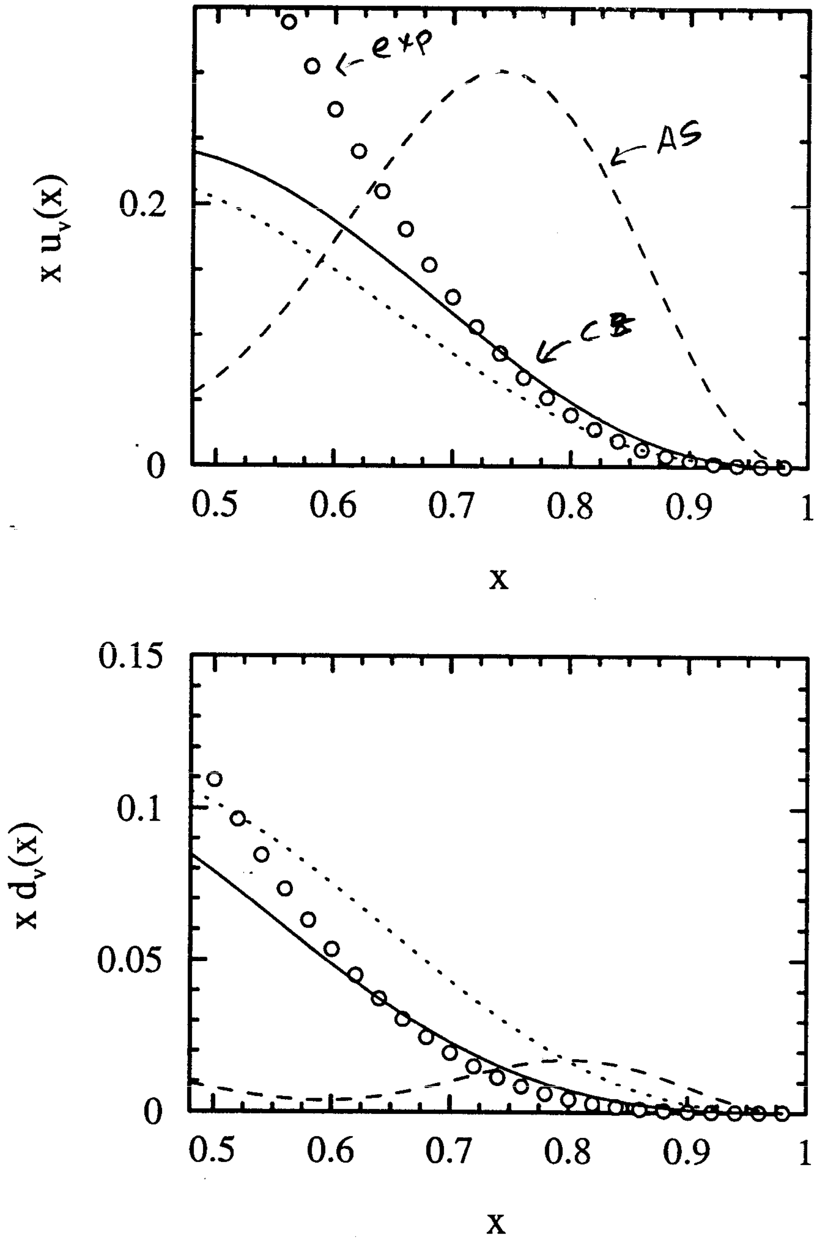


Fig. 4 (From P. Krdl)



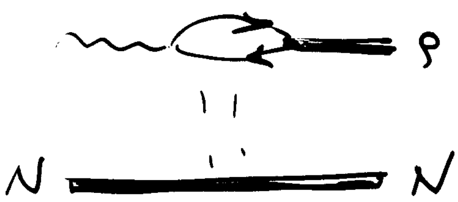
$\Psi \sim e^{-\sum_i \frac{k_\perp^2}{x_i \beta^2}}$

Figure Nucleon.

(From P. Kroll)

Hard diffractive electroproduction.

$\gamma^* N \to \rho N$ (Theory - see Brodsky review)



Power corrections?

$$\sqrt{T(Q^2)} = \frac{Q^4 \int \frac{dx}{x\bar{x}} \int d^2k_\perp \Psi(k_\perp x) \frac{\left(1 - 2\,\frac{\vec{k}_\perp^2/x\bar{x}}{Q^2 + k^2/x\bar{x}}\right)}{\left(Q^2 + \frac{\vec{k}_\perp^2}{x\bar{x}}\right)^2}}{\int \frac{dx}{x\bar{x}} \int d^2k_\perp^2 \, \Psi(k_\perp^2 x)}$$

$Q^2 \to \infty$, $T \to 1$

Deviations from $T = 1$ determine a region of applicability of the asymptotic formula

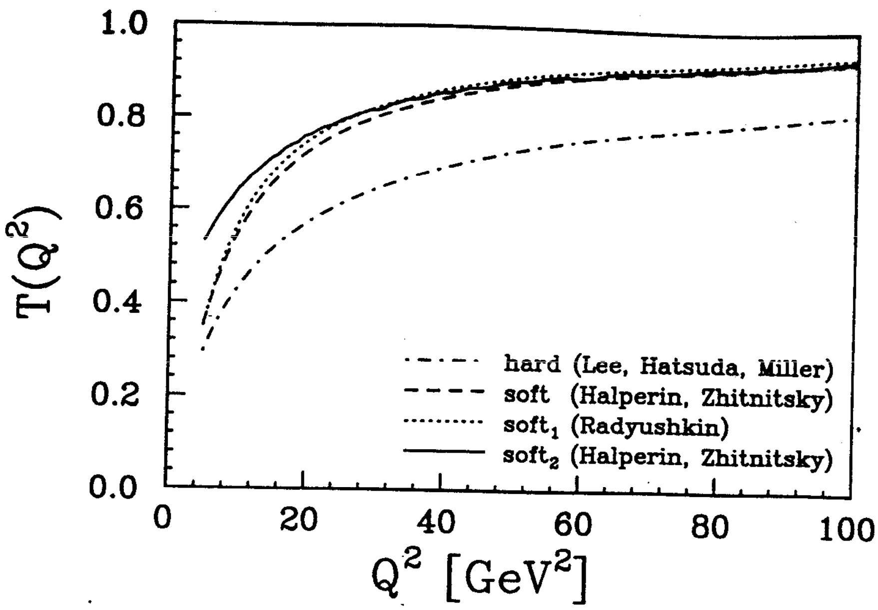


Figure ~~16~~ 5: The Fermi motion suppression factor $T(Q^2)$ of ~~Eq. (4)~~ for ρ^o electroproduction for various ρ-meson wave functions from ~~Refs. [38], [18] and [39]~~.

(from: Frankfurt, Koepf, Strikman).

Moral: 1. Pre-asymptotic behavior depends on properties $\Psi(x\ k_\perp^2)$

2. The asymptotic regime starts at rather low $Q^2 \simeq 10\,\mathrm{GeV}^2$, in drastic contrast to exclusive processes ($Q^2 \gtrsim 50 \div 100\,\mathrm{GeV}^2$)

3. In the intermidiate region $Q^2 \simeq 5 \div 10\,\mathrm{GeV}^2$ the amplitude is very sensitive to the fine structure of the QCD vacuum. (more interesting)

$\gamma^* \gamma \to \pi$

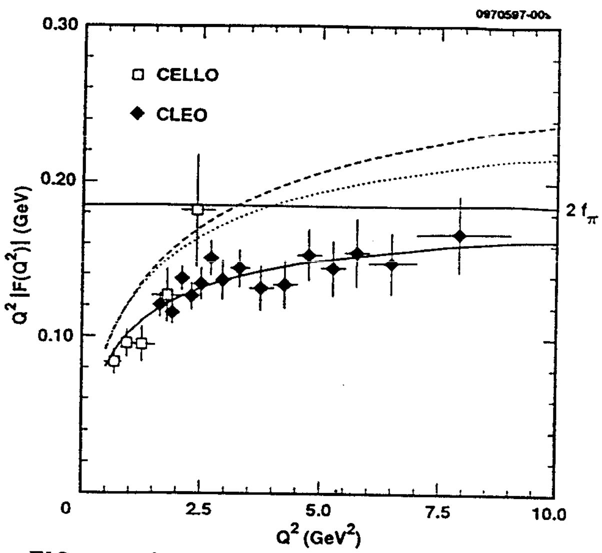


FIG. 18. Comparison of the results (points) for $Q^2|\mathcal{F}_{\gamma^*\gamma\pi^0}(Q^2)|$ with the theoretical predictions made by Jakob *et al.* [13] with the asymptotic wave function (solid curve) and the CZ wave function (dashed curve). The dotted curve shows the prediction made with the CZ wave function when its QCD evolution is taken into account.

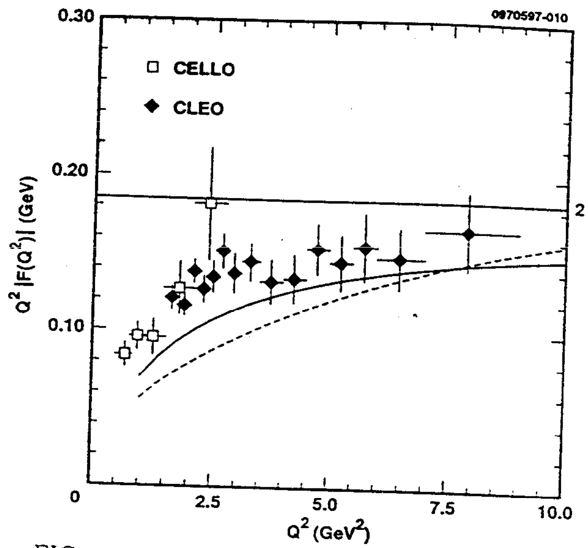


FIG. 19. Comparison of the results (points) for $Q^2|\mathcal{F}_{\gamma^*\gamma\pi^0}(Q^2)|$ with the theoretical predictions made by Cao *et al.* [16] with the asymptotic wave function (solid curve) and the CZ wave function (dashed curve).

Moral: 1. In the region $Q^2 \simeq 5 \div 10\ \mathrm{GeV}^2$ $\gamma^*\gamma\pi$ is sensitive to $\vec{k}_\perp^2$ distribution

2. One can not make a preferance regarding the WF (CZ or asymptotic one) without an understanding of power corrections.

XII. WF → Instantons → Constituent QM.

Instantons (introduction).

1. $S = +\frac{1}{4g^2}\int F_{\mu\nu}^a F_{\mu\nu}^a \, d^4x$ (Eucl.)

2. $Q = \frac{1}{32\pi^2}\int d^4x \, F_{\mu\nu}\tilde{F}_{\mu\nu}$, $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}F_{\lambda\sigma}^a$

$0 \leq \int d^4x \left(F_{\mu\nu}^a - \hat{F}_{\mu\nu}^a\right)^2 = \int d^4x \left(2F_{\mu\nu}^2 - 2F_{\mu\nu}\hat{F}_{\mu\nu}\right) =$

3. $= 8g^2 \cdot S - 64\pi^2 Q$

$S \geq \frac{8\pi^2}{g^2} Q$ $Q = 1$ - instanton with minimal action.

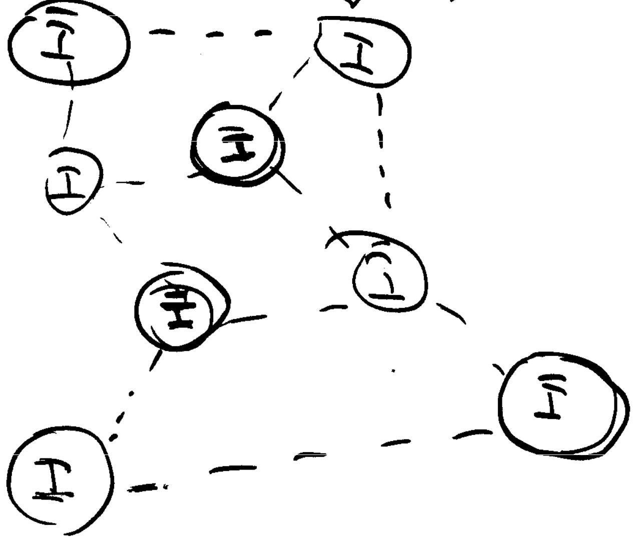
4. $F_{\mu\nu}^a = \hat{F}_{\mu\nu}^a$ self-duality eq.

5. $F_{\mu\nu}^a F_{\mu\nu}^a = \frac{192\rho^4}{(x^2+\rho^2)^4}$

6. $D\psi_R = 0.$ $\psi_R \sim \frac{1}{(x^2+\rho^2)^{3/2}}$. zero mode in the Instan. background

Instanton liquid model

(Shuryak, Diakonov, Petrov, Bjorken)



Calculated:

1. Static baryon characteristic
2. All possible correlation funct.
3. Lattice calculation has found instantons with those properties

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Parameters: $\left\langle \frac{F_{\mu\nu}^2}{32\pi^2} \right\rangle = \frac{1}{V} \langle Q \rangle = \frac{N}{V}$

$$\frac{\bar{\rho}}{\bar{R}} \simeq \frac{1}{3}$$

Packing fraction: $\frac{\pi^2 \bar{\rho}^4}{\bar{R}^4} \simeq 1/10 \ll 1$

(small parameter)

Remark: This parameter $\bar{\rho}/\bar{R} \simeq \frac{1}{3}$ describes unhomogeneous vacuum structure. It gives exactly correct magnitude for the nonfactoriza-bility ($R \sim 3$). This is new scale and new substructure.

Instantons and Chiral Symmetry Breaking

$$I \xleftrightarrow{\quad R \gg 1 \quad} \bar{I}$$

Two degenerate states with exactly zero eigenvalues

$$I - R - \bar{I}$$

The degeneracy is lifted through the diagonalization of the hamiltonian

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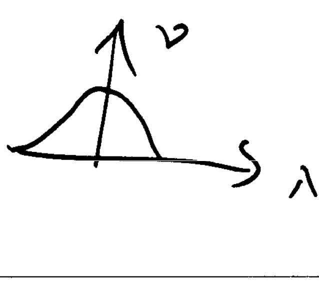
When one adds more I's and $\bar{I}$'s each of them brings in a would be zero mode. Eventually, for $I\bar{I}$ ensemble one gets a continuous band spectrum with a spectral density $\nu(\lambda \to 0) \neq 0$.



$$\langle \bar{\psi}\psi \rangle \sim \nu(0)$$

Instantons and Constituent Quark Propagator.

$$S^{-1}(p) = \frac{\not{p} + iM(p)}{p^2 + M^2(p)}$$

$$M(p^2) = \sqrt{\frac{\pi^2 N}{V}\frac{\bar{\rho}^2}{N_c}}\, F(p\bar{\rho}) \qquad M(p=0) = 350\,\text{MeV}$$

$$F(z) = 2z\left[I_0(z)K_1(z) - I_1(z)K_0(z) - \frac{1}{z}I_1(z)K_1(z)\right]$$

$$F(z\to\infty) \to \frac{6}{p^3\bar{\rho}^3} \qquad F(0) = 1$$

$F(z)$ is a combination of the modified Bessel functions and is related to the Fourier transform of the zero modes.

$$\langle\bar{\psi}\psi\rangle = i\int\frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\,S(p) = -4N_c\int\frac{d^4p}{(2\pi)^4}\frac{M(p)}{p^2+M^2(p)} \simeq -(255\,\text{MeV})^3$$

$$F_\pi = \frac{1}{\bar{\rho}}\left(\frac{\bar{\rho}}{\bar{R}}\right)^2\sqrt{\ln\frac{\bar{R}}{\bar{\rho}}} \simeq 100\,\text{MeV} \qquad (93\,\text{MeV} - \text{exp})$$

$$\langle r_\pi^2\rangle^{1/2} = \frac{\sqrt{N_c}}{2\pi F_\pi} \simeq (340\,\text{MeV})^{-1} \qquad (310\,\text{MeV})^{-1} - \text{exp}$$

Nonperturbative WF and

Constituent QM (through instantons)

1. Very high Q^2:

$\underline{\Psi}(\vec{k}^2 x) \sim e^{-\frac{\vec{k}^2}{x\bar{x}}}$

2. When Q^2 is getting smaller and smaller, gluon fluctuations are mainly instanton fluctuations.

3. When a quark propagate in the instanton background, it becomes constituent quark with $M \neq 0$.

4. $\Psi(\vec{k}_\perp^2 x) = A\, e^{-\frac{\vec{k}_\perp^2 + M^2(\frac{\vec{k}^2}{x\bar{x}})}{8\beta^2 \bar{x} x}} \left[1 + \ldots\right]$.

$M(\vec{k}^2 \to 0) \to M_0$

$M(k^2 \to \infty) \to 0$.

Therefore, all effects we have been discussed at $Q^2 \to \infty$ are the same; at $Q^2 \approx 0 \Rightarrow$ CQM. work.

5. Small parameter

$\bar{\rho}/\bar{R} \simeq \frac{1}{3}$ corresponds to the size of the constituent quark (in comparision with the hadron size)

6. Analogy with the harmonic oscillator problem in the external Electric field.

$$H = \frac{p^2}{2m} + \frac{\kappa x^2}{2} - eEx \Rightarrow H \to \frac{p^2}{2m} + \frac{\kappa\left(x - \frac{eE}{2\kappa}\right)^2}{2} - \frac{\kappa}{2}\frac{e^2E^2}{4\kappa^2}$$

$$x \to x - \frac{eE}{2\kappa} \qquad \Psi(x) \to \Psi(x - x_0)$$

In QCD-case, because of the external instanton fields, the variable $\vec{k}_\perp^{\,2}$ get shifted on the ammount $\vec{k}_\perp^{\,2} \to \vec{k}_\perp^{\,2} + M^2(\vec{k}_\perp^{\,2})$, where $M(\vec{k}_\perp^{\,2})$ is proportional to the instanton density $M^2(\vec{k}_\perp^{\,2}) \sim \frac{N}{V}$. $M^2(\vec{k}_\perp^{\,2})$ is not const. because the instanton field is not homogeneous.

Conclusion.

$Q^2 \gg 1$. Leading Twist Behavior

$\Psi(\vec{k}_\perp x)_{QCD}$

Instantons → Explicit mechanism for chiral symmetry breaking. $M(\vec{k}_\perp^2)$ through Instantons

$Q^2 \lesssim 1$

Description in terms of Quark Model Ψ_{CQM}