DYSON-SCHWINGER EQUATIONS FOR HEAVY QUARKS

[C.J.B. & D.-S. LIU : PHYS. REV. D55 (1997) 367
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DYSON-SCHWINGER EQUATIONS + **HEAVY QUARK EFFECTIVE THEORY**

DYSON-SCHWINGER EQUATIONS

→ LIGHT QUARK PROPAGATORS

- MECHANISM FOR DχSB
  CONFINEMENT
- APPLICATIONS TO MESON SPECTROSCOPY, π & K FORM FACTORS, etc.

HEAVY QUARK EFFECTIVE THEORY

→ $m_Q \to \infty$ LIMIT OF QCD

- HEAVY QUARK SYMMETRY
- APPLICATIONS TO B & D MESON PHYSICS.

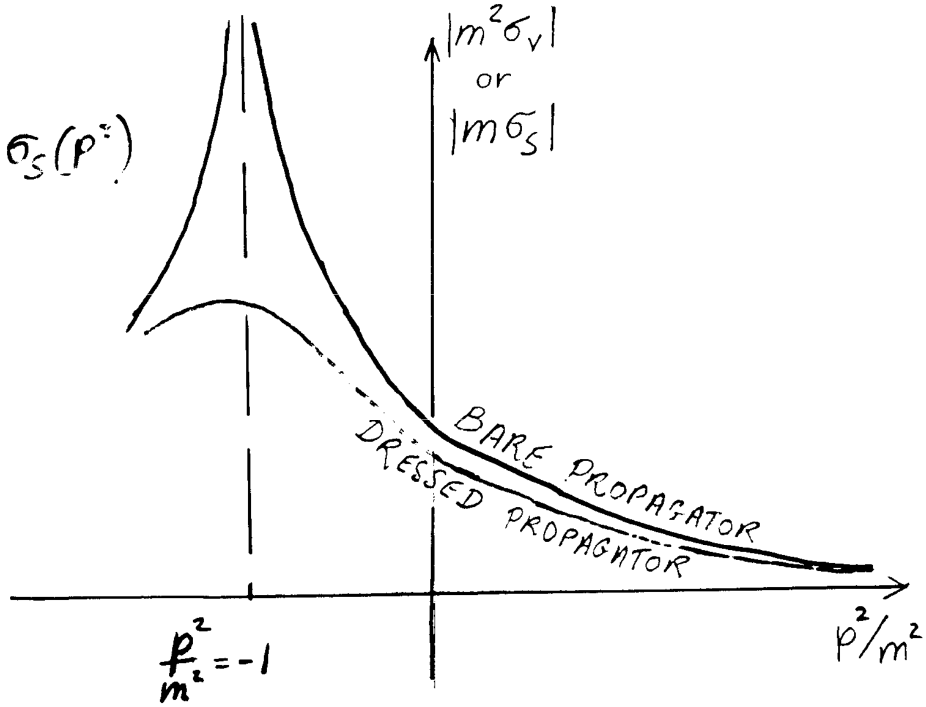
↓

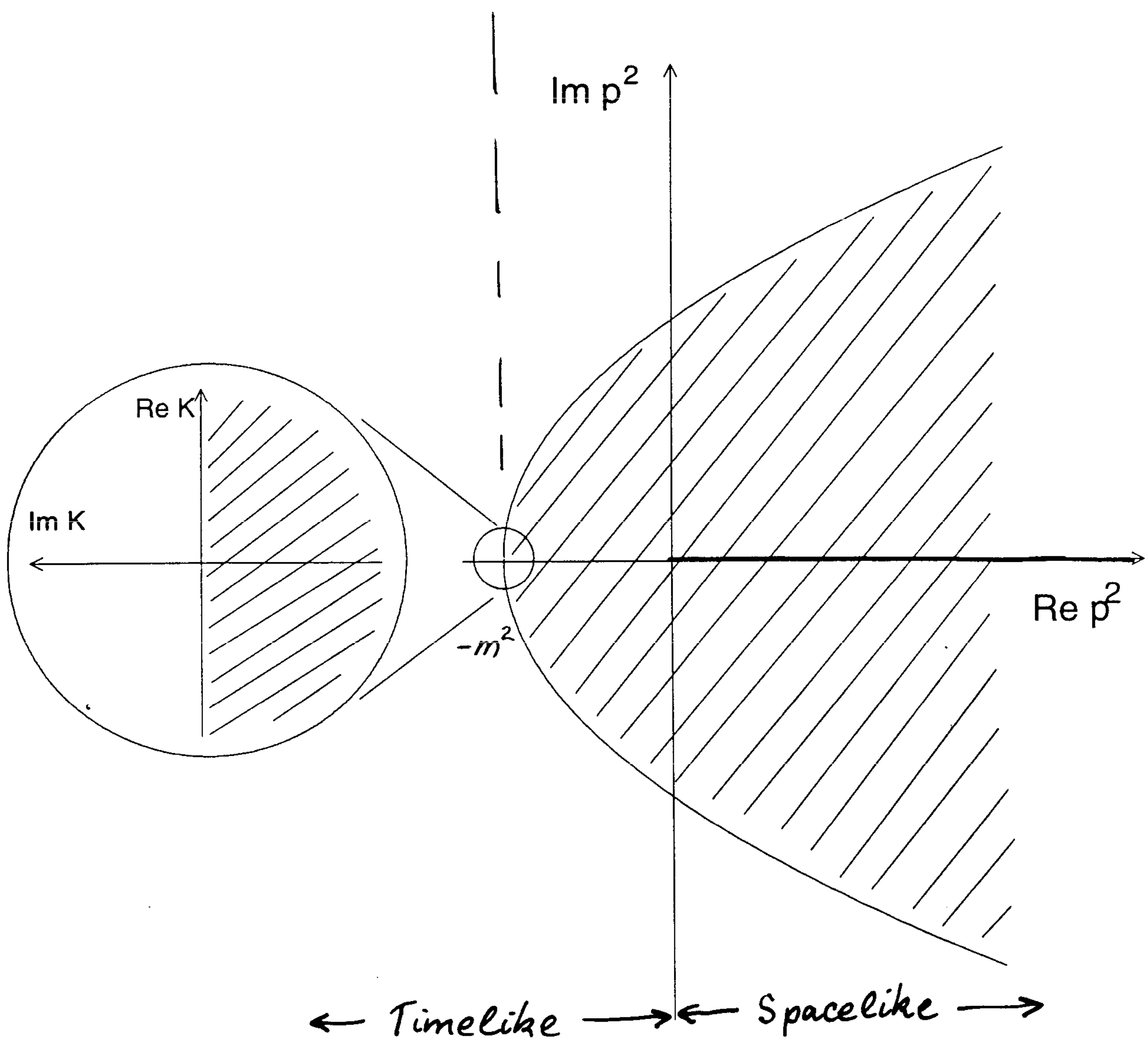
A NON-PERTURBATIVE TREATMENT OF HEAVY QUARK PROPAGATORS

- CONFINEMENT ?
- B & D MESON PHYSICS ?

$$S(p) = -i\not{p}\,\sigma_V(p^2) + \sigma_S(p^2)$$

$$S^{BARE}(p) = \frac{-i\not{p} + m}{p^2 + m^2}$$





Renormalised Dyson-Schwinger equation:

$$\boxed{\begin{aligned}&\Sigma'(p,\Lambda) = Z_1(\mu^2,\Lambda^2)\frac{4g^2}{3}\int^{\Lambda}\frac{d^4q}{(2\pi)^4}\,D_{\mu\nu}(p-q)\gamma_\mu S(q)\Gamma_\nu(q,p).\\ &\text{where}\\ &\qquad\Sigma'(p,\Lambda) = i\gamma\cdot p\left[A'(p^2,\Lambda^2)-1\right]+B'(p^2,\Lambda^2).\end{aligned}}$$

Renormalised quark propagator:

$$\boxed{S(p,\mu) = \frac{1}{i\gamma\cdot pA(p^2,\mu^2)+B(p^2,\mu^2)}}$$

$$= \frac{1}{Z_2(\mu^2,\Lambda^2)[i\gamma\cdot p+m_0(\Lambda)]+\Sigma'(p,\Lambda)}.$$

The renormalisation scale is set such that

$$S(p)\big|_{p^2=\mu^2} = \frac{1}{i\gamma\cdot p+m_R(\mu^2)}.$$

Choose Landau gauge

$$g^2 D_{\mu\nu}^{\text{Landau}}(k) = \left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\Delta(k^2).$$

and 'Abelian approximation':

$$Z_1 = Z_2.$$

Take heavy quark limit by setting

$$\boxed{p_\mu = i m_R v_\mu + k_\mu, \qquad v_\mu = (0,0,0,1),}$$

$$A(p^2,\mu^2) = 1 + \frac{\Sigma_A(K,\kappa)}{m_R}, \quad B(p^2,\mu^2) = m_R + \Sigma_B(K,\kappa),$$

where the new independent momentum variable is

$$K = \frac{p^2 + m_R^2}{2 i m_R} = k_4 + \frac{k^2}{2 i m_R},$$

and renormalisation point

$$\kappa = \frac{\mu^2 + m_R^2}{2 i m_R},$$

$\Rightarrow$ heavy quark propagator

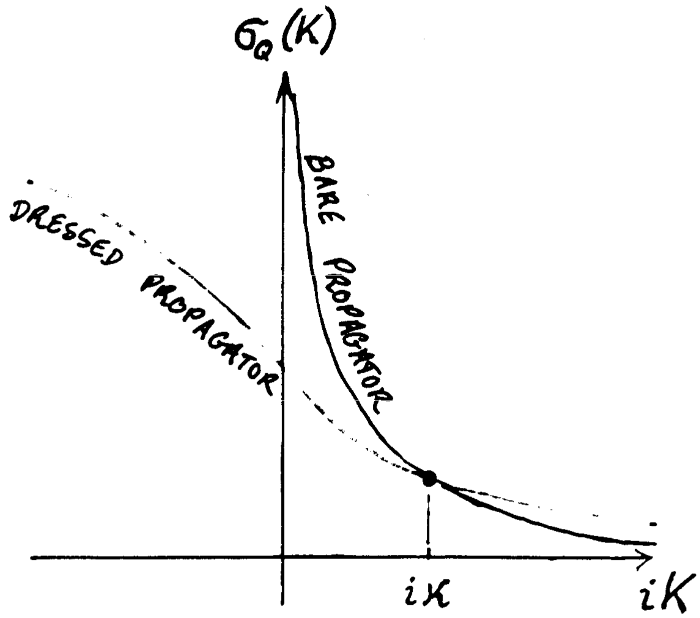
$$\boxed{S(p,\mu) = \frac{1+\gamma_4}{2}\frac{1}{iK + \Sigma(K,\kappa)} + O\left(\frac{1}{m_R}\right) = \frac{1+\gamma_4}{2}\sigma_Q(K,\kappa)}$$

where

$$\Sigma(K,\kappa) = \Sigma_B(K,\kappa) - \Sigma_A(K,\kappa).$$

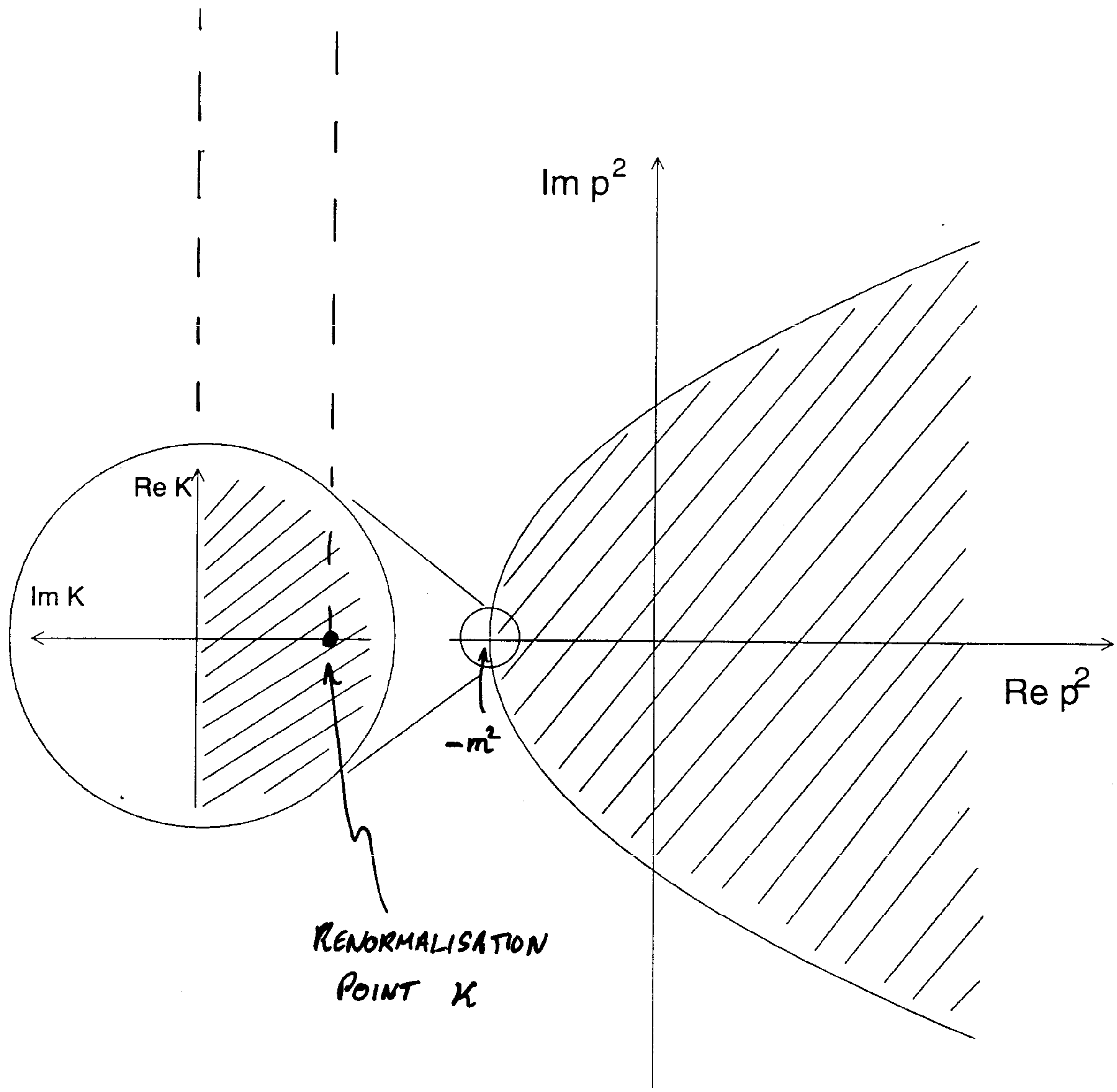
The renormalisation condition becomes

$$S(p,\mu)\big|_{p^2=\mu^2} = \frac{1+\gamma_4}{2}\frac{1}{iK}\bigg|_{K=\kappa} + O\left(\frac{1}{m_R}\right).$$



$$S(p) = \frac{1+\gamma_4}{2}\,\sigma_Q(K) + \cancel{O\left(\frac{1}{m}\right)}$$

$$S^{BARE}(p) = \frac{1+\gamma_4}{2}\,\frac{1}{iK} + \cancel{O\left(\frac{1}{m}\right)}$$



Renormalised, heavy quark Dyson-Schwinger equation (rainbow approximation, $\Gamma_\mu(q,p) = \gamma_\mu$):

$$\boxed{\begin{aligned}\Sigma(K,\kappa) &= \frac{4}{3}\int^{\Lambda} \frac{d^4k'}{(2\pi)^4} \frac{1}{ik_4' + k'^2/(2m_R) + \Sigma(k_4',\kappa)} \\ &\times |\mathbf{k}'|^2 \left\{ \frac{\Delta\left[(K-k_4')^2 + |\mathbf{k}'|^2\right]}{(K-k_4')^2 + |\mathbf{k}'|^2} - \frac{\Delta\left[(\kappa-k_4')^2 + |\mathbf{k}'|^2\right]}{(\kappa-k_4')^2 + |\mathbf{k}'|^2} \right\}.\end{aligned}}$$

Assuming gluon propagator decays as fast as

$$\Delta(k^2) \sim \frac{4\pi^2 d}{k^2} \quad \text{for } k^2 > m_t^2,$$

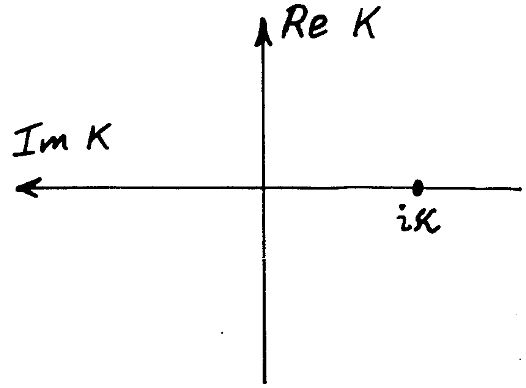
and assuming a hierarchy of scales

$$m_t,\ |K|,\ |\kappa| \ll m_R \ll \Lambda,$$

then Σ is independent of the cutoff Λ and

$$\Sigma(K,\kappa) \sim 2id(\kappa - K)\ln\left(\frac{m_R}{m_t}\right) \quad \text{as } m_R \to \infty.$$

The renormalisation point κ is chosen on negative imaginary K axis:



Recall renormalisation condition is

$$\Sigma(\kappa,\kappa)=0,\ \text{ i.e. } \sigma_Q(\kappa,\kappa)=\frac{1}{i\kappa}$$

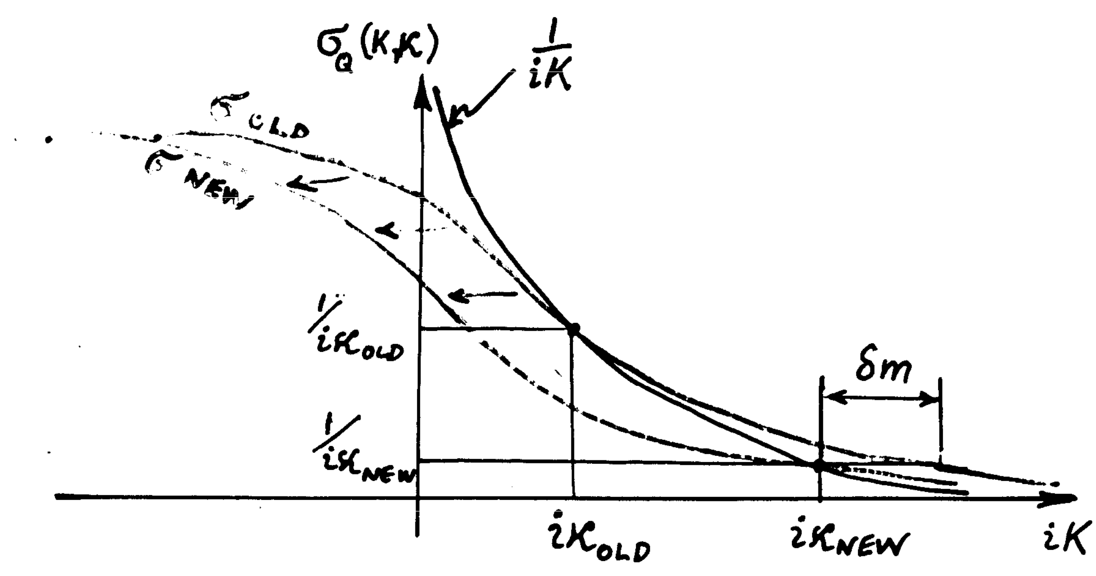
The effect of changing the renormalisation point $\kappa_{\mathrm{old}} \to \kappa_{\mathrm{new}}$ is a shift along the imaginary K axis:

$$\sigma_Q(K,\kappa_{\mathrm{new}})=\sigma_Q(K-i\delta m,\kappa_{\mathrm{old}})$$

where δm is the solution to

$$\sigma_Q(\kappa_{\mathrm{new}}-i\delta m,\kappa_{\mathrm{old}})=\frac{1}{i\kappa_{\mathrm{new}}}$$

... δm is the "residual mass" of HQET.



	Rainbow approx. $\Gamma_\mu(q,p) = \gamma_\mu$	Ball-Chiu vertex $\Gamma_\mu(q,p) = \Gamma_\mu^{\rm BC}+$ t'verse
Gaussian gluon propagator	Propagator poles interfere with solution to heavy-light BSE	Heavy quark DSE insensitive to t'verse piece; No improvement in analytic structure of σ_Q
Frank & Roberts gluon propagator	Significant shift in propagator poles deeper into complex plane; BSE not yet done	Not done

$$g^2 D_{\mu\nu}^{\rm Landau}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \Delta(k^2).$$

Gaussian gluon propagator:

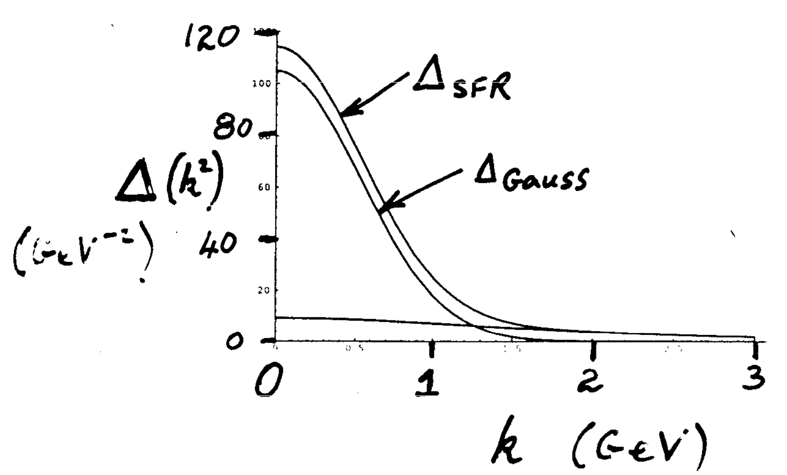
$$\Delta_{\rm Gauss}(k^2) = (2\pi)^4 \frac{m_t^2 d}{\alpha^2\pi^2} e^{-k^2/\alpha}.$$

Frank & Roberts propagator:

$$\Delta_{\rm FR}(k^2) = 4\pi^2 d \left[4\pi^2 m_t^2 \delta^4(k) + \frac{1 - e^{-k^2/(4m_t^2)}}{k^2}\right].$$

Smeared Frank & Roberts propagator:

$$\Delta_{\rm SFR}(k^2) = (2\pi)^4 \frac{m_t^2 d}{\alpha^2\pi^2} e^{-k^2/\alpha} + 4\pi^2 d \frac{1 - e^{-k^2/(4m_t^2)}}{k^2}.$$



$d = \dfrac{12}{33 - 2n_f}$

$m_t = 0.69$ GeV

$\alpha = 0.564$ GeV^2

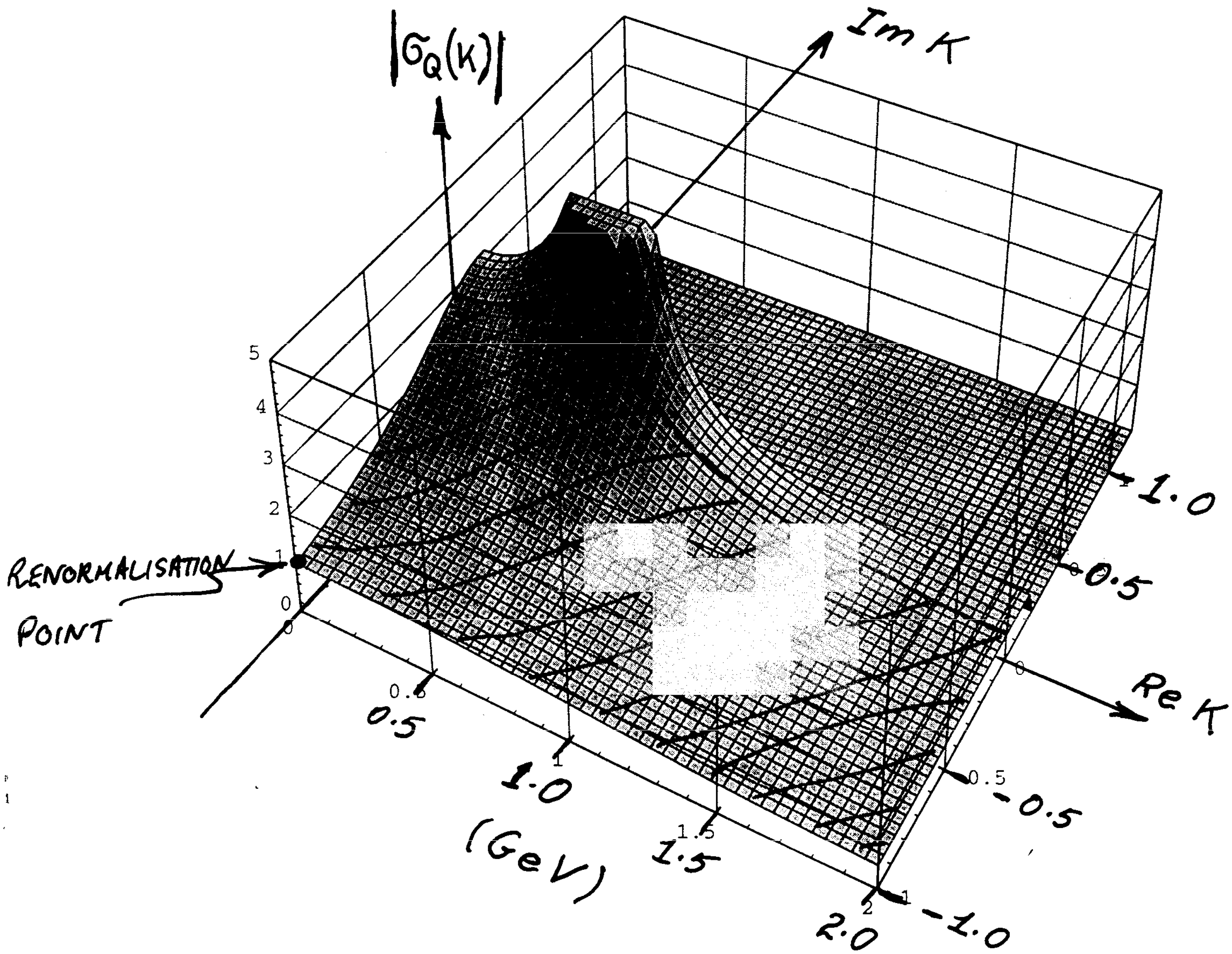
Gaussian gluon propagator:

$$\Delta_{\text{Gauss}}(k^2) = (2\pi)^4 \frac{m_t^2 d}{\alpha^2 \pi^2} e^{-k^2/\alpha}.$$

Rainbow approximation: $\Gamma(q,p) = \gamma(q,p)$

Renormalisation point: $i\kappa = 1$ GeV.

Ren. heavy quark mass: $m_R = 5$ GeV.



Frank & Roberts gluon propagator:

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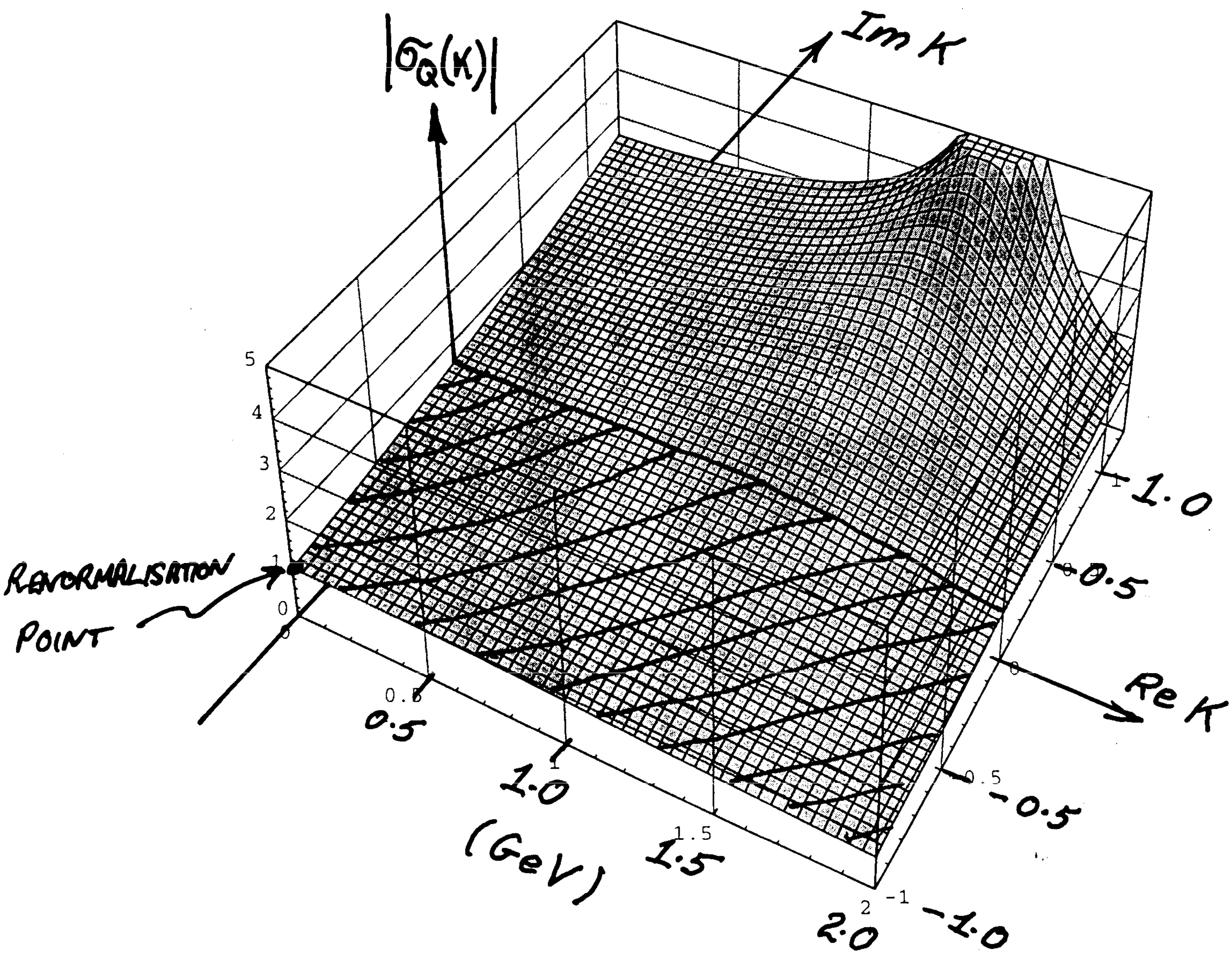

Future plans:

- Bethe Salpeter equation for heavy quark/light antiquark mesons; B and D meson masses and decay constants.
- Gauge parameter dependence of pole positions with Ball-Chiu vertex.
- Heavy fermion limit of confined heavy-heavy system as a Schrödinger equation; interpretation of propagator poles.