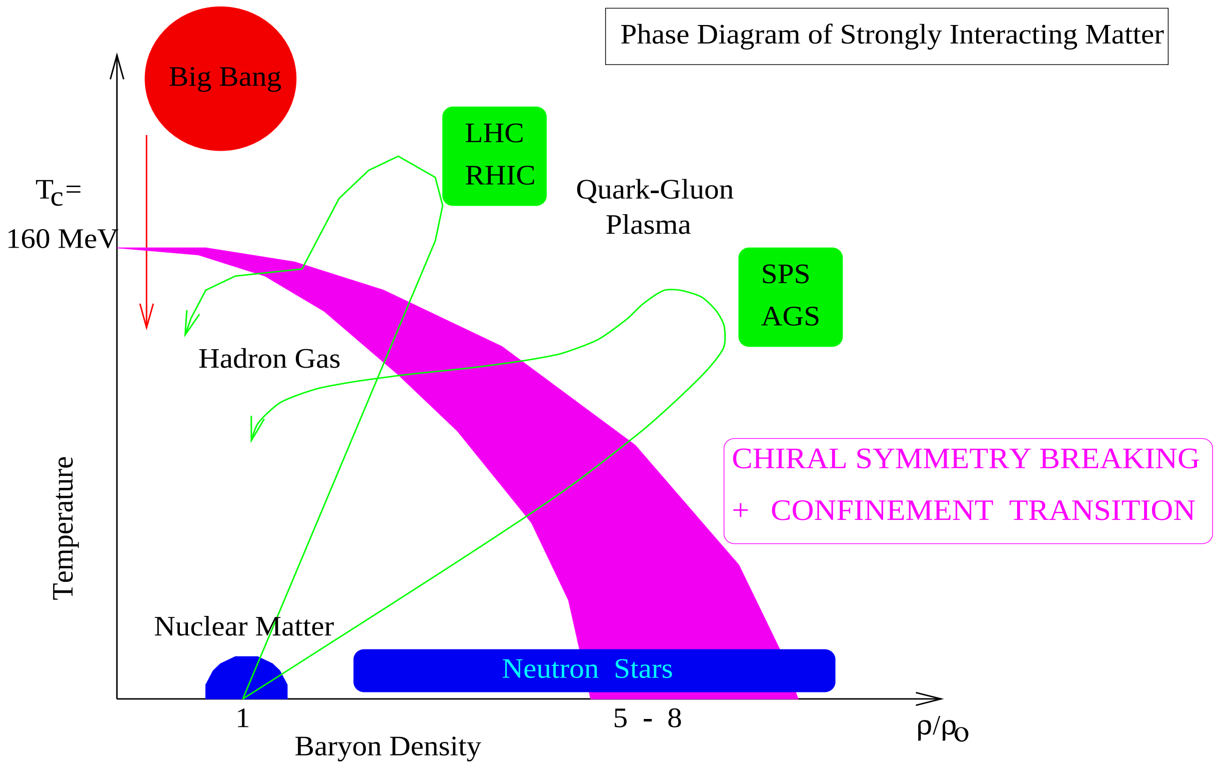
Thermodynamics of Quark Deconfinement

D. Blaschke, University of Rostock, Germany



1. Modelling QCD @ Finite T, μ
2. Order Parameter for Confinement and DχSB
3. Thermodynamics of a Simple, Confining Model

Lutsen, August 18, 1997

Effective QCD at finite Temperature

Generating functional (Euclidean: $\int_x = \int_0^{1/T} \int_V d^3x$)

$$Z[\eta,\bar\eta,j] = \int \mathcal{D}q\mathcal{D}\bar q \exp\{ \int_x [\bar q(i\gamma^\mu\partial_\mu - m_0)q + \bar q\gamma^0\mu q - \bar\eta q - \bar q\eta] + W[j]\}$$

Problem: Generating functional $W[j]$ - unknown,
Formal expansion:

$$W[j] = W_0 + \int_x D_\mu^a(x) j_\mu^a(x) + \underline{\int_{x,y} D_{\mu\nu}^{ab}(x,y) j_\mu^a(x) j_\mu^b(y)} + \dots$$

Color current: $j_\mu^a(x) = g\bar q(x)\gamma_\mu \frac{\lambda^a}{2} q(x)$
Gluon propagator - Model

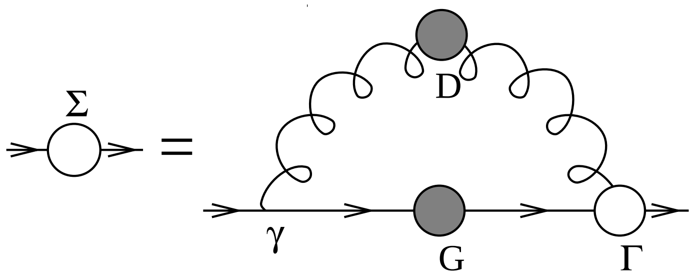
$$g^2 D_{\mu\nu}^{ab}(x,y) = \delta^{ab} P_{\mu\nu} D(x-y)$$

Model	$D(x)$	$D(k)$	χSB	Conf	AF	T,μ
NJL	$G\delta^4(x)$	const	+	-	-	[1]
MN	const	$\eta^2\delta^4(k)$	+	+	-	[2]
FR	$m^2 + \frac{e^{-m^2 x^2}}{x^2}$	$4\pi^2 m^2\delta^4(k) + \frac{1-e^{-k^2/4m^2}}{k^2}$	+	+	$+^*$	[3],-

[1] T. Hatsuda, T. Kunihiro, Phys Rep **247** (1994) 221
[2] D. B., S. Schmidt, C.D. Roberts, nucl-th/9706070
[3] A. Bender, D.B., Yu. Kalinovsky, C.D. Roberts, PRL **77** (1996) 3724

Dyson-Schwinger Eqn. (DSE) @ T$\neq$ 0

Self Energy



$$\Sigma(\vec{p},\omega_n) = \frac{1}{4}g^2 \int_{m,q}^{\bar{\Lambda}} \gamma_\mu D_{\mu\nu}(\vec{p}-\vec{q},\omega_n-\omega_m) S(\vec{q},\omega_m)\Gamma_\nu(\vec{p},\omega_n;\vec{q},\omega_m)$$

$$\int_{m,q}^{\bar{\Lambda}} \equiv T \sum_{m=-\infty}^{\infty} \int_0^{\bar{\Lambda}} \frac{d^3q}{(2\pi)^3}$$

Rainbow-Ladder-Approximation: $\Gamma_\nu \rightarrow \gamma_\nu$
Model Gluon Propagator (PRL **77** (96) 3724)

$$D^L(\vec{p},\Omega_n) \propto \frac{2\pi}{T} m_t^2 \delta^3(\vec{p})\delta_{0,n} + \frac{1 - \mathrm{e}^{\left[-(p^2+\Omega^2+m_D^2)/(4m_t^2)\right]}}{p^2+\Omega^2+m_D^2}$$

$$D^T(\vec{p},\Omega_n) \propto \frac{2\pi}{T} m_t^2 \delta^3(\vec{p})\delta_{0,n} + \frac{1 - \mathrm{e}^{\left[-(p^2+\Omega^2)/(4m_t^2)\right]}}{p^2+\Omega^2}$$

DSE $\rightarrow$ Renormalized (inverse) Quark Propagator

$$\begin{aligned} S^{-1}(\vec{p},\omega_n) &= i(\vec{\gamma}\vec{p}+\gamma_4\omega_n) + m_0 + \Sigma(\vec{p},\omega_n) \\ &= iA(\vec{p},\omega_n)\vec{\gamma}\vec{p} + iC(\vec{p},\omega_n)\gamma_4\omega_n + B(\vec{p},\omega_n) \end{aligned}$$

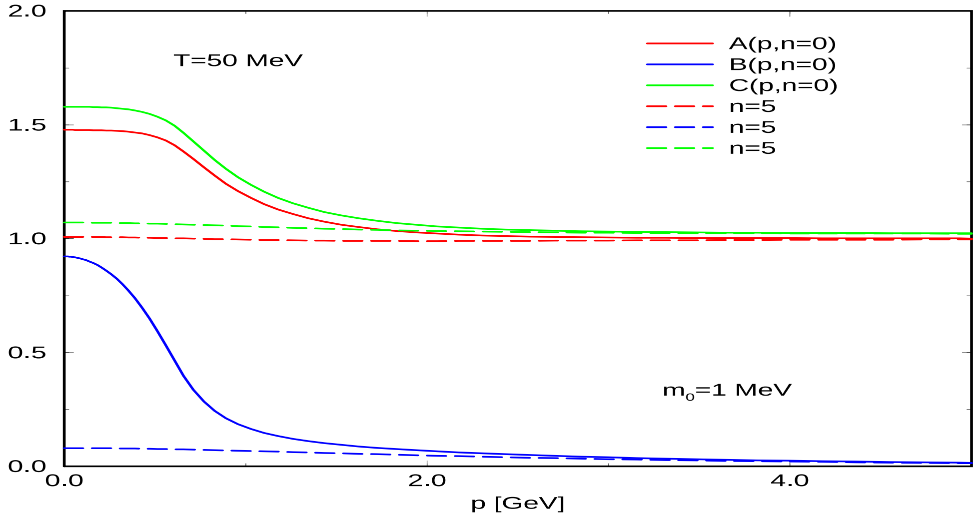
Analytic Properties of the A, B, C functions signal:

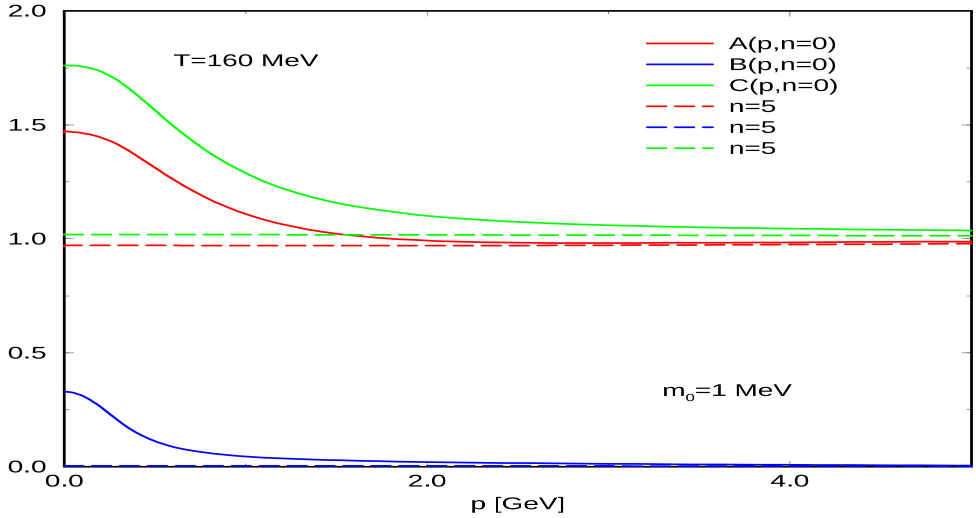
1. Quark Confinement:
   No Poles in S for real p^2! (sufficient condition)

2. Dynamical Chiral Symmetry Breaking (DχSB):
   $B(\vec{p}=0,\omega_0) \neq m_0$ (order parameter for DχSB)

Numerical Solutions for A, B, C @ $T \neq 0$

$T \neq 0$ generalization of the Frank/Roberts model





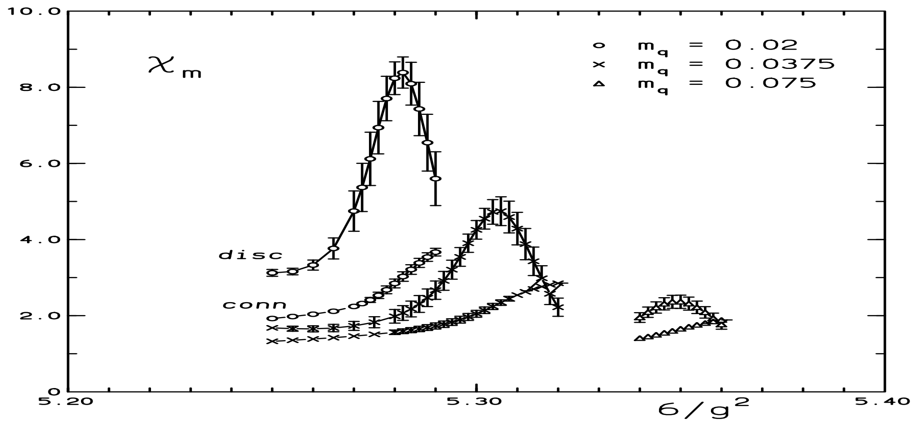
Note:

- ⊙ Nonperturbative effects dominated by n=0 Mode
- ⊙ $A(p, \omega_n) \neq C(p, \omega_n)$
- ⊙ Persistence of $A, C \neq 1$ above $T_c \sim 150$ MeV

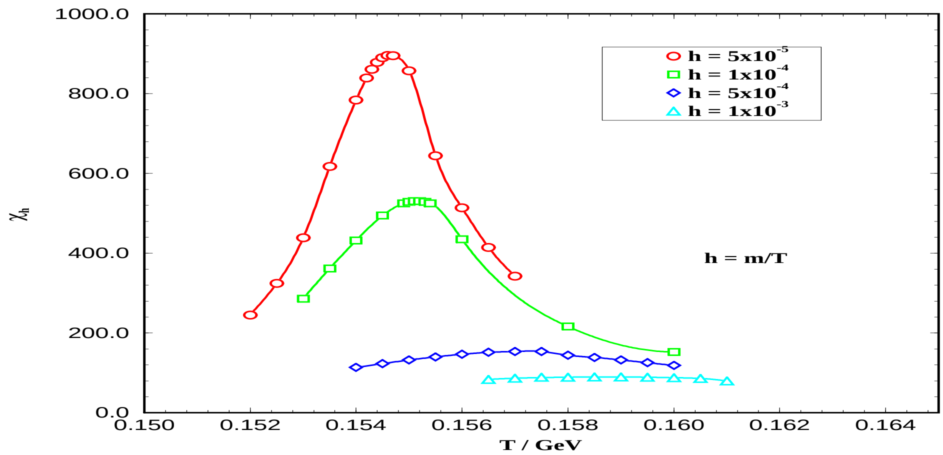
Susceptibilities at the Phase Transition

Chiral: $\chi_m = T \frac{\partial^2}{\partial m_q^2} \ln Z = \frac{4}{N_f} \frac{\partial}{\partial m_q} \langle \bar{q} q \rangle$

Lattice QCD: Karsch/Laermann, PRD **50** (1994) 6954.



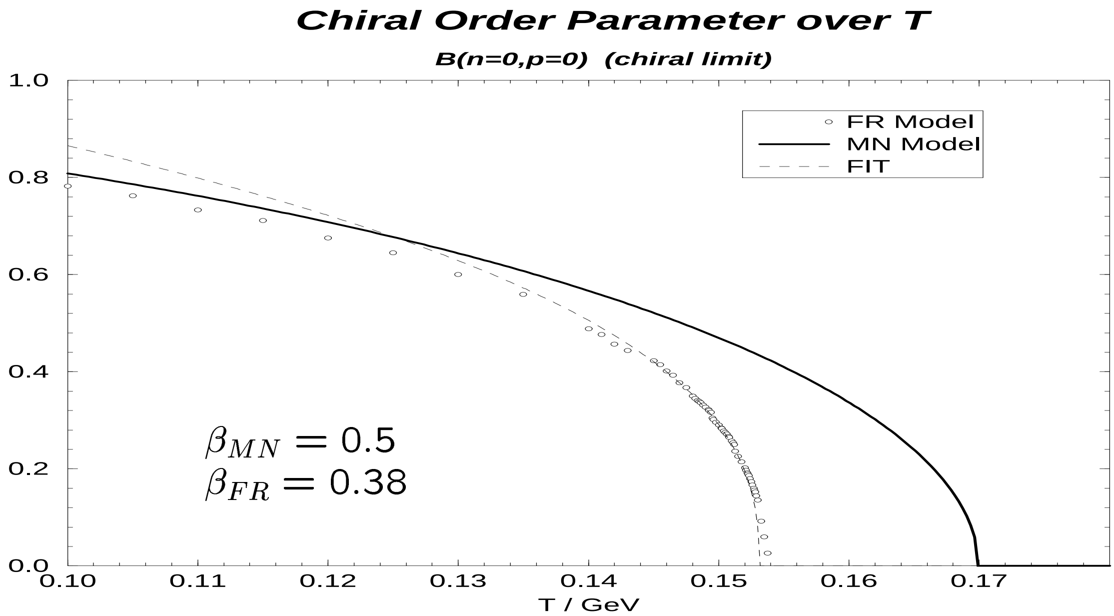
SDE-Approach to QCD:



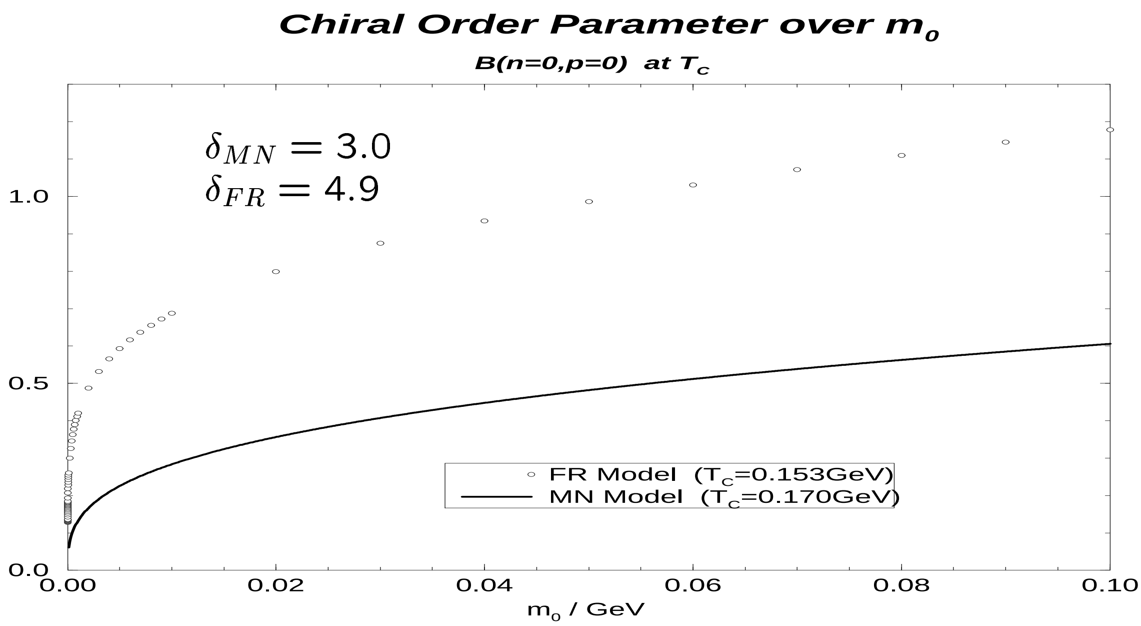
Thermal: $\chi_T = -T \frac{\partial^2}{\partial(1/T)\partial m_q} \ln Z = -\frac{4}{N_f} \frac{\partial}{\partial(1/T)} \langle \bar{q} q \rangle$

Chiral Order Parameter & Critical Exponents

Temperature dependence for $m_0 = 0$: $B = \alpha(1 - T/T_c)^{\beta}$

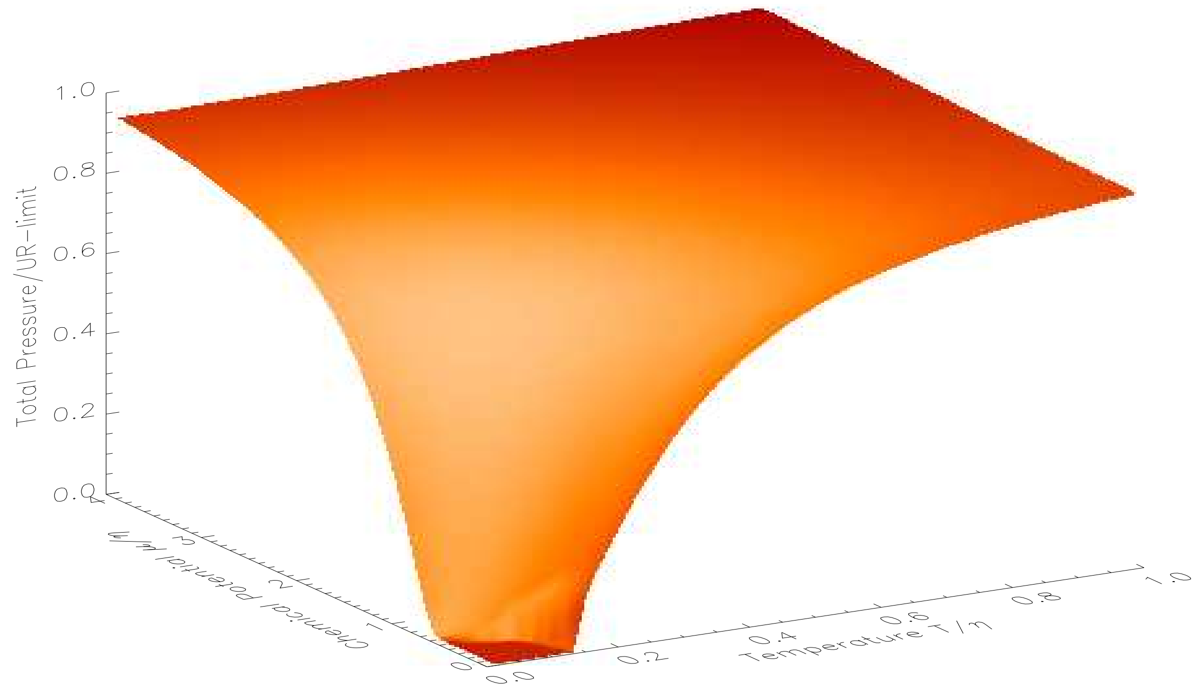


Bare mass dependence at $T = T_c$: $B = {m_0}^{1/\delta}$



Results: A. Höll, Diploma thesis, Rostock, 1997

Equation of State - Deconfined Quark Phase

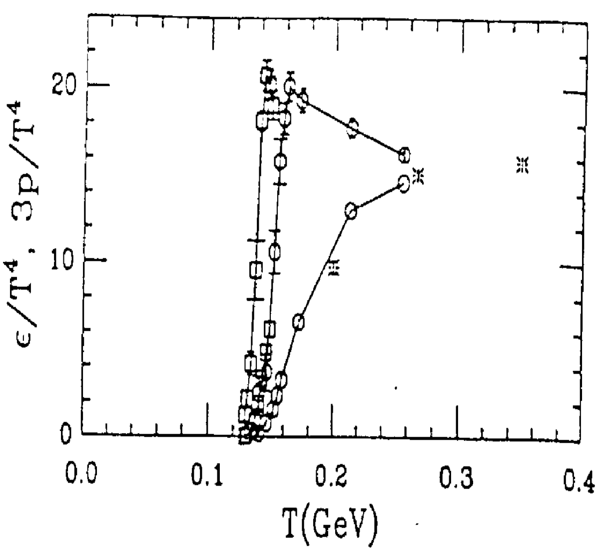


⊙ Large values of T and μ required in order to establish the ultrarelativistic limit of massless free quarks:

$$P_{UR} = N_c N_f \frac{1}{12\pi^2}\left(\mu^4 + 2\pi^2\mu^2T^2 + \frac{7\pi^4}{15}T^4\right)$$

⊙ Interactions modify the free quark pressure

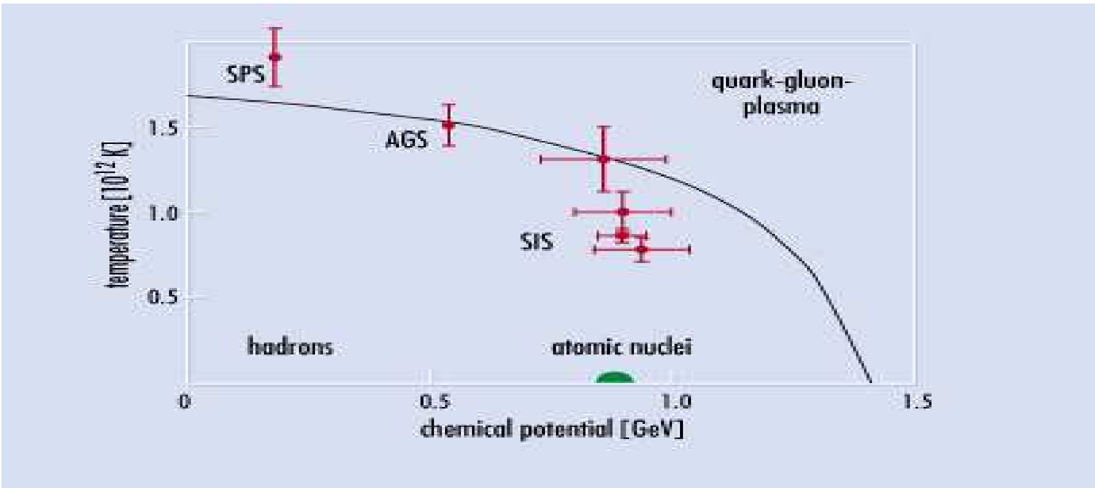
⊙ Extrapolation to the whole (T,μ) - plane! Lattice Simulations of QCD have access only to T-axis



Equation of State - Confinement Phase

Results:
⊙ second order phase transition at $\mu = 0$
⊙ first order at nonzero chemical potential
⊙ critical temparature at $\mu = 0$: $T_c \simeq 0.17$ GeV
⊙ critical μ at $T = 0$: $\mu_c \simeq 0.3$ GeV

Influence of the gluon propagator tail:
⊙ $T_c \simeq 0.15$ GeV (Bender et al: PRL 77 (1996) 3724)
⊙ $\mu_c \simeq 0.4$ GeV (G. Poulis, at "Few Body XV", 1997)



Resumé

- Confining gluon propagator is IR enhanced
  $\rightarrow$ Model: $\delta-$ function in p-space
  $\rightarrow$ parameter-free $T \neq 0$ generalization (Matsubara)

- Selfconsistent (iterative) solution of the quark SDE in rainbow approx.:
  $\rightarrow$ $A, B, C-$ functions: $A \neq C \neq 1$!

- Chiral and deconfinement transition temperature coincide: $T_\chi = T_d = 152$ MeV
  Critical index for the transitions NOT meanfield: $\beta \sim 0.38$

- Reproduces qualitatively Lattice QCD results (susceptibility, chiral order parameter, $T_c \sim 150$ MeV, coincident chiral and deconfinement transitions)

- Pion decay constant: $f_\pi(T \rightarrow T_\chi) \rightarrow 0$
  Pion bound state pole vanishes at temperatures $T \stackrel{<}{\sim} T_d$

- Equation of state for quark matter in the SDE approach (Munczek-Nemirovsky model)
  $\rightarrow$ T, μ $-$ dependent vacuum pressure (Bag function)
  $\rightarrow$ Dynamical correlations for $T > T_d$

Outlook

- Inclusion of hadronic states (pion, rho-meson, ...)

- Calculation of processes (loop integrals) within the confining SDE approach at $T, \mu \neq 0$, application: dilepton production

- Gluonic degrees of freedom: gluon deconfinement at $T, \mu \neq 0$

- Comparison with and discussion of Lattice QCD

Support by DAAD and DFG is acknowledged.