Hadronic Light Front Field Theory - Chiral symmetry, Zero Modes and new way to understand nuclei

Light Front Field Theory -
quantize at equal $x^+ = t+z$ "time"
Space $x^- = t-z$, $p^+ = p^0 + p^3$, momentum

Motivation - EMC effect in nuclear Deep Inelastic Scattering, $x_{Bj} = \frac{p^+}{P_A^+}$

Goal: redo nuclear physics with $p^+, \vec{p}_\perp$

LF is Hamiltonian formalism, useful for many body problem. Covariant results any problem with high momentum

Outline

1. Light Front quantization of hadronic Lagrangian
2. Infinite nuclear matter in mean field approximation
3. Nuclear momentum content - nucleon, meson
4. Chiral Symmetry in πN scattering
5. Chiral Symmetry in NN scattering

Goal – Light Front calculation of nuclear wave function –

Use $(k^+, \vec{k}_\perp)$ and/or $(x^-, \vec{x}_\perp)$ $\qquad x^+ =$ light front time

notation: $A^\pm = A^0 \pm A^3$

$A \cdot B = \frac{1}{2}(A^+B^- + A^-B^+) - \vec{A}_\perp \cdot \vec{B}_\perp$

Hadronic Lagrangian (no π's yet):

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2) - \frac{1}{4} V^{\mu\nu} V_{\mu\nu} + \frac{m_v^2}{2} V^\mu V_\mu$$

$$+ \bar{\psi}' \{\gamma^\mu (i\partial^\mu - g_v V^\mu) - (M + g_s \phi)\} \psi'$$

$$V^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu \qquad \begin{pmatrix} \phi \to \text{attraction} \\ V^\mu \to \text{repulsion} \end{pmatrix}$$

Low energy $\boxed{\text{effective}}$ theory (up to ~2-3 GeV)

Walecka model of nucl. physics

≈ good phenomenology (mean field approx)

(so far solved w. equal time formulation)

'NOW ↓

formalism Light Front Quantization ~ Soper 1971 Yan & Co. 1973

Glazek & Shakin GS nuclear matter w. scalar mesons PRC44, 1012 (91)

Light Front Field Equations - do it later

$$\gamma^\mu (i\partial_\mu - g_v V_\mu)\psi' = (M + g_s \phi)\psi'$$

$$\gamma^\mu i\partial_\mu = \gamma^\mu p_\mu = \tfrac{1}{2}(\gamma^+ i\partial^- + \gamma^- i\partial^+) - \gamma_\perp \cdot p_\perp$$

Only 2 (of 4) D. of F. Dynamical

$$\Lambda_\pm \equiv \frac{\gamma^0 \gamma^\pm}{2} \qquad \psi'_\pm \equiv \Lambda_\pm \psi'$$

ψ'_- is a constrained field

$$(i\partial^- - g_v V^-)\psi'_+ = (\vec{\alpha}_\perp \cdot (\vec{p}_\perp - g_v \vec{V}_\perp) + \beta(M + g_s\phi))\psi'_-$$

$$(i\partial^+ - \underline{g_v V^+})\psi'_- = (\vec{\alpha}_\perp \cdot (\vec{p}_\perp - g_v \vec{V}_\perp) + \beta(M + g_s\phi))\psi'_+$$

Can't set $V^+ = 0$ by simply choosing gauge $m_v^2 \neq 0$

Soper-Yan:

$$\psi' = e^{-ig_v \Lambda(x)}\psi$$

$$\partial^+ \Lambda(x) = V^+(x)$$

$$(i\partial^+ - g_v V^+)\psi'_- = e^{-ig_v \Lambda(x)}\, i\partial^+ \psi_-$$

$$\Rightarrow \quad (i\partial^- - g_v \bar{V}^-)\psi_+ = (\vec{\alpha}_\perp \cdot (\vec{p}_\perp - g_v \vec{\bar{V}}_\perp) + \beta(M + g_s\phi))\psi_-$$

$$i\partial^+\psi_- = (\vec{\alpha}_\perp \cdot (\vec{p}_\perp - g_v \vec{\bar{V}}_\perp) + \beta(M + g_s\phi))\psi_+$$

$$\bar{V}^\mu = V^\mu - \partial^\mu \Lambda = V^\mu - \frac{\partial^\mu}{\partial^+} V^+$$

$$\partial_\mu V^{\mu\nu} + m_v^2 V^\nu = g_v \bar{\psi}\gamma^\nu \psi$$

$$(\partial_\mu \partial^\mu + m_s^2)\phi = -g_s \bar{\psi}\psi$$

V^μ enters meson field eqns

$\bar{V}^\mu$ " fermion " eqn.

$$\partial_\mu V^{\mu\nu} + m_v^2 V^\nu = g_v \bar{\Psi} \gamma^\nu \Psi$$

$$(\partial_\mu \partial^\mu + m_s^2)\, \phi = g_s \bar{\Psi} \Psi$$

Mean Field Approx^{MFA}. for infinite nuclear matter, at rest

$g_v \bar{\Psi} \gamma^\nu \Psi$, $g_s \bar{\Psi} \Psi$ large $\Rightarrow$ many mesons; classical?

Volume $\to \infty$, all positions equivalent

MFA – nucleons in plane waves (Fermi gas)

$$g_v \bar{\Psi}(x) \gamma^\nu \Psi(x) \to g_v \langle \bar{\Psi}(x) \gamma^\nu \Psi(x) \rangle \Rightarrow \text{constant}^\nu$$

$$g_s \bar{\Psi}(x) \Psi(x) \to g_s \langle \bar{\Psi}(x) \Psi(x) \rangle \Rightarrow \text{constant}$$

constant ϕ, V^μ are solutions, zero modes

$$\phi(x) = -\frac{g_s}{m_s^2} \langle \bar{\Psi} \Psi \rangle$$

$$V^\mu(x) = -\frac{g_v}{m_v^2} \langle \bar{\Psi} \gamma^\mu \Psi \rangle = -\frac{g_v}{m_v^2} \rho_B \, \delta^{\mu 0}$$

$\bar{V} = V = V^0$, all other V's $= 0$

$$(i\partial^- - g_v \bar{V}^-)\psi_+ = \{ \vec{\alpha}_\perp \cdot \vec{P}_\perp + \beta(M + g_s\phi) \} \psi_-$$

$$i\partial^+\psi_- = \{\alpha_\perp \cdot \vec{P}_\perp + \beta(M + g_s\phi)\} \psi_+$$

$\psi \sim e^{ik\cdot x}$ choose mode functions such that

$$\boxed{(i\partial^- - g_v \bar{V}^-)\psi_+ = \frac{k_\perp^2 + (M + g_s\phi)^2}{k^+}\psi_+}$$

$\Rightarrow$ Plane waves Mass $= M + g_s\phi$

L.F. energy $i\partial^- =$ Kinetic term $+ g_v\bar{V}^-$

Solves Field eqns in MFA

consistent to build nucleus this way!

$$\boxed{\begin{aligned} \phi &= -\frac{g_s}{m_s^2}\langle\bar{\psi}\psi\rangle \\ V^\mu &= \delta^{\mu,0}\frac{g_v}{m_v^2}\rho_B \end{aligned}}$$

Theory solved in MFA

$$\to p^2 = p^- p^+ - p_\perp^2 = m^2 \Rightarrow p^- = \frac{(p_\perp^2 + m^2)}{p^+}$$

Momentum Content

Canonical symmetric energy momentum $T^{\mu\nu}$

$$P^\mu = \frac{1}{2}\int d^2x_\perp dx^- \;\langle T^{+\mu}\rangle ; \qquad \Omega = \frac{1}{2}\int d^2x_\perp dx^-$$

MFA

$$T^{+-} = m_s^2\phi^2 + 2\psi_+^\dagger\,(i\partial^- - g_v V^0)\,\psi_+$$

$$T^{++} = m_v^2 V_0^{\,2} + 2\psi_+^\dagger\, i\partial^+\,\psi_+ \;; \quad (V_0 = V^- = V^+)$$

↓ Field Eqns ↓

$$\frac{P^-}{\Omega} = m_s^2\phi^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+\,\frac{k_\perp^2 + (M+g_s\phi)^2}{k^+}$$

$$\frac{P^+}{\Omega} = m_v^2 V_0^{\,2} + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+\;k^+$$

$$\int_F \equiv |\vec{k}| < k_F \quad \text{with} \quad \boxed{k^+ = \sqrt{(M+g_s\phi)^2 + \vec{k}^2} + k^3}$$

$E = \frac{1}{2}(P^+ + P^-)$ same as Walecka

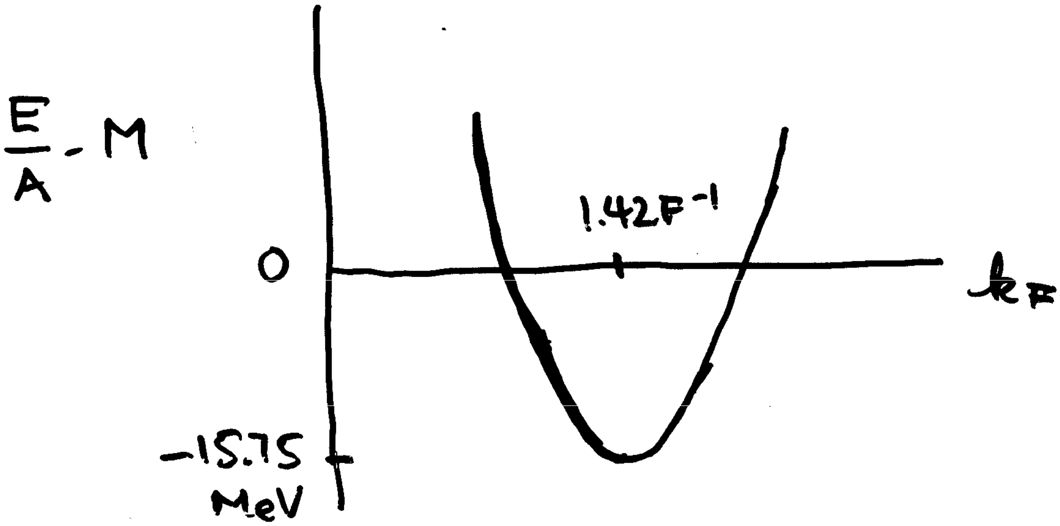
$\frac{\partial E}{\partial\phi} \Rightarrow$ field eq. for ϕ

For NM in rest frame

$P^+ = P^-$ follows from

$$\left(\frac{\partial E/A}{\partial k_F}\right)_\Omega = 0$$

$P^3 = 0$



Chin Walecka

Choose $g_v^2 M^2/m_v^2 = 195.9$, $g_s^2 M^2/m_s^2 = 267.1$

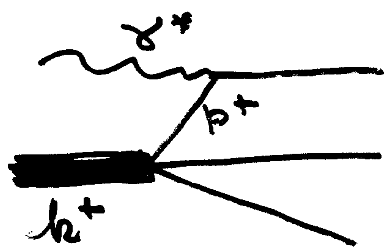
$\Rightarrow M + g_s\phi = 0.56\, M \quad \Rightarrow$

$$\frac{P^+}{\Omega} = m_v^2 V_0^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp\, dk^+\, k^+$$

$$= 0.35\,\frac{P^+}{\Omega} + 0.65\,\frac{P^+}{\Omega}$$

ONLY 65% of P^+ carried by nucleons (∞ matter)

(95% needed for EMC effect in Fe (F & S))

proton 

$$\Rightarrow \frac{F_{2A}(x)}{A} = \int dy\, f(y)\, F_{2N}(x/y)$$

$$\frac{x}{y} = \frac{p^+}{k^+}$$

+ Momentum of mesons

total $= 35\%$ $\bar{b}\bar{b}$ distribution?

ϕ, V^{μ} do NOT depend on x^-, $x_\perp$ (and x^+)

momentum distribution $\sim \delta(p^+)\,\delta(\vec{p}_\perp)$

Interpretation - infinite quanta at 0 Momentum - zero mode

$k^+_{meson} \Rightarrow 0$, quark in meson $p^+ = 0$

$x_{Bj} = 0 = \frac{Q^2}{2M\nu}$. requires ∞ γ^* energy

these MF mesons do <u>not</u> participate in deep inelastic scattering

Momentum sum rule - if p^+ momentum of nucleons is depleted in nuclei mesons are enhanced

Predictive power of Mom Sum Rule is WEAKENED

Is this <u>nuclear matter</u>, mean field approx result relevant?

Finite nuclei in mean field approx - P Blunden
Shell model - 2j+1 degeneracy
Beyond mfa, Bruckner theory LF - R Machleidt

↳ needs LF NN interaction

↳ needs LF πN interaction

Chiral NN, πN

Chiral Symmetry

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2) - \frac{1}{4} V^{\mu\nu} V_{\mu\nu} + \frac{m_V^2}{2} V^\mu V_\mu$$
$$+ \bar{\Psi}' (\gamma^\mu (i\partial_\mu - g_v V_\mu) - MU - g_s \phi) \Psi'$$
$$+ \frac{f^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{4} m_\pi^2 f^2 \mathrm{Tr}(U + U^\dagger - 2)$$
$$U \equiv e^{-i\gamma_5 \underset{\sim}{\tau} \cdot \underset{\sim}{\pi}/f}$$

Non-derivative πN coupling *

Invariance under $\quad \psi' \to e^{i\gamma_5 \underset{\sim}{\tau} \cdot \underset{\sim}{a}} \psi'$
$\qquad U \to e^{-i\gamma_5 \underset{\sim}{\tau} \cdot \underset{\sim}{a}}\, U\, e^{-i\gamma_5 \underset{\sim}{\tau} \cdot \underset{\sim}{a}}$

$\Rightarrow$ PCAC

* Start $\bar{\psi}' (\gamma_5 \gamma^\mu \underset{\sim}{\tau}) \psi' \partial_\mu \underset{\sim}{\pi} + \cdots$

transform to get rid of $\gamma^5 \gamma^+$ term in

$$(i\partial^+ - \gamma_5 \underset{\sim}{\tau}\, \partial^+ \underset{\sim}{\pi}) \psi_- = \cdots$$

$\Rightarrow$ $\underline{\mathcal{L}}$ $\underline{\text{above}}$ with U coupling

· Quantize – derive P^-

Example: $\underline{\text{Low energy}}$ πN scattering $\underline{\text{low energy}}$

pair suppression – must emerge

Chiral symm $\Rightarrow$ $\mathcal{M}_{if} \approx \delta_{if} \frac{m_\pi^2}{M_N} + i \epsilon_{ifn} \tau_n \frac{m_\pi}{M_N} (\,)$

$\pi_i + N \to \pi_f + N$ $\qquad$ soft π thm. with γ_5 coupling

Interaction

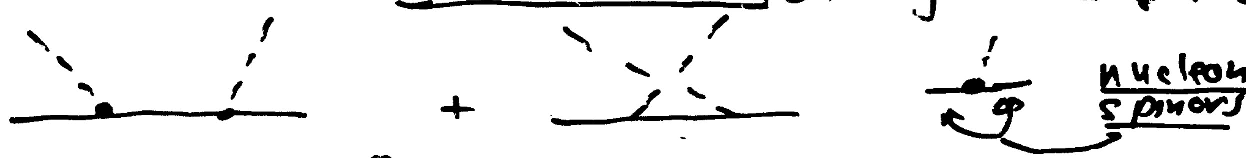
$$M U = M e^{-i\gamma_5 \underset{\sim}{\tau}\cdot\underset{\sim}{\pi}/f} \to M\left(1 - \frac{i\gamma_5}{f}\underset{\sim}{\tau}\cdot\underset{\sim}{\pi} - \frac{1}{2}\frac{\pi^2}{f^2}\right)$$

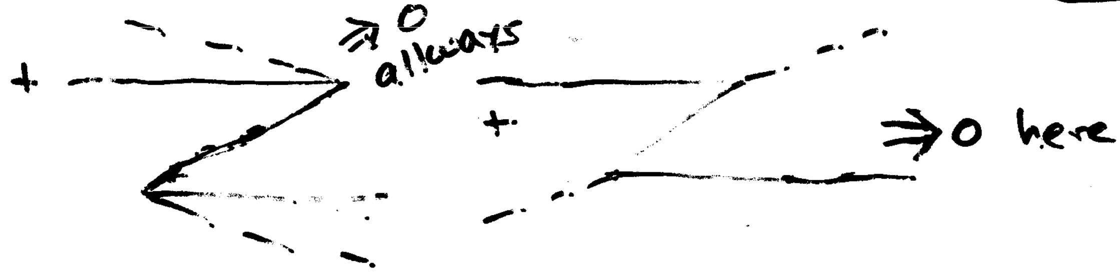
$$T^{+-} = \bar{\psi}\Big(-iM\frac{\gamma_5}{f}\underset{\sim}{\tau}\cdot\underset{\sim}{\pi} - \frac{M}{2f^2}\pi^2$$

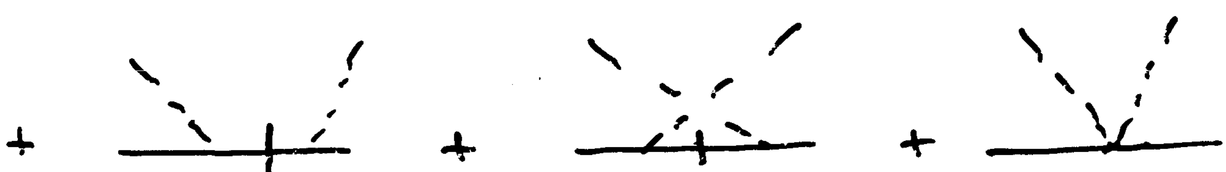
$$+\, i\gamma_5\frac{M}{f}\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\;\frac{\gamma^+}{2p^+}\;i\gamma_5\frac{M}{f}\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\Big)\psi$$

eliminate $\psi_- \propto \frac{1}{p^+}$ ...

Evaluate m – $\boxed{\text{time } x^+ \text{ order}}$ diagrams (tree)

$m =$ 

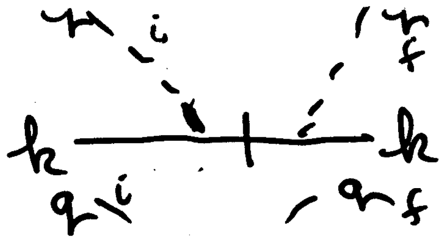




$$+ \Rightarrow \quad \frac{\gamma^+}{2p^+}\frac{M^2}{f^2} \qquad \frac{\gamma^+}{2p^+}\frac{M^2}{f^2} \qquad \frac{-M}{f^2}$$

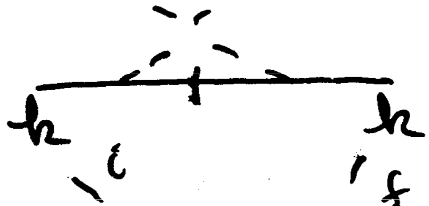
Red diagrams are p-wave scattering $\to 0$
Green diagrams are 0, by p^+ mom. conservati
at low energy

$\mathcal{M} =$



$$\tau_i \tau_f \, i\gamma_5 \frac{M}{f} \frac{\gamma^+}{2(q^+ + k^+)} \, i\gamma_5 \frac{M}{f}$$

$+$



$$\tau_f \tau_i \, i\gamma_5 \frac{M}{f} \frac{\gamma^+}{2(k^+ - q^+)} \, i\gamma_5 \frac{M}{f}$$

$+$

$$-\delta_{fi} \frac{M}{f^2}$$

Low energy limit $k^+ = M$, $q^+ = m_\pi$, $\gamma^+ \to 1$

$$\mathcal{M} = \delta_{if} \left[\left(\frac{1}{2(M+m_\pi)} + \frac{1}{2(M-m_\pi)} \right) \frac{M^2}{f^2} - \frac{M}{f^2} \right]$$

$$+ i\epsilon_{ifn}\tau_n \left[\frac{1}{2(M+m_\pi)} - \frac{1}{2(M-m_\pi)} \right] \frac{M}{f^2}$$

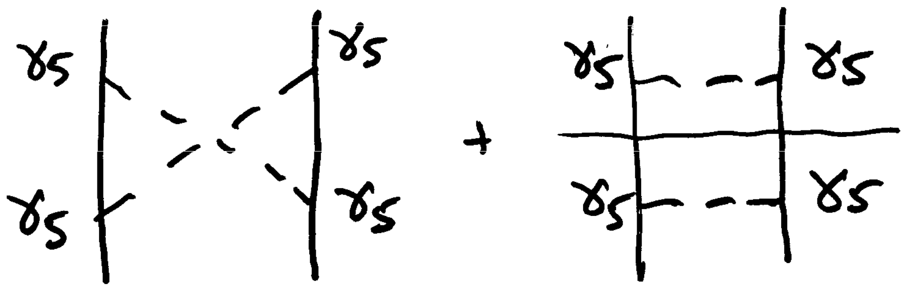
$$\approx \delta_{if} \frac{M^2}{f^2} \frac{m_\pi^2}{M^3} - i\epsilon_{ifn}\tau_n \frac{M^2}{f^2} \frac{m_\pi}{M^2}$$

= OLD SOFT π THEOREMS !

(for πN)

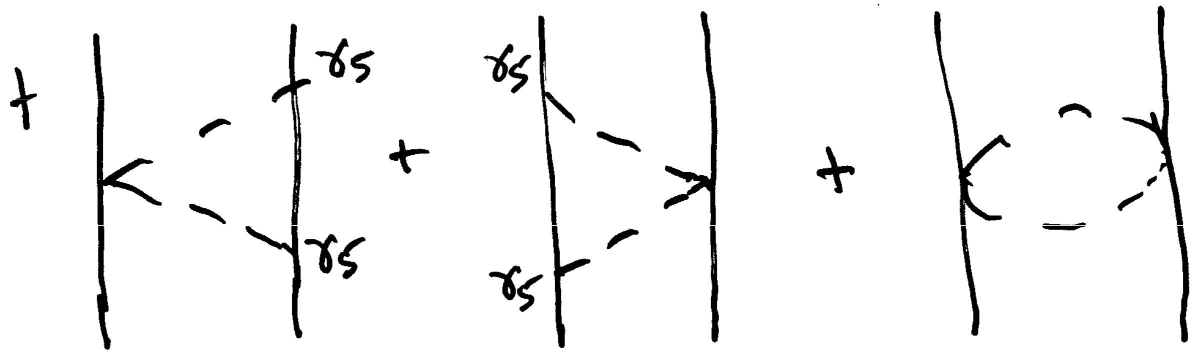
Two-pion exchange potential

(w. π, N) Sum of T.O. diagrams = Feynman diagram



Pantovi-Lomon

PL



Influence of chiral symmetry - reduces the attraction of PL

Summary

Light front field theory can handle πN scattering.

Can do πN, NN, Chiral then go to nuclear physics with correlations & relativity