SPECTRUM AND FORM FACTORS OF LIGHT MESONS

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npQCD modeling via Dyson-Schwinger Eqn Approach

▷ Questions:

- Properties of light mesons $\pi, K, \eta, \rho, \omega, \cdots$
- Interactions/decays as governed by their substructure
- Indirect information on how QCD must work in this non-perturbative region

Recent applications include :

1. Soft chiral observables $m_{\pi/K}$, $f_{\pi/K}$, $\pi\pi$ scattering lengths, $r^{em}_{\pi/K}$, ...
2. Charge form factors $F^{em}_{\pi}(Q^2)$ and $F^{em}_{K}(Q^2)$
3. $g_{\pi\gamma\gamma}$, $g_{\gamma\pi\pi\pi}$, $g_{\rho\pi\pi}$, $g_{\omega\pi\pi}$, $g_{\gamma\pi\rho}$ and form factors.
4. Pion loop effects e.g. $m_\rho - m_\omega$, r^{em}_{π}.
5. $g_{\rho NN}$, $f_{\rho NN}$, $g_{\omega NN}$, $f_{\omega NN}$; spacelike $\bar{q}q$ "mesons".
6. Vector Meson Dominance ($\gamma - \rho$—hadron) ?
7. Semi-Leptonic transitions + decays: $K^+ \to \pi^0 \ell^- \nu_\ell$, $\pi^+ \to \pi^0 e^+ \nu_e$, $B^0 \to D^- \ell^+ \nu_e$; + f_B f_D
   + form factors

* 8. Electroproduction of ρ, ϕ... on nucleon — Mike Pichowski
   9^{00} am FRIDAY.

Collaborators:

C.D. Roberts (ANL)

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Lu Qian (KSU)

S. Banerjee (KSU)

K.L. Mitchell (TRIUMF)

C.J. Burden (ANU)

P. Maris (ANL)

D. Blaschke (Rostock)

Results also borrowed from:

Alkofer

Bender

Cahill

Meissner

Thomson

Kalinovsky

Pichowski

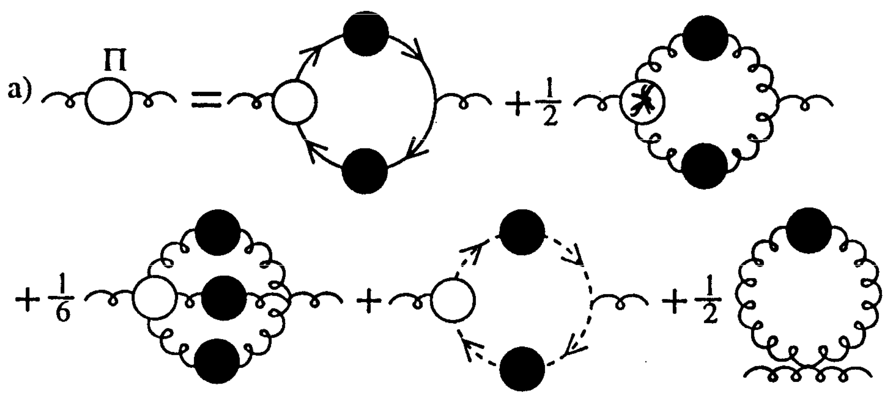
▷ **npQCD modeling** guided by :

- Dyson-Schwinger Eqn approach (DSE)
- Solve Eqns of Motion with effective 2-point gluon fn

$$S_{\rm QCD}[\bar{q}, q, A_\mu] = \int d^4x \left\{ \bar{q}(\gamma_\mu D_\mu + m)q + \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right\}$$

$$D_\mu = \partial_\mu - ig\tfrac{\lambda^a}{2} A^a_\mu \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu \boxed{+g\, f^{abc} A^b_\mu A^c_\nu} *$$

DSE for GLUON 2-point fn:



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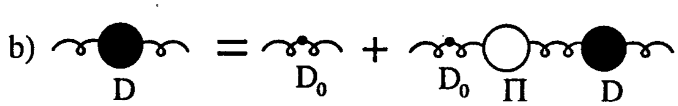


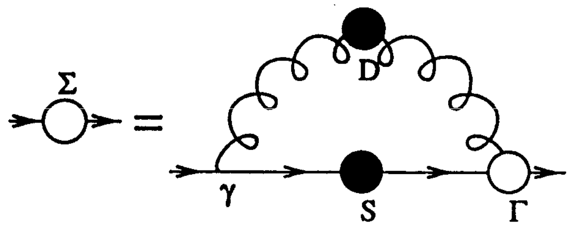
* Dominant (~ singular) IR enhancement ~ confinement ~ no mass-shell for quarks or gluons—

Brown and Pennington, Phys. Rev. **D39**, 2723 (89)....

CALCULATED SOFT π, K OBSERVABLES IN DSE APPROACH†

	Calculated	Experiment
f_π	0.0924 GeV	0.0924 $\pm$ 0.001
f_K	0.113	0.113 $\pm$ 0.001
m_π	0.1385	0.1385
m_K	0.4936	0.4937
$m^{\rm ave}_{1\,{\rm GeV}^2}$	0.0051	0.0075
$m^{s}_{1\,{\rm GeV}^2}$	0.128	0.1 $\sim$ 0.3
$-\langle\bar{u}u\rangle^{\frac{1}{3}}_{1\,{\rm GeV}^2}$	0.221	0.220
$-\langle\bar{s}s\rangle^{\frac{1}{3}}_{1\,{\rm GeV}^2}$	0.205	0.175 $-$ 0.205
$r_{\pi^\pm}$	0.56 fm	0.663 $\pm$ 0.006
$r_{K^\pm}$	0.49	0.583 $\pm$ 0.041
$r^2_{K^0}$	-0.020 fm^2	-0.054 $\pm$ 0.026
$g_{\pi^0\gamma\gamma}$	0.505 (dimensionless)	0.504 $\pm$ 0.019
$F^{3\pi}(4m_\pi^2)$	1.04	1 (Anomaly)
a_0^0	0.17	0.26$\pm$ 0.05
a_0^2	-0.048	-0.028$\pm$ 0.012
a_1^1	0.030	0.038 $\pm$ 0.002
a_2^0	0.0015	0.0017 $\pm$ 0.0003
a_2^2	-0.00021	0.00013 $\pm$ 0.0003
f_K/f_π	1.22	1.22 $\pm$ 0.01
$r_{K^\pm}/r_{\pi^\pm}$	0.87	0.88 $\pm$ 0.06

DRESSED QUARK AND GLUON PROPAGATORS



$$S^{-1}(p) = Z_2[i\gamma\cdot p + m_0(\Lambda)] + Z_1\,\frac{4}{3}\int^{\Lambda}\frac{d^4k}{(2\pi)^4}g^2 D_{\mu\nu}(p-k)\gamma_\mu S(k)\Gamma^g_\nu(k,p)$$

$$\begin{aligned} g^2 D_{\mu\nu}(q) &= \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{g^2}{q^2[1+\Pi(q^2)]} + \xi\frac{q_\mu q_\nu}{q^4} \\ &\sim \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{4\pi\alpha_s(q^2)}{q^2} \end{aligned}$$

A generic model

$$\frac{4\pi\alpha_s(q^2)}{q^2} = \frac{4\pi^2\chi^2}{\Delta^2}\exp\left(-q^2/\Delta\right) + \frac{4\pi}{q^2}\frac{12\pi/(33-2N_f)}{\ln(1+\epsilon+q^2/\Lambda^2_{QCD})}$$

$$\boxed{S(p) = -i\gamma\cdot p\;\sigma_V(p^2) + \sigma_S(p^2) = (i\gamma\cdot k\;A(p^2) + B(p^2) + m)^{-1}}$$

To survey a variety of hadron physics for feasibility:–

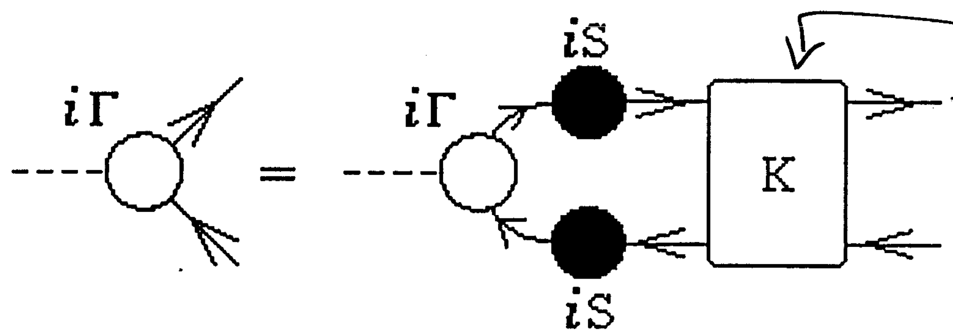
Model functions σ_V and σ_S consistent with:

- DχSB and Dyson-Schwinger equation experience
- pQCD (up to log corr) for UV behavior
- $\boxed{\text{Quark confinement}}$ taken to mean entire fns of p^2 as produced in an IR dominant model DSE (Burden, etal, Phys.Lett. **B285**,347(1992))

Calculating Hadron couplings via the DSE Approach

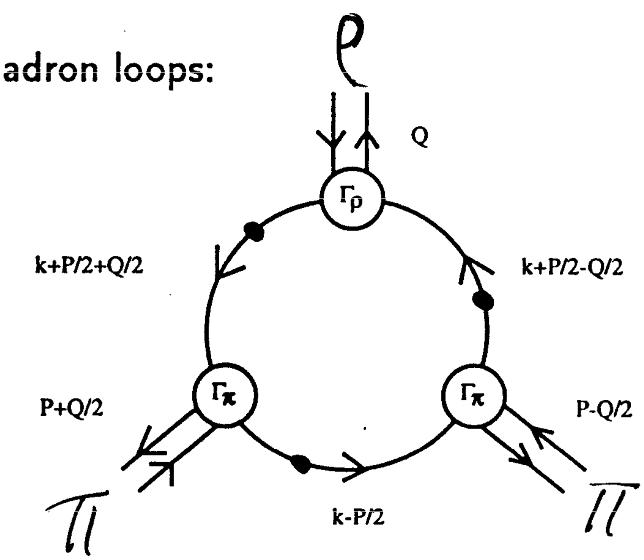
▷ Goldstone Boson modes (π, K, η $\cdots$) obtained from quark propagator or BSE

▷ Other mesons (ρ, ω, ϕ, K^* $\cdots$) require explicit BSE solution ; e.g. ladder



▷ Hadron n-point functions at 0^{th} level in hadron loops:

- Euclidean metric
- Loop integration mom real
- Hadron mass-shell $Q^2 < 0$
- Complex quark p^2
- Analytic $S(p)$ helpful



$\rho\pi\pi$

PARAMETERIZED CONFINING QUARK PROPAGATOR

$$S(p) = -i\gamma\cdot p\ \sigma_V(p^2) + \sigma_S(p^2) = (i\gamma\cdot p\ A(p^2) + B(p^2) + m)^{-1}$$

- C. D. Roberts, Nucl. Phys. **A605**, 475 (96).

$$\begin{aligned}
\bar{\sigma}_S(x) &= \frac{1-e^{-b_1 x}}{b_1 x}\frac{1-e^{-b_3 x}}{b_3 x}\left(b_0 + b_2\frac{1-e^{-\Lambda x}}{\Lambda x}\right) + \frac{\bar{m}}{x+\bar{m}^2}\left(1-e^{-2(x+\bar{m}^2)}\right) \\
\bar{\sigma}_V(x) &= \frac{2(x+\bar{m}^2) - 1 + e^{-2(x+\bar{m}^2)}}{2(x+\bar{m}^2)^2}
\end{aligned}$$

where $x = p^2/\lambda^2$, $\bar{m} = m/\lambda$, $\bar{\sigma}_S = \lambda\sigma_S$, $\bar{\sigma}_V = \lambda^2\sigma_V, \lambda = \sqrt{2D}$

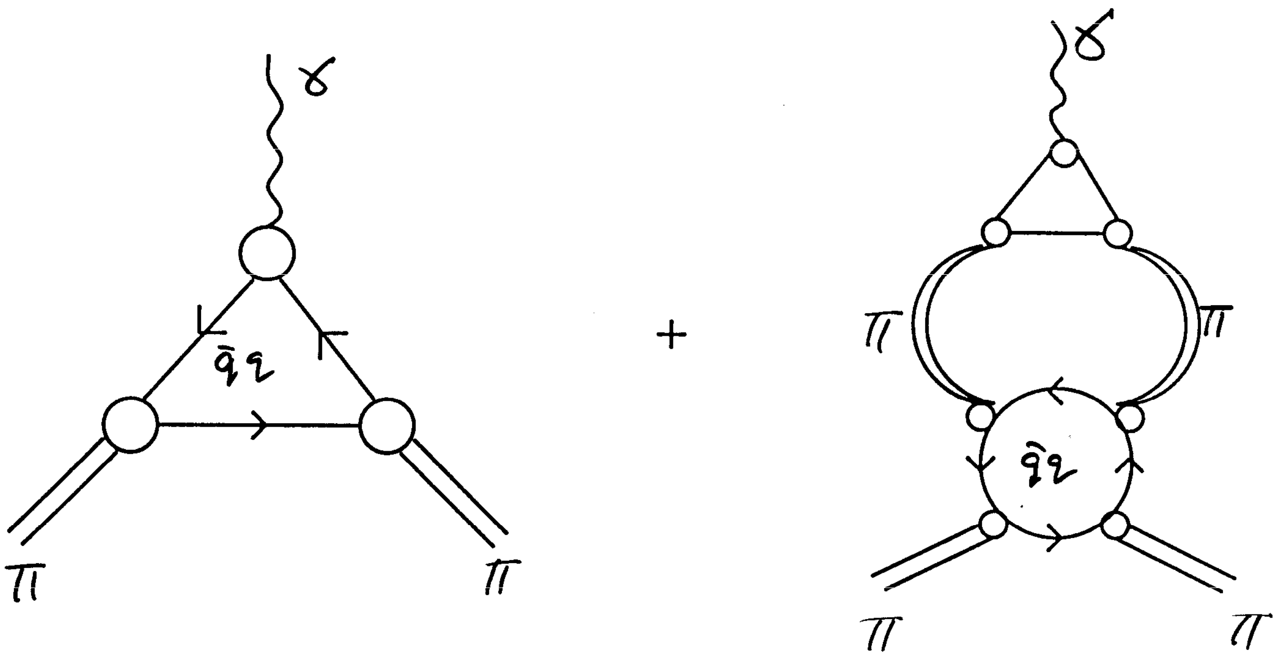
- 5 adjustable parameters
- $\boxed{\text{Entire function—confining}}$

	u/d-quark	s-quark
$\bar{m}$	0.00897	0.224
b_0	0.131	0.105
b_1	2.90	2.90
b_2	0.603	0.740
b_3	0.185	0.185
D (GeV^2)	0.160	0.160

Well-defined at complex quark momenta req. by loops for hadron coupling

- Implicitly ties some IR aspects of gluon propagator/vertex to NG boson octet
- A 1-parameter gluon propagator with ladder approx. can
  reproduce essential features [Frank & Roberts PRC C53, 390 (96)]

PION CHARGE FORMFACTOR



DRESSED QUARK-PHOTON VERTEX

▷ **Ball-Chiu Ansatz** $[\hat{Q} = \frac{1}{2}(\tau_3 + \frac{1}{3}),\ \ k_\pm = k \pm \frac{Q}{2}]$ PR D22, 2542 (80) + CURTIS + PENNINGTON...

$$\begin{aligned}\Gamma_\mu(k;Q) &= -i\gamma_\mu \hat{Q}[A(k_+) + A(k_-)]/2 \\ &+ \frac{k_\mu\, \hat{Q}}{k\cdot Q}\{i\gamma\cdot k[A(k_-) - A(k_+)] + [B(k_-) - B(k_+)]\}\end{aligned}$$

- Satisfies Ward-Takahashi Id. $\Rightarrow$ EM current consv.
- Free of kinematic singularities.
- Approaches bare vertex consistent with pQCD in UV.
- Same transformation properties as $\hat{Q}\gamma_\mu$ under CPT.
- Efficient: totally determined by quark propagator

BSE AMPLS FOR PIONS, KAONS

$$\left.\begin{aligned}\Gamma^j_\pi(k;P) &= \tau^j\gamma_5\,[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) \\ &\quad + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)]\end{aligned}\right\}$$ Maris + Roberts Aug 97

- Chiral limit: Axial Ward Takahashi Id $\Rightarrow \boxed{E_\pi(q;P^2=0) = B_0^u(q^2)/f_\pi}$
- Ladder BSE (for E_π) = DSE (for B) $\Rightarrow m_\pi = 0$ (Goldstone Thm)
- $\Gamma_\pi(k;P^2=-m_\pi^2) \approx i\gamma_5 B_0^u(k^2)/f_\pi$

The approx^n used in (all) hadron physics applications to present.

PION, KAON MASS CALCULATION

$$\hat{\Delta}_b^{-1}(P^2) = \mathrm{tr}\int \frac{d^4q}{(2\pi)^4}\,[i\gamma_5\lambda_b^\dagger S(q_+) i\gamma_5\lambda_b S(q_-)]\,\hat{\Gamma}_b^2(q;P) + \frac{9}{2}\int d^4r\,\frac{\hat{\Gamma}_b^2(r;P)}{D(r)}$$ *

- The chiral limit DSE $\Rightarrow B_0(r)/D(r) = \frac{4}{3}\mathrm{tr}_s S_0(r) \Rightarrow$ (Eliminate D)

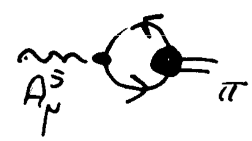
$$\hat{\Delta}_\pi^{-1}(P^2) = 2N_c\,\mathrm{tr}_s\int\frac{d^4q}{(2\pi)^4}[S_0(q)B_0(q^2) - S(-q_+)S(q_-)\,B_0^2(q^2)]$$

- Quark propagator alone $\Rightarrow$

$$\Delta_\pi^{-1}(P^2 = -m_\pi^2) = 0 \qquad N_\pi^2 = \frac{d}{dP^2}\Delta_\pi^{-1}(P^2)\Big|_{P^2=-m_\pi^2} \approx f_\pi^2$$

- Weak decay

$$\delta^{ij} f_\pi P_\mu = Z_2\mathrm{tr}\int^\Lambda \frac{d^4q}{(2\pi)^4}[\frac{\tau^i}{2}\gamma_5\gamma_\mu S(q_+)\Gamma_\pi^j(q;P)S(q_-)]$$



- Kaon generalization $\Gamma_K(k;P^2=-m_K^2) \approx i\gamma_5 B_0^s(k^2)/f_K$
- Explicit χSB in $S(q) \Rightarrow m_{\pi/K} \neq 0$

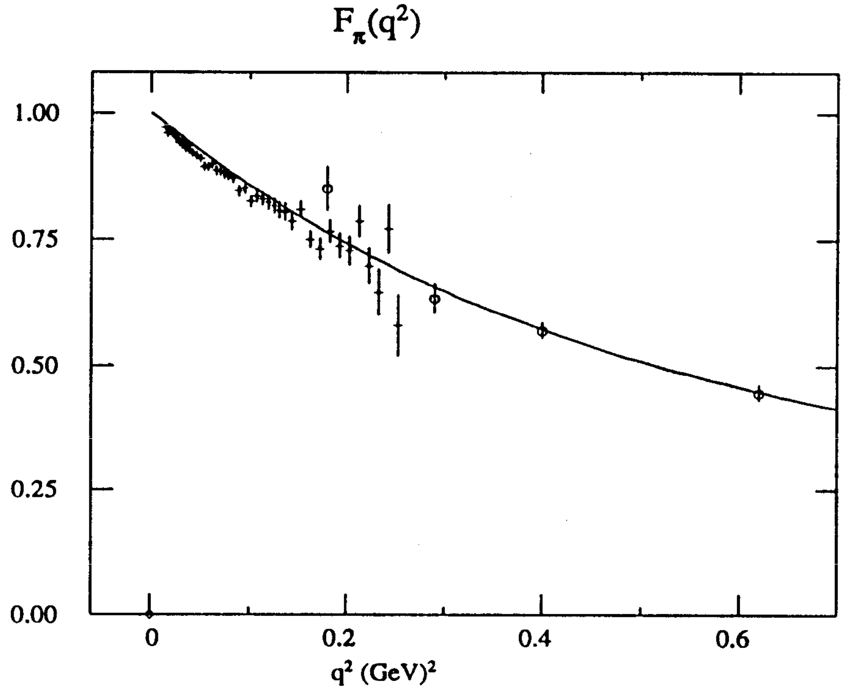
$\bar{q}q$:

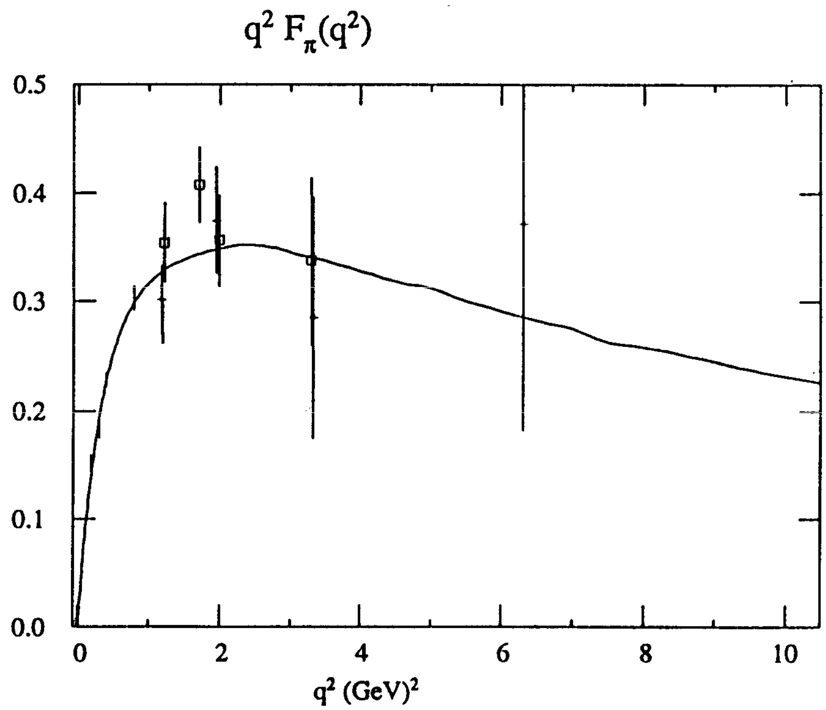
$T = D - DSST$

* $\boxed{T^{-1} = SS + D^{-1}}$

PION CHARGE FORMFACTOR–GENERALIZED IMPULSE APPROX

Parameterized quark propagator, γ_5 pion BS ampl only

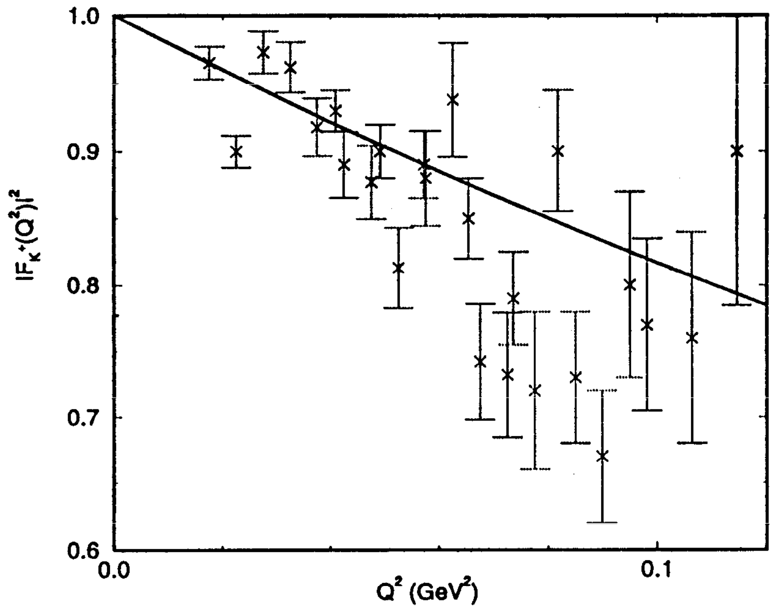


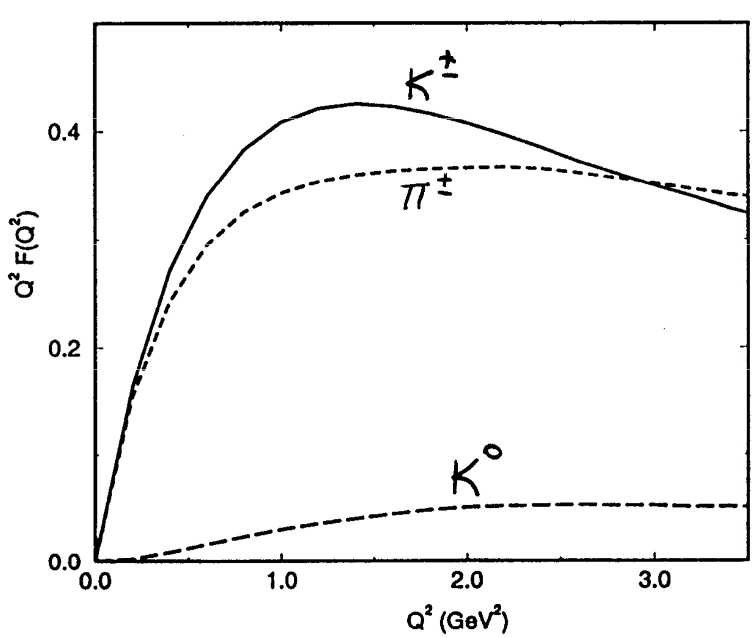


Roberts 94, 96

KAON CHARGE FORMFACTORS–GENERALIZED IMPULSE APPROX

Parameterized quark propagator, γ_5 kaon BS ampl only





Burden, Roberts
+ Thomson (96)

$\boxed{\text{CHIRAL OBSERVABLES}}$

$\gamma^* \pi \to \pi\pi$

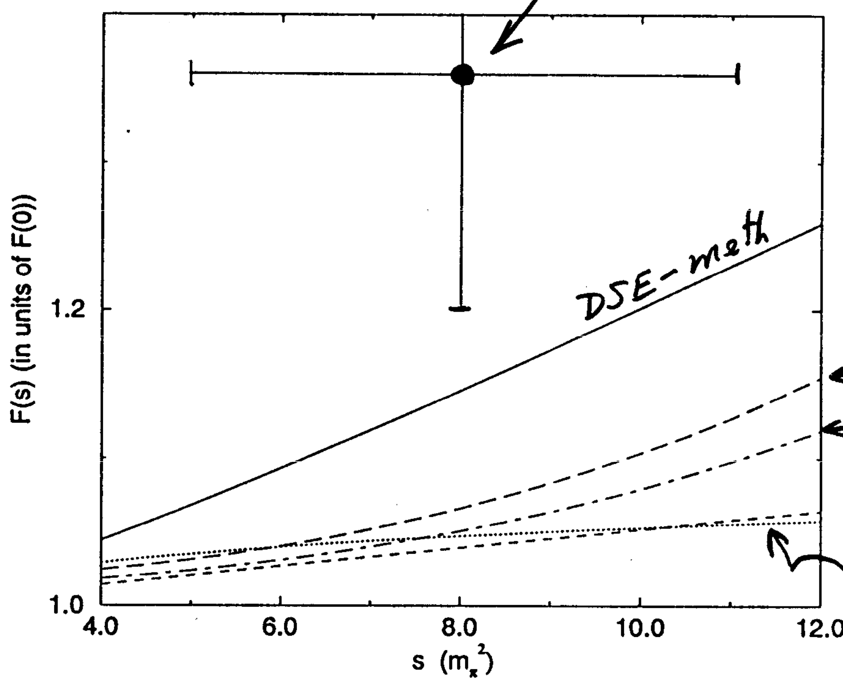
$\tilde{F}^{3\pi}(0,0,0)$

$\tilde{F}^{3\pi}(4m_\pi^2) = 1.04$ [Expt = 1 (anomaly)]

at Serpukhov

P.R. D36 21(87)

Primakov – $\pi^- A \to \pi^- \pi^0 A'$

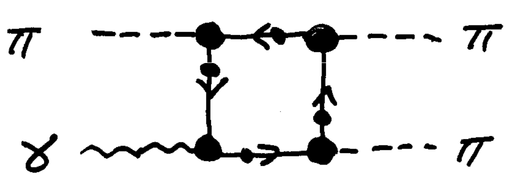


VMD Rudaz (74)

ChPT + VMD Sat^n. (Bijnens et al (90)) incl $\pi\pi$ FSI.

π_3 obs.

DSE: [Alkofer and Roberts, Phys. Lett. **B369**, 101 (96)]



$$\left\{\begin{matrix}\text{EM-WTI}\\ \text{D}\chi\text{SB}\\ \text{AV-WTI}\end{matrix}\right\} \Rightarrow F^{3\pi}(0,0,0) = \frac{e N_c}{12\pi^2 f_\pi^3}$$

$= $ Adler et al

$\Downarrow$

$i\varepsilon_{\mu\nu\rho\sigma}\, p_{1\nu}\, p_{2\rho}\, p_{3\sigma}\, F^{3\pi}(s,t,u)$.

$\boxed{\text{indep}^n \text{ of form of } S(p)}$

$\boxed{\text{THE } \gamma\pi^0\gamma \text{ VERTEX FUNCTION}}$

- Off-shell behavior of axial anomaly (form factor).
- Very basic test of NpQCD model (one hadronic vertex).
- Eucl. metric, ext momenta OMS $\longrightarrow$ complex quark p.

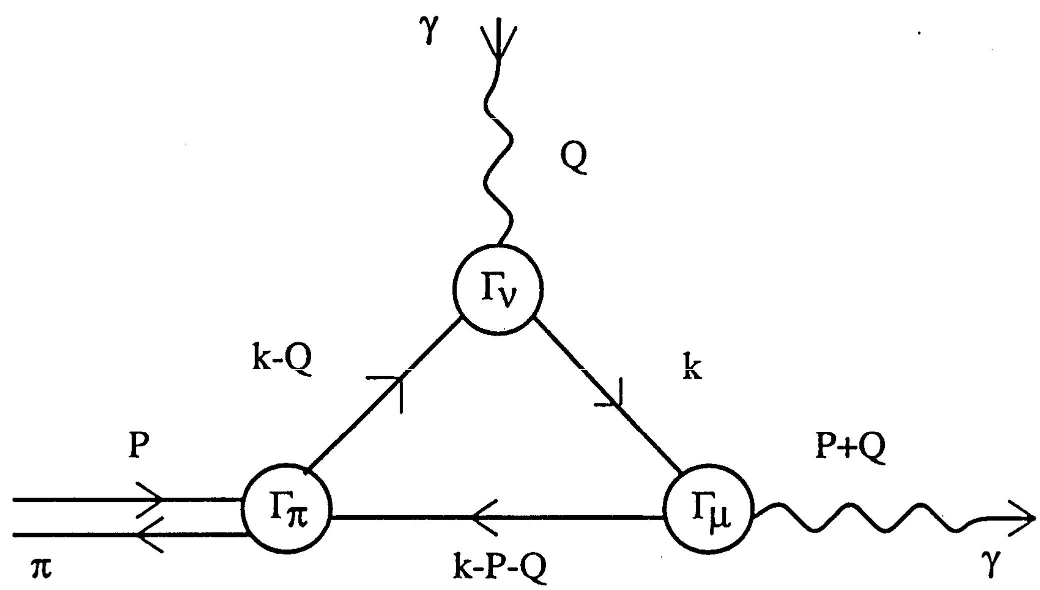
$$S[\pi^0\gamma\gamma] = \int \frac{d^4P d^4Q}{(2\pi)^8} A_\mu(-P-Q)A_\nu(Q)\pi^0(P)\Lambda_{\mu\nu}(P,Q)$$

General form of Λ allowed by CPT is

$$\Lambda_{\mu\nu}(P,Q) = -i\frac{\alpha}{\pi f_\pi}\epsilon_{\mu\nu\alpha\beta}P_\alpha Q_\beta \; g(Q^2, P^2, P\cdot Q)$$

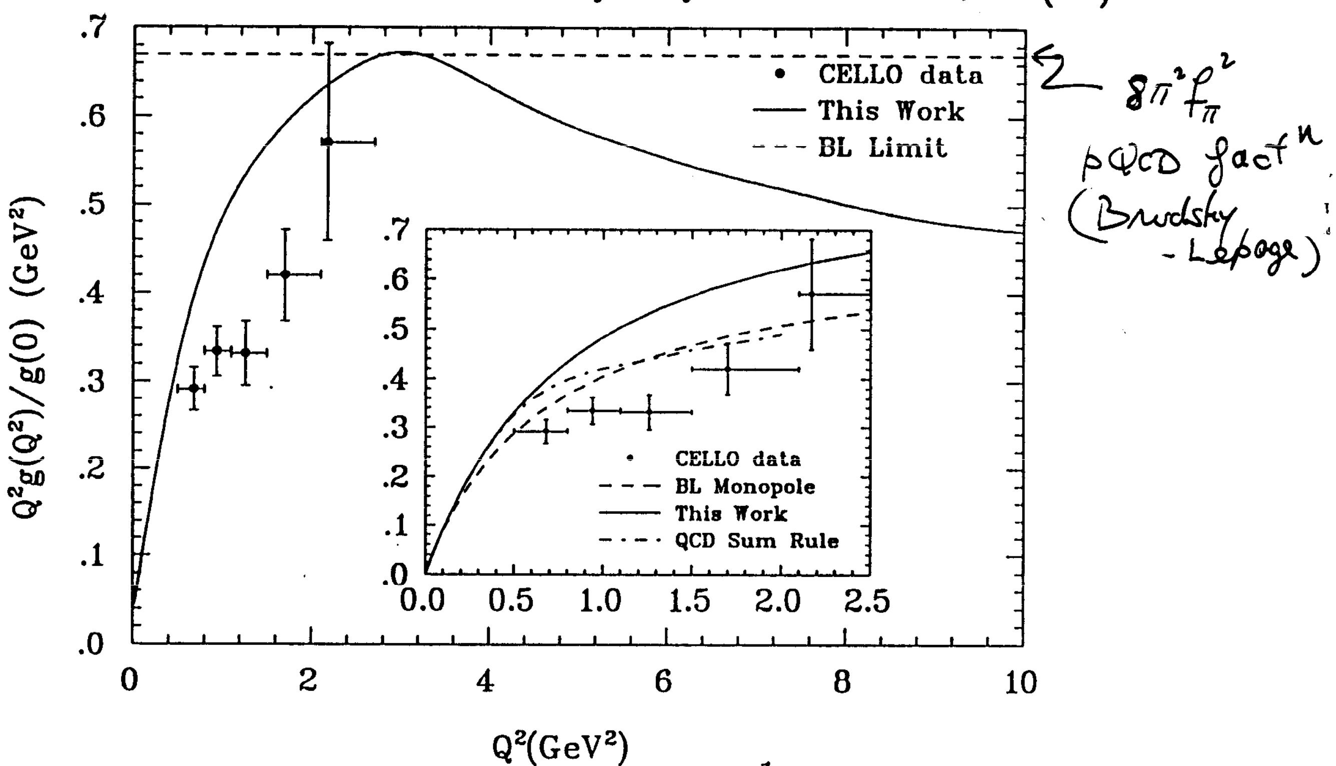
Lowest order in meson loop expn $\Rightarrow$ Gen. Impulse Approx:

$$\Lambda_{\mu\nu}(P,Q) = -\int \frac{d^4k}{(2\pi)^4} \quad \mathrm{tr}\Big[S(k-P-Q)\Gamma_\mu\left(k-\frac{P}{2}-\frac{Q}{2};-P-Q\right)S(k)$$
$$\times\Gamma_\nu\left(k-\frac{Q}{2};Q\right)S(k-Q)i\gamma_5\tau_3\Gamma_\pi\left(k-\frac{P}{2}-Q;P\right)\Big]$$

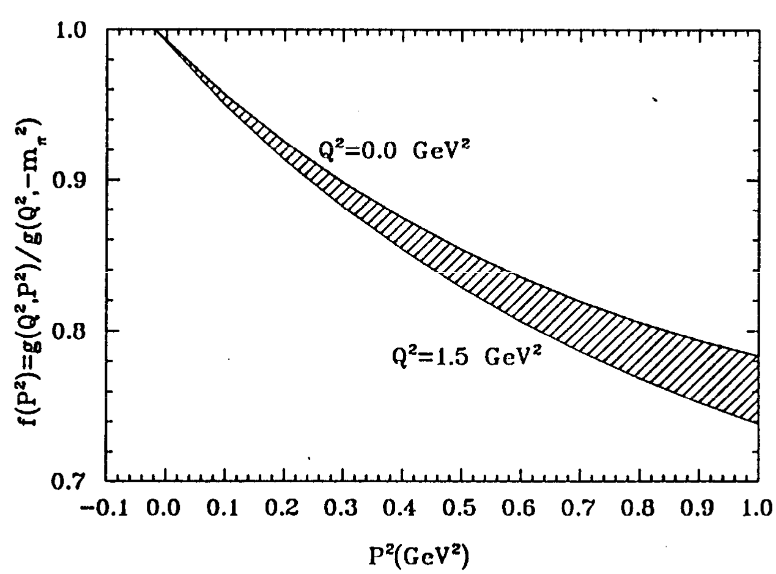


$\boxed{\gamma^*\pi^0 \to \gamma \text{ Transition Form Factor}}$

Frank, Mitchell, Roberts ,Tandy, Phys. Lett. **B359**, 17 (95)



$g_{\pi^0\gamma\gamma} = 0.497$, (expt$= 0.504 \pm 0.02$), $g(0,0) = \frac{1}{2}(\Gamma = 7.7 \text{ eV})$



$\boxed{\text{ASYMPTOTIC FORM OF } \gamma^*\pi_0\gamma \text{ FORM FACTOR}}$

- pQCD factorization limit (Brodsky-Lepage):

  $Q^2F(Q^2) \rightarrow 8\pi^2 f_\pi^2 = 0.67\ \mathrm{GeV}^2$

PRESENT WORK:

- UV behavior consistent with pQCD because propagators and vertices are.
- Loop integral: $\boxed{\text{on-shell photon vertex remains soft}}$

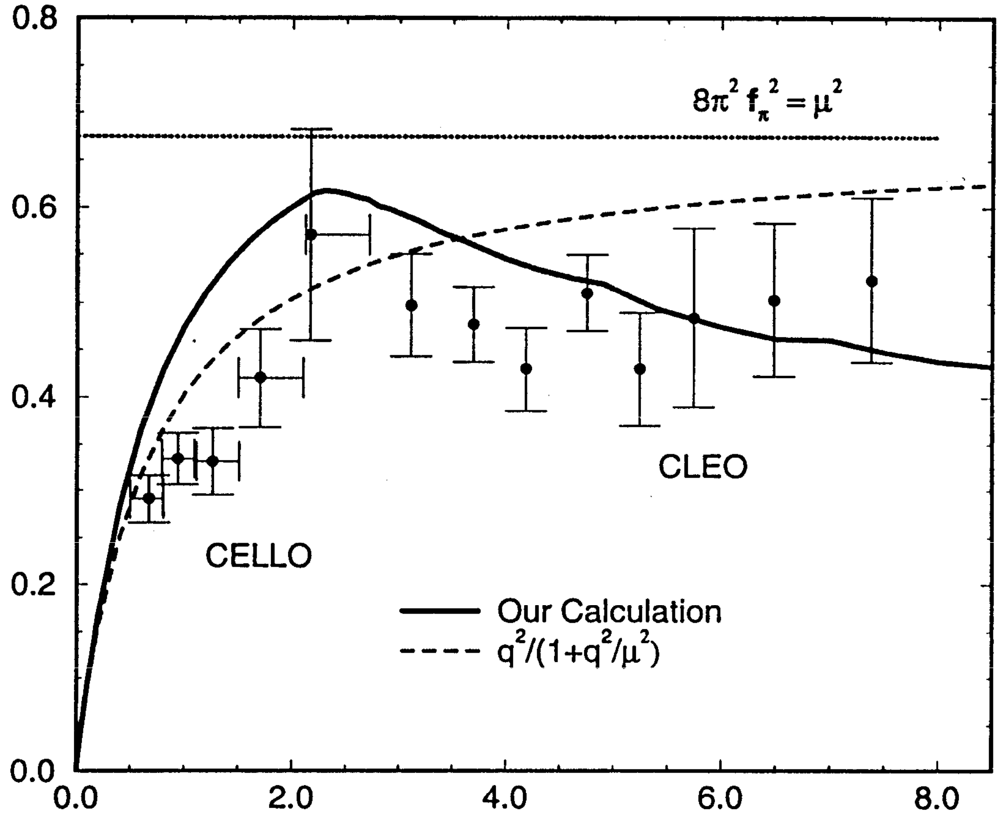
  $\Rightarrow$ Log corrections to pQCD factorization result.

$$F(Q^2) \rightarrow A[1.0 + BQ^2 ln(CQ^2)]^{-1}$$

$$A = 1.021, \qquad B = \frac{0.461}{m_\rho^2} = 0.777\ \mathrm{GeV}^2, \qquad C = \frac{1.16}{m_\rho^2} = 1.45\ \mathrm{GeV}^2$$

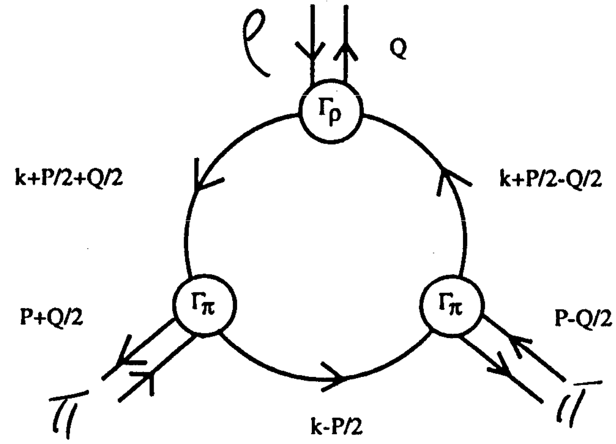
- Bare coupling for both photons $\rightarrow$ pQCD factn result.
- Similar UV log correction in π EM form factor.

Include CELLO + CLEO $\gamma\pi\gamma$ data to constrain quark propagator



Q^2 GeV^2

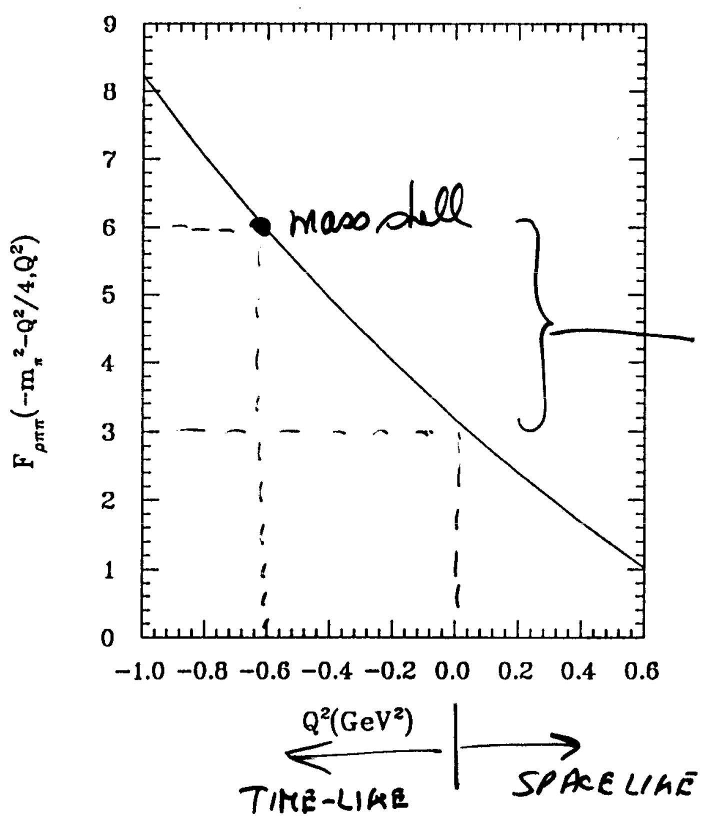
$\boxed{\rho\pi\pi}$ FORM FACTOR



- ρ BS Ampl: $\Gamma_\mu(q;P) = iN\,(\gamma_\mu - \frac{P_\mu \gamma\cdot P}{P^2})\; e^{-q^2/a^2}$

fit
- $a \Rightarrow g_{\rho\pi\pi} = 6.05$, *Strength set by norm:*

$$2P_\mu = \frac{\partial}{\partial P_\mu}\, N_c \mathrm{tr}_D \int \frac{d^4q}{(2\pi)^4} \overline{\Gamma}(q,-K) S_{f_1}(q_+) \Gamma(q,K) S_{f_2}(q_-) \Big|_{K=P=OMS}$$



factor of 2 decrease

$\Downarrow$

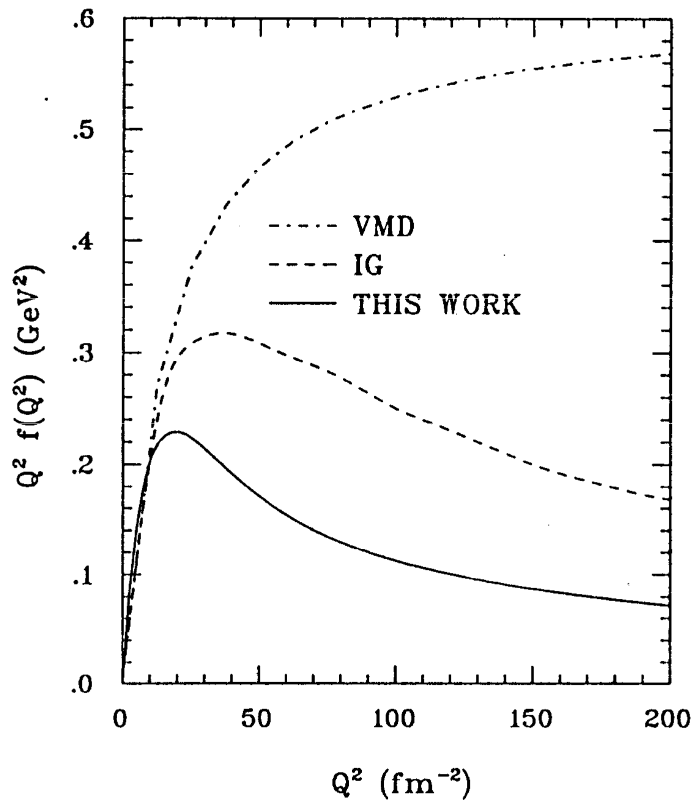
[implication for ρ "VMD" contrib^n to Γ_π^{em}]

$\boxed{\text{THE } \gamma^* \pi \rho \text{ VERTEX FUNCTION}}$

- Important meson exch current for deuteron EM structure. (d(e,e)d)
  TJLAB

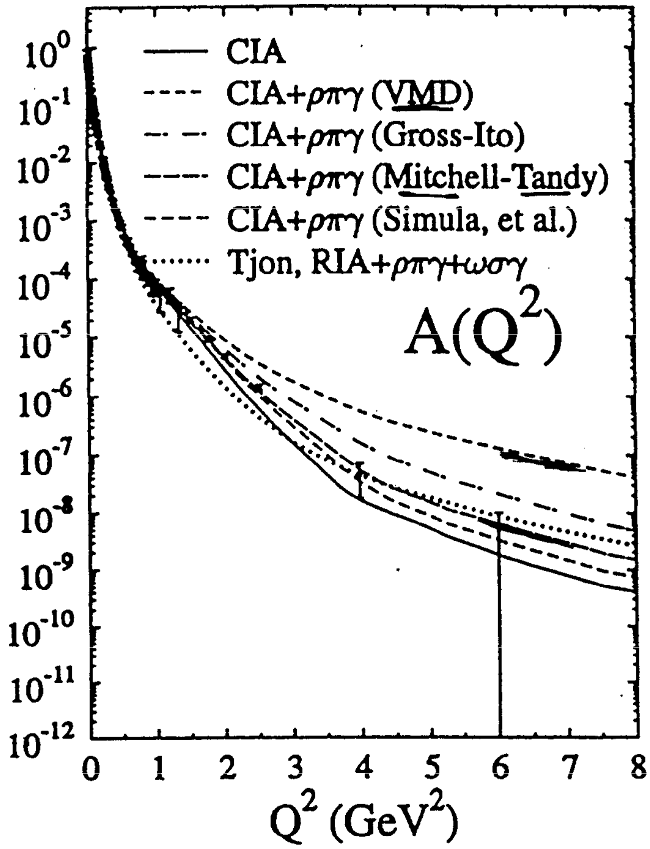
General form allowed by CPT is

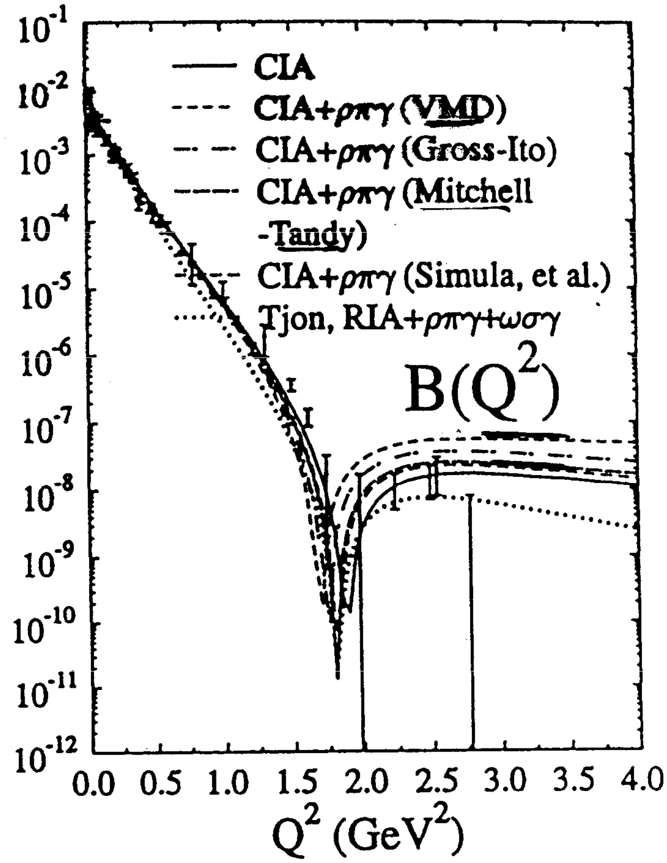
$$\Lambda_{\mu\nu}(P,Q) = -i\frac{e}{m_\rho}\epsilon_{\mu\nu\alpha\beta}P_\alpha Q_\beta \; g_{\gamma\pi\rho} \; f(Q^2, P^2, P\cdot Q)$$

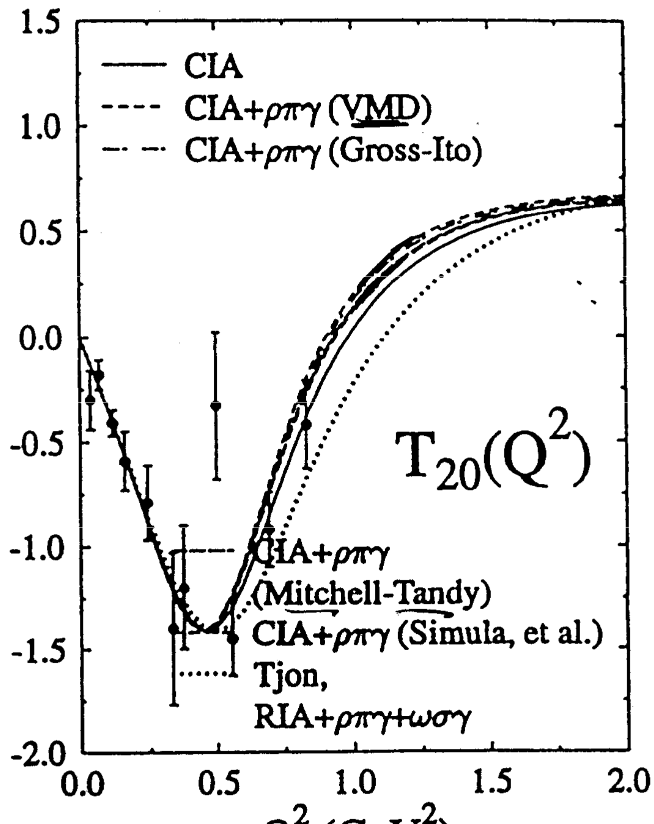


- On shell π and ρ
- Coupling constant $\boxed{g_{\gamma\pi\rho} = 0.5, \text{ (expt}= 0.54 \pm 0.03) \quad \text{PREDICTION}}$
- Softer than VMD, or with free quark propagator.

$^2H(e,e')$ ANALYSIS — F. GROSS (1996).

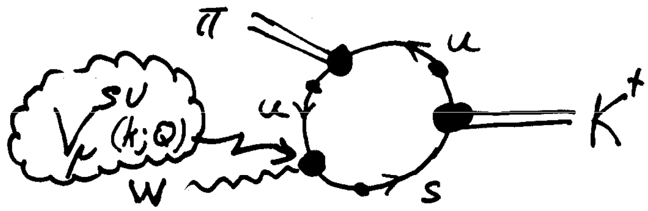






SEMI-LEPTONIC TRANSITIONS AND DECAYS

- $\overset{+0}{K} \to \pi l \nu_l$ $[K_{l3}]$
- $\pi \to \pi l \nu_l$ $[\pi_{l3}]$



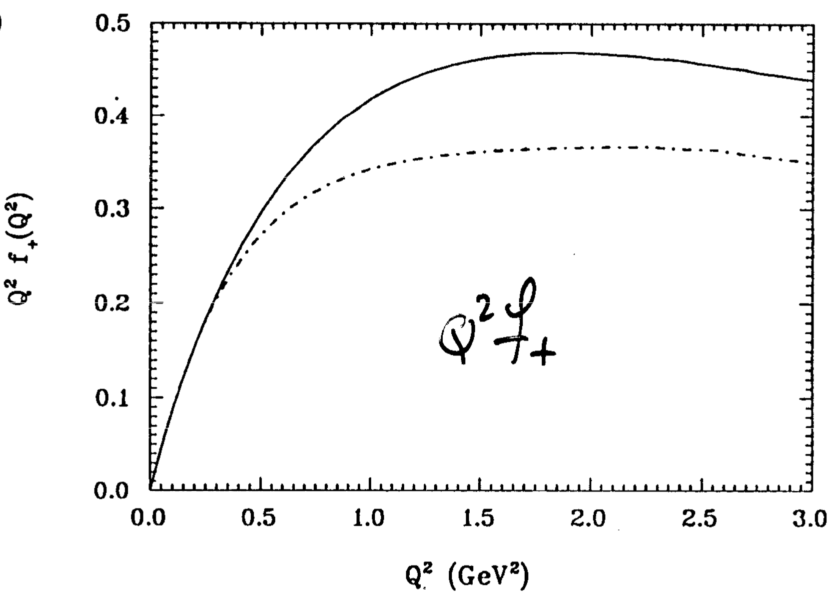
Kalinowsky, Mitchell + Roberts
P.L. B399, 22 (97)

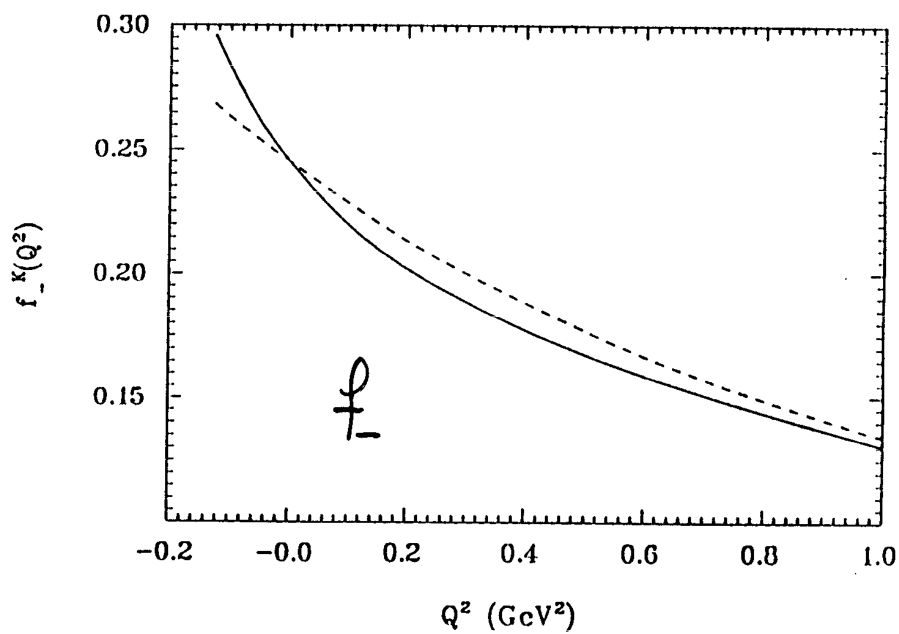
$$\langle \overset{0}{\pi}(p)|\bar{s}\gamma_\mu u|\overset{+}{K}(k)\rangle = \frac{1}{\sqrt{2}}\left(f_+^{\overset{+}{K}}(t)K_\mu + f_-^{\overset{+}{K}}(t)Q_\mu\right) \quad , \quad K = k+p, \quad Q = k-p$$

PROBES $SU_f(3)$ VIOLATION

$\partial_\mu J_\mu^{ff'} \propto m_f - m_{f'} \neq 0$

CKM Matrix info.





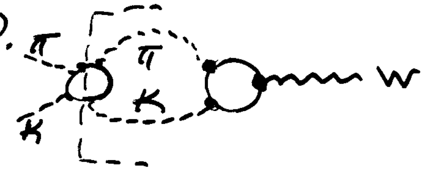
SEMI-LEPTONIC TRANSITIONS AND DECAYS

WTI: $q_\mu \, i V^{su}_\mu(p;q) = S_s^{-1}(p_+) - S_u^{-1}(p_-) - (m_s - m_u)\,\Gamma_I^{su}(p;q)$.

ANSATZ-1 $V^{su}_\mu(p;q) \approx \frac{1}{2}\left(\Gamma^s_\mu(p;q) + \Gamma^u_\mu(p;q)\right)$, Γ = Ball-Chiu.

ANSATZ-2 Add non-analytic term due to K-π loop in a form given by Gasser + Leut. (85).

[Reconciles pert. q-model and meson model results.]



Consistent with Ademollo-Gatto Theorem (≈ 1)

Supports f_- as measure of non-pert. enhance M_s/M_u.

SCALAR FORM FACTOR of K.

	V^{su} approx 1	V^{su} approx 2	Experiment: $K^+_{\mu 3}$ / $K^0_{\mu 3}$
$-f_+(t_m)$	1.11	1.22	
$f_-(t_m)$	0.27	0.29	
$-f_+(0)$	0.98	0.98	
$f_-(0)$	0.24	0.24	
$-\xi(0)$	0.25	0.25	0.35 ± 0.15 0.11 ± 0.09
λ_+^e	0.018	0.028	0.0286 ± 0.0022 0.0300 ± 0.0016
λ_+^μ	0.018	0.029	0.033 ± 0.008 0.034 ± 0.005
λ_-^e	0.012	0.023	
λ_-^μ	0.012	0.023	
$-f_0(\Delta)$	0.95	$\boxed{1.18}$ ⇐ CURRENT ALGEBRA	
λ_0^e	-0.0024	0.0067	
λ_0^μ	-0.0024	0.0073	0.004 ± 0.007 0.025 ± 0.006
$r_{\pi K}$ (fm)	0.47	0.58	

$\Delta = M_K^2 - M_\pi^2$

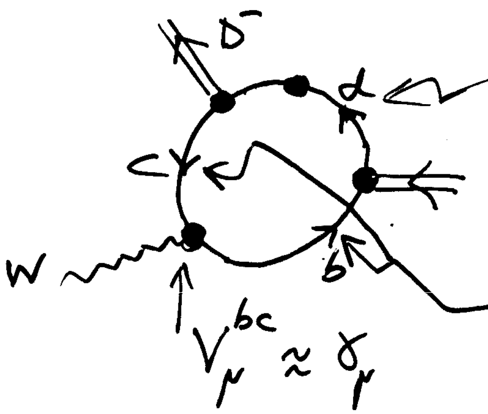
(CALLAN-TREIMAN PT)

$f_0^K(\Delta) = -\frac{f_K}{f_\pi}$

HEAVY MESON ($q\bar{Q}$) STUDY

Ivanov, Kalinovsky, Maris & Roberts PREPRINT 4/97

- SEMI-LEPTONIC DECAY: $B^0 \to D^- \ell^+ \nu_\ell$ — FORM FACTOR
- LEPTONIC (WEAK) DECAY: $B^+/D \to \ell^+ \nu_\ell$ — Decay constants (f_B, f_D)



keep full dynamics

$= f_+(t)(K+P)_\mu - f_-(t)(K-P)_\mu$

heavy quark limit $\Sigma(k) \sim$ const

$$f_\pm(t) \to \frac{1}{2} \frac{M_D \pm M_B}{\sqrt{M_D M_B}} \, \xi(\omega) \Big|_{\omega = v_B \cdot v_D}$$

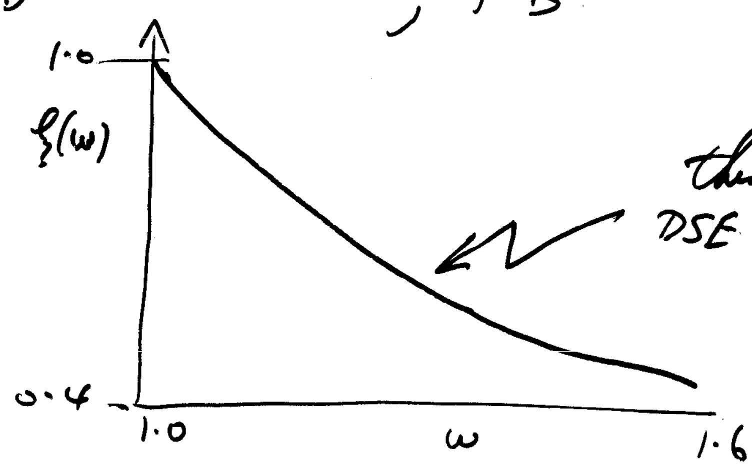
Loop integral calculation

BS Amp:

$$\Gamma_{B,D}(k;P) \sim \gamma_5 \left(1 - \frac{1}{2} i\gamma\cdot v\right) \frac{\varphi(k^2)}{N}$$

$\Rightarrow$ $\boxed{\text{Solid predictions indep}^{\text{n}} \text{ of form of } \varphi(k^2)}$

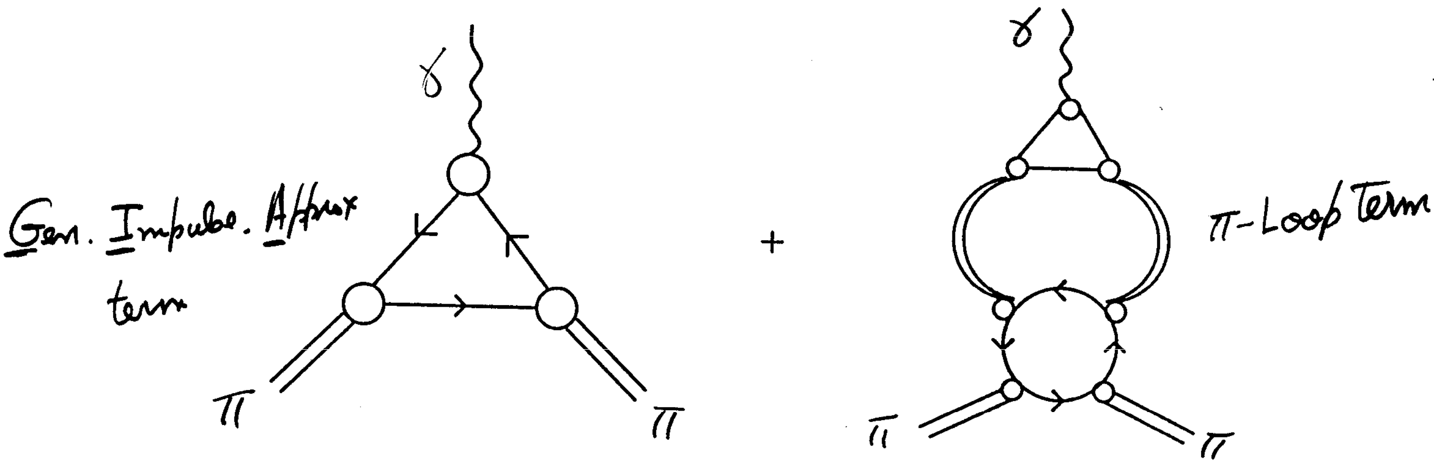
$$f_D \sim 0.227 \text{ GeV}, \quad f_B \sim 0.135 \text{ GeV}.$$



this DSE calc AND EXPERIMENTAL FIT Duboscq et al (96).

BEYOND TREE LEVEL— PION LOOPS

- Pion Loop part of Pion Charge Radius



$$F_\pi(Q^2) = F_\pi^{GIA}(Q^2)\,[1 + L(Q^2; m_\pi)] \qquad r_\pi^2 = (r_\pi^{GIA})^2 + (r_\pi^{loop})^2$$

[Alkofer, Bender and Roberts, Int. J. Mod. Phys. **A10**, 3319 (95)] $\Rightarrow$

$$\boxed{r_\pi^{loop} \leq 10 - 15\%\ r_\pi^{GIA}}$$

One loop ChPT, $\mathcal{O}(p^4)$, [Gasser and Leutwyler, Nucl. Phys. **B250**, 517 (85)] $\Rightarrow$

$$r_\pi^2 = \frac{12\,L_9^r}{f_\pi^2} - \frac{1}{32\pi^2\,f_\pi^2}\left[2\ln\left(\frac{m_\pi^2}{\mu^2}\right) + \ln\left(\frac{m_K^2}{\mu^2}\right) + 3\right]$$

$$L_9^r \Rightarrow 85 - 90\%$$

	$m_M^{\text{calc.}}$ GeV	$m_M^{\text{calc. Dom}}$	Expt.
π (0^{-+})	0.139 (fit)	0.116	$\pi^{\pm}(140), \pi^0(135)$
f_0/a_0 (0^{++})	0.715	0.743	$f_0(980)/a_0(982)$
0^{+-}	1.082	1.092	Not Seen
0^{--}	1.319	1.299	Not Seen
K	0.494 (fit)	0.412	$K^{\pm}(494), K^0(498)$
K_0^*	unbound	unbound	$K_0^*(1430)$
$\eta(\theta_P = 5^0)$ $\eta(\theta_P = 0^0)$	0.549 0.513	0.472 0.441	$\eta(547)$
ω/ρ	0.736	0.755	$\omega(782)/\rho(770)$
a_1/f_1	1.34	1.37	$a_1(1260)/f_1(1285)$
K^*	0.854	0.866	$K^*(892)$
K_1	1.39	1.39	$K_1(1270)$, $K_1(1400)$
ϕ $(\bar{s}s\ 1^-)$	0.950	0.957	$\phi(1020)$
$\bar{s}s\ 1^+$	1.60	1.60	$f_1(1510)$

	$f_M^{\text{calc.}}$ GeV	$f_M^{\text{calc. Dom}}$ GeV	Expt.
π	0.0924 (fit)	0.056	$\pi^+(0.0924)$
$K^{\pm}$	0.113 (fit)	0.76	$K^+(0.113)$
$\eta(\theta = 5^0)$ $\eta(\theta = 0^0)$	0.114 0.111	0.086 0.082	0.094±0.007 or 0.091±0.006

- HERE: No adjustible parameters; all set by π, K properties
- Nucleon model $\Rightarrow \mu_P = 3.08$ $[expt = 2.79]$, $\mu_N = -2.10$ $[expt = -1.91]$
- Meson $\bar{q}q$ BS ampls provide significant strength (~ 5)

	$\Gamma = 3$ covs	$\Gamma =$ dom. cov.	NJL†	Empirical-OBE
$F_1^\omega(0)$ $(g_{\omega NN})$	7.2 (9.9)	7.7	(14-18)	11.7 (16; 7-10.5††)
$F_2^\omega(0)$ $(f_{\omega NN})$	0.08 (0.11)	-1.2	(-)	0 (0)
$F_1^\rho(0)$ $(g_{\rho NN})$	2.4 (3.3)	2.6	(5.6)	2.6 (3.5)
$F_2^\rho(0)$ $(f_{\rho NN})$	9.7 (13.3)	8.3	(-)	16.1 (22)
$\kappa_\rho = \frac{f_{\rho NN}}{g_{\rho NN}}$	4.0	3.3	3.7 (VMD)	6.1
$\kappa_\omega = \frac{f_{\omega NN}}{g_{\omega NN}}$	0.01	-0.16	-	0
$\frac{\kappa_\rho}{\kappa_\gamma}$	1.09	0.9	1	1.78

†† T. Sato, T.-S. H. Lee, Phys. Rev. **C54**, 2660 (97).

† Gao, Shakin et al., Phys. Rev. **C53**, 1936 (96).

* C.M. Shakin and W.-D. Sun, Phys. Rev. **D55**, 2874 (97).

SUMMARY

- Have presented/reviewed most of the applications of DSE-based QCD phenomenology to hadron physics.
- __Capabilities__: Covariant, $D\chi SB$, $E\chi SB$, Confinement; Continuum treatment, preserves current algebra results if preserve Ward Identities and avoid arbitrary form factors/cutoffs.
- __Intrinsic properties__ of mesons $M \lesssim 1.5$ GeV
- __Dynamical__ properties of mesons — interactions, __coupling constants__ and __form factors__
- Can relate to + provide some inputs for effective hadronic field theory models in nuclear physics.
- # Parameters $\lesssim 6$ $\left\{ \begin{array}{l} \text{tell about} \\ \text{replace} \end{array} \right\}$ IR behavior of  ,  for hadron physics.
- IT IS __ROBUST__ — SAME PARAMETER SET IS WIDELY APPLICABLE AND __PREDICTS__ WELL (SO FAR.)
- __FUTURE__ — explore connections to Lattice, ChPT, QCD sum rules, gauge sector DSEs; $(T, \mu)\uparrow$.