Low Energy QCD and the Quark-Quark-Interaction

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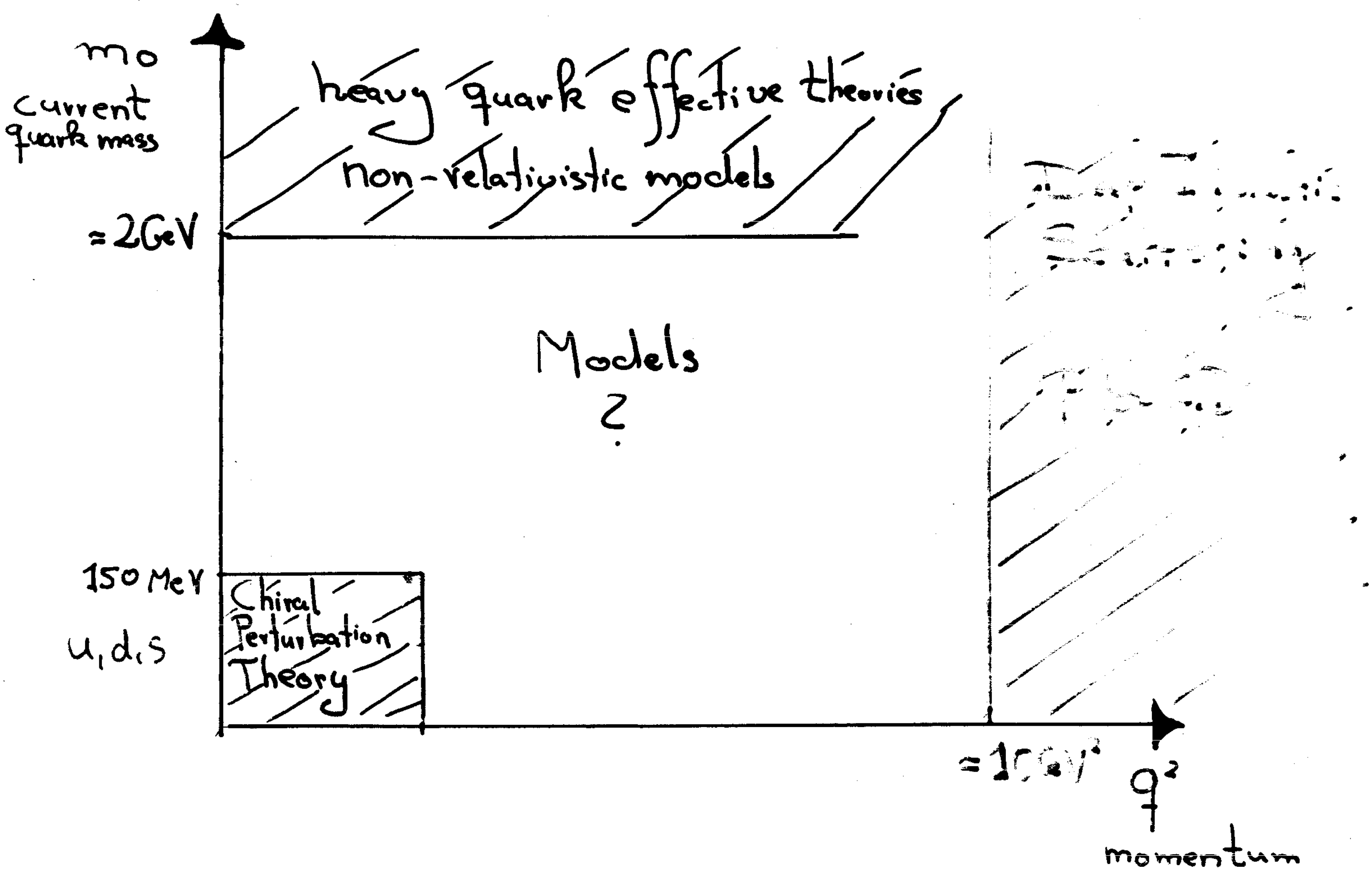
August 97



Low Energy QCD

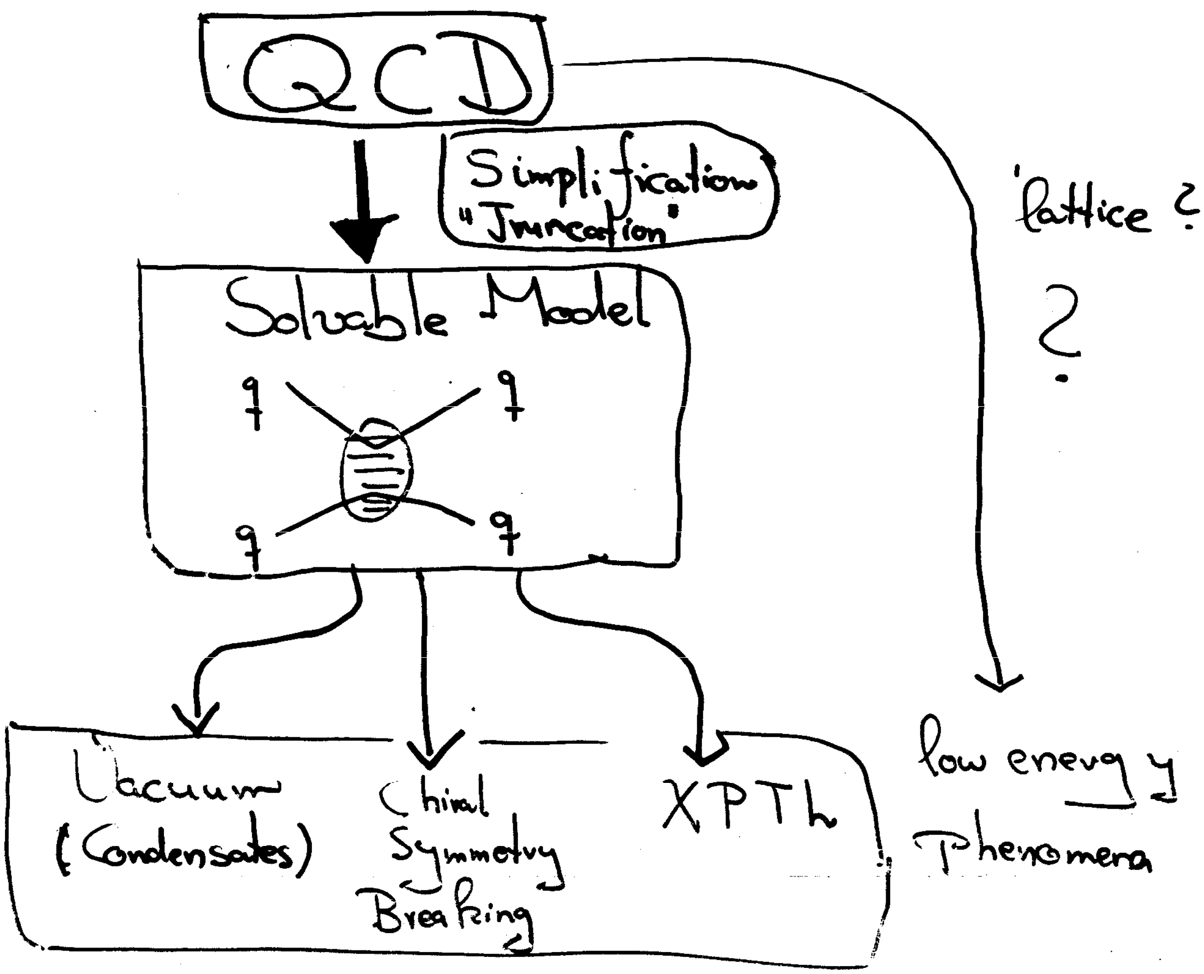
- Confinement ?
- "Non-Trivial" Vacuum (Condensates)
- Dynamical (Spontaneous) Chiral Symmetry Breaking
  → "Massless" π (Goldstone Bosons)
- Effective Degrees of freedom: Goldstone-Bosons (π, K ...)
  + Nucleons
  (+ Δ ... ?)

  $\boxed{\text{XPTh}}$

Effective hadronic theories are by construction designed to describe low energy phenomena → forfeit information about QCD

Idea: Formulate an "effective" theory at the quark level and derive hadronic theory

"Solvable" Model of QCD based on an effective quark-quark interaction



Outline

(1) Chiral Perturbation Theory

(2) Global Color Model
(Model Truncation of QCD) @ quark level

(3) Bosonization
$\Rightarrow$ Mean Field Vacuum
$\Rightarrow$ Hadronic Degrees of Freedom
$\Rightarrow$ Long Wave Length Expansion

(4) Condensates
$U_A(1)$ (η' mass)

Effective Chiral Lagrangian   Chiral Perturbation Theory

Construct effective Theory at low energy

Goldstone degrees of freedom (π^a)

(+ baryon $N, \Delta \ldots$)

which is the most general compatible with χ symm.

- Expansion in the momenta of the Goldstone Bos.
- Expansion in the χ symm. breaking mass

  $m_\pi^2, m_K^2 \sim M_0$

$$U \equiv e^{i\pi^a \lambda^a / f_\pi}$$

λ^a: SU(3) generators

f_π: Pion decay

$$U \to V_L U V_R^+ \quad \text{chiral transf.}$$

$\Longrightarrow$ Exp. in (∂U)

Exp. in m_π

Gasser- Leutwyler (1983, 1985)

$$\mathcal{L}[U] = \mathcal{L}^{(2)}[U] + \mathcal{L}^{(4)}[U] + \ldots$$

$$\mathcal{L}^{(2)}[U] \equiv \frac{f_\pi^2}{4}\, tr\,[\partial_\mu U \partial^\mu U^\dagger] - \frac{f_\pi^2}{4} tr[U\chi^\dagger + \chi U^\dagger]$$

$\hookrightarrow \chi = -2\langle\bar{q}q\rangle \frac{M_c}{f_\pi^2} \sim m_\pi^2 \ldots$

$$\mathcal{L}^{(4)}[U] = \sum_{i=1}^{8} L_i \mathcal{L}_i^{(4)} + \ldots$$

external gauge field

$$\mathcal{L}_{i=1}^{(4)} = [tr\ \partial_\mu U \partial^\mu U^\dagger]^2$$

$$\mathcal{L}_{i=2}^{(4)} = [tr\ \partial_\mu U \partial_\nu U^\dagger][tr\ \partial^\mu U \partial^\nu U^\dagger]$$

$$\mathcal{L}_{i=3}^{(4)} \quad tr\,[\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger]$$

$$\mathcal{L}_{i=4}^{(4)} = tr[\partial_\mu U \partial^\mu U^\dagger] \cdot tr\,[\chi U^\dagger + \chi^\dagger U]$$

$$\mathcal{L}_{i=5}^{(4)} = tr\,[\partial_\mu U \partial^\mu U^\dagger (\chi U^\dagger + \chi^\dagger U)]$$

$$\mathcal{L}_{i=6}^{(4)} = [tr\,(\chi U^\dagger + U\chi^\dagger)]^2$$

$$\mathcal{L}_{i=7}^{(4)} = [tr\,(\chi^\dagger U - U\chi^\dagger)]^2$$

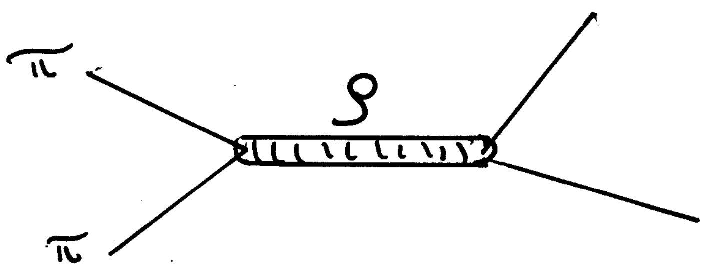
$$\mathcal{L}_{i=8}^{(4)} = tr\,[\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger]$$

L_i : dimensionless coefficients (low energy chiral coeff.)

Chiral Symmetry determines uniquely the form of the low energy Lagrangian $\mathcal{L}[U]$ but not the values of the coefficients L_i

→ L_i fitted to experimental data

<u>alternative:</u> Resonance - Saturation



higher resonance exchange $(\rho, \omega, \sigma, \dots)$

→ needs hadronic model $(\rho\pi\pi, \dots)$

If the L_i are known the $\boxed{\text{whole physics}}$ involving soft Pions is completely determined up to $\mathcal{O}(p^4)$

L_i	Value $\times 10^3$	Input
L_1	0.7 ± 0.5	$K \to \pi\pi e\bar{\nu}_e$, $\pi - \pi$ scattering
L_2	1.2 ± 0.4	$K \to \pi\pi e\bar{\nu}_e$, $\pi - \pi$ scattering
L_3	-3.6 ± 1.3	$K \to \pi\pi e\bar{\nu}_e$, $\pi - \pi$ scattering
L_4	~ 0	Vanishes in the large N_c limit
L_5	1.4 ± 0.5	f_K/f_π + Meson masses
L_6	~ 0	Vanishes in the large N_c limit
L_7	-0.4 ± 0.2	Meson masses
L_8	0.9 ± 0.3	Meson masses

L_9, L_{10}
external
gauge field
(weak + electromagn.)

Large N_c limit predicts

N_c: # of colors

$$L_7 = \mathcal{O}\left(N_c^2\right)$$

$$L_1, L_2, L_3, L_5, L_8 = \mathcal{O}\left(N_c\right)$$

$$2L_1 - L_2, L_4, L_6 = 0$$

The N_c behavior of L_7 is due to the $U_A(1)$ anomaly through its effect on the η' mass – $L_7 \propto N_c/m_{\eta'}^2$

η' is like a Goldstone mode for large N_c – $m_{\eta'}^2 \sim \frac{1}{N_c}$

Can we derive these results from a simple model of QCD?

euclidean space-time

Cahill + Roberts
Munczek
Kleinert, Shrauner

$$S_{QCD} = \int dx\, \bar{q}\,[\partial\!\!\!/ + M_0 + i g_s A\!\!\!/\,] - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

$$Z_{QCD}[\bar{\eta},\eta] = \int \mathcal{D}\bar{q}\,\mathcal{D}q\,\mathcal{D}A\, \exp\left[-S_{QCD}(\bar{q},q,A^a_\mu) + \bar{\eta}q + \bar{q}\eta\right]$$

can be rewritten

$$Z_{QCD}[\bar{\eta},\eta] = \int \mathcal{D}\bar{q}\,\mathcal{D}q\, \exp\left[-\int dx\, \bar{q}(\partial\!\!\!/ + M_0)q + \bar{\eta}q + \bar{q}\eta\right] \cdot \exp\left[W\left(i g_s \bar{q} \tfrac{\lambda^a}{2}\gamma_\mu q\right)\right]$$

with: $W[J]$ defined through:

$$\exp[W(J)] = \int \mathcal{D}A\, \exp\left(-\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + J^a_\mu A^a_\mu\right)$$

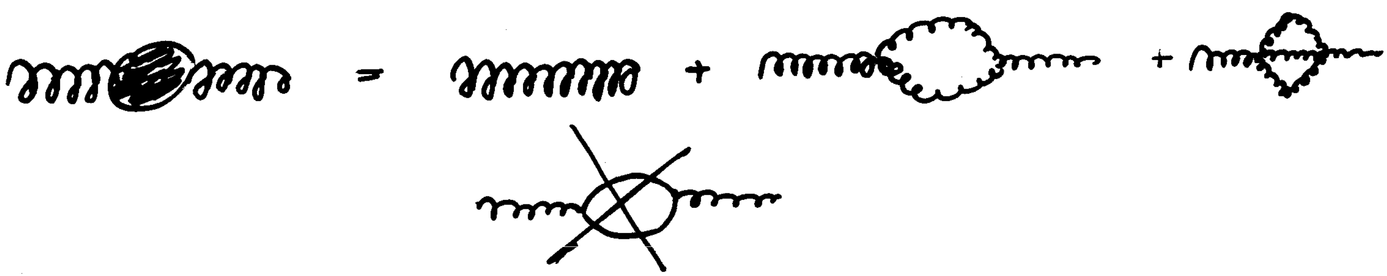
Expansion of $W[J]$ in gluon n-point functions

$$W[J] = \underbrace{W[0]}_{=\text{const}} + \underbrace{\int dx\, J^a_\mu(x) \frac{\delta W}{\delta J^a_\mu}\Big|_{J=0}}_{\sim \int \mathcal{D}A\,[A^a_\mu]\,e^{-\cdots} = \langle A^a_\mu\rangle = 0} + \frac{1}{2}\iint dx\,dy\, J^a_\mu(x) \frac{\delta^2 W}{\delta J^a_\mu(x)\,\delta J^b_\nu(y)}\Big|_{J=0} J^b_\nu(y) + \ldots$$

$$\left.\frac{\delta^2 W[J]}{\delta J^a_\mu(x)\,\delta J^b_\nu(y)}\right|_{J=0} = \int \mathcal{D}A\; A^a_\mu(x)\, A^b_\nu(y)\, e^{-\int dx\, \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}}$$

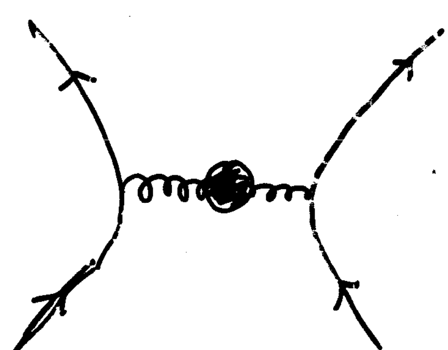
$$= D^{ab}_{\mu\nu}(x,y)$$

Gluon 2-point function (without quarks)

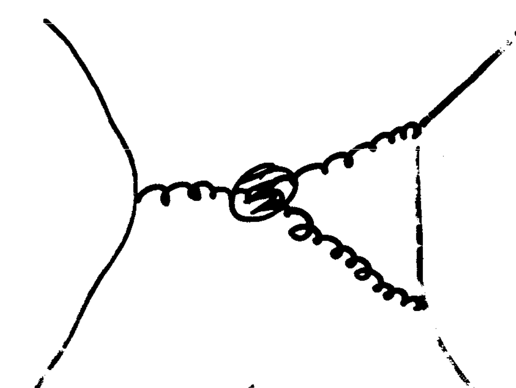


$$W[J] = \frac{1}{2}\int J^a_\mu(x)\, D^{ab}_{\mu\nu}(x,y)\, J^b_\nu(y) \;+\; \underbrace{W_n[J]}_{n\geq 3 \text{ point fcts.}}$$

Truncation: Only include $n=2$ point fcts., $\cancel{W_n}[J]$



Yes



No

$\Rightarrow$

Global Color Model GCM

$$Z_{GCM}[\bar\eta,\eta] = \int \mathcal{D}\bar q\, \mathcal{D}q\, e^{-\left\{S_{GCM}(\bar q, q) + \bar\eta q + \bar q \eta\right\}}$$

$$S_{GCM}[\bar q, q] = \int dx\,dy \left\{ \bar q(x)\left[\not\partial_x + M_0 \delta(x-y)\right] q(y) + \underbrace{\frac{g^2}{2} j^a_\mu(x) D^{ab}_{\mu\nu}(x-y) j^b_\nu(y)}\right\}$$

with: $j^a_\mu(x) = \bar q(x) \gamma_\mu \frac{\lambda^a}{2} q(x)$

Bi-Local
Quark-Quark
Interaction

All global symmetries of QCD
Chiral Symmetry !

lost: local color symmetry : not gauge invariant
renormalizability

Confinement ?

Model "Parameter":

Gluon 2 point function $D^{ab}_{\mu\nu}(x,y)$

$\Rightarrow$ Characterizes Quark-Quark Interaction

$$D^{ab}_{\mu\nu}(q^2) = \delta^{ab}\, \delta_{\mu\nu}\, \frac{\alpha_s(q^2)}{q^2}$$

"running coupling"

QED type

Model form for $\alpha_s(q^2)$

Special case: Contact form (Nambu-Jona-Lasinio)

$$D^{ab}_{\mu\nu}(x,y) = \delta_{\mu\nu}\, \delta^{ab}\, \delta(x-y)$$

Bosonization of the quark-quark interaction

· Fierz-Reordering of the quark-quark interaction:

$$\left[\bar{q}(x)\gamma_\mu \tfrac{\lambda^a}{2} q(x)\right] D(x-y) \left[\bar{q}(y)\gamma_\nu \tfrac{\lambda^b}{2} q(y)\right] =$$

$$= -\left[\bar{q}(x)\,\Lambda^\theta q(y)\right] D(x-y) \left[\bar{q}(y)\,\Lambda^\theta q(x)\right]$$

$$\Lambda^\theta = \frac{1}{2}\left(\mathbb{1}, i\gamma_5, \frac{i}{\sqrt{2}}\gamma_\nu, \frac{i}{\sqrt{2}}\gamma_\nu\gamma_5\right) \otimes \left(\mathbb{1}_F, \frac{\lambda_F^a}{\sqrt{2}}\right) \otimes \left(\frac{4}{3}\mathbb{1}_c, \frac{i}{\sqrt{3}}\lambda_c^a\right)$$

Hubbard-Stratonovich Transformation

$\Rightarrow$ bilocal meson fields ("auxiliary field")

$$\left[\bar{q}(x)\,\Lambda^\theta q(y)\right] \longrightarrow B^\theta(x,y)$$

$$\mathbb{1} \equiv \text{const.} \int \mathcal{D}B \; e^{-\int dx\,dy \,\frac{B^\theta(x,y)\,B^\theta(y,x)}{2g^2 D(x-y)}}$$

into $Z_{GCM} =$

$$= \int \mathcal{D}q\,\mathcal{D}\bar{q}\; e^{-\int dx\,dy \left\{\bar{q}(x)(\partial\!\!\!/ + M_0)q(y)\,\delta(x-y) - \frac{g^2}{2}\left[\bar{q}(x)\Lambda^\theta q(y)\right] D(x-y)\left[\bar{q}(y)\Lambda^\theta q(x)\right]\right\}}$$

Shift: $B^\theta_{(x,y)} \mapsto B^\theta(x,y) + g^2 D(x-y)\left[\bar{q}(y)\Lambda^\theta q(x)\right]$

$$Z_{GCM} = \int \mathcal{D}\bar{q}\, \mathcal{D}q\, \mathcal{D}B\; e^{-S_{GCM}[\bar{q},q,B]}$$

$$S_{GCM} = \int dx\, dy \left\{ \bar{q}(x) \left[\not{\partial}_x + M_0\, \delta(x-y) + \Lambda^{\theta} B^{\theta}(x,y) \right] q(y) + \frac{B^{\theta}(x,y)\, B^{\theta}(y,x)}{2g^2\, D(x-y)} \right\}$$

$\Rightarrow$ Quadratic in quark fields

$\Rightarrow$ Integrate over quark fields

$$Z_{GCM} = \int \mathcal{D}B\; e^{-S[B]}$$

$$S[B] = -\underbrace{\mathrm{Tr} \log G^{-1}[B]}_{\text{fermion determinant}} + \int dx\, dy\, \frac{B^{\theta}(x,y)\, B^{\theta}(y,x)}{2g^2\, D(x-y)}$$

$$G^{-1}(x,y) = (\not{\partial} + M_0)\, \delta(x-y) + \Lambda^{\theta} B^{\theta}(x,y)$$

= quark inverse Greens-Fct. with background B^{θ}

Mean Field

"Classical" Vacuum (Stationary Phase Approximation)

$$\boxed{\left.\frac{\delta S[B^\theta]}{\delta B^\theta}\right|_{B_0^\theta} = 0}$$

$O(N_c)$ in the quark

($D(x-y)$ fixed)

$\Rightarrow$ Schwinger-Dyson-Eq.

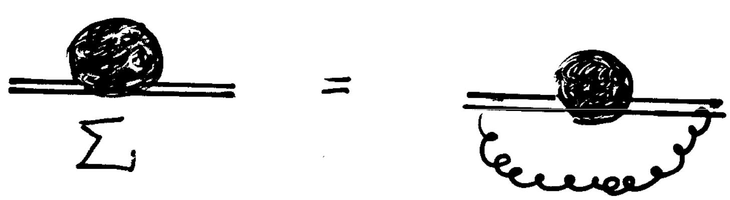
$$B_0^\theta(x-y) = g^2 D(x-y)\, \mathrm{tr}[\Lambda^\theta G_0(x-y)]$$

$$G_0 \equiv G[B_0]$$

$\Rightarrow$ Quark-Self Energy $\Sigma(p) = \Lambda^\theta B_0^\theta(p) = i\not{p}[A(p^2)-1] + B$

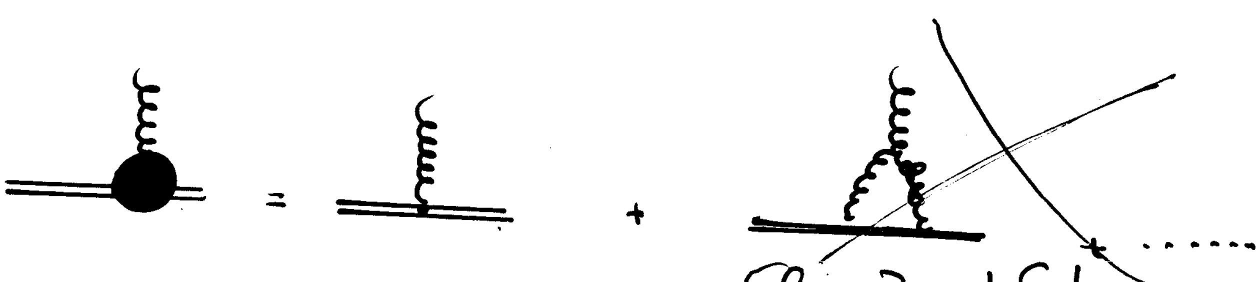
("Constituent" Quarks, dynamical χSB) $\mathbf{M(p^2) = \frac{B(p^2)}{A(p^2)}}$

"Constituent mass"



"Rainbow" SDE for quark propagator

No Dressing of the quark-gluon vertex !



Small fluctuations around Mean Field Vacuum $\mathcal{B}_0^\Theta$

$$\mathcal{B}^\Theta(x,y) = \mathcal{B}_0^\Theta(x-y) + \hat{\mathcal{B}}^\Theta(x,y)$$

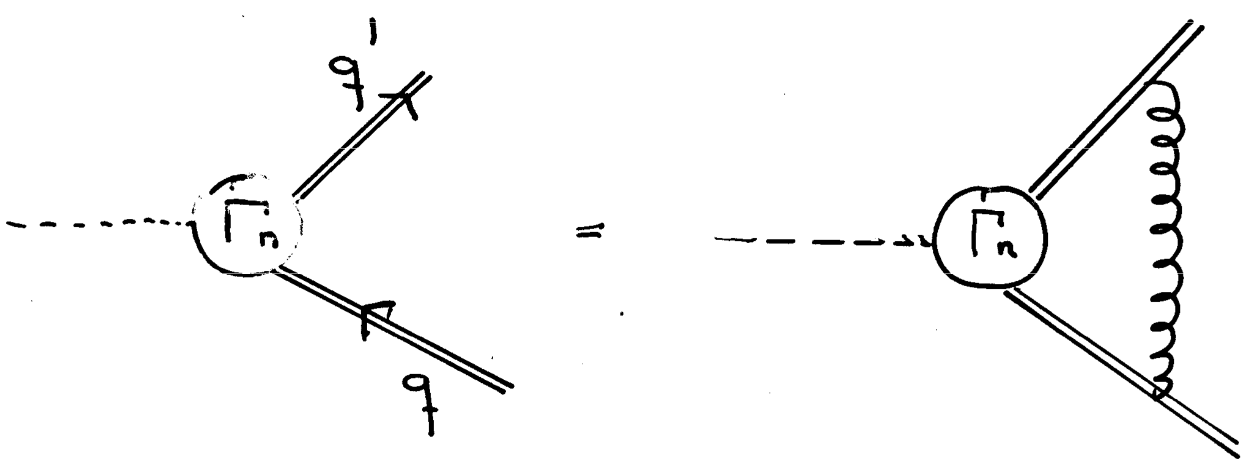
$\Rightarrow$ 2nd order

$$S^{(2)}[\mathcal{B}^\Theta] = \frac{1}{2}\int dP\,dq\,dq'\;\hat{\mathcal{B}}^\Theta(P,q)\,\underbrace{\mathcal{D}^{-1}_{\Theta\Theta'}(P,q,q')}_{\text{inverse meson propagator}}\,\hat{\mathcal{B}}^{\Theta'}(P,q')$$

Eigenmodes $\quad \mathcal{D}^{-1}\,\Gamma_n = \Delta_n^{-1}\,\Gamma_n$

on shell : $\quad \Delta_n(P^2 = -M_n^2) = 0 \Rightarrow$ bound meson with mass M_n

$$\boxed{\text{Ladder BSE}}$$



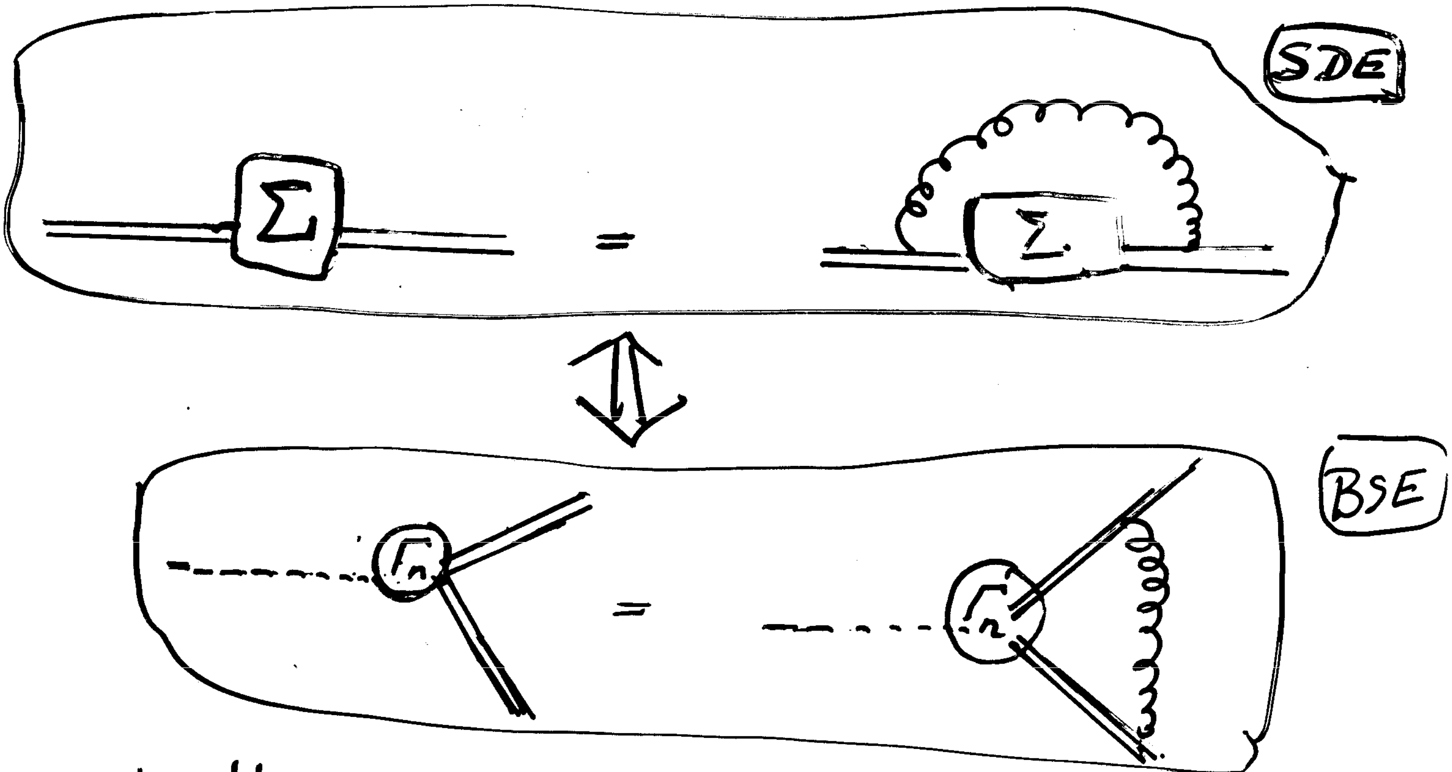
$\boxed{\text{Goldstone Boson} : \pi, K, \eta}$

Dynamical Chiral Symmetry Breaking

$\Longrightarrow$ Existence of massless bound state in the Pseudoscalar channel

$$\boxed{\Delta^{-1}_{\gamma_5} (P^2 = -M_\pi^2 = 0) = 0}$$

(Nambu - Goldstone - Theorem)



With: $\Gamma_5 = B$

$\Sigma = i\not{p}(A-1) + B$

Effective Interaction for Goldstone Bosons

at low energy : neglect all higher mass fluctuations
(e.g. $\rho, \omega, \sigma, \pi', \dots$)

$$\Lambda^{\epsilon} \hat{B}^{\epsilon}(x,y) = B(x-y) \left[U_5\left(\tfrac{x+y}{2}\right) - 1 \right]$$

Bethe Salpeter Ampl.

$$U = e^{i \lambda^a \pi^a / f_\pi} \qquad U_5 = P_R U + P_L U^+$$

$\Rightarrow$

$$\boxed{S[U] = -\mathrm{Tr}\log\left[G^{-1}[U]\right]}$$

$$G^{-1}[U] = \underbrace{(\gamma \cdot \partial_x) A(x-y) + M_0 \delta(x-y) + B(x-y)}_{\to\, G_0^{-1}} + B(x-y)\left[U_5\left(\tfrac{x+y}{2}\right) - 1\right]$$

free constituent quarks

inverse quark Propagator with U_5 - background

Low Energy Expansion

$$U_5 = P_L U^\dagger + P_R U$$

$$U = e^{i\pi^a\lambda^a/f_\pi}$$

$$S[U_5] = -\mathrm{Tr}\log\Big\{\underbrace{\big[\gamma\cdot\partial_x A(x-y) + M_0\,\delta(x-y) + B(x-y)\big]}_{G_0^{-1}} +$$

$$+ \underbrace{B(x-y)\left(U_5\left(\tfrac{x+y}{2}\right) - 1\right)}_{=V}\Big\}$$

Expansion simultaneously in M_0 and ∂U_5

(highly non-local operator: ∂_x, A, B)

$\Longrightarrow$ Gasser-Leutwyler Lagrangian

$$
\begin{aligned}
\mathcal{S} = \int d^4x \{ & \frac{f_\pi^2}{4}\mathrm{tr}\left[(\partial_\mu U)(\partial_\mu U^\dagger)\right] - \frac{f_\pi^2}{4}\left[U\chi^\dagger + \chi U^\dagger\right] \\
& - L_1\left(\mathrm{tr}\left[(\partial_\mu U)(\partial_\mu U^\dagger)\right]\right)^2 - L_2 \mathrm{tr}\left[(\partial_\mu U)(\partial_\nu U^\dagger)\right]\cdot \mathrm{tr}\left[(\partial_\mu U)(\partial_\nu U^\dagger)\right] \\
& - L_3 \mathrm{tr}\left[(\partial_\mu U)(\partial_\mu U^\dagger)(\partial_\nu U)(\partial_\nu U^\dagger)\right] + L_5 \mathrm{tr}\left[(\partial_\mu U)(\partial_\mu U^\dagger)(U\chi^\dagger + \chi U^\dagger)\right] \\
& - L_8 \mathrm{tr}\left[\chi U^\dagger \chi U^\dagger + U\chi^\dagger U \chi^\dagger\right]\},
\end{aligned}
$$

$$
\left.\begin{aligned} L_4 = L_6 = 0 \\ L_2 = 2L_1 \end{aligned}\right\} \text{Exact in l.c. } 1/N_c \text{ (consistent with phenomenology,}
$$

$L_7 = 0 \not\propto N_c^2$? left out $U_A(1)$, $\eta - \eta'$ mix

$\Rightarrow$ Predictions for low energy coefficients

f_π^2, L_i

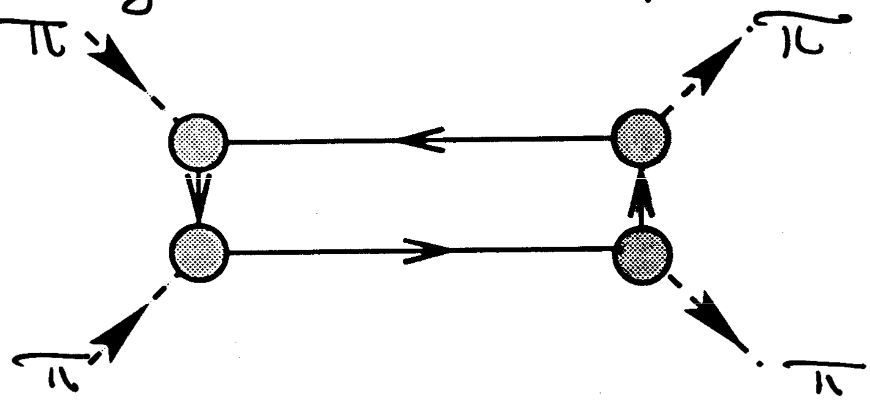
determined as complicated analytic expressions of the quark self energy $\Sigma(p) = i\not{p}[A(p^2)-1] + B(p^2)$ (and their derivatives)

$$
f_\pi^2, L_i = \int d^4p \;\text{fcts.}\left(A(p^2), B(p^2), A'(p^2), \dots\right)
$$

$\Sigma(p) \Leftarrow$ Gluon 2 point function $D(x-y)$

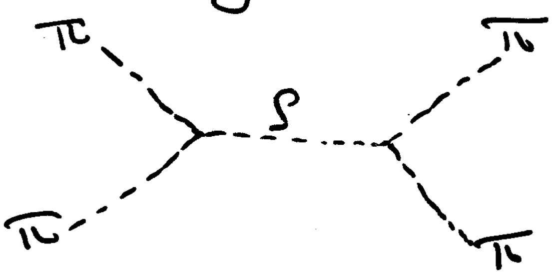
$\boxed{\text{Dyson-Schwinger Eq.}}$

π-π scattering via constituent quark loops

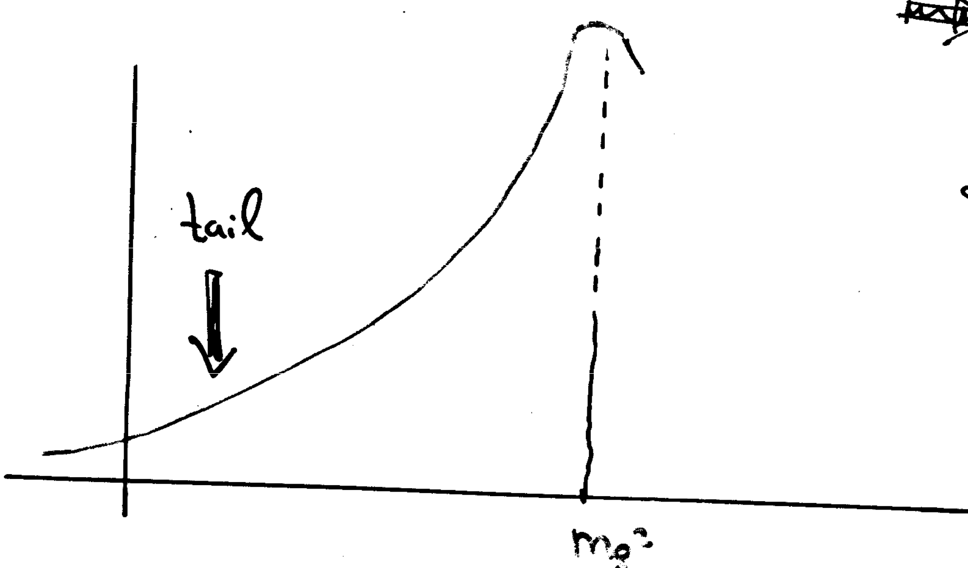


"Constituent Quark Mass" $M(p^2) = \frac{B(p^2)}{A(p^2)}$ Peaked at around 300-400 MeV

$\Rightarrow$ heavy meson exchange ($\rho, \sigma \ldots$) tail



at low energy



$\Rightarrow$ Low Energy Coefficients determined at $\mu^2 = m_\rho^2$

π- Loops ?

Meson Dressing of quark propagator
of higher order $1/N_c$
not present @ mean field level
finite corrections

diffent in χPTh

π- loops ∞ $\Rightarrow$ Renormalize

Goldstone field in χPTh at first unphysical
Parameters infinite
after renormalization: infinities from Loops cancel other infinities
$\Rightarrow$ Physical Parameters

our approach:

π- field Physical field (Solution of BSE)
Meson dressings finite $1/N_c$ corrections

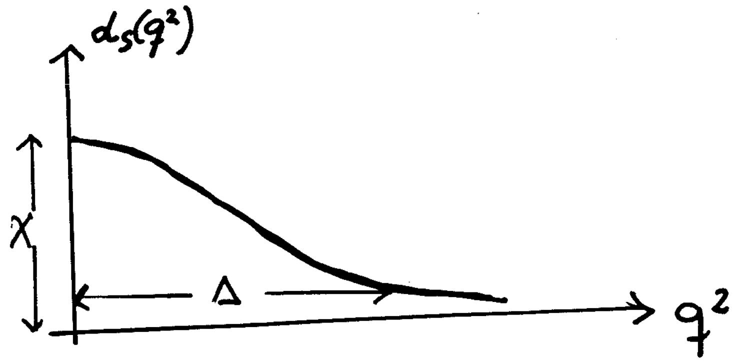
Gluon Propagator $D(q^2)$

"QED" like: $\quad g^2 D(q^2) = 4\pi \dfrac{\alpha_s(q^2)}{q^2}$

infrared model

χ: "strength"

Δ: "width"



$$\alpha_1(s) = \frac{3\pi}{4}\, s\, \frac{\chi^2}{\Delta^2}\, e^{-\frac{s}{\Delta}} + \frac{\pi d}{\left(\ln s/\Lambda^2 + e\right)}$$

$$\alpha_2(s) = \pi d \left[\frac{s}{s^2+\Delta}\chi^2 + \frac{1}{\left(\ln s/\Lambda^2 + e\right)} \right]$$

$$\alpha_3(s) = \pi d \left[\frac{1 + \chi e^{-s/\Delta}}{\ln\left(s/\Lambda^2 + e\right)} \right]$$

asyptotic freedom: $\quad \alpha_s(s) \overset{s\to\infty}{\sim} \dfrac{\pi \cdot d}{\ln s/\Lambda^2}$

$d = \dfrac{12}{33-2N_f}$

$\Lambda = 200\ \mathrm{MeV}$

$\alpha_1(s) = 3\pi s\chi^2 e^{-s/\Delta}/(4\Delta^2) + \pi d/\ln(s/\Lambda^2+e)$							
Δ (GeV^2)	χ (GeV)	$-\langle\bar{q}q\rangle^{1/3}$ (MeV)	L_1(0.7±0.5)	L_3(-3.6±1.3)	L_5(1.4±0.5)	L_8(0.9±0.3)
0.002	1.4	150	0.84	-4.4	1.0	0.88
0.02	1.5	160	0.82	-4.4	1.14	0.84
0.2	1.65	173	0.81	-4.0	1.66	0.83
0.4	1.84	177	0.80	-3.8	2.0	0.87

π-π
K-decay

Decay constants
meson masses

$\alpha_2(s) = \pi d(s\chi^2/(s^2+\Delta) + 1/\ln(s/\Lambda^2+e))$						
Δ (GeV^4)	χ (GeV)	$-\langle\bar{q}q\rangle^{1/3}$ (MeV)	L_1(0.7±0.5)	L_3(-3.6±1.3)	L_5(1.4±0.5)	L_8(0.9±0.3)
10^{-7}	0.83	162	0.82	-4.4	1.28	0.87
10^{-4}	1.02	167	0.81	-4.2	1.60	0.91
10^{-1}	1.83	173	0.79	-3.8	2.36	1.00
1	2.73	173	0.79	-3.5	3.0	1.17

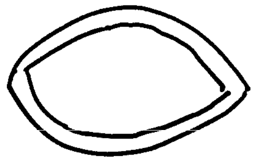
π-π
K-decay

decay constants
meson masses

$\alpha_3(s) = \pi d(1+\chi e^{-s/\Delta})/\ln(s/\Lambda^2+e)$						
Δ (GeV^2)	χ	$-\langle\bar{q}q\rangle^{1/3}$ (MeV)	L_1(0.7±0.5)	L_3(-3.6±1.3)	L_5(1.4±0.5)	L_8(0.9±0.3)
0.1	61.0	163	0.82	-4.3	1.22	0.83
0.4	24.0	169	0.81	-4.2	1.48	0.84
1.0	15.3	171	0.80	-4.1	1.73	0.88
2.0	12.2	172	0.80	-4.0	1.97	0.95

⇒ L_1, L_3 insensitive to form of $\alpha(s)$
L_5 more sensitive

Vacuum Structure

$\langle \bar{q} q \rangle$  dressed quark loop

Action is quadratic in gluon field

$\Rightarrow$ Perform Integration over Gluon field

$$\int \mathcal{D}A(A_\mu A_\nu \ldots)\, e^{-\int A D^{-1} A + j \cdot A}$$

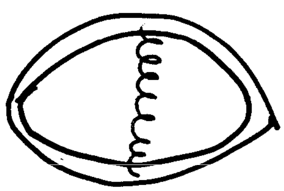
Quark color vector current

Gluon Propagator

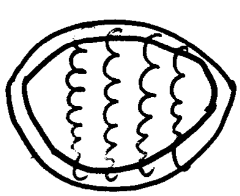
Mixed Condensate $\langle \bar{q}\, G_{\mu\nu} \sigma^{\mu\nu} q \rangle$

UV divergent integrals

$\Rightarrow$ [illegible]

 + ...

Gluon Condensate $\langle G_{\mu\nu} G^{\mu\nu} \rangle$

 + ...

	$-\langle\bar{q}q\rangle^{\frac{1}{3}}$ [MeV]	$-\langle g_s\bar{q}\sigma Gq\rangle^{\frac{1}{5}}$ [MeV]
this calculation	150 – 180	400 – 460
QCD sum rules	210 – 230	375 – 395
quenched lattice	225	402 – 429
instanton liquid model	272	490

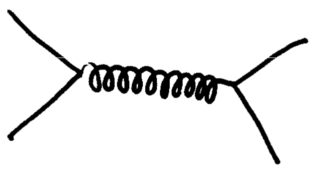
Summary

QCD

⇓

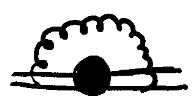
Truncation: only gluon 2 point functions

Quark-Quark Interaction



 = Model-Gluon 2 point function

⇓ Hadronize @ Mean-Field Level $O(N_c)$

"Dressed" Quarks 

Meson Bound States (Bethe-Salpeter)

⇓

Effective Chiral Hadronic Theory ≡ XPTH

... underlying quark theory

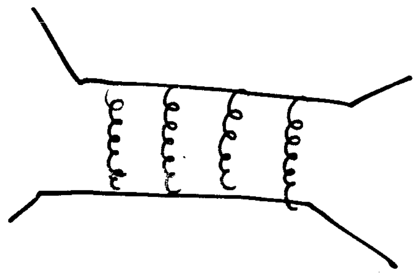
⇒ good description of low energy phenomena (Chiral Coefficients, Condensates)

* Talks on Monday:
Meson Formfactors, Higher Resonances, Heavy-Light Quark, finite T

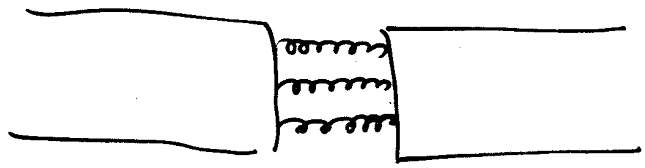
Next Step: Include higher resonances $(\delta, \ldots)$
Integrate them out

$\rightarrow$ Generates

"Triangle Type"



$O(N_c)$



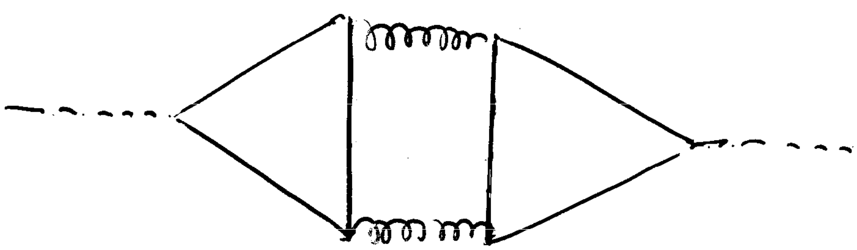
$O(1)$

Triangle Type Diagram breaks $U_A(1)$

$\boxed{\text{Mass Generation for } \eta'}$

Frank + Meissner
hep-ph 9703270

Chiral Limit $m_q \to 0$



Triangle
Diagram
2 gluons exchanged

Necessary and sufficient: infrared singular $\sim \frac{1}{q^4}$
(Kogut + Susskind)

$\{\mathrm{Ln\ Det}\ U\}^2$

alternative to Instanston ('t Hooft)