Chiral Condensates in the N-flavor Massive Schwinger Model

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Strong – weak coupling

small – large fermion masses

$T = 0$ and $T \neq 0$

θ

anomaly around $\theta = \pi$

1. QED$_2$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{a=1}^{N} \overline{\psi}_a \{ \gamma^\mu (i\partial_\mu - eA_\mu) - m_a \} \psi_a$$

$$(\; e \; , \; m \; , \; \theta \; , \; T \;)$$

$$\begin{cases} \dfrac{e}{m} \gg 1 & \text{strong coupling} \\ \dfrac{e}{m} \ll 1 & \text{weak coupling} \end{cases}$$

$\boxed{T \neq 0 \text{ on } R^1}$

$$\psi_a(\tau + \frac{1}{T}, x) = -\psi_a(\tau, x)$$
$$A_\mu(\tau + \frac{1}{T}, x) = \quad A_\mu(\tau, x)$$

$$\Updownarrow$$

$\boxed{T = 0 \text{ on } S^1}$

$$\psi_a(t, x+L) = -\psi_a(t, x)$$
$$A_\mu(t, x+L) = \quad A_\mu(t, x)$$

Manton 85, Hetrick-Hosotani 88
PR D38 (88) 2621

2. N-flavor Massless Fermions

Chiral $U(1)$: broken by anomaly

Chiral $SU(N)$: intact

$$\langle \overline{\psi} \psi \rangle = 0 \quad \text{for} \quad N \geq 2$$

Bosonization on S^1

$$\psi_\pm^a(t,x) = \frac{1}{\sqrt{L}} C_\pm^a \, e^{\pm i \{ q_\pm^a + 2\pi p_\pm^a (t \pm x)/L \}} \, : e^{\pm i \sqrt{4\pi} \phi_\pm^a(t,x)} :$$

$$e^{2\pi i p_\pm^a} \, | \, \text{phys} \, \rangle = \, | \, \text{phys} \, \rangle$$

– *exact operator identity* –

Strategy

Interaction picture defined by free massless fermions

⇓

Schrödinger/Heisenberg picture

Halpern 75
Coleman 76

3. Hamiltonian

anomaly

$$H_0 = \frac{\pi\mu^2 L}{2N} P_W^2 + \frac{1}{2\pi L} \sum_{a=1}^{N} (\Theta_W + 2\pi p_a)^2$$

$$+ \int_0^L dx \, \frac{1}{2} \{ \sum_{a=1}^{N} ({\Pi_a}^2 + \phi_a'^2) + \mu^2 \, (\frac{1}{\sqrt{N}} \sum_a \phi_a)^2 \}$$

coulomb int.

(Θ_W, P_W) : Wilson line phase

— *Exactly solved.* —

$\Phi = \frac{1}{\sqrt{N}} \sum_a \phi_a$: massive $\mu^2 = \frac{Ne^2}{\pi}$

$\chi_1 \sim \chi_{N-1}$: massless

$SU(N)$ current algebra

$$Q_5 = \underbrace{2\Sigma\, p_a}_{\tilde{Q}_5} + \frac{N\Theta_W}{\pi}$$

4. θ vacuum

Invariance under $\Theta_W \to \Theta_W + 2\pi \quad , \quad p_a \to p_a - 1$

$$U = e^{2\pi i P_W + i\sum q_a} \quad , \quad UHU^\dagger = H$$

$$\Downarrow$$

$$U \,|\Psi_{\rm vac}(\theta)\,\rangle = e^{i\theta}\,|\Psi_{\rm vac}(\theta)\,\rangle$$

Coleman 1976
Hetrick-Hosotani 1988

Martinovic
Srivastava

Wave function

$$|\Psi_{\rm vac}(\theta)\,\rangle = \sum_n \int dp_W[d\varphi]\;|p_W, n, \varphi_a\,\rangle$$

$$\times\; e^{-in\theta + 2\pi i n p_W}\; f(p_W, \varphi_a + \frac{2\pi p_W}{N} + \delta_a)$$

for massless fermions $\quad f = {\rm const}\cdot e^{-\pi\mu L p_W^2/2N}$

$\Gamma_a + \frac{N}{\pi}\Theta_W$

5. Condensates ($m = 0$)

$$\psi^\dagger_{a+}\psi_{a-} \sim -\frac{1}{L}\, e^{iq_a} N_0[e^{\sqrt{4\pi} i\phi_a}]$$

but $\quad N_0[e^{i\beta\Phi(x)}] = B(\mu L)^{\beta^2/4\pi}\, N_\mu[e^{i\beta\Phi(x)}]$

$$B(\mu L) = \frac{\mu L}{4\pi} \exp\{\gamma + \frac{\pi}{\mu L} - 2\int_0^\infty \frac{dx}{e^{\mu L \cosh x} - 1}\}$$

Hetrick-Hosotani 88, Sachs-Wipf 92

$$\langle \overline{\psi}_a \psi_a \rangle_\theta = \begin{cases} -\dfrac{2}{L}\, B(\mu L)\, e^{-\pi/\mu L} \cos\theta & \text{for } N = 1 \\ 0 & \text{for } N \geq 2 \end{cases}$$

$$\langle \psi^\dagger_{N+} \cdots \psi^\dagger_{1+} \psi_{1-} \cdots \psi_{N-} \rangle_\theta = -\frac{B(\mu L)^N}{L^N} e^{-i\theta} e^{-N\pi/\mu L}$$

$$\sim -(\frac{\mu e^\gamma}{4\pi})^N e^{-i\theta}$$

Chiral condensates arise as the mass of Φ _changes_ from 0 to μ.

θ vacuum ··· zero modes

Spontaneous Breakdown

of Chiral Symmetry

fermion picture … N-J (60)

massless fermions → massive
chiral pair BCS states

Bogoliubov transformation

boson picture in $d=2$

Bogoliubov transf.

6. Massive fermions

$$m_1 = m_2 = \cdots = m_N$$

$$H = H_0 + H_{\rm mass}$$

$$\mu_\Phi^2 = \mu^2 + \delta\mu_\Phi^2$$

$$\mu_\chi^2 = \qquad \delta\mu_\chi^2$$

$$*** \qquad \langle \overline{\psi}\psi \rangle_\theta^{\rm physical} = \langle \overline{\psi}\psi \rangle_{\theta,e} - \langle \overline{\psi}\psi \rangle_{e=0} \qquad ***$$

Mass perturbation theory (N=1) — Adams
Reduction to QM — (Y.H. et al)
Lattice gauge theory
Light-cone method

$$H\, f(p_W,\varphi) = \epsilon\, f(p_W,\varphi)$$

$$H = -\left(\frac{N}{2\pi}\right)^2 \frac{\partial^2}{\partial p_W^2} - (N-1)\triangle_\varphi + V_N(p_W,\varphi)$$

$$\triangle_N = \sum_{a=1}^{N-1} \frac{\partial^2}{\partial\varphi_a^2} - \frac{2}{N-1}\sum_{a<b}^{N-1}\frac{\partial^2}{\partial\varphi_a\partial\varphi_b}$$

$$V_N = \frac{(\mu L p_W)^2}{4} - \frac{N}{\pi}\sum_{a=1}^{N} m_a L\overline{B}_a \cos\left(\varphi_a - \frac{2\pi p_W}{N}\right)$$

$$\varphi_N = \theta_{\rm eff} - \sum_{a=1}^{N-1}\varphi_a$$

QUANTUM MECHANICS of N degrees

$$V(p_W,\varphi) \to f(p_W,\varphi) \to \mu_\alpha^2 \to V(p_W,\varphi)$$

7. 1-flavor

$$V = \frac{(\mu L)^2}{4} p_W^2 - \frac{mLB(\mu_1 L)}{\pi} \cos(2\pi p_W - \theta)$$

For small m

$$\mu_1^2 = \mu^2 + \frac{8\pi m B(\mu_1 L)}{L} \langle \cos(2\pi p_W - \theta) \rangle_f$$

$$\langle P_q \rangle_{\theta,T} = \begin{cases} 0 & \text{for } \frac{q}{e} \notin \mathbb{Z} \\ \int_{-\infty}^{\infty} dp_W \, f(p_W)^* f(p_W - \frac{q}{e}) & \text{for } \frac{q}{e} \in \mathbb{Z} \end{cases}$$

As $L \to \infty \quad (T \to 0)$

$$\mu_1^2 = \mu^2 + 2e^\gamma m \mu_1 \langle \cos(2\pi p_W - \theta) \rangle_f$$

Naively

$$\langle \overline{\psi} \psi \rangle_\theta = -\frac{2B(\mu_1 L)}{L} \langle \cos(2\pi p_W - \theta) \rangle_f$$

In the free theory ($\mu \to 0, L \to \infty$)

$$\langle \cos(2\pi p_W - \theta) \rangle_f \to 1$$

$\mu_1 = 2m$ since ϕ is the breather mode.

Hence $$\langle \overline{\psi} \psi \rangle_{e \to 0, L \to \infty} = -\frac{e^\gamma}{\pi} m = -0.567 \ m$$

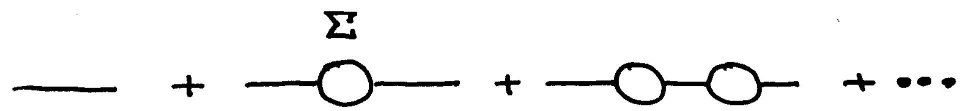
numerically $-0.562 \ m$ at $T/\mu = 0.005$.

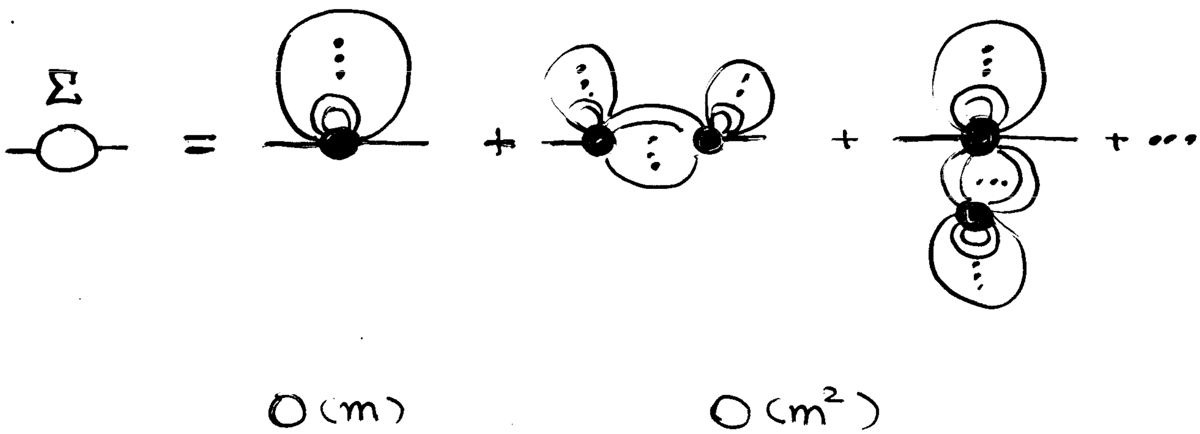
In the bosonization method

$$\boxed{\langle \overline{\psi} \psi \rangle_\theta^{\text{physical}} = -\frac{2B(\mu_1 L)}{L} \langle \cos(2\pi p_W - \theta) \rangle_f + \frac{e^\gamma}{\pi} m}$$

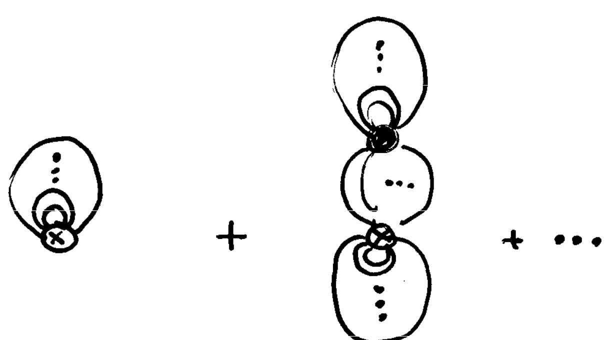
Mass perturbation

$\langle \phi \phi \rangle =$





$\langle \bar{\psi} \psi \rangle =$



$T=0$ Adam

$T \neq 0$ Y.H.

For large m

$$\frac{m}{e} \to \infty \text{ corresponds to the free theory.}$$

$$\mu_1 \sim 2m$$

$$\boxed{\mu_1 = g(\mu, m, L, \theta)}$$

— Interpolation formula —

$$\mu_1^2 = \mu^2 + \frac{8\pi m B(\mu_1 L)}{L} \langle \cos(2\pi p_W - \theta) \rangle_f - 4(e^\gamma - 1)\, m^2 h(mL)$$

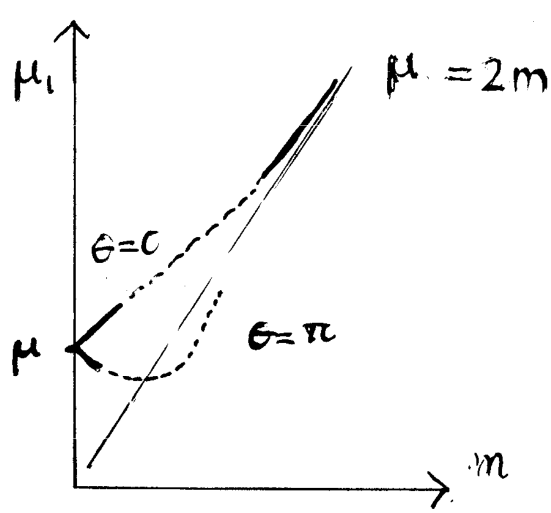
$h(\infty) = 1$

??

$T = 0,\ \theta = 0$
$(L = \infty)$

$\mu_1^2 \simeq \mu^2 + 2e^\gamma m + 4m^2$

Harada, Heinzl, Stern



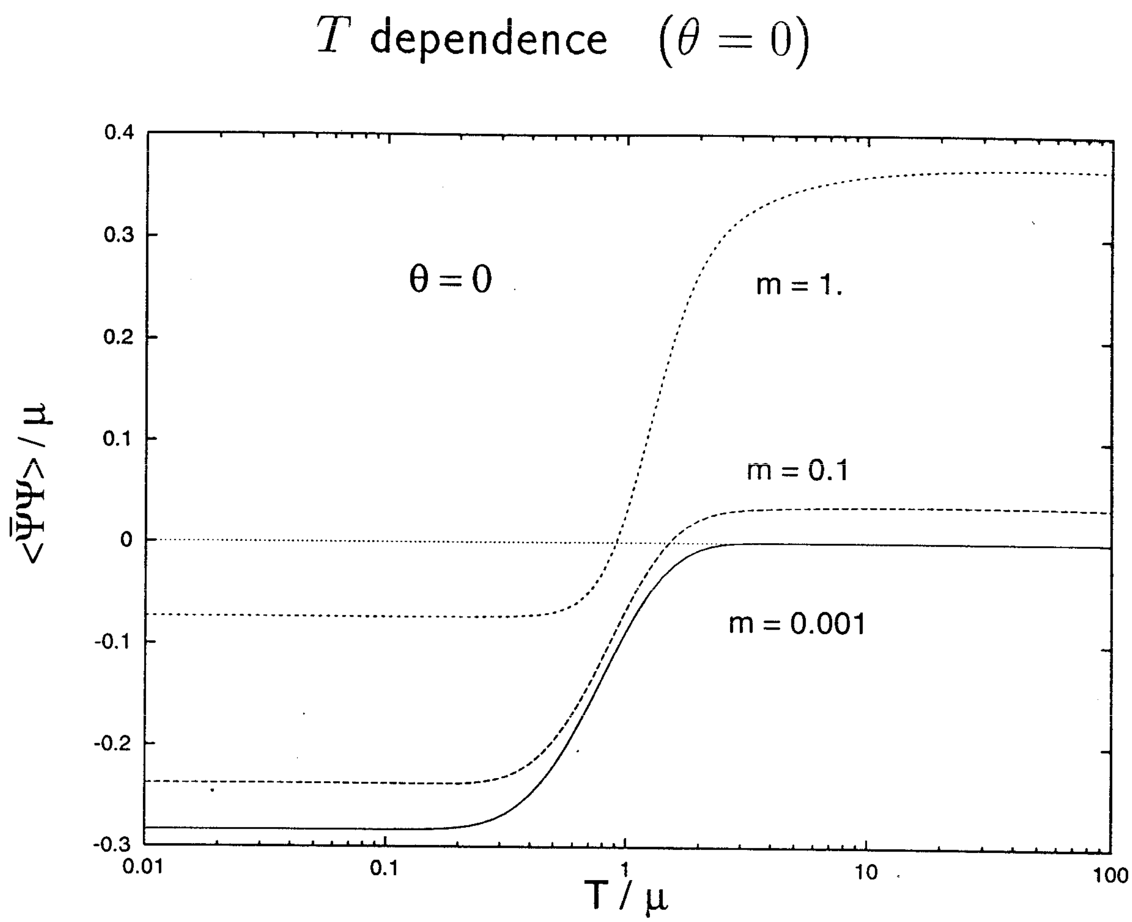


Figure 1 : T dependence of the chiral condensate in the $N = 1$ model at $\theta = 0$. The mass m in the figure is measured in the unit of μ.

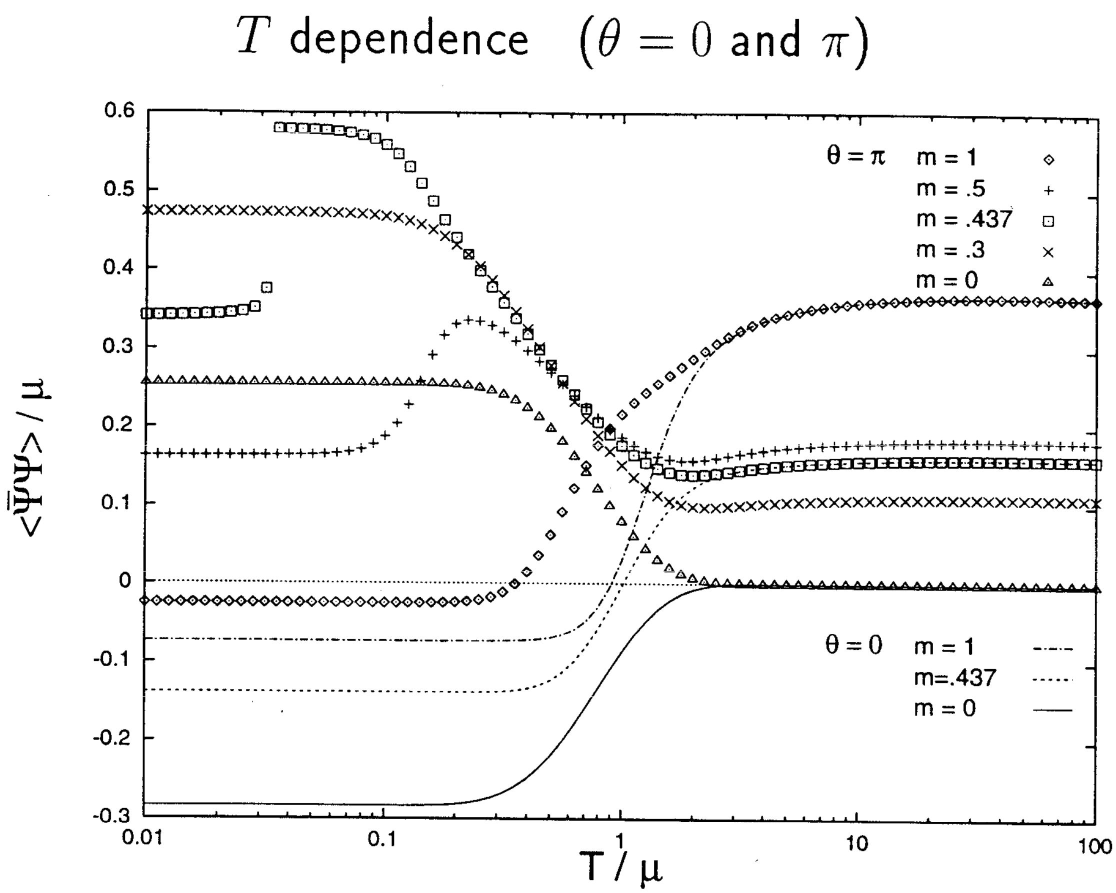
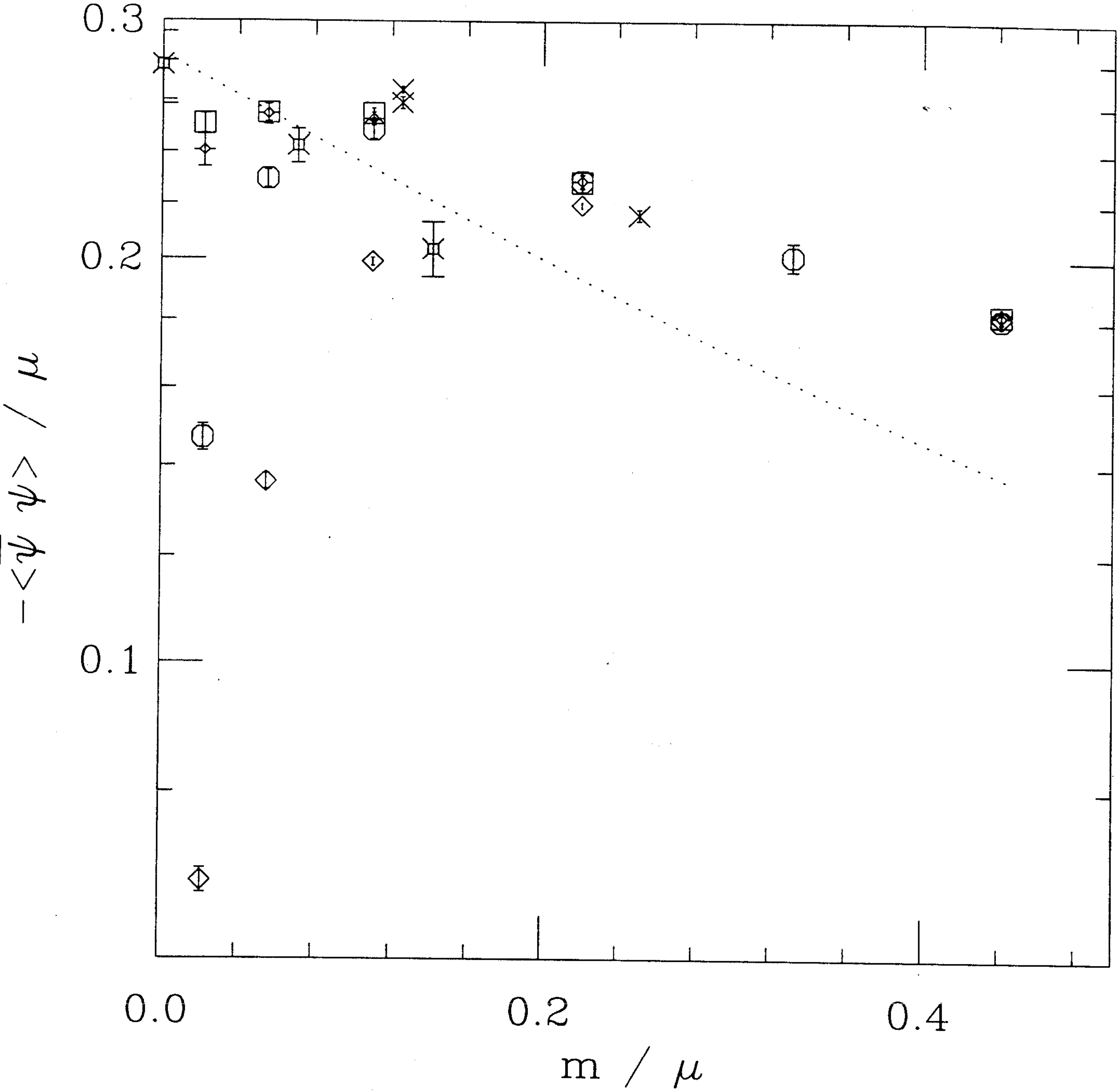


Figure 2 : T dependence of the chiral condensate in the $N = 1$ model. Lines and points are for $\theta = 0$ and π, respectively. The mass m in the figure is measured in the unit of μ.

Lattice gauge theory



de Forcrand, Arjan, Hetrick

6 × 16 ~ 48 × 128

$\beta = \frac{1}{(ea)^2} = 2$

8. Anomalous behavior around $\theta = \pi$

Y.H., Rodriguez
PLB 389 (96) 121

Near $\dfrac{m}{\mu} = .44$

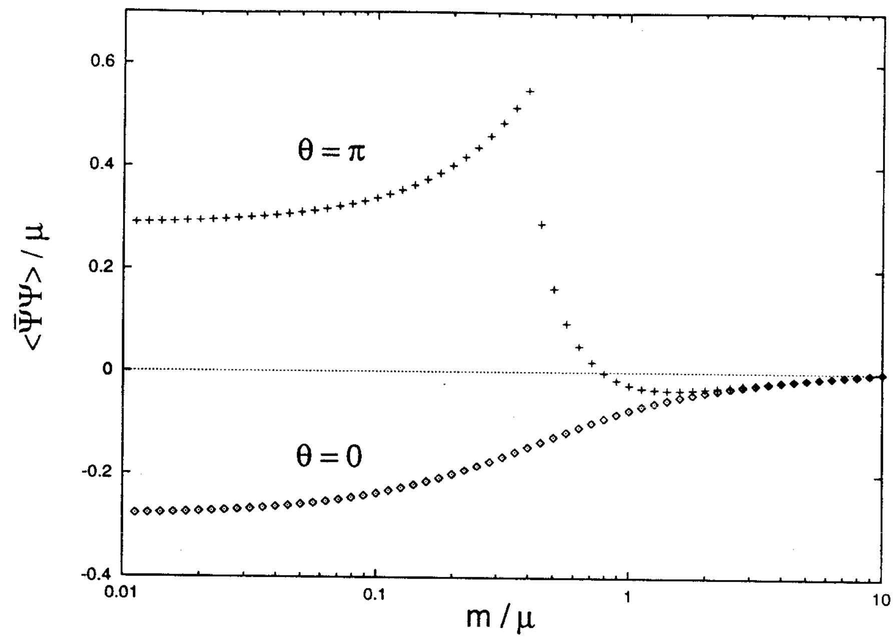


Figure 4: m dependence of the chiral condensate in the $N = 1$ model.

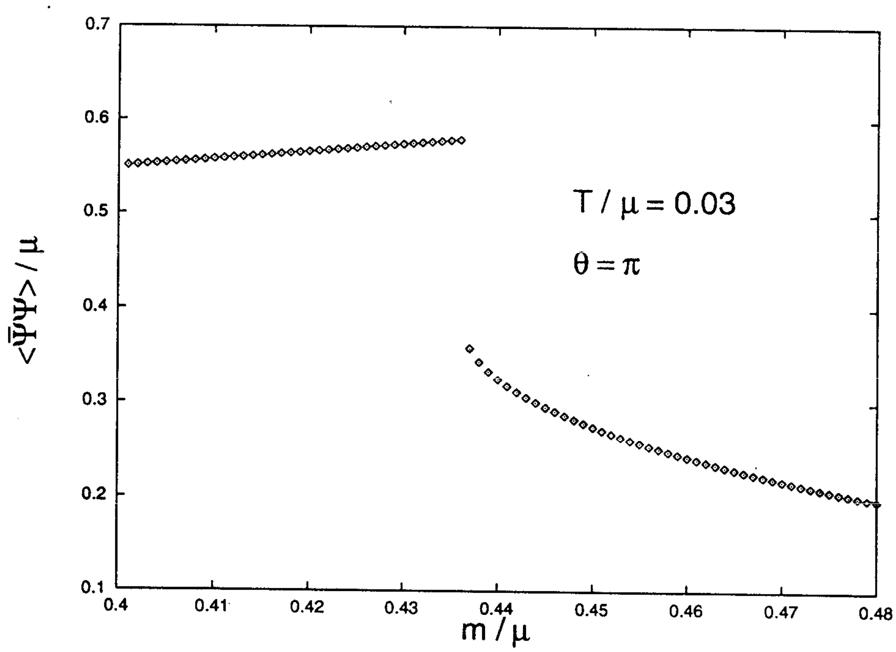


Figure 5: m dependence of the chiral condensate in the $N = 1$ model.

Why?

$$\{ -\frac{d^2}{dp_W^2} + (\pi\mu L\, p_W)^2 - k\cos(2\pi p_W + \theta)\} f(p_W) = \epsilon f(p_W)$$

$$\boxed{k_{\rm in}} \to V(p_W) \to f(p_W) \to \mu_1 \to \boxed{k_{\rm out}}$$

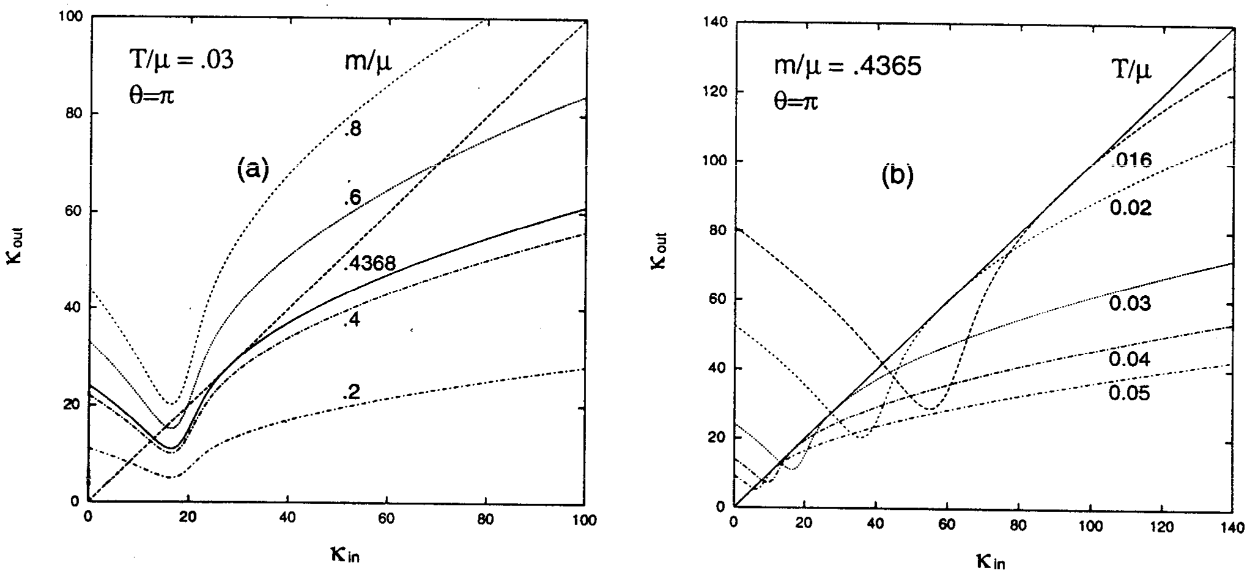


Figure 6: $\kappa_{\rm in}$ vs $\kappa_{\rm out}$.

Consistent with Mermin-Wagner theorem?

8. N flavor

$$\mu_\chi^2 \sim m \, \langle \overline{\psi} \psi \rangle \sim m \, \mu_\Phi^{\frac{1}{N}} \, \mu_\chi^{1-\frac{1}{N}}$$

$$\mu_\chi \sim m^{\frac{N}{N+1}} \quad \text{for small } m$$

$$\frac{1}{\mu} \langle \overline{\psi} \psi \rangle_\theta =$$

$$\begin{cases} -\frac{1}{4\pi}(2e^\gamma \cos \frac{\bar{\theta}}{N})^{\frac{2N}{N+1}} (\frac{m}{\mu})^{\frac{N-1}{N+1}} & \text{for } T \ll m^{\frac{N}{N+1}} \mu^{\frac{1}{N+1}} \\ -\frac{2N}{\pi(N-1)} \frac{m}{\mu} (\frac{\mu e^\gamma}{4\pi T})^{2/N} & \text{for } m^{\frac{N}{N+1}} \mu^{\frac{1}{N+1}} \ll T \ll \mu \\ -\frac{2N}{\pi(N-1)} \frac{m}{\mu} e^{-2\pi T / N\mu} & \text{for } T \gg \mu \end{cases}$$

$N=2$, $T=0$ Coleman 76

$N=2$, $T \neq 0$ Hetrick, YH, Iso PLB 350 (95) 92

3 PRD 53 (96) 7255

⋮ Rodriguez, YH PLB 375 (96) 273

T dependence

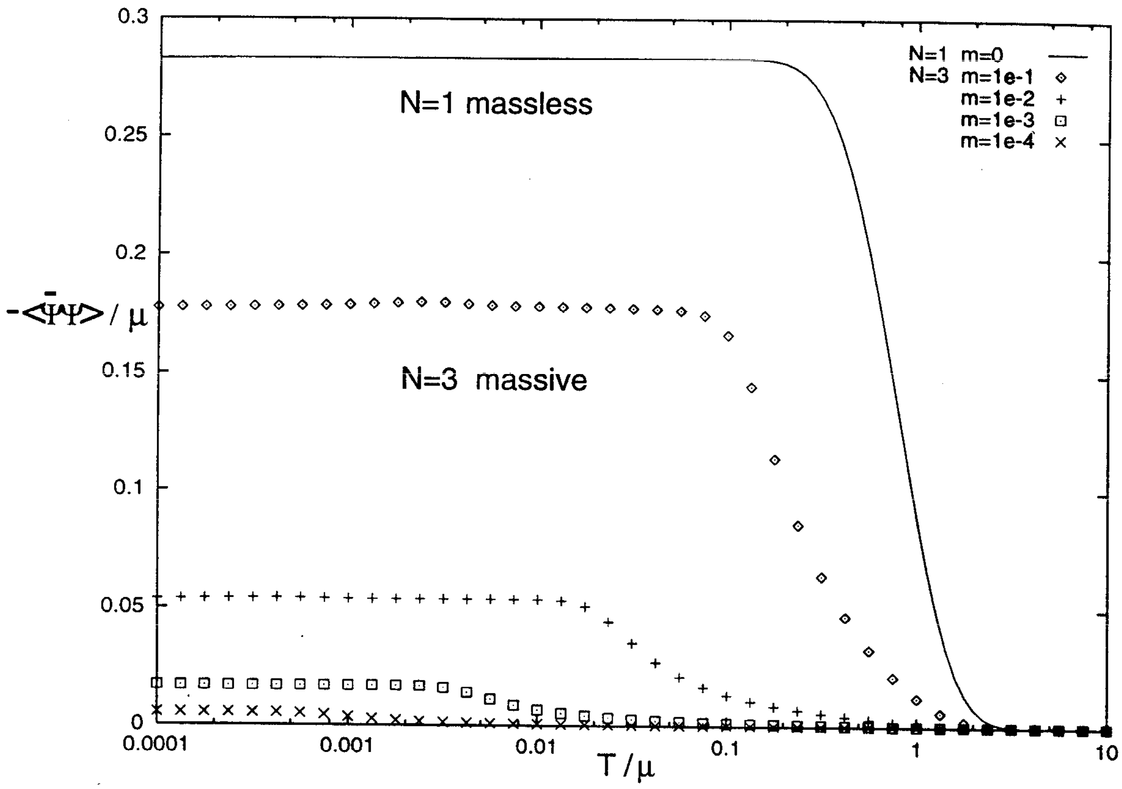


Figure 7: Chiral condensates in the 3 flavor model.

$\boxed{T \to 0}$

$\boxed{\text{Discontinuity at } \theta = \pm\pi}$ $\Rightarrow$ $\boxed{\text{Cusp}}$ in $\langle \overline{\psi}\psi \rangle_\theta$

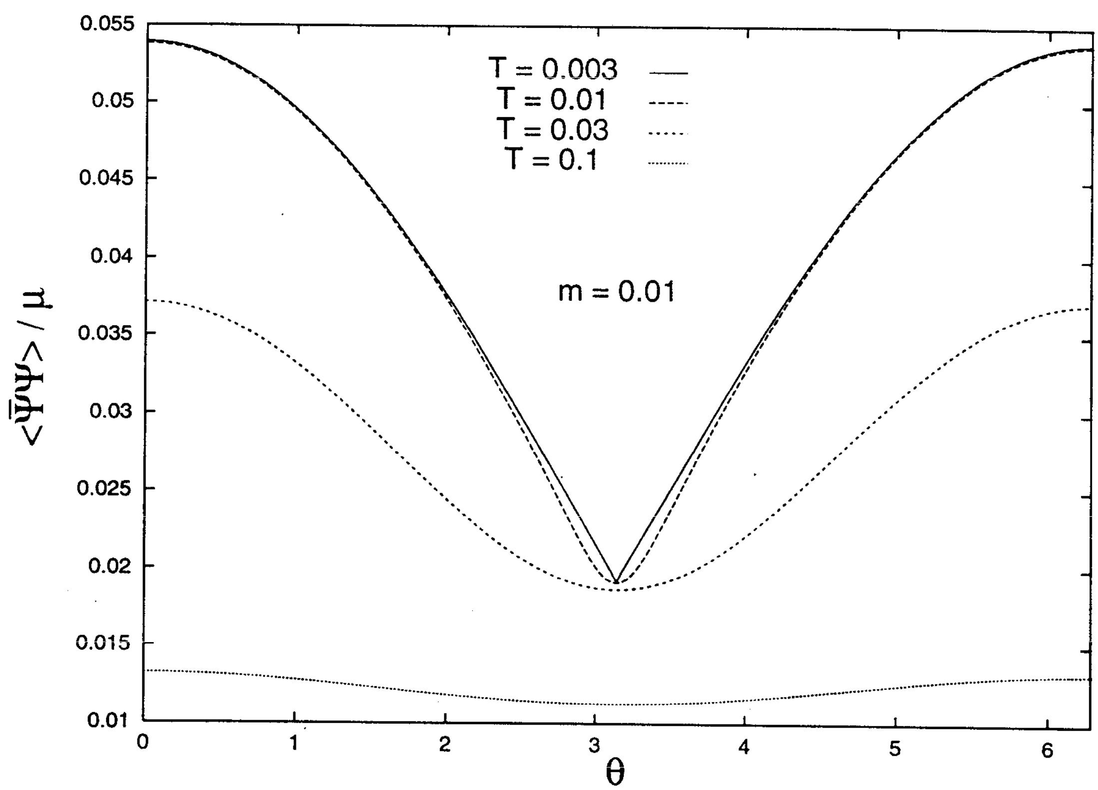


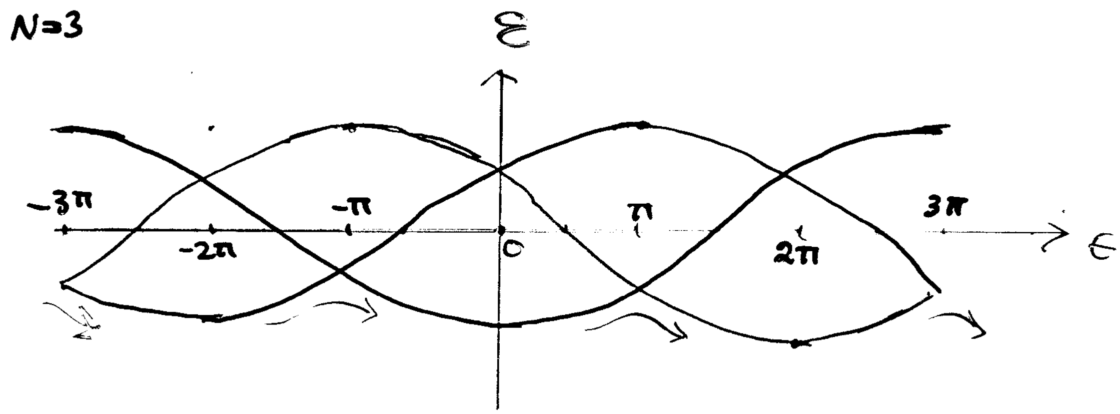
Figure 8 : The θ dependence. There appears a cusp at $\theta = \pi$ at $T = 0$.
$N_f = 3$ model.

θ-dependence

$$\langle \bar\psi \psi \rangle_\theta \propto \left(\cos \frac{\bar\theta}{N} \right)^{2N/N+1}$$

$$\bar\theta = \theta \mod 2\pi$$

$$-\pi < \bar\theta < \pi$$



~ Witten's picture of QCD

Summary

QED$_2$

$\Downarrow$

N-degree Quantum Mechanics

input $V[p_W, \varphi]$ = output $V[p_W, \varphi]$

$\langle \bar{\psi}\psi \rangle$ at arbitrary m, θ, T

Anomalous behavior at N=1, $\theta \sim \pi$, $\dfrac{m}{\mu} \sim 0.44$

m (fermion) – μ_α (bosons) : needs improvement