BRS-BFT QUANTIZATION OF CHIRAL SCHWINGER MODEL ON THE LIGHT-FRONT

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- SSB on LF
- Condensate vacua (θ-vacua) in the LF quantized Schwinger Model (SM)
- Vacuum in LF quantized Chiral Schwinger Model (CSM)
- BRS-BFT quantization of CSM and First class Hamiltonian and First class Lagrangians.

SSB

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Separation:

$$\boxed{\phi(\tau, x^-, x^\perp) = \omega(\tau, x^\perp) + \varphi(\tau, x^-, x^\perp)} \quad \tau = \frac{1}{\sqrt{2}}(x^0 + x)$$

Dynamical "condensate" variable — quantum fluctuations

⊙ φ (and ω) Fourier transformable in spatial variables x^- and $x^\perp$

followed by the Dirac procedure for constrained dynamical system to construct Hamiltonian frame work on the LF which may then be quantized.

Eg: 1+1 dim: $\mathcal{L} = \dot{\varphi}\varphi' - V(\omega+\varphi)$; $\phi = \omega(\tau) + \varphi$

$\rightarrow H^{LF} = \int V(\phi)\,dx^-$, $[\varphi(\tau, x^-), \varphi(\tau, y^-)] = -\frac{i}{4}\epsilon(x^- - y^-)$

plus Constraint eq.: $\int \left(\frac{\delta V(\phi)}{\delta \phi}\right) dx^- = 0$

For $V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2$

$$\omega(\lambda\omega^2 - m^2) + \lim_{L\to\infty} \frac{\lambda}{L}\left[\int_{-L/2}^{L/2} \varphi^2 dx^- + \omega \int_{-L/2}^{L/2} \varphi^3 dx^-\right] = 0$$

- ~~The~~ 2^{nd} term drops at Tree Level $\rightarrow$
  $\omega(\lambda\omega^2 - m^2) = 0$, $\omega = 0, \pm\frac{m}{\sqrt{\lambda}}$ + Stability OK.
- quantum corrections to tree level value of ω.
- ∞-volume limit of DLCQ exists and
  $[\omega(\tau), \varphi(\tau, x^-)] = 0$ only in ∞-vol. limit
  i.e. ω becomes a c-number.

- Extension to (3+1) dim and continuous symmetry
- Coleman's theorem on the absence of Goldstone boson's in two dim: a new proof on LF.

⊙ Theory quantized on $x^+ =$ const. plane already contains equal x^- commutators in it.

⊙ The same physical results emerge on the LF as in the conventional equal-time frame work though obtained through different mechanisms.

Needed: Topological considerations on LF?
Decay of False Vacuum on LF?
Chirality on the LF vs Chirality in instant form?
...
...

Schwinger Model. Condensate (or Theta) Vacua:

(1+1 dim) hep-th/9610149

Lowenstein + Swieca: conventional equal-time formulation in Lorentz gauge $\partial_\mu A^\mu = 0$

- $\rightarrow$ theta-vacua ~'72
- use ~ "Bosonization"

$$\mathcal{L} = \bar{\psi} i \gamma^\mu \partial_\mu \psi - e \bar{\psi} \gamma^\mu \psi A_\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad QED_2$$

- $j_5^\mu = \epsilon^{\mu\nu} j_\nu$ 1+1 dim

<u>On LF</u>: $j_5^+ = \epsilon^{+-} j_- = -j^+$; $\epsilon^{-+} = 1$

- $\rightarrow$ $Q_5^{l.f} = -Q^{lf}$

(cf $j_5^0 = j_1 = -j^1$ + $j^0 = j_5^1$)

Bosonized Version (to study vacuum state):

$$\bar{\psi}\psi \leftrightarrow K : \cos 2\sqrt{\pi}\phi :$$

$$\bar{\psi}\gamma_\mu \psi \leftrightarrow \epsilon_{\mu\nu} \frac{\partial^\nu \phi}{\sqrt{\pi}} \quad \text{etc.}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - g A_\mu \epsilon^{\mu\nu}(\partial_\nu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$U(1)_5$: $\psi \rightarrow e^{i\gamma_5 \alpha}\psi \leftrightarrow \phi \rightarrow \phi + \text{const.}$

Separation: $\phi(\tau, x) = \omega(\tau) + \varphi(\tau, x)$

Chiral transf:

$$\begin{cases} \omega \rightarrow \omega + \text{const} \\ \varphi \rightarrow \varphi \\ A_\mu \rightarrow A_\mu \end{cases}$$

• Mathematical framework "well posed":

$$S = \int_{-\infty}^{\infty} dx \left[\dot{\varphi}\varphi' + g(A_+\varphi' - A_-\dot{\varphi}) + \frac{1}{2}(\dot{A}_- - A_+')^2 \right] - g\dot{\omega}h$$

where $h(\tau) = \int dx\, A_-(\tau, x)$ and $g = \frac{e}{\sqrt{\pi}}$

Dirac Procedure & Hamiltonian framework:

After implementing the constraints we are left with

- $\omega(\tau)$, $\pi_\omega(\tau) \equiv g h(\tau)$, $\varphi(\tau, x)$
- $[\omega, \omega] = 0$, $[\omega, \varphi] = 0$, $[\pi_\omega, \varphi] = 0$

$[\pi_\omega, \omega] = i$, $[\varphi(\tau, x), \varphi(\tau, y)] = -\frac{i}{4}\epsilon(x-y)$

$\omega(\tau)$ is now a q-no. operator

<u>Condensate or Theta-Vacua</u>:

$\bar{\psi}\psi \leftrightarrow K : \cos(2\sqrt{\pi}\varphi)$:

→ $U(1)_5$: $\omega \to \omega + \frac{\beta}{\sqrt{\pi}}$ (angular variable (period π))

$\beta = \beta_0 + n\pi$ $n = 0, \pm 1, \pm 2, \ldots$

where $0 \le \beta_0 \le \pi$

$$Q_5(\beta) = e^{i\frac{1}{\sqrt{\pi}}(\beta\pi_\omega)}$$

• $Q_5(\pi) = Q_5(0) = I$

Π_ω has discrete spectrum

$$\Pi_\omega |n\} = 2n\sqrt{\pi}\,|n\}$$

$$\{m|n\} = \delta_{mn} \qquad n = 0, \pm 1, \pm 2, \ldots$$

Fock space: $\quad |\varphi) \otimes |n\}$

Chiral vacua

$$|0) \otimes |n\} \qquad n = 0, \pm 1, \pm 2, \ldots$$

Chirally symmetric vacuum $|0) \otimes |0\}$

→ violation of cluster decomposition property (Lowenstein - Swieca)

<u>avoided</u> by employing

Condensate vacua:

$$\omega |\omega'\} = \omega' |\omega'\}$$

$$|\omega'\} \otimes |\varphi) = \frac{1}{\pi^{1/4}} \sum_{n=-\infty}^{\infty} e^{2i\sqrt{\pi}\, n\omega'} |n\} \otimes |\varphi)$$

where $\quad 0 \leq \omega' \leq \sqrt{\pi}$ or $\quad 0 \leq \theta' = 2\sqrt{\pi}\,\omega' \leq 2\pi$

(<u>physical range</u>)

$$\{\theta'' | \theta'\} = \delta(\theta'' - \theta')$$

Vacua

$$|\omega'\} \otimes |0) = \frac{1}{\pi^{1/4}} \sum_{-\infty}^{\infty} e^{2i\sqrt{\pi}\, n\omega'} |n\} \otimes |0)$$

Chiral Schwinger Model:

Bosonized version

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \underbrace{e A_\nu (\eta^{\mu\nu} - \epsilon^{\mu\nu}) \partial_\mu \phi + \frac{1}{2} a e^2 A_\mu A^\mu}_{} - \frac{1}{4} F$$

$$= \frac{1}{2}(\dot{A}_- - A_+')^2 + \dot{\phi}\phi' + 2e A_- \dot{\phi} + a e^2 A_+ A_-$$

Separation: $\phi = \omega + \varphi$

$$\mathcal{L} = \frac{1}{2}(\dot{A}_- - A_+')^2 + \dot{\varphi}\varphi' + a e^2 \left[A_+ + \frac{2}{ae}(\dot{\varphi} + \dot{\omega})\right] A_-$$

- A_-, φ dynamical fields
- A_+ auxiliary field; no kinetic term
- $U(1)$ symmetry broken by the mass term — arising from the regularization ambiguity.

Redefine: $A_+ \longrightarrow \left[A_+ - \frac{2}{ae}\dot{\omega}(\tau)\right]$

$$\mathcal{L} = \frac{1}{2}(\dot{A}_- - A_+')^2 + \dot{\varphi}\varphi' + 2e\dot{\varphi}A_- + a e^2 A_+ A_-$$

The condensate variable $\omega(\tau)$ removed! Vacuum structure different from Schwinger model.

Lagrange Eqns:

$$\partial_+\partial_-\varphi = -e\partial_+ A_-$$

$$\partial_+\partial_+ A_- - \partial_+\partial_- A_+ = ae^2 A_+ + 2e\,\partial_+\varphi$$

$$\partial_-\partial_- A_+ - \partial_+\partial_- A_- = ae^2 A_-$$

$$\rightsquigarrow \quad \left.\begin{aligned} &[(a-1)\,\Box + a^2e^2]\,A_- = 0 \\ &(2-a)\,\partial_+ A_- = a\,\partial_- A_+ \\ &\Box\varphi = -2e\,\partial_+ A_- \end{aligned}\right\}$$

$\rightsquigarrow$ For $a > 1$:

$$\Box\varphi = -\frac{ea}{(a-1)}\,(F_{+-})$$

$$\left[\Box + \frac{a^2e^2}{(a-1)}\right](F_{+-}) = 0$$

$\rightsquigarrow$ Both massive and massless excitations (not a Goldstone boson) are present in the model.

- The vacuum structure is different from Schwinger model case – no θ-vacua.

Primary constraints:

$$\pi^+ \approx 0 \qquad (1^{st} \text{ class})$$

$$\Omega_1 \equiv (\pi_\varphi - \varphi') \approx 0 \qquad \text{and } \pi^- = \dot{A}_- - A_+' - 2e\varphi,$$

$$\mathcal{H}_c^{lt} = \frac{1}{2}(\pi^- + 2e\varphi)^2 + (\pi^- + 2e\varphi)A_+' - a e^2 A_+ A_-$$

Secondary constraint:

$$\Omega_2 = \partial \pi^- + 2e\varphi' + a e^2 A_- \approx 0$$

- A_+ absent from Ω_1 and Ω_2.
- Convenient to add $A_+ \approx 0$ as gauge-fixing (1^{st} class $\pi^+ \approx 0$) constraint.

  Ω_1, Ω_2 are 2^{nd} class

- Def $\{\}_{DB(\pi^+, A_+)}$ w.r.t the pair:

(π^+, A_+) — trivial pair –

For the surviving variables $\{\}_{DB(\pi^+, A_+)}$ $\equiv \{,\}_{PB}$.

Then

$$\mathcal{H}_c^{lt} = \frac{1}{2}(\pi^- + 2e\varphi)^2$$

$\oplus$

$$\Omega_1 \approx 0$$

$$\Omega_2 \approx 0$$

Diagonalization of the constraints: $(a>1)$

$$(\Omega_1, \Omega_2) \longrightarrow (T_1, T_2)$$

$$T_1 = c_1 \left(\Omega_1 + \frac{1}{M}\Omega_2\right)$$

$$T_2 = c_2 \left(\Omega_1 - \frac{1}{M}\Omega_2\right)$$

where $$c_1 = \frac{1}{\sqrt{2(1-\frac{e}{M})}}, \quad c_2 = \frac{1}{\sqrt{2(1+\frac{e}{M})}},$$

$M^2 = ae^2$, $a>1$ (and $x \equiv x$ etc).

$$\{T_i, T_j\} = \delta_{ij}\left(-2\partial_x \delta(x-y)\right)$$

BFT Procedure:

Introduce auxiliary fields: Φ^j, $j=1,2$

in order to convert the 2^{nd} class T_i into 1^{st} class ones in the extended phase space.

$$\{A^\mu (\text{or } \Pi_\mu), \Phi^j\} = 0, \quad \{\varphi (\text{or } \Pi_\varphi), \Phi^j\} = 0$$

$$\{\Phi^i(x), \Phi^j(y)\} = \omega^{ij}(x,y) = -\omega^{ji}(y,x)$$

Construct $\tilde{T}_i$ such that

$$\{\tilde{T}_i, \tilde{T}_j\} = 0$$

$$\tilde{T}_i \Big|_{\Phi^i=0} = T_i$$

ansatz

$$\tilde{T}_i(A^\mu, \pi_\mu, \varphi, \pi_\varphi; \Phi^i) = T_i + \sum_{n=1}^{\infty} \tilde{T}_i^{(n)}$$

where $\quad T_i^{(n)} \sim (\Phi^j)^n$

and set

$$T_i^{(1)}(x) = \int dy \; X_{ij}(x,y)\, \Phi^j(y)$$

We are led to

$$\{T_i, T_j\} + \{\tilde{T}_i^{(1)}, \tilde{T}_j^{(1)}\} = 0$$

or

$$\left(-2\partial_x \delta(x-y)\right)\delta_{ij} + \int dy' dz \; X_{ik}(x,y')\, \omega^{k\ell}(y',z)\, X_{j\ell}(y,z) = 0$$

Solution (simplest)

$$\boxed{\begin{aligned} \omega^{ij}(x,y) &= -\delta^{ij}\,\epsilon(x-y) \\ X_{ij}(x,y) &= \delta_{ij}\,\partial_x \delta(x-y) \end{aligned}}$$

Inverses:

$$\omega^{-1}_{ij}(x,y) = -\frac{1}{2}\,\delta_{ij}\,\partial_x \delta(x-y)$$

$$(X^{-1})^{ij}(x,y) = \frac{1}{2}\,\delta^{ij}\,\epsilon(x-y)$$

$\rightarrow$

$$\tilde{T}_i = T_i + \partial_x \Phi^i$$

Already with the 1st order correction we get the 1st class algebra

$$\{T_i + \tilde{T}_i^{(1)}, T_j + \tilde{T}_j^{(1)}\} \approx 0$$

Higher order correction terms vanish!

"Gauge-invariant" modified dynamical variables:

$$\tilde{F} = (\tilde{A}_\mu, \tilde{\Pi}^\mu, \tilde{\varphi}, \tilde{\Pi}_\varphi) \quad \ni$$

$$\{\tilde{T}_i, \tilde{F}\} = 0$$

Set $\tilde{F} = F + \sum_{n=1}^{\infty} \tilde{F}^{(n)}$; $\tilde{F}^{(n)} \sim (\Phi^i)^n$

Then

$$\tilde{F}^{(1)} = -\int du\,dv\,dz\; \Phi^j(u)\,\omega^{-1}_{jk}(u,v)\,(X^{-1})^{kl}(v,z) \cdot$$

$$\{T_l(z), F(x)\}_{(A,\Pi,\varphi,\Pi_\varphi)}$$

where $F \equiv (A_\mu, \pi^\mu; \varphi, \pi_\varphi)$. Again only the 1st order correction is enough

$$\tilde{A}_-^{(1)} = \frac{1}{2M} \partial_x [c_1 \Phi^1 - c_2 \Phi^2]$$

$$\tilde{\pi}^{-(1)} = \frac{M}{2} (c_1 \Phi^1 - c_2 \Phi^2)$$

$$\tilde{\varphi}^{(1)} = -\frac{1}{2} (c_1 \Phi^1 + c_2 \Phi^2)$$

$$\tilde{\pi}_\varphi^{(1)} = \frac{1}{2} \partial \left[(c_1 \Phi^1 + c_2 \Phi^2) - \frac{2e}{M} (c_1 \Phi^1 - c_2 \Phi^2) \right]$$

Dirac Brackets:

$$\{f, g\}_D = \{\tilde{f}, \tilde{g}\}_{\Phi^i = 0}$$

$\rightsquigarrow$

$$\{\varphi, \varphi\}_D = \{\tilde{\varphi}^{(1)}, \tilde{\varphi}^{(1)}\} = \frac{a}{(a-1)} \left(-\frac{1}{4} \epsilon(x-y)\right)$$

$$\{\varphi, \pi^-\}_D = \frac{ae}{(a-1)} \left(-\frac{1}{4} \epsilon(x-y)\right)$$

$$\{\pi^-, \pi^-\}_D = \frac{a^2 e^2}{(a-1)} \left(-\frac{1}{4} \epsilon(x-y)\right)$$

and

$$H_D^{ef} = \frac{1}{2} \int dx^- (\pi^- + 2e\varphi)^2$$

plus $\Omega_1 = 0$, $\Omega_2 = 0$

First class Hamiltonian $\tilde{H}$

$$\tilde{H}\Big|_{\Phi^i=0} = \bar{H}_D^{\,H} \text{ (above)}$$

$$\{\tilde{T}_i, \tilde{H}\} = 0$$

(and $\{\tilde{T}_i, \tilde{T}_j\} = 0$)

May be obtained systematically by a procedure given by BFT or simply guessed in our case

$$\tilde{H} = \frac{1}{2}\int dx \, (\tilde{\pi} + 2e\tilde{\varphi})^2$$

We do check
$$\{\tilde{H}, \tilde{H}\} = 0$$

Completes the operational conversion of 2^{nd} class dynamical system with H_c and constraints $\Omega_1 = 0, \Omega_2 = 0$ to a <u>1^{st} class one</u> with $\tilde{H}$ and <u>abelian</u> constraints $\tilde{T}_i$.

First class Lagrangian:

- $$\bar{\Phi}^i = \frac{1}{2}\pi^i - \int du\, \epsilon(x-u)\, \theta^i(u)$$

with
$$\{\pi^i, \theta^j\} = -\delta^{ij}\,\delta(x-y) \quad \text{etc.}$$

$$Z = \int \mathcal{D}A_- \mathcal{D}\pi^- \mathcal{D}\varphi\, \mathcal{D}\pi_\varphi\, \mathcal{D}\theta^1 \mathcal{D}\theta^2 \mathcal{D}\pi^1 \mathcal{D}\pi^2$$
$$\prod_{i,j=1}^{2} \delta(\tilde{T}_i)\,\delta(\Gamma_j)\, \det|\{\tilde{T}_i, \tilde{T}_j\}|\, e^{iS}$$
where
$$S = \int d^2x \,(\pi^- \dot{A}_- + \pi_\varphi \dot{\varphi} + \pi^1\dot{\theta}^1 + \pi^2\dot{\theta}^2 - \tilde{\mathcal{H}})$$

- $\Gamma_i = 0$ are gauge-fixing conditions chosen appropriately $\ni \det|| \ne 0$ etc.

Unitary Gauge: $\Gamma_i \equiv \Omega_i = 0$

has $$\delta(\pi_\varphi - \varphi')\,\underbrace{\delta(\pi^{-\prime} + 2e\varphi' + M^2 A_-)}_{}\,\delta(\pi^1 - 4\theta^{1\prime})\,\delta(\pi^2 - 4\theta^{2\prime})$$
(exponentiate it to bring back A_+)

Shift: $\pi^- \to \pi^- - 2e\varphi$ etc.

$\to$ leads to original Lagrangian

- Different acceptable choices of gauge-fns. Γ_i give different effective Lagrangians.
- $\tilde{H}$ is not unique

- Batalin & Tyutin: Int. Jl. Mod Phys. A6, 3225 ('91)
- Batalin & Fradkin: Nucl. Phys. B279 ('87) 514