θ - Dependence of "Observables" in the Light-Front Massive Schwinger Model

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dynamical zero modes (ZM) and
the vacuum structure on the light front (LF)

2 parts / topics

1. LF Weyl-gauge formulation of the massive bosonized Schwinger model and its vacuum structure
2. mass perturbation theory calculation: θ-dependence of $O(m^2)$ corrections of some "observables" (Schwinger boson mass)

Motivation:

1. we need to better understand the dynamics of zero modes and their role in making the LF vacuum "non-trivial"

   old question: where are symmetry-breaking effects in the LF framework?

2-dim LF models studied - dynamical gauge ZM A_0^+
massless QED(1+1) - G. McCartor, Th. Heinzl

not all aspects fully understood yet

a possible explanation: we miss some degrees of fr.

fixing of the gauge at classical level typically

try instead: partial gauge fixing plus

unitary transformations to a new repres'n

in equal-time quantization: K. Haller, F. Lenz et al.

an example: fermionic LF Schwinger model in $A^- = 0$ gaug

2. subtleties of the fermionic model ($\psi_-(x)$ comp.)

→ look at bosonized formulation first

(massless: Heinzl, Krusche & Werner,

Kalloniatis & Robertson, L.M.)

massive model: not exactly solvable

new parameter θ at quantum level

physical quantities depend on θ

weak and strong coupling limits analyzed by Coleman ('7

θ-dependent number of (stable) bound states

- massive (bosonized) Schwinger model is a good testing ground for LF methods and its physical contents interesting by itself

Related work: T. Fields, H. Pirner, J. Vary, Phys. Rev. D '96

Ch. Adam, preprint BUTP 95-27, '95

Outline:
1. LF bosonized massive Schwinger model in the Weyl gauge
2. Residual gauge freedom and ZM of A^+
3. θ-vacuum in terms of coherent states of the ZM and P^-_θ
4. Mass PT for $O(m^2)$ corrections to μ^2 and momentum densities
5. Conclusions

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1. LF bosonized massive Schwinger model in $A^-=0$ gauge

conventions: $x^\pm = x^0 \pm x^1$, $\partial^\pm = 2\partial_\mp \equiv 2\frac{\partial}{\partial x^\mp}$

$p^\pm = p^0 \pm p^1$

$px = \frac{1}{2}p^+x^- + \frac{1}{2}p^-x^+$, $p^2 = p^+p^- = m^2$, $p^+ \geq 0$

LF Hamiltonian P^-: Fock vacuum is an eigenstate of the full interacting P^- (ZM neglected)

- finite box: $-L \leq x^- \leq L$ (IR regularization)

periodic fields $\phi(\bar{x}) = \phi_0 + \phi_n(\bar{x})$, $A^\mu(\bar{x}) = A^\mu_0 + A^\mu_n(\bar{x})$

$k^+_n = \frac{2\pi}{L}n$, $n = 1, 2, \ldots$ $\quad \phi_0 = \frac{1}{2L}\int_{-L}^{L} d\bar{x}\,\phi(\bar{x})$, ϕ_n - normal mode (NM)

$\delta_p(\bar{x}) = \frac{1}{L} + \delta_n(\bar{x})$, $\delta_n(\bar{x}) = \frac{1}{L}\sum_{n=\pm1,\pm2,\ldots} \exp[i\frac{\pi}{L}n\bar{x}]$, $\epsilon_p(\bar{x}) = \frac{\bar{x}}{L} + \epsilon_n(\bar{x})$

- Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)\partial^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e j_\mu A^\mu - m\tilde{K} :\cos c\phi:$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \epsilon^{+-}=2$$

$$j^\mu = \frac{\epsilon^{\mu\nu}}{\sqrt{\pi}}\partial_\nu\phi\,, \quad \bar{\Psi}\Psi = \tilde{K}:\cos c\phi:\,, \quad \tilde{K} \equiv \frac{\mu}{2\pi}e^{\gamma_E}, \quad \mu = \frac{e}{\sqrt{\pi}}, \ c = 2\sqrt{\pi}$$

- gauge fixing

$A^- = 0$ (easy to construct the corresp. gauge func

reason: less restrictive choice than standard $\partial_- A^+ = 0$

$\Rightarrow$ residual gauge freedom w.r. to x^+-indep. gauge trans

$$A^-(x^-) \to A^-(x^-) - 2\partial_+ \tilde{\chi}(x^-) = A^-(x^-)$$

e.a. redundant variables present

$$\mathcal{L}_W = 2(\partial_+\phi)\partial_-\phi + \frac{1}{2}(\partial_+ A^+)^2 + \mu A^+\partial_+\phi - m\tilde{K}:\cos c\phi:$$

Euler-Lagrange eqs.:

I. $4\partial_+\partial_-\phi_n + \mu\partial_+ A^+ = cm\tilde{K}:\sin c\phi:$

II. $\partial_+^2 A^+ = \mu\partial_+\phi$

no Gauß's law, only $\partial_+ G_w = 0$ (taking ∂_- II.)

$\to$ in QT: $\qquad G_w(x) = \partial_-\partial_+ A_n^+(x) - \mu\partial_-\phi_n(x)$

condition on states

- canonical momenta $\quad \Pi_{\varphi_i} = \dfrac{\delta \mathcal{L}_w}{\delta \partial_+ \varphi_i}$

$\Pi_\phi = 2\partial_- \phi + \mu A^+ \qquad$ NM: $\Pi_{\phi_n} = 2\partial_- \phi_n + \mu A^+_n$

$\Pi_{A^+} = \partial_+ A^+ \qquad \Pi_{A^+_n} = \partial_+ A^+_n$

ZM: $\Pi_{\phi_0} \equiv (\Pi_\phi)_0 = \mu A^+_0$

Primary constraints: $\qquad \Pi_{A^+_0} \equiv (\Pi_{A^+})_0 = \partial_+ A^+_0$

$\overset{n}{\varphi}_1 = \Pi_{\phi_n} - 2\partial_- \phi_n - \mu A^+_n$

$\overset{0}{\varphi}_1 = \Pi_{\phi_0} - \mu A^+_0 \qquad \rightarrow$ constrained quantization

- Dirac-Bergmann procedure

canonical Hamiltonian

$$P^-_c = \frac{1}{2}\int_{-L}^{L} dx^- \left[\Pi^2_{A^+} + 2m\tilde{K} : \cos c\phi : \right],$$

primary Hamiltonian

$$P^-_p = P^-_c + L\, \overset{0}{u}_1 \overset{0}{\varphi}_1 + \frac{1}{2}\int_{-L}^{L} dx^- \, \overset{n}{u}_1(x) \overset{n}{\varphi}_1(x)$$

<u>Normal-mode sector</u>:

- fundamental Poisson brackets (PB's)

$\{\phi_n(x^-), \Pi_{\phi_n}(y^-)\} = \delta_n(x^- - y^-)$

$\{A^+_n(x^-), \Pi_{A^+_n}(y^-)\} = \delta_n(x^- - y^-)$

- consistency of the primary constraint: $\partial_+ \overset{n}{\varphi}_1 = \{P^-_p, \overset{n}{\varphi}_1\} \approx 0$
  $\rightarrow$ eq. for the Lagrange multiplier $\overset{n}{u}_1$

"matrix" of the constraints $C_{ij}(x^-, y^-)$

$$C^{-1}(x^-, y^-) = -\frac{1}{4}\,\frac{1}{2}\,\epsilon_n(x^- - y^-)$$

Dirac brackets

$$\{A(x^-), B(y^-)\}^* = \{A(x^-), B(y^-)\} - \int_{-L}^{L}\frac{du^-}{2}\frac{dv^-}{2}\{A(x^-), \varphi_i(u^-)\}\, C^{-1}_{ij}(u^-, v^-)\{\varphi_j(v^-), B(y^-)\}$$

non-vanishing:

$$\{\phi_n(x^-), \Pi_{\phi_n}(y^-)\}^* = \frac{1}{2}\,\delta_n(x^- - y^-)$$

$$\{\phi_n(x^-), \phi_n(y^-)\}^* = -\frac{1}{8}\,\epsilon_n(x^- - y^-)$$

$$\{\Pi_{\phi_n}(x^-), \Pi_{\phi_n}(y^-)\}^* = \partial_-^x\,\delta_n(x^- - y^-)$$

$$\{\phi_n(x^-), \Pi_{A_n^+}(y^-)\}^* = -\mu\,\frac{1}{8}\,\epsilon_n(x^- - y^-)$$

$$\{\Pi_{\phi_n}(x^-), \Pi_{A_n^+}(y^-)\}^* = \mu\,\frac{1}{2}\,\delta_n(x^- - y^-)$$

$$\{\Pi_{A_n^+}(x^-), \Pi_{A_n^+}(y^-)\}^* = -\mu^2\,\frac{1}{8}\,\epsilon_n(x^- - y^-)$$

$$\{A_n^+(x^-), \Pi_{A_n^+}(y^-)\}^* = \delta_n(x^- - y^-)$$

They can be simplified by defining

$$\Pi_n^-(x^-) = \Pi_{A_n^+}(x^-) - \mu\,\phi_n(x^-)$$
$$\Pi_n(x^-) = \Pi_{\phi_n}(x^-) - \mu\,A_n^+(x^-)$$

$$\{\phi_n(x^-), \phi_n(y^-)\}^* = -\frac{1}{8}\,\epsilon_n(x^- - y^-)$$

$$\{\phi_n(x^-), \Pi_n(y^-)\}^* = \frac{1}{2}\,\delta_n(x^- - y^-)$$

$$\{\Pi_n(x^-), \Pi_n(y^-)\}^* = \partial_-^x\,\delta_n(x^- - y^-)$$

$$\{A_n^+(x^-), \Pi_n^-(y^-)\}^* = \delta_n(x^- - y^-)$$

Zero-mode sector :

fundamental PB's

$$\{\phi_0, \Pi_{\phi_0}\} = \frac{1}{L} \quad , \quad \{A_0^+, \Pi_{A_0^+}\} = \frac{1}{L}$$

consistency of the primary constraint $\overset{\circ}{\varphi}_1$

$$\partial_+ \overset{\circ}{\varphi}_1 = \{\overset{\circ}{\varphi}_1, P_p^-\} \approx 0 \longrightarrow \text{secondary constraint}$$

$$\overset{\circ}{\varphi}_2 = m\tilde{K}c \int_{-L}^{L} \frac{dx^-}{2L} :\sin c\phi: - \mu \Pi_{A_0^+} \approx 0$$

Matrix of the ZM constraints

$$\{\overset{\circ}{\varphi}_1, \overset{\circ}{\varphi}_2\} = \frac{\mu^2}{L} - m\tilde{K}c^2 \frac{1}{L} \int_{-L}^{L} \frac{dx^-}{2L} :\cos c\phi:$$

has the inverse

$$\overset{\circ}{C}{}^{-1}_{ij} = \begin{pmatrix} 0 & -\frac{L}{\mu^2}\frac{1}{1-\alpha} \\ \frac{L}{\mu^2}\frac{1}{1-\alpha} & 0 \end{pmatrix} \quad , \alpha \equiv \frac{m\tilde{K}c^2}{\mu^2} \int_{-L}^{L} \frac{dx^-}{2L} :\cos c\phi:$$

Non-zero Dirac brackets

$$\{A_0^+, \Pi_{A_0^+}\}^* = \frac{1}{L}\left(1 - \frac{1}{1-\alpha}\right) \qquad \{\phi_0, \Pi_{\phi_0}\}^* = \frac{1}{L}\frac{1}{1-\alpha}$$

$$\{A_0^+, \phi_0\}^* = -\frac{1}{L\mu}\frac{1}{1-\alpha} \qquad \{\Pi_{A_0^+}, \Pi_{\phi_0}\}^* = \frac{\mu}{L}\frac{\alpha}{1-\alpha}$$

can be simplified by defining $\boxed{\Pi_0^- = \Pi_{A_0^+} - \mu\phi_0}$:

$$\{A_0^+, \Pi_0^-\}^* = \frac{1}{L} \qquad \{\phi_0, \Pi_{\phi_0}\}^* = \frac{1}{L}\frac{1}{1-\alpha}$$

$$\{\Pi_{\phi_0}, \Pi_0^-\}^* = \frac{\mu}{L} \qquad \{\phi_0, A_0^+\}^* = \frac{1}{\mu L}\frac{1}{1-\alpha}$$

At this stage one can implement the constraints

$$\overset{n}{\varphi}_1 : \quad \Pi_{\phi_n} = 2\partial_-\phi_n + \mu A_n^+ \quad \rightarrow \quad \Pi_n = 2\partial_-\phi_n$$

$$\overset{\circ}{\varphi}_1 : \quad \Pi_{\phi_0} = \mu A_0^+$$

$$\overset{\circ}{\varphi}_2 : \quad \Pi_{A_0^+} = \frac{m\tilde{K}c}{\mu}\int_{-L}^{L}\frac{dx^-}{2L} :\sin c\phi:$$

to obtain the commutators in new variables

$\{A,B\}^* \to -i[A,B]$

$$[\phi_n(\bar{x}), \phi_n(\bar{y})] = \frac{-i}{8}\epsilon_n(\bar{x}-\bar{y})$$

$$[\phi_n(\bar{x}), \Pi_n(\bar{y})] = [\phi_n(\bar{x}), 2\partial_-\phi_n(\bar{y})] = \frac{i}{2}\delta_n(\bar{x}-\bar{y})$$

$$[\Pi_n(\bar{x}), \Pi_n(\bar{y})] = [2\partial_-\phi_n(\bar{x}), 2\partial_-\phi_n(\bar{y})] = i\partial_-^x\delta_n(\bar{x}-\bar{y})$$

$$[A_n^+(\bar{x}), \Pi_n^-(\bar{y})] = i\delta_n(\bar{x}-\bar{y})$$

$$[A_0^+, \Pi_0^-] = \frac{i}{L} \qquad [\phi_0, A_0^+] = \frac{i}{L\mu}\frac{1}{1-\alpha}$$

($m=0$, coincide in $\alpha=0$ limit, also $\Pi_{A_0^+}=0$ ✓)

and the LF Hamiltonian

$$P_W^- = L\frac{m^2\tilde{K}c^2}{\mu^2}(:\sin c\phi:)_0^2 + \frac{1}{2}\int_{-L}^{L}dx^-[\Pi_{A_n^+}^2 + 2m\tilde{K}:\cos c\phi:]$$

↗

In the "old" variables:

Gauß's law as a condition on states:

$$G_n(\bar{x}) = \partial_-\Pi_{A_n^+}(\bar{x}) - \mu\partial_-\phi_n(\bar{x}), \quad G_n|\Phi\rangle = 0 \quad (G^{(+)}|\Phi\rangle = 0)$$

One has $[P_W^-, G_n(\bar{x})] = 0$

and G_n is closely related to the generator of residual GT in the Weyl gauge:

$$\Omega[\beta] = \exp\left[-i\int_{-L}^{L}\frac{dx^-}{2}\left(\Pi_{A^+} - \mu\phi\right)\partial_-\beta\right]$$

$$\beta(x^-) = \beta_n(x^-) + \beta_e \,, \quad \beta_e = \frac{2\pi}{eL}x^- m$$

↖ BC for A^μ

$$\Omega[\beta] = \exp\left[i\int_{-L}^{L}\frac{dx^-}{2}\, G_n(x^-)\,\beta_n(x^-)\right]\Omega_0$$

$$\Omega_0 = \exp\left[-i\,\frac{2\pi m}{e}\left(\Pi_{A_0^+} - \mu\phi_0\right)\right] \,, \quad m = \pm 1, \pm 2, \ldots$$

Indeed,

$$\Omega[\beta]\,\phi(y^-)\,\Omega^\dagger[\beta] = \phi(y^-) - i\int_{-L}^{L}\frac{dx^-}{2}\,\partial_-\beta\left[\Pi_{A^+}(x^-) - \mu\phi(x^-), \phi(y^-)\right] = \phi(y^-)$$

$$\Omega[\beta]\,\Pi_\phi(y^-)\,\Omega^\dagger[\beta] = \Pi_\phi(y^-)$$

$$\Omega[\beta]\,A^+(y^-)\,\Omega^\dagger[\beta] = A^+(y^-) - \partial_-\beta(y^-) \Rightarrow \boxed{A_0^+ \to A_0^+ - \frac{2\pi}{eL}m}$$

$$\Omega[\beta]\,\Pi_{A^+}(y^-)\,\Omega^\dagger[\beta] = \Pi_{A^+}(y^-)$$

and also

$$\Omega[\beta]\,P_W^-\,\Omega^\dagger[\beta] = P_W^-$$

The next step usually:

unitary transformation to a new representation ("quantum-mechanical gauge fixing") via operator

$$U[\vartheta] = \exp\left[-i\int_{-L}^{L}\frac{dx^-}{2L}\, g(x^-)\,\vartheta(x^-; A_n^+)\right]$$

$$\vartheta = \int_{-L}^{L}\frac{dy^-}{2}\,\frac{1}{2}\,\epsilon_n(x^- - y^-)\,A_n^+(y^-) \,, \quad g(x^-) = G(x^-) - \partial_-\Pi_{A_n^+}(x^-)$$

- in fermionic Schw. model it induces nontrivial transf[illegible]

In the present context, the change of variables did the job:

$$G(\bar{x}) \to \tilde{G}(\bar{x}) = \partial_- \Pi_n^-(\bar{x})$$

$$\tilde{g}(\bar{x}) = \tilde{G}(\bar{x}) - \partial_- \Pi_n^-(\bar{x}) = 0 \Rightarrow U[v] = \mathbb{1}$$

$$\Omega[\beta] \to \tilde{\Omega}[\beta] = \underbrace{\exp\left[i\int_{-L}^{L} \frac{d\bar{x}}{2L} (\partial_- \Pi_n^-)\beta\right]}_{\text{identity in the space of phys. states since } \partial_-\Pi_n^-|\Phi\rangle = 0} \underbrace{\exp\left[-i\frac{2\pi}{e}\Pi_0^-\right]}_{\text{non-trivial symmetry}}$$

- P_ω^- in the new variables

insert $\Pi_{A_n^+} = \Pi_n^- + \mu\phi_n$

$$P_\omega^- = P_A^- + P_{un}^-$$

$$P_A^- = L\frac{m^2\tilde{K}^2c^2}{\mu^2}(:\sin c\phi:)_0^2 + \frac{1}{2}\int_{-L}^{L} d\bar{x}\left[\mu^2\phi_n^2(\bar{x}) + 2m\tilde{K}:\cos c\phi(\bar{x}):\right]$$

$$P_{un}^- = \frac{1}{2}\int_{-L}^{L} d\bar{x}\left[\Pi_n^{-2} + 2\mu\phi_n\Pi_n^-\right], \quad P_{un}^-|\Phi\rangle = 0$$

2. Residual gauge freedom and θ-vacuum in terms of coherent states of A_0^+

non-trivial symmetry in the finite-volume formulation

$\mathcal{U}_m P_W^- \mathcal{U}_m^\dagger = P_W^-$ with $\mathcal{U}_m = \exp\left[-i \frac{2\pi}{eL} m \hat{\Pi}_0^-\right]$, $m = \pm 1, \pm 2, \ldots$

and $\hat{\Pi}_0^-$ obeys the commutation relation ($\hat{\Pi}_0^- = L \Pi_0^-$)

$[A_0^+, \hat{\Pi}_0^-] = i$ $\qquad b = \frac{2\pi}{eL}$

A_0^+ transforms as $A_0^+ \to \mathcal{U}_m A_0^+ \mathcal{U}_m^\dagger = A_0^+ - bm$ (large GT)

We know that invariance of $\mathcal{L}(x)$ under constant shifts of a field : $\phi(x) \to \phi(x) + \lambda$ generates the new (transformed, displaced) vacuum

$$e^{-\frac{\lambda}{2}[a^\dagger(0) - a(0)]} |0\rangle \quad , \quad a(0) = a(k=0) \qquad (12)$$

which is the coherent state (CS) of zero mode of $\phi(x)$

In general, CS for an overcomplete set of states

$|\alpha\rangle = \exp[\alpha a^\dagger - \alpha^* a] |0\rangle \equiv \hat{D}(\alpha)|0\rangle \qquad \alpha$ - complex n.

are eigenstates of a

$a|\alpha\rangle = \alpha|\alpha\rangle$

$\hat{D}$ displace a : $\hat{D}^\dagger(\alpha) a \hat{D}(\alpha) = a + \alpha$, $\hat{D}^\dagger(\alpha) a^\dagger \hat{D}(\alpha) = a^\dagger + \alpha^*$

Also: $|\alpha\rangle = \sum_n C_n(\alpha)|n\rangle$, $C_n(\alpha) = \exp\left(-\frac{|\alpha|^2}{2}\right)\frac{\alpha^n}{\sqrt{n!}}$, $|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle$

The same situation in the present model

$\longrightarrow$ realize the algebra $[A_0^+, \hat{\Pi}_0^-] = i$ in Fock represent'n

L.M., PLB 400, 335 ('97)

$$\left.\begin{array}{l} A_0^+ = \frac{1}{\sqrt{2\omega L}}(a_0 + a_0^+) \\ \hat{\Pi}_0^- = -i\sqrt{\frac{\omega L}{2}}(a_0 - a_0^+) \end{array}\right\} \quad [a_0, a_0^+] = 1 \Longrightarrow \boxed{|\alpha\rangle = \exp[\alpha(a_0^+ - a_0^+)]|0\rangle}$$

α - real

$$\mathcal{U}_1 = \exp\left[-\sqrt{\frac{\omega L}{2}}\, b\,(a_0^+ - a_0)\right], \quad \mathcal{U}_1|\alpha\rangle = |\alpha - \sqrt{\tfrac{\omega L}{2}}\, b\rangle$$

Choose $\omega = \frac{e^2 L}{2\pi^2}$

to obtain the lowering and raising oper. acting on $|\alpha\rangle$

$$\hat{\mathcal{U}}_1|\alpha\rangle = |\alpha - 1\rangle$$

$$\hat{\mathcal{U}}_1^+|\alpha\rangle = |\alpha + 1\rangle, \quad \hat{\mathcal{U}}_1 = \exp[-(a_0^+ - a_0)]$$

The <u>gauge invariant</u> ground state will be a superposition

$$\boxed{|\theta\rangle = \int_{-\infty}^{\infty} d\alpha\, e^{-i\theta\alpha}|\alpha\rangle}$$

with the required property

$$\hat{\mathcal{U}}_1|\theta\rangle = e^{-i\theta}|\theta\rangle$$

gauge invar. vacuum (up to phase)

I had $[P_\omega^-, \hat{\mathcal{U}}_1] = 0 \rightarrow |\theta\rangle$: eigenstates of P_ω^-

3. The effective Hamiltonian P_θ and mass perturbation theory for some "observables"

The light-front Hamiltonian for the bosonized massive Schwinger model

$$P_A^- = L\,\frac{m^2\tilde{K}^2c^2}{\mu^2}\,(:\sin c\phi:)_0^2 + \frac{1}{2}\int_{-L}^{L} dx^-\left[\mu^2\phi_n^2 + 2m\tilde{K}:\cos c\phi:\right]$$

~ nonlinear selfinteraction of the real scalar field of mass μ^2

- Can one obtain the θ-dependent P_A^- ?

Look at linear mass term $2m\tilde{K}:\cos c\phi:$, $\phi = \phi_0 + \phi_n$

$$:\cos(c\phi_n + c\phi_0): \;=\; :\cos\Big(c\phi_n + \underbrace{\tfrac{c}{\mu}\Pi_{A_0^+} - c\,\tfrac{\Pi_0^-}{L\mu}}\Big):$$

$$= \frac{1}{2}\Big\{:\exp\big[i(c\phi_n + \tfrac{c}{\mu}\Pi_{A_0^+})\big]:\underbrace{\exp\big[-i\tfrac{c\Pi_0^-}{L\mu}\big]}_{=\exp[a_0 - a_0^+] = \hat{u}_1} + :\exp\big[-i(c\phi_n + \tfrac{c}{\mu}\Pi_{A_0^+})\big]:\underbrace{\exp\big[i\tfrac{c\Pi_0^-}{L\mu}\big]}_{\hat{u}_1^+}$$

Thus

$$\langle\theta|:\cos(c\phi_n + c\phi_0):|\theta\rangle = \tfrac{1}{2}\langle\theta|\Big\{:\overbrace{\exp\big[i(c\phi_n + \tfrac{c}{\mu}\Pi_{A_0^+})\big]}^{1 + ic\phi_n + \cdots}:e^{i\theta} + h.c.\Big\}|\theta\rangle$$

$$= \langle\theta|\cos\big[c\phi_n + \tfrac{c}{\mu}\Pi_{A_0^+} - \underline{\underline{\theta}}\big]|\theta\rangle$$

, similarly for $m^2(:\sin c\phi:)_0^2$

- All matrix elements $\langle\theta|a_{k_1}a_{k_2}\cdots P_A^- \cdots a_{p_2}^+ a_{p_1}^+|\theta\rangle$ will pick up the θ-dependence

One can thus directly work with the effective P_Θ:

$$P^-_\Theta = L\frac{m^2\tilde{K}^2c^2}{\mu^2}\left(:\sin\left[c\phi_n + \frac{c}{\mu}\Pi_{A_0^+} - \underline{\underline{\Theta}}\right]:\right)^2_0 + \frac{1}{2}\int_{-L}^{L} dx^- \mu^2\phi_n^2$$

$$+\frac{1}{2}\int_{-L}^{L} dx^- \, 2m\tilde{K} :\cos\left[c\phi_n + \frac{c}{\mu}\Pi_{A_0^+} - \underline{\underline{\Theta}}\right]$$

$$\Pi_{A_0^+} = \frac{m\tilde{K}c}{\mu}\int_{-L}^{L}\frac{dx^-}{2L} :\sin c\phi:$$

In the mass perturbation theory to $O(m^2)$,

$\frac{c}{\mu}\Pi_{A_0^+}$ term will contribute only partly: $P = \frac{\tilde{K}c^2}{\mu^2}\int_{-L}^{L}\frac{dx^-}{2L}:\sin c\phi:$

$$\tilde{K}m:\cos\left[c\phi_n + \frac{c}{\mu}\Pi_{A_0^+} - \Theta\right]: = \underbrace{m\tilde{K}\cos(c\phi_n - \Theta)}\cos mP: + \underbrace{m:\sin(c\phi_n - \Theta)\sin mP:}$$

$$+\frac{c^2\tilde{K}m^2}{\mu^2}\sin^2(c\phi_n - \Theta)$$

Thus effectively to $O(m^2)$ one can work with

$$P^-_\Theta = \frac{1}{2}\int_{-L}^{L} dx^- \mu^2\phi_n^2 + P^-_m$$

$$P^-_m = 2L\frac{m^2\tilde{K}^2c^2}{\mu^2}\left(:\sin(c\phi_n - \Theta):\right)^2_0 + \frac{1}{2}\int_{-L}^{L} dx^- \, 2m\tilde{K} :\cos(c\phi_n - \Theta)$$

$$\equiv P_{m_1} + P_{m_2}$$

$$P_{m_2} = m\tilde{K}\left[\cos\Theta\int_{-L}^{L} dx^- :\cos c\phi_n: + \sin\Theta\int_{-L}^{L} dx^- :\sin c\phi:\right]$$

Expansions:

$$:\cos c\phi_n: = 1 - \frac{c^2}{2}:\phi_n^2: + \frac{c^4}{4}:\phi_n^4: - \dots$$

$$:\sin c\phi_n: = c:\phi_n: - \frac{c^3}{3!}:\phi_n^3: + \dots$$

field expansion:

$$\phi_n(x^-) = \frac{1}{\sqrt{4\pi}} \sum_{k=1,2,\dots} \frac{1}{\sqrt{k}} \left[a_k e^{-idkx^-} + a_k^\dagger e^{idkx^-} \right], \quad d \equiv \frac{\pi}{L}$$

$$[a_k, a_{k'}^\dagger] = \delta_{kk'}$$

One finds (δ_Σ – momentum conservation)

$$\hat{C} = \int_{-L}^{L} dx^- : \cos c\phi_n : = 2L \Big\{ 1 - \sum_{k_1,k_2}{}' \frac{1}{\sqrt{k_1 k_2}} a_1^\dagger a_2 \delta_\Sigma +$$

$$+ \frac{1}{4!} \sum_{k_1,\dots k_4}{}' \frac{1}{\sqrt{k_1 k_2 k_3 k_4}} \left[(4 a_1^\dagger a_2 a_3 a_4 \delta_\Sigma + h.c.) + 6 a_1^\dagger a_2^\dagger a_3 a_4 \delta_\Sigma \right] - \dots$$

$$\hat{S} = \int_{-L}^{L} dx^- : \sin c\phi_n : = 2L \Big\{ -\frac{1}{3!} \sum_{k_1,\dots k_3}{}' \frac{1}{\sqrt{k_1 k_2 k_3}} \left[3 a_1^\dagger a_2 a_3 \delta_\Sigma + h.c. \right]$$

$$+ \frac{1}{5!} \sum_{k_1,\dots k_5}{}' \frac{1}{\sqrt{k_1 k_2 \dots k_5}} \left[(5 a_1^\dagger a_2 a_3 a_4 a_5 \delta_\Sigma + h.c.) + (10 a_1^\dagger a_2^\dagger a_3^\dagger a_4 a_5 \delta_\Sigma + h.c.) \right] + \dots$$

- Mass corrections to the Schwinger boson mass μ^2

a) $O(m)$ shift

$$\delta\mu^2 = \delta(P^+ P^-) = P^+ \delta P^- = P^+ \langle P^+ | P_m^- | P^+ \rangle = \frac{2\pi}{L} K \langle K | P_m^+ | K \rangle$$

$P^+ = \frac{2\pi}{L} K$ $\quad |P^+\rangle$ – 1-boson state $a_K^\dagger |0\rangle$

$$\langle K | P_m^- | K \rangle = +2L m \tilde{K} \cos\theta \left[1 - \frac{1}{K} \right]$$

$$\boxed{\delta\mu^2 = -2m\mu e^{\gamma_E} \cos\theta + O(K)}$$

b) $\mathcal{O}(m^2)$ shift

$$\delta\mu^2 = P^+ \delta P^-$$

$$\delta P^- = \sum_{\substack{I \\ P_K^- \neq P_I^-}} \frac{\langle\theta| a_K P^-_{m_2} |I\rangle\langle I| P^-_{m_2} a_K^\dagger |\theta\rangle}{P_K^- - P_I^-} + \langle\theta| a_K P^-_{m_1} a_K^\dagger |\theta\rangle$$

$$\sum_I = \sum_n \sum_{k_1^+, \dots k_n^+} \quad , \quad P_K^- = \frac{\mu^2}{P^+} \quad , \quad P_K^- - P_I^- = \frac{L}{2\pi}\mu^2\left(\frac{1}{K} - \frac{1}{K_I}\right)$$

Evaluate the matrix elements

$$\langle k_L, k_M, k_N | P^-_{m_2} a_K^\dagger |0\rangle = m\tilde{K} L \cos\theta \frac{c^4}{4!} \frac{1}{(2\pi)^2} \frac{4!}{\sqrt{k_K k_L k_M k_N}} \delta_{K, L+M+N}$$

$$\langle k_L, k_M | P^-_{m_2} a_K^\dagger |0\rangle = m\tilde{K} L \sin\theta \frac{c^3}{3!} \frac{1}{(2\pi)^{3/2}} \frac{3!}{\sqrt{k_K k_L k_M}} \delta_{K, L+M}$$

generalization to arbitrary # of bosons in $|I\rangle$

$$\delta\mu^2 = \frac{4\pi^2 m^2 \tilde{K}^2}{\mu^2}\left[A \sin^2\theta + B\cos^2\theta\right]$$

$$A = \sum_{k=1} \sum_{l_1, \dots l_{2k}} \frac{1}{l_1 \cdots l_{2k}} \frac{1}{\frac{1}{K} - \frac{1}{l_1} - \dots \frac{1}{l_{2k}}} \delta_{K, l_1 + \dots l_{2k}}$$

$$B = \sum_{k=1} \sum_{l_1, \dots l_{2k+1}} \frac{1}{l_1 l_2 \dots l_{2k+1}} \frac{1}{\frac{1}{K} - \frac{1}{l_1} - \dots - \frac{1}{l_{2k+1}}} \delta_{K, l_1 + \dots l_{2k+1}}$$

e.a.

$$A = \sum_{l_1, l_2} \frac{1}{l_1 l_2} \frac{1}{\frac{1}{K} - \frac{1}{l_1} - \frac{1}{l_2}} + \sum_{l_1, \dots l_4} \frac{1}{l_1 l_2 l_3 l_4} \frac{1}{\frac{1}{K} - \frac{1}{l_1} - \frac{1}{l_2} - \frac{1}{l_3} - \frac{1}{l_4}}$$

- Numerical evaluation of the coefficients A, B

combinatorics of

$$|I\rangle = \frac{1}{\sqrt{n_1!}}\left(a^+_{k_1}\right)^{n_1} \frac{1}{\sqrt{n_2!}}\left(a^+_{k_2}\right)^{n_2} \dots |0\rangle$$

choose $K = n_1 k_1 + n_2 k_2 + \cdots n_N k_N$

and study convergence with K and # of interm. bosons

B / K	S_3	S_5	S_7	S_9
32	−.340	−.101	−.014	−.000
64	−.370	−.155	−.036	−.005
128	−.388	−.204	−.069	−.014
256	−.398	−.243	−.108	−.030
⋮	⋮	⋮	⋮	⋮
∞	−.4103	−.319	−.221	−.108

T. Fields et al.,
Ch. Adam: 1.067

Extrapolations based on K = 200, 300, 400 : B = 1.058 ± 0.

A / K	S_2	S_4	S_6	S_8
32	−.589	−.201	−.041	−.004
64	−.600	−.253	−.081	−.014
128	−.601	−.300	−.127	−.033
256	−.603	−.315	−.172	−.061
⋮				
∞	−.605	−.353	−.281	−.148

Ch. Adam: 2.3876

A = 1.387 ± 0.05

- $O(m^2)$ correction to the momentum densities

$$f_K(k) = N^{-1} \langle K | a_k^\dagger a_k | K \rangle$$

$$N = \sum_k \langle K | a_k^\dagger a_k | K \rangle$$

Correction to the state vector of 1 boson:

$$|K\rangle_2 = a_K^\dagger |0\rangle + \sum_{\substack{I \\ P_K^- \neq P_I^-}} \frac{\langle I | P_{m_2}^- | K \rangle}{P_K^- - P_I^-} |I\rangle$$

Calculate matrix elements, operator algebra →

$${}_2\langle K | a_k^\dagger a_k | K \rangle_2 = \frac{m^2}{\mu^2} e^{2\gamma_E} \left[C(x) \cos^2\theta + S(x) \sin^2\theta \right]$$

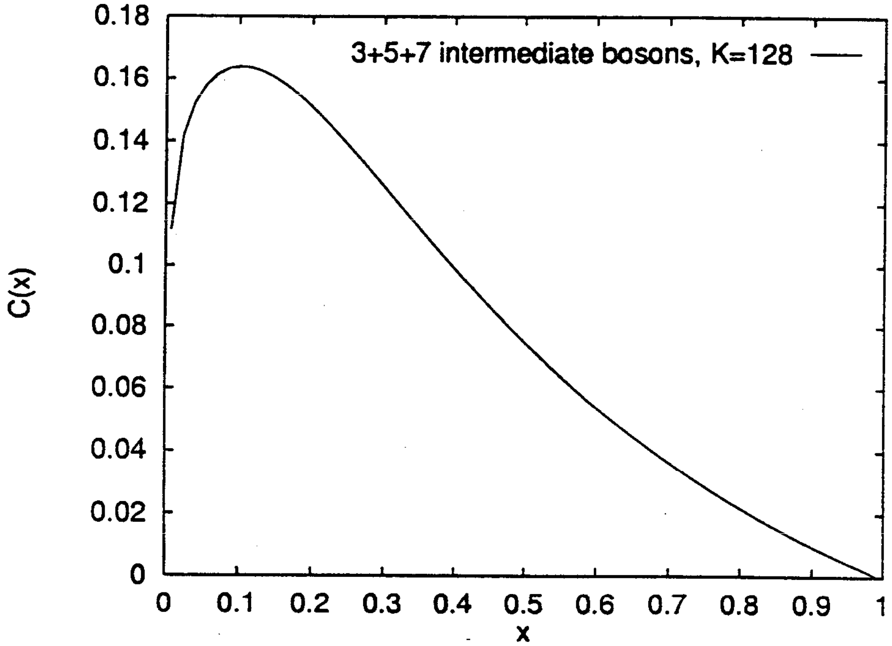
$$x = \frac{k}{K}$$

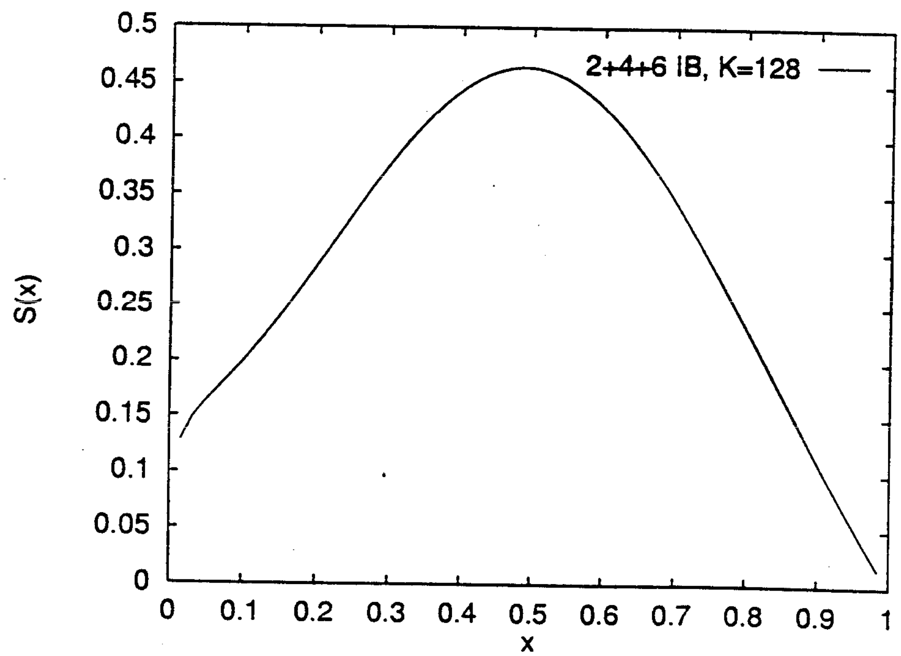
$$C(k) = 3 \sum_{\ell_1, \ell_2} \frac{\delta_{K, k+\ell_1+\ell_2}}{\left(\frac{1}{K} - \frac{1}{k} - \frac{1}{\ell_1} - \frac{1}{\ell_2}\right)^2} \frac{1}{k \ell_1 \ell_2} + 5 \sum_{\ell_1, \dots \ell_4} \dots$$

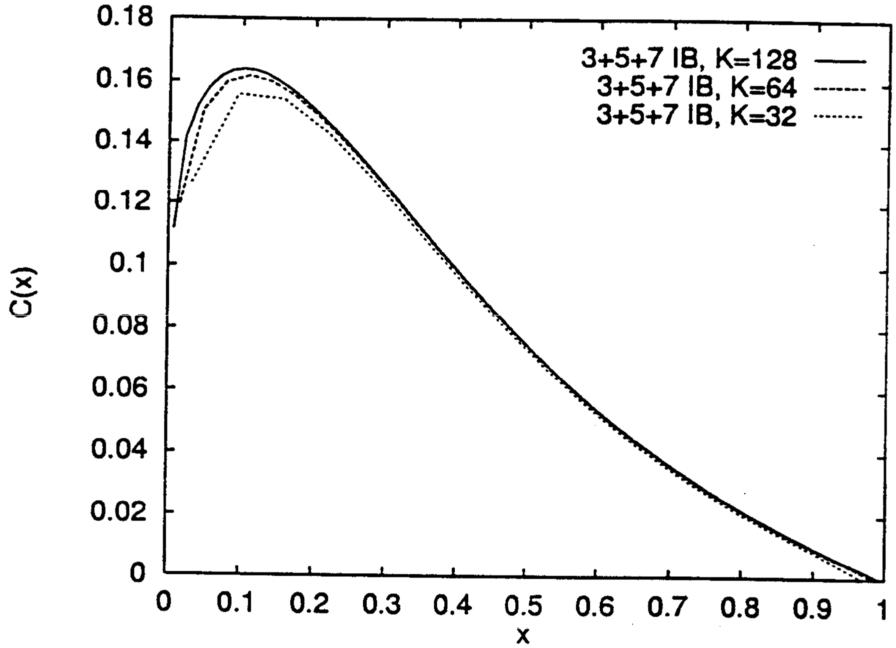
$$S(k) = 2 \sum_{\ell_1} \frac{\delta_{K, \ell_1 + k}}{\left(\frac{1}{K} - \frac{1}{k} - \frac{1}{\ell_1}\right)^2} \frac{1}{k \ell_1} + 4 \sum_{\ell_1, \ell_2, \ell_3} \frac{\delta_{K, k+\ell_1+\ell_2+\ell_3}}{\left(\frac{1}{K} - \frac{1}{k} - \frac{1}{\ell_1} - \frac{1}{\ell_2} - \frac{1}{\ell_3}\right)^2} \frac{1}{k \ell_1 \dots \ell_3} + \dots$$

→ Figures
- fast convergence with K and # of I states

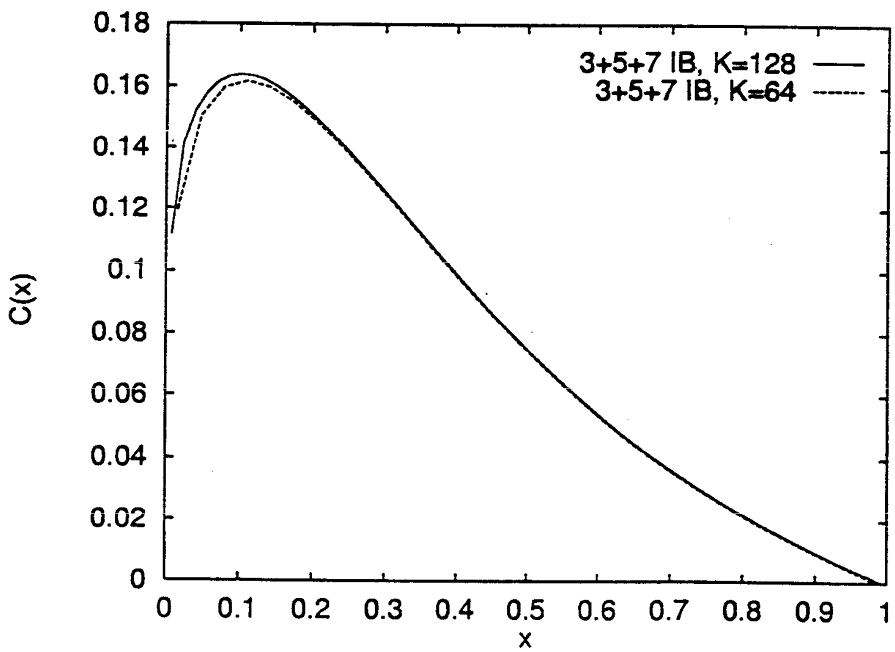
- <u>Conclusions</u>
  - compare with near L.C − many I states
  - θ-dependence calculable on the LF
  - Future work: DLCQ?, Weyl-gauge fermionic Schw. m, QCD_2


3+5+7 intermediate bosons, K=128
C(x)
x
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

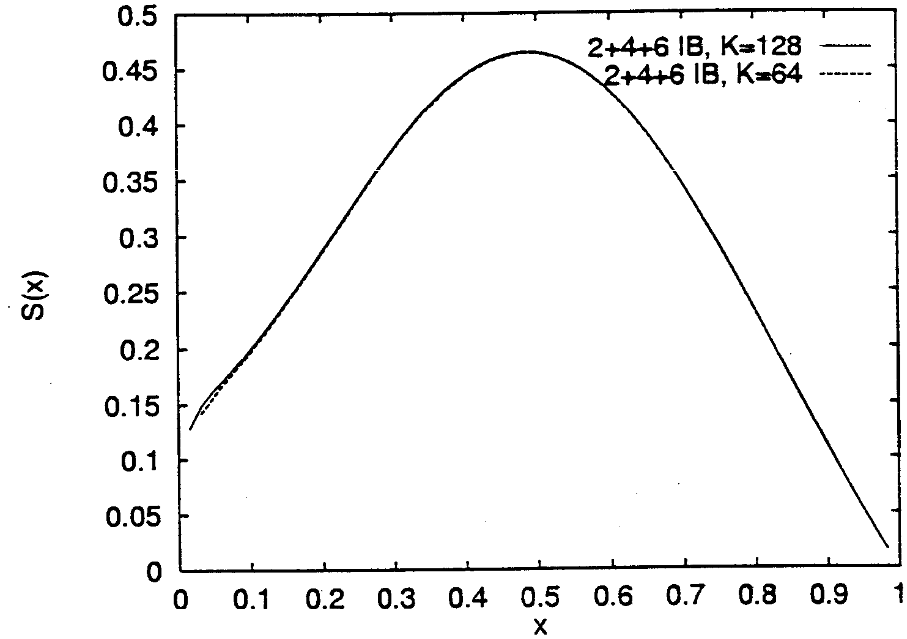

2+4+6 IB, K=128
S(x)
x
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1



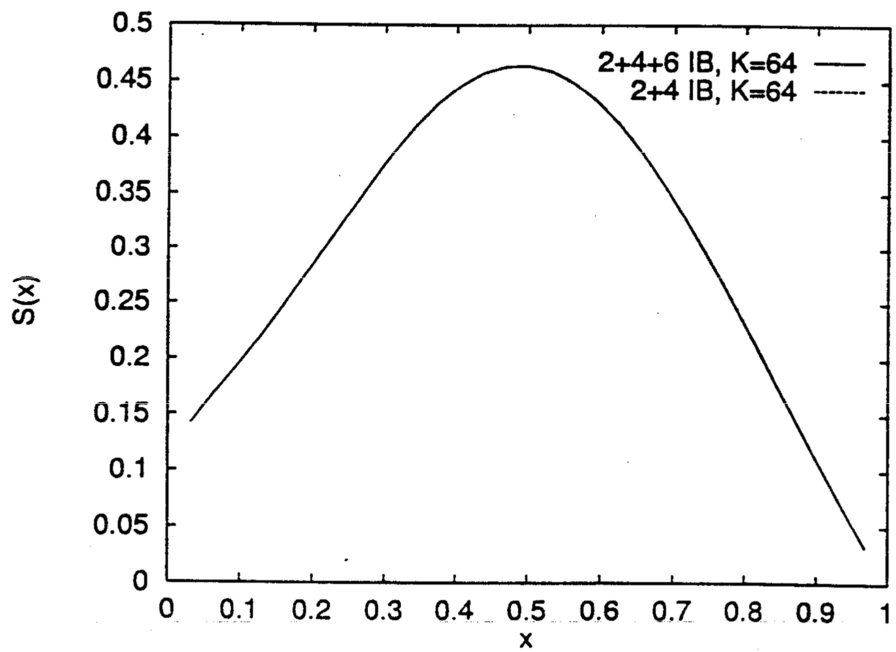
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
C(x)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
3+5+7 IB, K=128
3+5+7 IB, K=64
3+5+7 IB, K=32



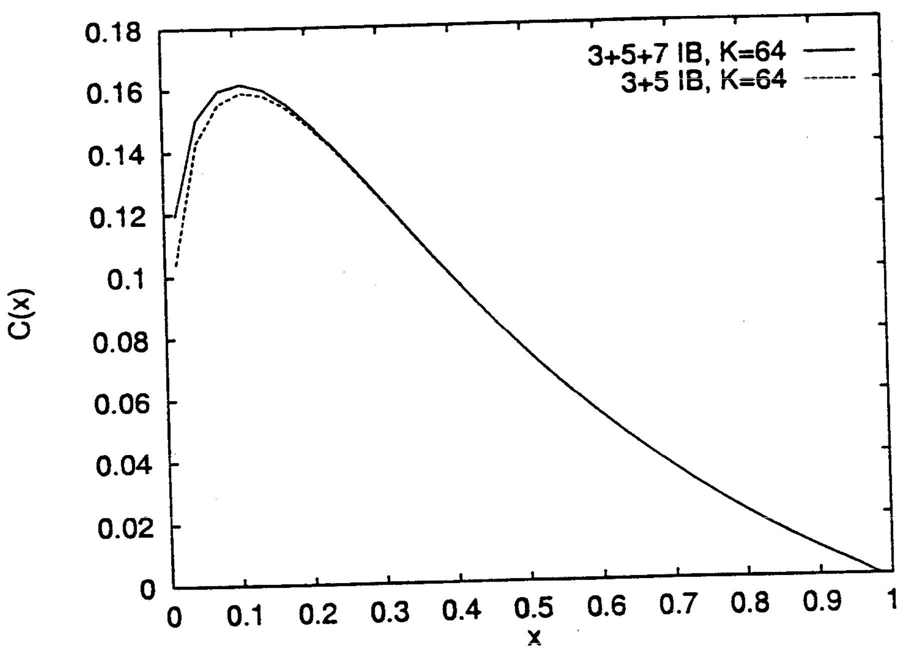
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
C(x)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
3+5+7 IB, K=128
3+5+7 IB, K=64

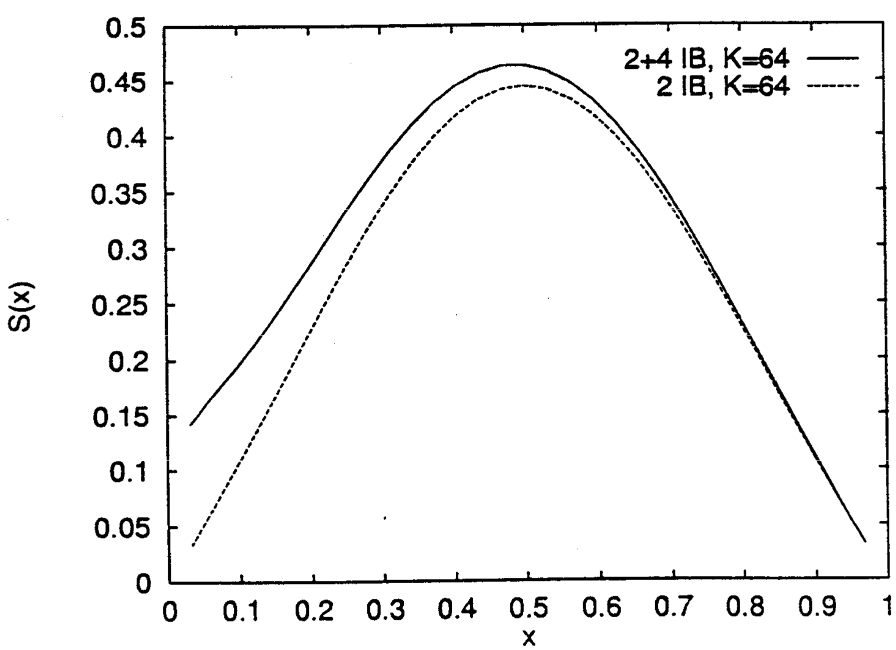


2+4+6 IB, K=128
2+4+6 IB, K=64
S(x)
x
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5



2+4+6 IB, K=64
2+4 IB, K=64
S(x)
x
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5


3+5+7 IB, K=64
3+5 IB, K=64
C(x)
x
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


2+4 IB, K=64
2 IB, K=64
S(x)
x
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1