LIGHT-CONE WAVE FUNCTIONS AND CHIRAL SYMMETRY

Lutsen, August '97

OUTLINE:

1. Introduction : LC Wave Functions

2. D = 1+1 : 't HOOFT and SCHWINGER model *)

3. D = 3+1 : NAMBU-JONA-LASINIO model

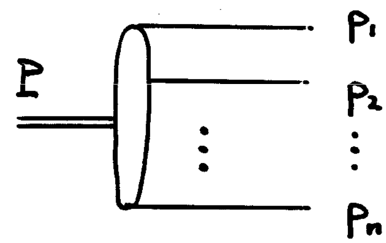
4. Conclusions + Outlook

*) with K. HARADA, C. STERN

1. INTRODUCTION

Q. : " How to describe a hadron ? "

A. : " Determine its wave function ! "

$\psi_n(P; p_1, \dots, p_n) \sim$ 

→ non-perturbative many-body bound state problem

→ most intuitive : Fock space approach

e. g. pion :

$|\pi\rangle = \psi_2 \, |q\bar{q}\rangle + \psi_3 \, |q\bar{q}g\rangle + \psi_4 \, |q\bar{q}q\bar{q}\rangle + \dots$

with WF's $\psi_n \equiv \langle n | \pi \rangle$

→ Hamiltonian formulation: no explicit covariance

(i) quantizing at equal time : $t = 0$

$\hat{=}$ disconnected vacuum diagrams : $\langle 0 | H_{int} | n \rangle \neq 0$

$\hat{=}$ boosts dynamical ('complicated') →

WF's frame dependent

(ii) Light-Cone (LC) quantization: $x^+ \equiv t + z = 0$

$\ominus$ vacuum 'trivial': $\langle 0 | H_{int} | n \rangle = 0$

$\oplus$ boosts kinematic: frame dependence of WF's trivial

$\ominus$ rotations/parity dynamical

technically: have to solve LC bound-state equation (LC BSE)

e.g. for pion ($\vec{P}_\perp = 0$)

$$\hat{M}^2 |\pi\rangle \equiv P^+ \hat{P}^- |\pi\rangle = m_\pi^2 |\pi\rangle$$

$\rightarrow$ determine WF's $\psi_n = \langle n | \pi \rangle$ and m_π^2

difficulties:

(i) determination of Hamiltonian $P^- = P^0 - P^3$

$\rightarrow$ solve constraints

(ii) infinite set of coupled integral eq.s

$\rightarrow$ truncate

(iii) renormalization

lack of explicit covariance + rotational invariance →

abundance of counterterms :

- different for k^+ and $\vec{k}_\perp$
- non-local $(1/k^+)$

have to trade cutoff dependence for

scale dependence : $\psi_n(\Lambda) \longrightarrow \psi_n(Q)$

two regimes :

$\psi_n(Q \approx \Lambda_{QCD})$ constituent quarks	$\xleftrightarrow{\text{RG ?}}$	$\psi_n(Q \gg \Lambda_{QCD})$ partons

↓

this talk : particular aspect of 'constituent regim

χSB with 'trivial vacuum'

(drastic) simplification

- D = 1+1 : no cutoff dependence
- D = 3+1 : eff. field theory → cutoff = parameter

2. D = 1+1 : 'T HOOFT + SCHWINGER MODEL

't Hooft model (tHM): QCD_{1+1}, $N_c \to \infty$

Schwinger model (SM): QED_{1+1} (here: $\Theta = 0$)

features:

- no elementary fermions in spectrum
  (linear Coulomb potential)

- χSB

  tHM: 'almost' spontaneous (→ Kosterlitz/Thouless)

  SM: anomalous

- lowest bound state: 'pion'

  $$|\pi\rangle = \psi_2 \, |q\bar{q}\rangle + \psi_4 \, |q\bar{q}q\bar{q}\rangle + \dots$$

  tHM: $\psi_4, \psi_6, \dots = 0$ $(N_c \to \infty)$

  SM: $\psi_4, \psi_6, \dots$ small $(m \ll e)$

- chiral limit: $m = 0$

  tHM: $m_\pi = 0$

  $m_\pi = \alpha = 1$ (anomaly)

• 'pion' mass squared ($m \neq 0$)

$$m_\pi^2 = \alpha + M_1 m + M_2 m^2 + M_3 m^3 + \dots$$

in units of basic scale μ_0

tHM: $\mu_0^2 = g^2 N_c / 2\pi$; $\alpha = 0$ (no anomaly)

SM: $\mu_0^2 = e^2/\pi$; $\alpha = 1$ (anomaly)

important: coefficients M_i are related to

vacuum structure (χSB) :

tHM: ('t HOOFT, ZHITNITSKY)

$$\langle 0|\bar\psi\psi|0\rangle / N_c = - M_1/4\pi = -0.28868$$

SM: (HAMER et al.; ADAM; FIELDS et al.)

$$\langle 0|\bar\psi\psi|0\rangle = - M_1/4\pi = -\frac{1}{2\pi}\sqrt{M_2/1.0678} = -0.28347$$

→ use these models as test laboratory for

LC techniques; in particular:

solve LC bound-state equation for

amplitude $\phi(x)$ to find quark with longitudinal momentum fraction x in 'pion':

$$\underbrace{m_\pi^2}_{\text{eigenvalue}} \phi(x) = \underbrace{\frac{m^2}{x(1-x)}}_{\text{kin. energy}} \phi(x) + \underbrace{\int_0^1 dy \, \frac{\phi(x)-\phi(y)}{(x-y)^2}}_{\text{COULOMB pot.}} + \underbrace{\int_0^1 dx \, \phi(x)}_{\text{anomaly}}$$

note: restriction to 2-particle sector; solve via

(i) variational procedure with trial WF

$$\phi(\beta, a, b, \dots) = \underbrace{x^\beta (1-x)^\beta}_{\text{'t HOOFT's ansatz}} + \varphi(a, b, \dots)$$

$\rightarrow$ β determines end point behaviour $x \rightarrow 0, 1$

(ii) expand around chiral limit ($\rightarrow$ ADAM)

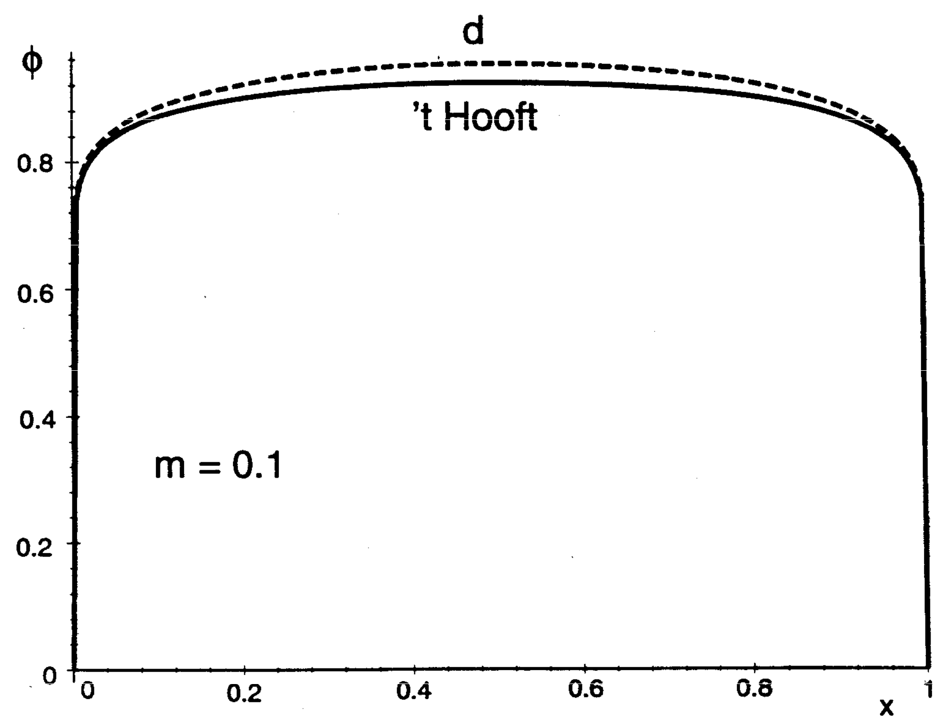
$$\beta = \beta_1 m + \beta_2 m^2 + \dots \qquad \text{etc.}$$

$\rightarrow$ calculate variational parameters $\beta, a, b, \dots$

$\rightarrow$ wave function ϕ, eigenvalue (mass) m_π

LC wave function of 'pion'

(tHM: $\alpha = 0$)



- endpoint behaviour ($\rightarrow \beta_1$) stable under improvements a, b, c, d
- slight enhancement in intermediate region

$m_\pi\ (m)$:

tHM :

TABLE I. The expansion coefficients M_i for the 't Hooft model ($\alpha = 0$).

	M_1	M_2	M_3
variational 2PTD	$2\pi/\sqrt{3} = 3.627599$	$3.581055 \pm 3 \cdot 10^{-6}$	$0.061793 \pm 3 \cdot 10^{-6}$
numerical 2PTD	$3.62758 \pm 2 \cdot 10^{-5}$	$3.5829 \pm 3 \cdot 10^{-4}$	$0.0636 \pm 1 \cdot 10^{-3}$
't Hooft	$2\pi/\sqrt{3} = 3.627599$	–	–
Li	3.64 ± 0.06	–	–
Li et al.	3.64 ± 0.03	3.60 ± 0.06	0.04 ± 0.04
Burkardt	$2\pi/\sqrt{3}$	3.5812	–

- $M_1 = 2/\beta_1 = 2\pi/\sqrt{3}$ ('t Hooft - Bergknoff)

  independent of improvements a, b, c, ...

- good convergence for M_2, M_3

- consistency with other results, but

- much higher accuracy

- $\langle\bar{\psi}\psi\rangle = -M_1/4\pi = -1/2\pi\beta_1 = -0.28868$

  $\uparrow$

  endpoint behaviour

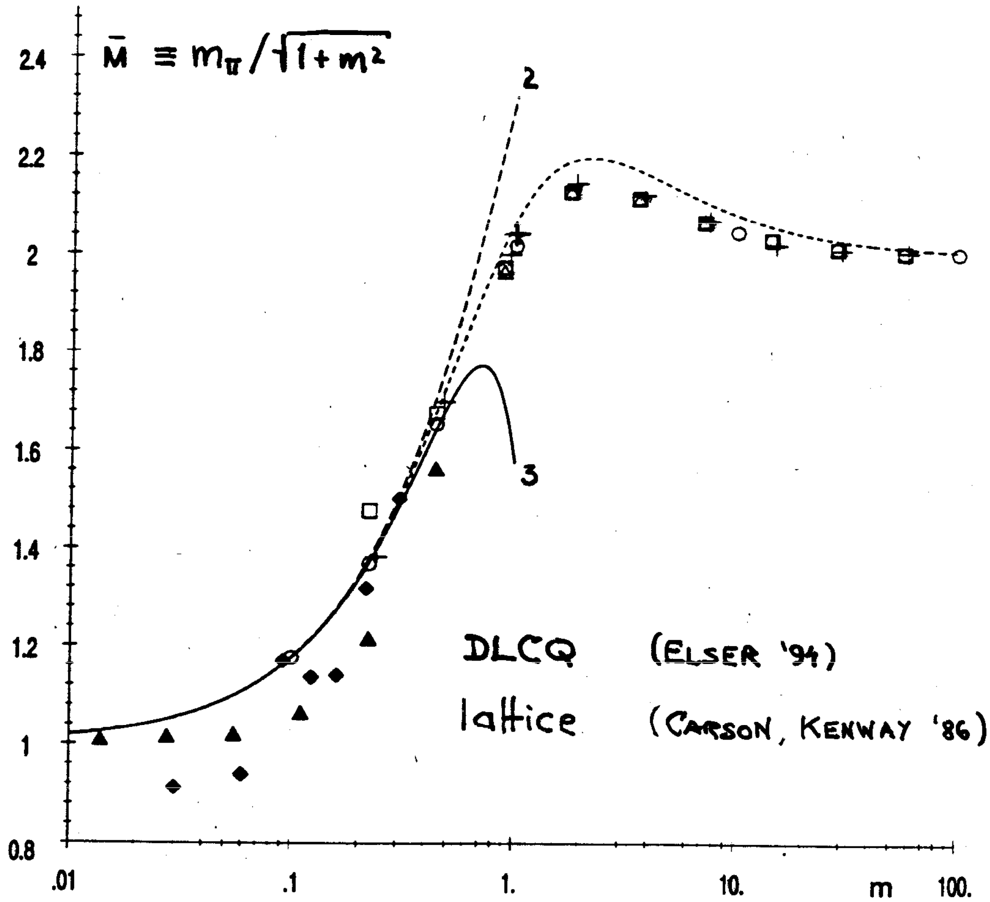
SM:

TABLE II. The expansion coefficients M_i for the Schwinger model ($\alpha = 1$).

	M_1	M_2	M_3
variational 2PTD	$2\pi/\sqrt{3} = 3.627599$	$3.308608 \pm 4 \cdot 10^{-6}$	$0.348204 \pm 1 \cdot 10^{-6}$
numerical 2PTD	$3.6268 \pm 8 \cdot 10^{-4}$	3.32 ± 0.02	0.28 ± 0.06
numerical 6PTD	$3.6267 \pm 4 \cdot 10^{-4}$	3.22 ± 0.02	0.5 ± 0.1
numerical 4PTD	3.62 ± 0.07	3.2 ± 0.3	0.3 ± 0.2
DLCQ	3.7 ± 0.02	3.5 ± 0.3	–
lattice	3.5 ± 0.2	3.7	0.02
Adam	$2e^{\gamma} = 3.562146$	3.3874	–
Fields et al.	$2e^{\gamma} = 3.562146$	3.387399	–

- $M_1 = 2\pi/\sqrt{3}$ (as for tHM)
- 2 % discrepancy in M_1, M_2 compared to bosonization results →
- inclusion of higher Fock states (numerically)
  - very small contributions ; at $m = 0.1$:
    $|\psi_2|^2 : |\psi_4|^2 : |\psi_6|^2 \simeq 1 : 10^{-5} : 10^{-7}$
  - discrepancy remains
- still, do better than other approx.s (small m)

discussion: (SM: $\alpha = 1$)



------ $\bar{M}$ with $m_\pi^2 = 1 + 2e^\gamma m + 4m^2$ (exact for $m \to 0, \infty$)

--- 2^{nd} order mass pert. theory

— 3^{rd} order mass pert. theory

} obtained analytically! (MAPLE)

+ CREWTHER, HAMER (lattice)

□ PAULI, BRODSKY (DLCQ)

○ MO, PERRY (LFTDA)

3. D = 3+1 : NAMBU-JONA-LASINIO MODEL

features:

- chirally invariant 4-fermion interaction $\sim g\,(\bar{\psi}\Gamma\psi)^2$
- not renormalizable $\rightarrow$ cutoff $\Lambda \simeq 1$ GeV = parameter

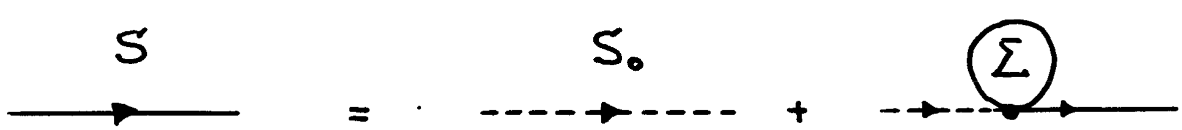
  ($\rightarrow$ effective field theory)
- beyond critical coupling g_c :

  dynamical mass generation $\rightarrow$ SBχS

alternative way of deriving LC BSE:

(i) solve SCHWINGER - DYSON equation

for quark self-energy $\Sigma \equiv m$

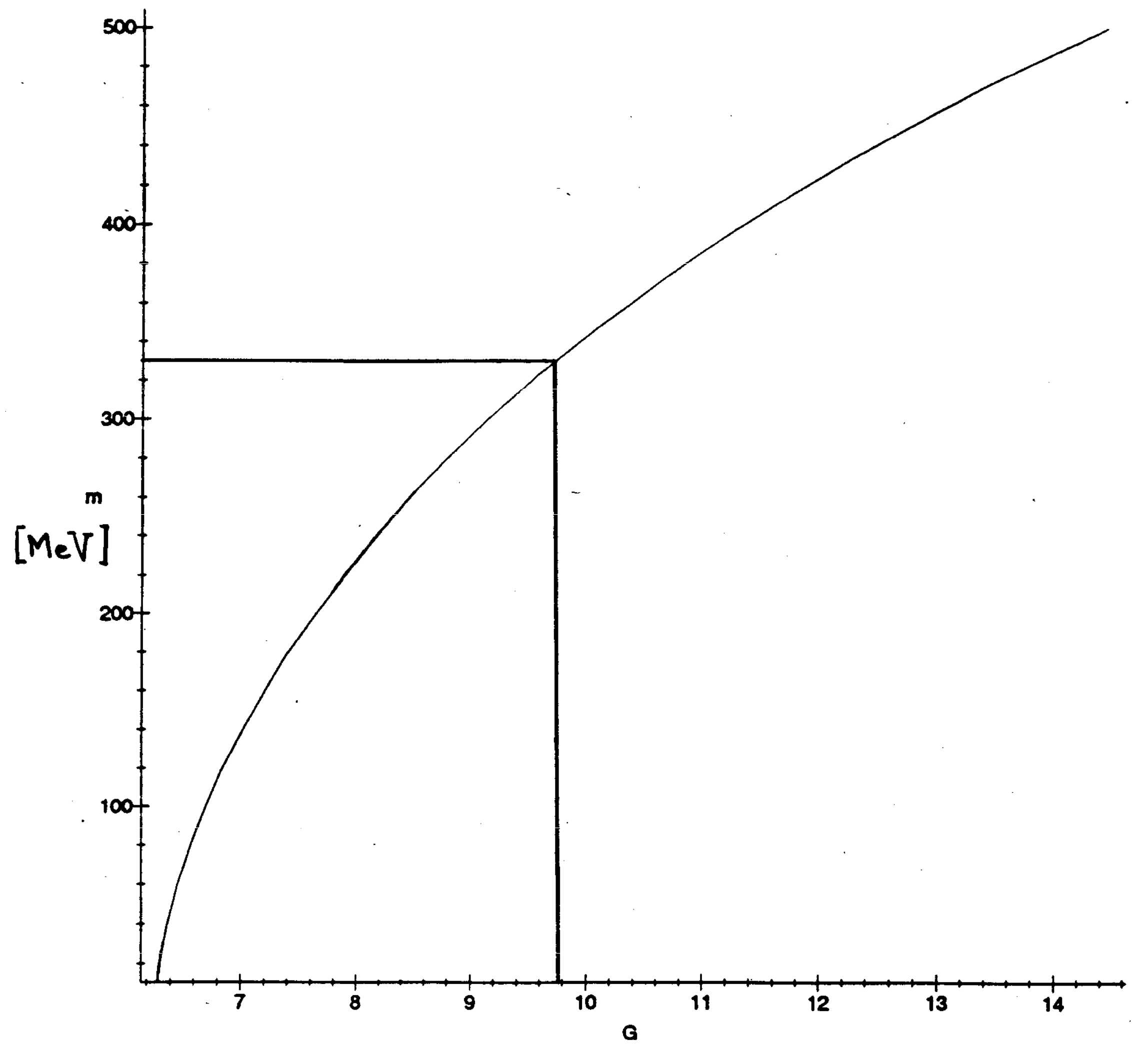


$\rightarrow$ gap - equation

$$m = -\,2g\,\langle\bar{\psi}\psi\rangle_{m,\Lambda}$$

dynamical fermion mass m as function of coupling G

(invariant mass cutoff, $m_0 = 0$)



dimensionless coupling: $G = g\Lambda^2$

critical value: $G_c = 2\pi$

CUTOFF :

rotational invariance ($|\vec{k}| < \Lambda$) + parity invariance $\rightarrow$

$$M_0^2 \equiv \frac{k_\perp^2 + m^2}{x(1-x)} < \Lambda^2 \quad ; \quad x = k^+/\Lambda$$

(invariant mass cutoff) $\longrightarrow$

$$\frac{m^2}{\Lambda^2} < x < 1 - \frac{m^2}{\Lambda^2} \qquad \text{(small - x cutoff)}$$

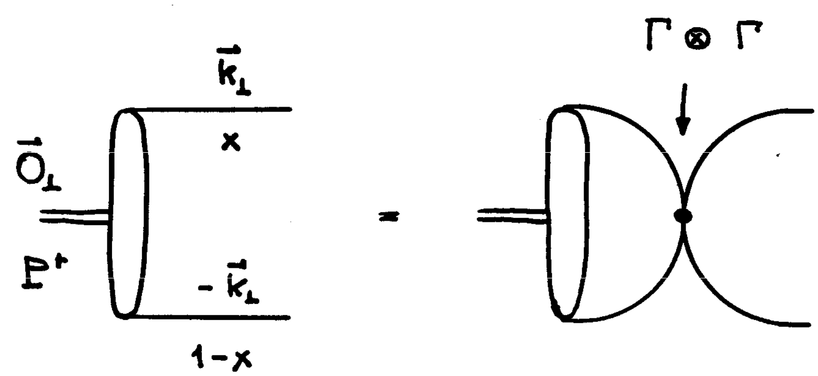
(ii) BETHE - SALPETER equation

here : Kernel $K = \Gamma \otimes \Gamma$ momentum independent $\rightarrow$

projection onto $x^+ = 0$ via $\int dk^- \ldots$ trivial $\rightarrow$

immediately get LC BSE

$\psi(x, \vec{k}_\perp)$ =



can show : NJL mechanism

get massless GOLDSTONE pion iff

gap equation holds $\rightarrow$ LC BSE o.k. ✓

$\rightarrow$ SBCS ✓

r.h.s. of LC BSE contains only normalization integrals $\rightarrow$

solution:

$$\psi(x, \vec{k}_\perp, \lambda, \lambda') = N_\Lambda \; \phi(D_0) \times \underbrace{\bar{u}_\lambda \, (m_\pi + \not{P}) \gamma_5 \, v_\lambda}_{\text{'spin structure'}}$$

- N_Λ : cutoff dependent normalization
- $\phi(D_0)$: scalar function of 'off-shell-ness'

$$D_0 = m_\pi^2 - M_0^2 = m_\pi^2 - \frac{k_\perp^2 + m^2}{x(1-x)}$$

- here:

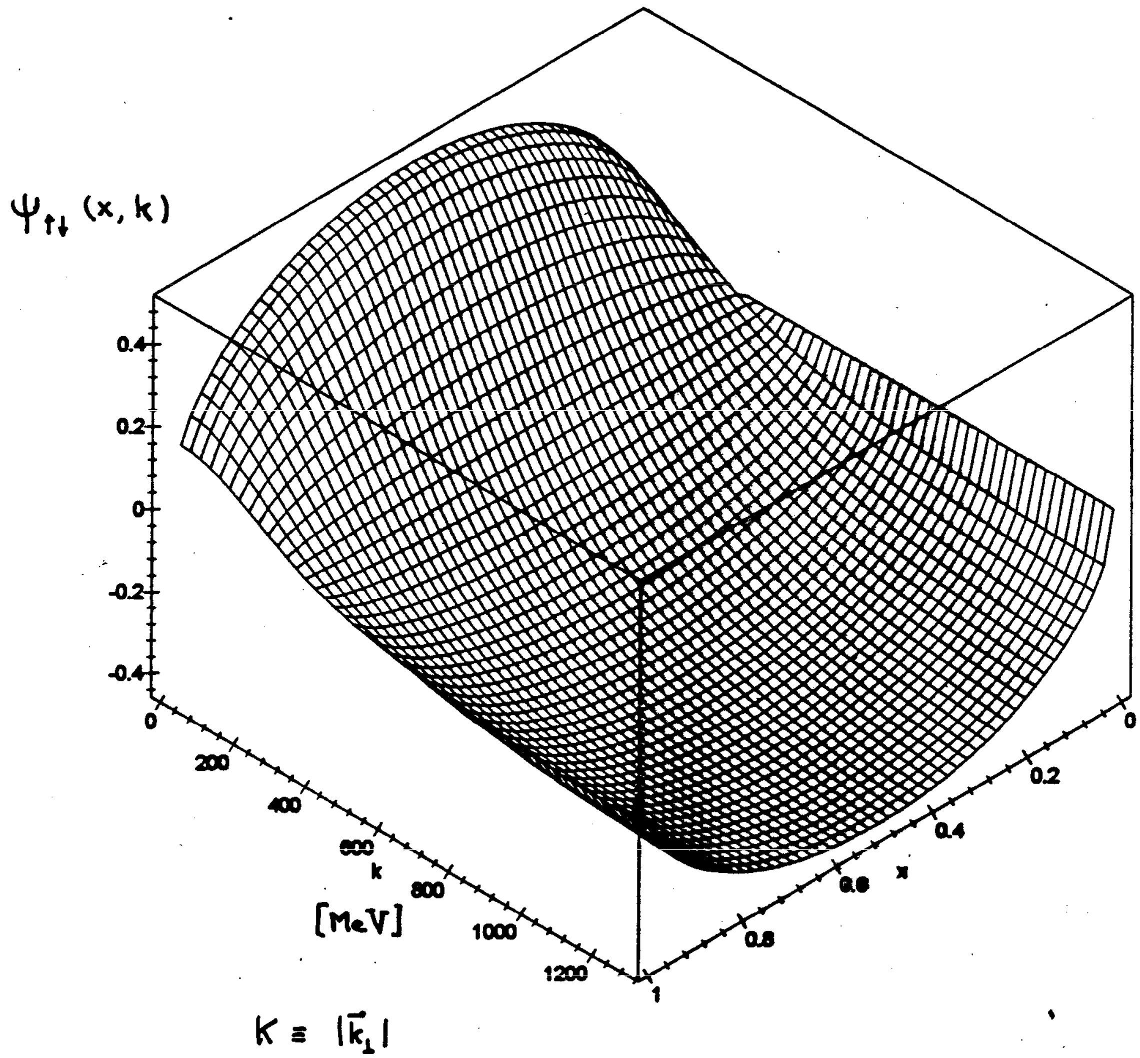
$$\phi(D_0) = 1/D_0$$

- relativistic constituent quark model:

(Terent'ev; Karmanov; Coester et al.; Dziembowski, Mankiewicz; Jaus; Ji et al.; Weber et al.; ...)

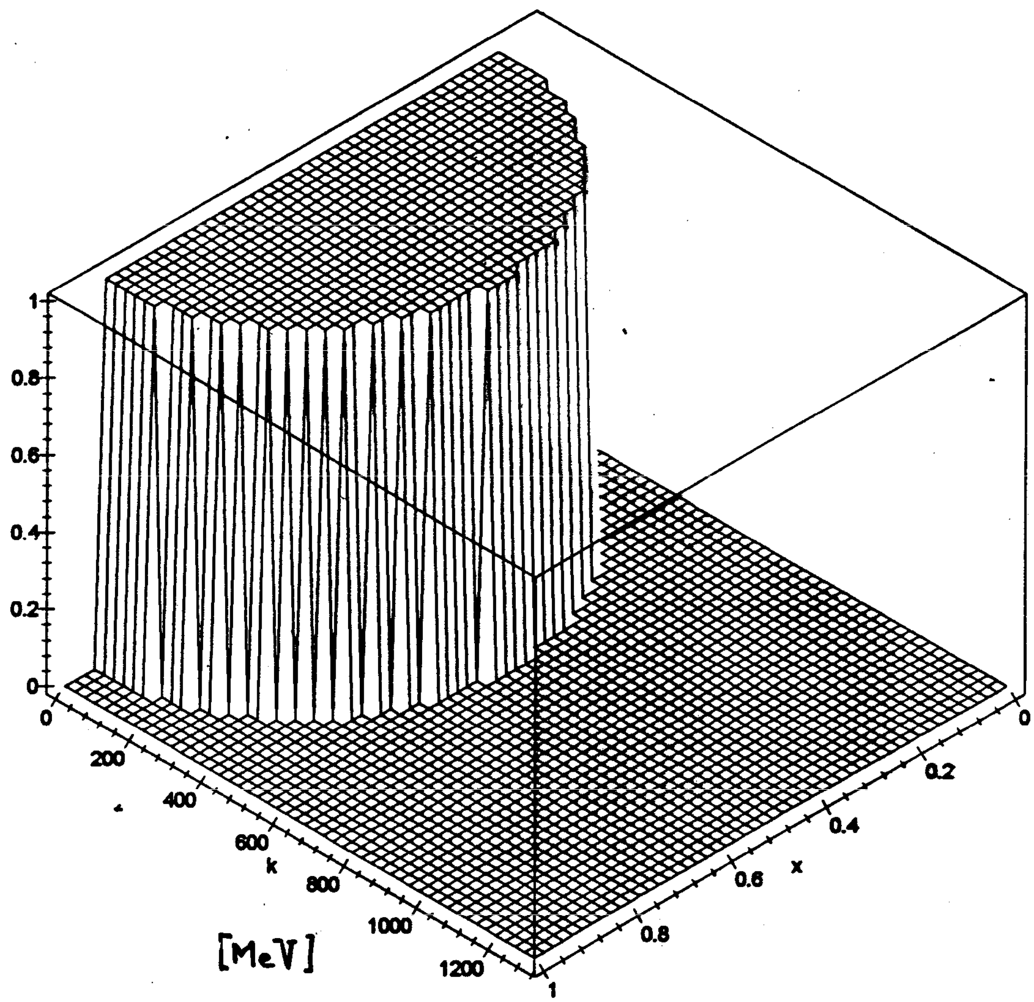
$$\phi(D_0) = \exp(D_0/\beta^2)$$

β: confinement scale parameter; absent for NJL π - WF without cutoff :



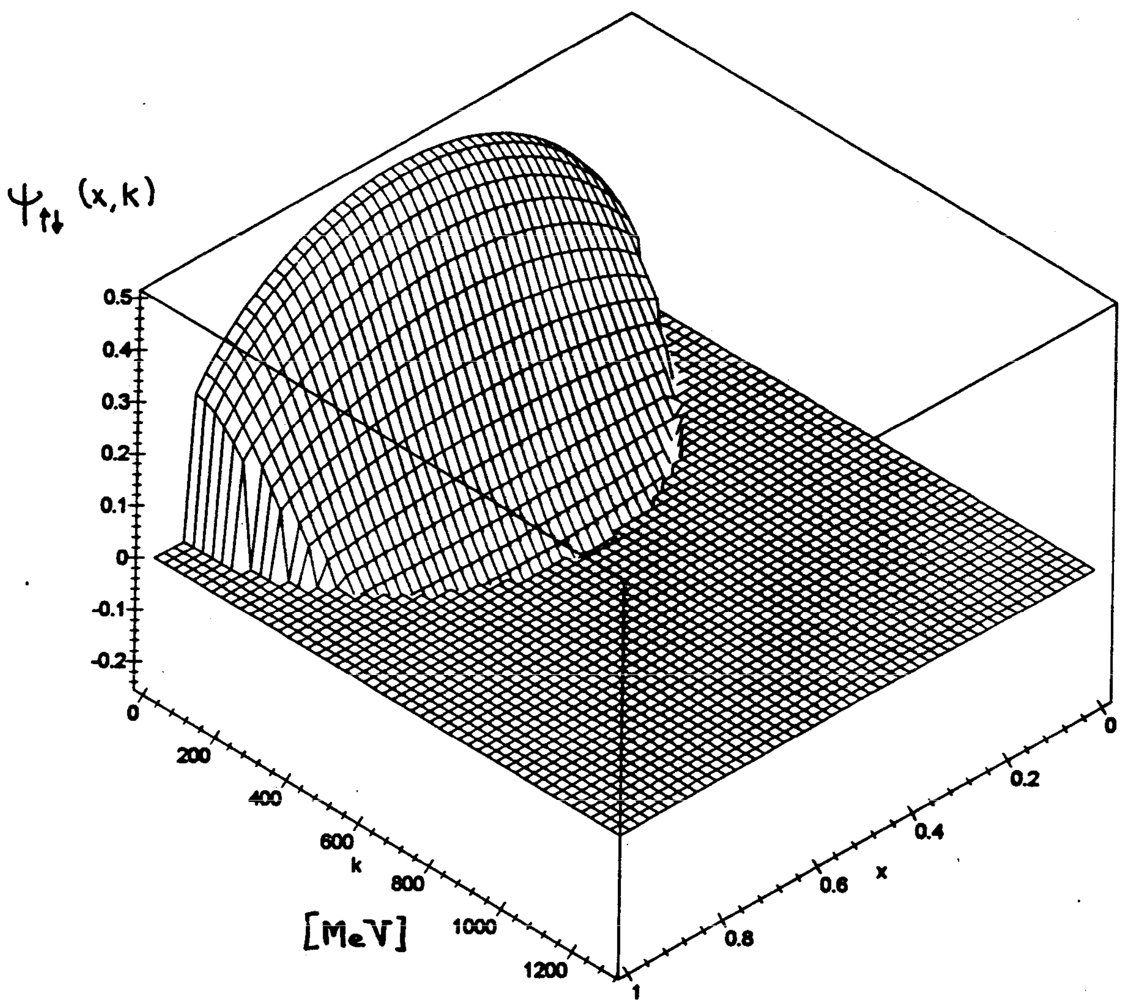
'invariant mass cutoff' :

$$k_\perp^2 < \Lambda^2 x(1-x) - m^2 \; ; \qquad m = 330 \text{ MeV}$$



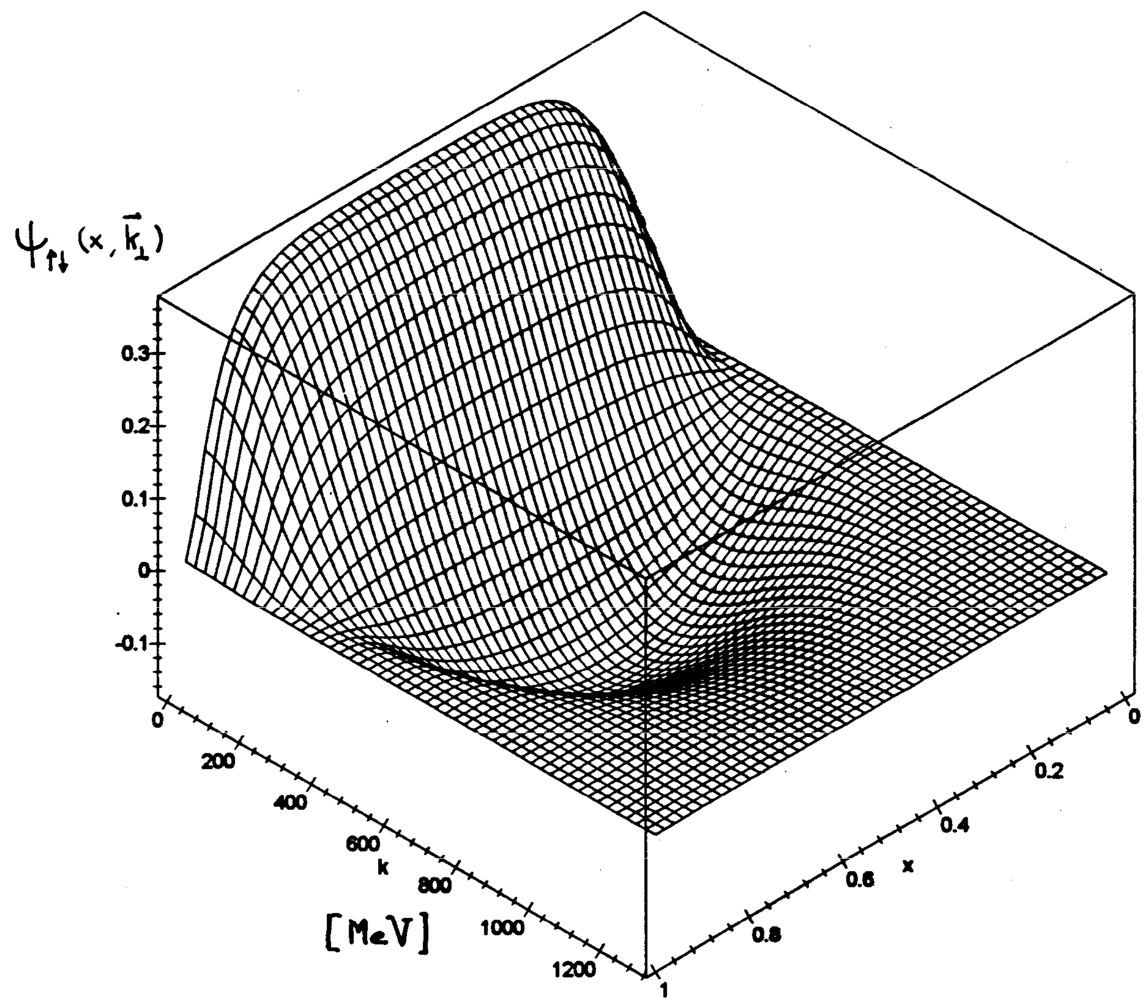
$k \equiv |\vec{k}_\perp|$

π - WF with invariant - mass cutoff



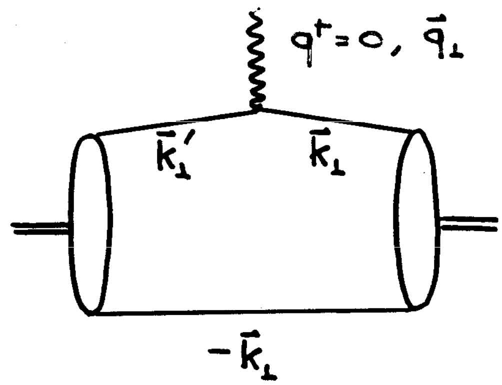
$k \equiv |\vec{k}_\perp|$

π - WF in the relativistic CQM (Gaussian)



$k \equiv |\vec{k}_\perp|$

determine N_Λ, Λ analytically from pion form factor:



$$F(q_\perp^2) \quad = \quad P^+,\ \vec{P}_\perp + \vec{q}_\perp \quad [\text{diagram}] \quad P^+,\ P_\perp$$

$$= \sum_{\lambda,\lambda'} \int dx\, d^2k_\perp\, \psi^*(x, \vec{k}_\perp', \lambda, \lambda')\, \psi(x, \vec{k}_\perp, \lambda, \lambda')$$

assume hierarchy : $m_\pi^2 \ll m^2 \ll \Lambda^2$

lowest order : chiral limit, $m_\pi = 0$

(i) normalization : $F(\vec{q}_\perp = 0) \overset{!}{=} 1$

$$|N_\Lambda|^2 = \frac{15}{\pi}\frac{1}{\Lambda^2} + \mathcal{O}(m^2/\Lambda^4)$$

(ii) cutoff via form factor and charge radius :

$$F(q_\perp^2) = 1 - 3q_\perp^2/\Lambda^2 + \mathcal{O}(q_\perp^2 m^2/\Lambda^4) \quad \rightarrow$$

$$\langle r^2 \rangle = 0.44\ \text{fm}^2 = 18/\Lambda^2 \quad \rightarrow$$

$\Lambda \simeq$ 1.[illegible] GeV

4. CONCLUSIONS AND OUTLOOK

LC wave functions of hadrons can be calculated

- analytically
- for relativistic QFT's
- near the chiral limit, $m_q \to 0$

prospects for QCD:

- use covariant equations:
  - SCHWINGER - DYSON
  - BETHE - SALPETER
- renormalize
- project onto light-front $x^+ = 0$