Dynamical Fermion Masses in Light-Front QCD_{3+1}

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SOME QUESTIONS ......

..... and answers ....

- **How can we see dynamical fermion mass generation ?**

  → Mass gap equations (e.g. Dyson-Schwinger)

- **What do we need to derive mass gap equations ?**

  (i) RENORMALISATION (one loop)

  (ii) LADDER SUMMATION TECHNIQUES
  (e.g. rainbow approximation

CLAIM : In LC field theory, (i) & (ii) may be addressed by studying small-x behaviour of LC wave functions ....

RENORMALISATION : A New Paradigm?

Conventional Wisdom ......

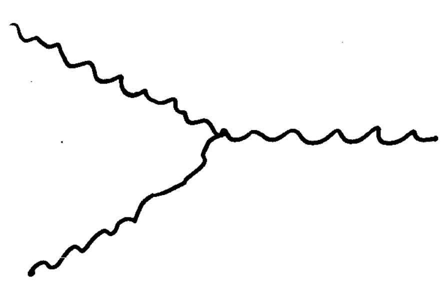
A SIMPLE EXAMPLE:

Large N ϕ^3 Matrix Field Theory

mass term: $m^2\phi^2$

interaction term: $\frac{\lambda}{3\sqrt{N}}\phi^3$

$\vec{k}_1 = (x_1, k_1^\perp)$

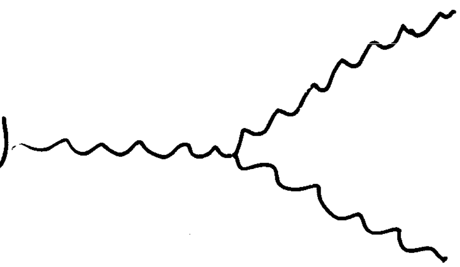


$\vec{k}_1 + \vec{k}_2$

$\vec{k}_2 = (x_2, k_2^\perp)$

$(\alpha x_1, p^\perp)$

$\vec{k}_1 = (x_1, k_1^\perp)$



$(\beta x_1, q^\perp)$

$$\begin{cases} \alpha + \beta = 1 \\ p^\perp + q^\perp = k_1^\perp \end{cases}$$

BOUND STATE EQUATIONS .....

$$M^2 \chi_n(\vec{k}_1, \dots, \vec{k}_n) =$$

$$\left(\sum_{i=1}^{n} \frac{m^2 + |k_i^\perp|^2}{x_i} \right) \chi_n(\vec{k}_1, \dots, \vec{k}_n)$$

$$- g \left[\frac{\chi_{n-1}(\vec{k}_1 + \vec{k}_2, \vec{k}_3, \dots, \vec{k}_n)}{x_1 x_2} + \cdots \right]$$

$$- g \left[\int_0^1 d\alpha \, d\beta \, \delta(\alpha + \beta - 1) \int dp^\perp dq^\perp \, \delta(p^\perp + q^\perp - k_1^\perp) \right.$$

$$\times \chi_{n+1}((\alpha x_1, p^\perp), (\beta x_1, q^\perp), \vec{k}_2, \dots, \vec{k}_n)$$

$$\left. + \cdots \cdots \right]$$

Notation: $\vec{k} = (x, k^\perp)$, $x = k^+/p^+$

CANCELATION CONDITIONS.....

$$\left(m^2 + |k_1^\perp|^2\right) \lim_{x_1 \to 0^+} \psi_n(\vec{k}_1, \ldots, \vec{k}_n)$$

$$=$$

$$g \lim_{x_1 \to 0^+} \left[\frac{\psi_{n-1}(\vec{k}_1 + \vec{k}_2, \vec{k}_3, \ldots, \vec{k}_n)}{x_2} + \frac{\psi_{n-1}(\vec{k}_n + \vec{k}_1, \vec{k}_2, \ldots, \vec{k}_{n-1}}{x_n} \right.$$

$$+$$

$$g \int_0^1 d\alpha\, d\beta\; \delta(\alpha + \beta - 1) \int dp^\perp dq^\perp\, \delta(p^\perp + q^\perp - k_1^\perp) \times$$

$$\lim_{x_1 \to 0^+} x_1 \cdot \psi_{n+1}\left((\alpha x_1, p^\perp), (\beta x_1, q^\perp), \vec{k}_2, \ldots, \vec{k}_n\right)$$

N.B. Independent of M^2!

Alternatively......

APPLICATIONS ......

Schematically :

$$\boxed{\begin{aligned}(m^2 + |k_1^\perp|^2)\, \chi_n(0^+, \ldots) &= \\ g \cdot \chi_{n-1}(\ldots) \\ + \; g \cdot \int \chi_{n+1}(0^+, 0^+, \ldots)\end{aligned}}$$

After one iteration ......

$$(m^2 + |k_1^\perp|^2)\, \chi_n \longrightarrow (m^2 + |k_1^\perp|^2 - g^2 I(k_1^\perp))\, \chi_n$$

$$\boxed{I(k_1^\perp) = \int_0^1 d\alpha\, d\beta\, \delta(\alpha+\beta-1) \int dp^\perp dq^\perp\, \delta(p^\perp + q^\perp - k_1^\perp) \frac{1}{\alpha(m^2 + |q^\perp|^2) + \beta(m^2 + |p^\perp|^2)}}$$

RESULTS ......

D = 3 :

$$\Gamma(k^\perp) = \frac{2\pi}{|k^\perp|} \tan^{-1} \frac{|k^\perp|}{2m}$$

D = 4 :

$$\Gamma(k^\perp) = \left(\pi \ln \frac{m^2 + \Lambda_\perp^2}{m^2} \right) + (\text{finite}) |k^\perp|^2 + \cdots\cdots$$

D = 5 :

$$\Gamma(k^\perp) = \left(4\pi \Lambda_\perp - 4\pi m \tan^{-1} \frac{\Lambda_\perp}{m} \right) + (\text{finite}) |k^\perp|^2 + \cdots\cdot$$

D = 6 :

$$\Gamma(k^\perp) = \left(\pi^2 \Lambda_\perp^2 - \pi^2 m^2 \ln \frac{m^2 + \Lambda_\perp^2}{m^2} \right) + \left(\frac{\pi^2}{6} \ln \frac{m^2 + \Lambda_\perp^2}{m^2} + \text{finite} \right) |k^\perp|^2 + (\text{finite}) |k^\perp|^4 + \cdots\cdots$$

LIGHT-FRONT QCD_{3+1} .....

$$\Psi = \Psi_R + \Psi_L$$

CONSTRAINTS ($A^+ \equiv 0$)

$$i\partial_- \Psi_L = (i\gamma^\perp D_\perp + m)\Psi_R$$ (FERMION CONSTRAINT

$$\partial_-^2 A_+ = J$$ (GAUSS' LAW)

$$\Downarrow$$

$$\int dx^- J \equiv 0$$ (colour singlets)

$$\int dx^- (i\gamma^\perp D_\perp + m)\Psi_R \equiv 0$$ (Small-x ladder relations.

($\Rightarrow$ FINITESS CONDITIONS for LC Hamiltonian)

$$m\,\chi_{\mp\pm\alpha_1\ldots\alpha_n}(\vec{k},\vec{k}_1,\ldots,\vec{k}_{n+1})$$

$$\pm(k^1 \pm i k^2)\,\chi_{\pm\pm\alpha_1\ldots\alpha_n}(\vec{k},\vec{k}_1,\ldots,\vec{k}_{n+1})$$

$$=$$

$$\frac{g\sqrt{N}}{(2\pi)^{3/2}}\Bigg[\frac{\chi_{\pm\alpha_1\ldots\alpha_n}(\vec{k}+\vec{k}_1,\vec{k}_2,\ldots,\vec{k}_{n+1})}{\sqrt{k_1^+}}$$

$$+$$

$$\int_0^\infty \frac{dp^+dq^+}{\sqrt{q^+}}\,\delta(p^+ + q^+ - k^+)\int dp^\perp dq^\perp\,\delta(p^\perp + q^\perp - k^\perp)\,\times$$

$$\chi_{\pm\mp\pm\alpha_1\ldots\alpha_n}(\vec{p},\vec{q},\vec{k}_1,\ldots,\vec{k}_{n+1})\Bigg]$$

[Notation: $\vec{k} = (k^\perp \to 0^+, k^+)$]

Anticipated results......

When I ~~did~~ do this calculation ....

CONCLUSIONS......

Future looks bright.......

....even at small-x .....