Chiral Symmetry (Breaking) on the light-front

1) Canonical Considerations (tree level)

2) Light-front Chiral Charge

3) Explicit Breaking in the Hamiltonian

4) Spontaneous Symmetry Breaking?

5) Perturbative Renormalization (one loop)

6) Non trivial effect of explicitly breaking term at high energy

7) Conclusion

Chiral Symmetry (Breaking) on the light front

Canonical Considerations

Start with fermion field:

Not all four components of ψ are dynamical.

Define the projection operators $\Lambda^{\pm} = \frac{1}{4}\gamma^{\mp}\gamma^{\pm}$

$\gamma^{\pm} = \gamma^0 \pm \gamma^3$ $\qquad\qquad \psi^{\pm} = \Lambda^{\pm}\psi$

With the choice of the gauge $A^+ = 0$, it follows from the Dirac equation that ψ^- is constrained.

Explicitly $\psi^- = \frac{1}{i\partial^+}\left[\alpha^{\perp}\cdot(i\partial^{\perp} + gA^{\perp}) + \gamma^0 m\right]\psi^{(+)}$

$\partial^+ = 2\frac{\partial}{\partial x^-}$ longitudinal spatial derivative

$$\psi^-(x^-, x^{\perp}) = \frac{-i}{4}\int dy^- \,\epsilon(x^- - y^-)\left[\alpha^{\perp}\cdot(i\partial^{\perp} + gA^{\perp}(y^-, x^{\perp})) + \gamma^0 m\right] \psi^+(y^-, x^{\perp})$$

Fermion mass enters the Hamiltonian P^- and the currents J^{μ}, J_5^{μ} through ψ^-.

$P^-, J^{\perp}, J^-, J_5^{\perp}, J_5^-$ depends on ψ^-

Same m term in ψ^- gives rise to m^2 and m terms in P^-, J^-, J_5^-.

Free fermion Hamiltonian $P^-_{free} = \int dx^- d^2x^\perp \, \psi^{(+)\dagger} \frac{(-\partial_\perp^2 + m^2)}{i\partial^+} \psi^{(+)}$

Quadratic in the fermion mass.

Chiral transformation $\psi^{(+)} \to \psi^{(+)} + \delta\psi^{(+)}$

$$\delta\psi^{(+)} = -i\theta\gamma_5 \psi^{(+)}$$

P^-_{free} is invariant.

<u>Folklore</u>: Free massive fermions on the light-front do not break chiral symmetry.

Define axial vector charge

$$Q_{5_{LF}} = \frac{1}{2}\int dx^- d^2x^\perp \; J_5^+$$

$$= \sum_\lambda \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} \lambda\left[b^\dagger(k,\lambda)b(k,\lambda) + d^\dagger(k,\lambda)d(k,\lambda)\right]$$

Same as helicity (Even for massive fermions)

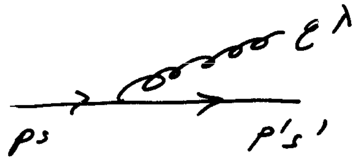
Light-front chirality $\equiv$ Light front helicity

$Q_{5_{LF}}$ does not create pairs from the free vacuum in contrast to $Q_{5_{ET}}$ for the massive case.

Interactions

Same m in ψ^- gives rise to m^2 and m term in P^-.

Quark Gluon vertex



$$= \chi^\dagger_{s'}\left[-2\frac{q^\perp}{q^+} + \frac{\sigma^\perp\sigma^\perp\cdot p^\perp}{p^+} + \frac{\sigma^\perp\cdot p'^\perp\sigma^\perp}{p'^+} - im\left(\frac{1}{p'^+}-\frac{1}{p^+}\right)\sigma^\perp\right]\chi_s \cdot \epsilon^{\perp *}_\lambda$$

Linear mass term → helicity flip → Explicit chiral symmetry breaking term.

Not caused by soft (small q^+) gluon

Emission of soft gluon does not flip the helicity of the quark.

Helicity flip enhanced by transition to a 'soft fermion state

↳ fermion near zero longitudinal momentum mode

Folklore: Spontaneous chiral symmetry breaking has to d with fermion zero modes.

How to go about it?

* Explicit consideration of fermion zero modes

* Effective Hamiltonian approach →

  Cut off soft longitudinal momenta

  Large effect on the light front ($P^- \sim \frac{1}{P^+}$)

  Generate effective interactions through renormalization

In the second game, the words 'spontaneous symmetry breaking' is misleading.

Vacuum is trivial.

Hamiltonian explicitly breaks the symmetry even in the chiral limit.

In QCD, the needed term is expected (Wilson) to be non canonical. Why?

In the canonical theory, in the chiral limit (no helicity flip) expect π and ρ to be degenerate.

→ Very bad phenomenology

Need to have a Large separation between π and ρ in the chiral limit.

Is there any constraint on the non canonical term?

Chiral current should be conserved.

Room for the non canonical term?

Equation for ψ^- → Sensitive to small k^+ fermion

However,

if we motivate the non-canonical effective interaction term as arising from the equation for ψ^-, not only ρ^- but $J_5^\perp$ and J_5^- also affected.

The equation for the conservation of chiral current involves all three.

Is this bad news? (too many parameters to be fixed)

Aspects of perturbative renormalization

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m^2 term : Not forbidden by chiral symmetry, not protected by it either.

Power counting says quadratic divergences. They do show up.

Second order perturbation theory

$$\delta m^2 = -g^2 C_f \int \frac{d^2 k^\perp}{2(2\pi)^3} \int \frac{dx}{1-x} \left\{ \frac{4}{x^2} - \frac{4}{x} + 2 \right\}$$

One may cancel it by carefully adjusting the contribution by "self inertias." But it is there.

m term : Protection by chiral symmetry

Find $\delta m \propto m$

Even with the brutal cut off regularization

$$(k_i^\perp < \Lambda, \quad k_i^+ > \epsilon)$$

find $\delta m = C_f \frac{3g^2}{8\pi^2} m \ln \frac{\Lambda}{\mu}$

Exactly the same as Feynman perturbation theory.

One interesting complication:

m occurs in the quark gluon vertex in the combination mg. Thus m renormalization comes as a part of coupling renormalization.

In non abelian theory there are linear divergences due to $k^+ \to 0$ belonging to a gluon (soft gluon divergences) in different time ordered diagrams.

Cancellation between different time ordered diagrams. Otherwise new non-canonical counterterms.

Linear divergences occur only in the renormalization of the helicity flip vertex.

Example of soft gluon affecting explicit chiral symmetry breaking interaction beyond tree level.

Not to be completely lost in the wilderness of renormalization,
Useful to develop chiral quark models for baryons and mesons to gain insight from phenomenology.

Why phenomenological chiral quark models are interesting?

Not only from low energy aspects.

Polarized deep inelastic scattering –
High Energy side of things.

Usually masses appear power suppressed in high energy scattering.

Not so in some polarized scatterings.

Helicity distribution. (leading terms)

$$g_1(x) = \frac{1}{8\pi S^+} \int_{-\infty}^{\infty} dy^- \, e^{-i\eta x}$$

$$\langle PS | 2\, \psi^{(+)\dagger}(y^-)\, \gamma_5\, \psi^{(+)}(0) + h.c \,| PS \rangle$$

$$\eta = \tfrac{1}{2} P^+ y^-$$

Helicity flip distribution

$$g_T(x) = \frac{1}{8\pi S^\perp} \int_{-\infty}^{\infty} dy^- \, e^{-i\eta x} \langle PS | [\mathcal{O}_m + \mathcal{O}_{k^\perp} + \mathcal{O}_g] + h.c | P\ldots$$

$$= g_T^{m}(x) + g_T^{k^\perp}(x) + g_T^{g}(x)$$

$$\mathcal{O}_m = m\, \psi^{(+)\dagger}(y^-)\, \gamma^\perp \left(\frac{1}{i\overrightarrow{\partial}^+} - \frac{1}{i\overleftarrow{\partial}^+} \right) \gamma_5\, \psi^{(+)}(0)$$

$$\mathcal{O}_{k^\perp} = -\psi^{(+)\dagger}(y^-) \left[\gamma^\perp \frac{\overrightarrow{\not\partial}^\perp}{\overrightarrow{\partial}^+} + \frac{\overleftarrow{\not\partial}^\perp}{\overleftarrow{\partial}^+} \gamma^\perp \right] \gamma_5 \psi^{(+)}(0)$$

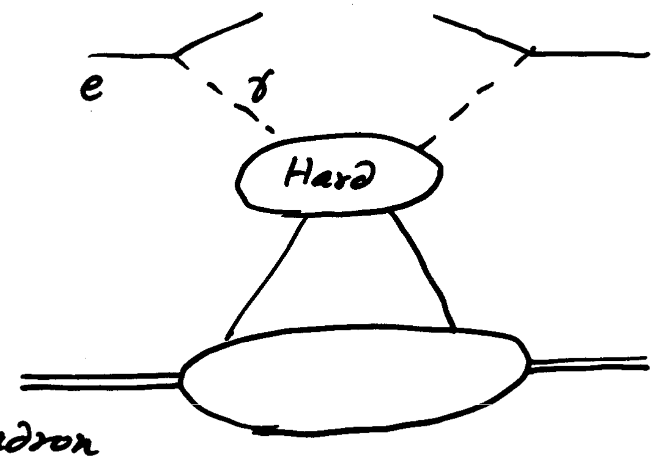
$$\mathcal{O}_g = g\, \psi^{(+)\dagger}(y^-) \left[\not{A}^\perp(y^-) \frac{1}{i\overleftarrow{\partial}^+} \gamma^\perp - \gamma^\perp \frac{1}{i\overrightarrow{\partial}^+} \not{A}^\perp(0) \right] \gamma^5\, \psi^{(+)}(0)$$

Why light-front:

- Equal "time" correlation functions only on the LF
- Non trivial phase factor absent in $A^+ = 0$ gauge
- Parton picture

Perturbative Aspects

Folklore of pQCD:



"Interference between amplitudes with left and right handed quarks is zero for perturbative hard scattering coefficients"

[J. Collins, High Energy Spin Physics, 1996]

quark mass zero

What goes wrong?

g_T in pQCD

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g_T in pQCD

For a single quark state with helicity λ

$$|k\lambda\rangle = b^+(k,\lambda)|0\rangle \phi_1$$

$$+ \sum_{\sigma_1 \lambda_2} \int \frac{dk_1^+ d^2k_1^\perp}{\sqrt{2(2\pi)^3 k_1^+}} \int \frac{dk_2^+ d^2k_2^\perp}{\sqrt{2(2\pi)^3 k_2^+}} \phi_2(k\lambda|k_1\sigma_1 k_2\lambda_2)$$

$$\sqrt{2(2\pi)^3 p^+}\, \delta^3(p - k_1 - k_2)\, b^+(k_1\sigma_1)\, a^+(k_2\lambda_2)|0\rangle$$

$$\psi^{(+)}(x) = \sum_{s_1} \chi_{s_1} \int \frac{dp_1^+ d^2p_1^\perp}{2(2\pi)^3 \sqrt{p_1^+}} [b(p_1 s_1) e^{-ip_1\cdot x} + d^+(p_1,-s_1) e^{ip_1\cdot x}]$$

$$A^\perp(x) = \sum_\lambda \int \frac{dp^+ d^2p^\perp}{2(2\pi)^3 p^+} [a(p,\lambda)\, \epsilon_\lambda^\perp\, e^{-ip\cdot x} + a^+(p,\lambda)\, \epsilon_\lambda^{*\perp}\, e^{ip\cdot x}]$$

Need to know ϕ_2, the quark-gluon wave function

$$\phi_2 = \frac{1}{\sqrt{p^+}} \psi_2$$

$$\psi_{2\lambda}(\kappa^\perp, x; \sigma_1\lambda_2) = \frac{g}{\sqrt{2(2\pi)^3}} \frac{x(1-x)}{(\kappa^\perp)^2} (-) \frac{1}{\sqrt{1-x}}$$

$$\chi_{\sigma_1}^+ \left\{ \frac{2\kappa^\perp}{1-x} + \frac{\sigma^\perp\cdot\kappa^\perp \sigma^\perp}{x} - im\,\sigma^\perp \left(\frac{1-x}{x}\right) \right\} \chi_\lambda \cdot \epsilon_{\lambda_2}^{*\perp}$$

Helicity Flip term

Without loss of generality, let the target be transversely polarized in the x direction.

The target state can be expressed as

$$|k, s'\rangle = \frac{1}{\sqrt{2}} \left(|k \uparrow\rangle \pm |k, \downarrow\rangle \right)$$

$$S' = \pm m_q^R, \quad \text{Renormalized quark mass}$$

<u>Free theory</u>

Only the quark mass term contributes.

Then

$$g_T(x) = g_T^m(x) = \frac{1}{2} \frac{m_q}{S'} \delta(1-x)$$

$$= \frac{1}{2} \delta(1-x) \quad \text{since } m_q = m_q^R \text{ in free theory}$$

Result independent of the quark mass

For the longitudinally polarized state

$$g_1(x) = \frac{1}{2} \delta(1-x)$$

Thus for a free quark $g_2(x) = g_T(x) - g_1(x)$
$= 0$

$O(g^2)$ correction

$$g_T^m(x, Q^2) = \frac{1}{2}\frac{m_q}{S'}\left\{\delta(1-x) + \frac{\alpha_s}{2\pi} C_f \ln Q^2 \left[\frac{2}{1-x} - \delta(1-x)\int_0^1 dy\, \frac{1+y^2}{1-y}\right]\right\}$$

Wavefunction Renormalization Contribution

$$g_T^{k_\perp}(x, Q^2) = -\frac{1}{2}\frac{m_q}{S'}\frac{\alpha_s}{2\pi} C_f \ln Q^2 \,(1-x)$$

$$g_T^{g}(x, Q^2) = \frac{1}{2}\frac{m_q}{S'}\frac{\alpha_s}{2\pi} C_f \ln Q^2 \,\frac{1}{2}\delta(1-x)$$

Adding everything,

$$g_T(x, Q^2) = \frac{1}{2}\frac{m_q}{S'}\left\{\delta(1-x) + \frac{\alpha_s}{2\pi} C_f \ln Q^2 \left[\frac{1+2x-x^2}{(1-x)_+} + 2\,\delta(1-x)\right]\right\}$$

$+$ prescription : $\int_0^1 dz\, \frac{f(z)}{(1-z)_+} = \int_0^1 dz\, \frac{f(z)-f(1)}{(1-z)}$

Not the whole story!

m_q is the bare quark mass

$S' = m_q^R$

To order α_s, $m_q^R = m_q \left(1 + \frac{3}{4\pi}\alpha_s C_f \ln Q^2\right)$

mass renormalization

Finally,

$$g_T(x, Q^2) = \frac{1}{2}\left\{\delta(1-x) + \frac{\alpha_s}{2\pi} C_f \ln Q^2 \left[\frac{1+2x-x^2}{(1-x)_+} + \frac{1}{2}\delta(1-x)\right]\right\}$$

Again final answer independent of quark mass!

But finite quark mass essential to start the calculation!!

By a similar calculation,

$$g_1(x, Q^2) = \frac{1}{2}\left\{\delta(1-x) + \frac{\alpha_s}{2\pi} C_f \ln Q^2 \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]\right\}$$

Then

$$g_2(x, Q^2) = \frac{1}{2}\frac{\alpha_s}{2\pi} C_f \ln Q^2 \left[2x - \delta(1-x)\right]$$

Comparison with OPE:

In the standard OPE method one calculates the moments of the structure functions

$$\int_0^1 dx\, x^{n-1}\, g_1(x, Q^2) = g_1^n \,, \qquad n = 1, 3, 5, \cdots$$

$$\int_0^1 dx\, x^{n-1}\, g_2(x, Q^2) = g_2^n \,, \qquad n = 3, 5, \cdots$$

Once we have g_1 and g_2, we can calculate all the moments.

We get

$$g_1^n = \frac{1}{2}\left\{1 + \frac{\alpha_s}{2\pi} \ln Q^2 \left[-\frac{1}{2} + \frac{1}{n(n+1)} - 2\sum_{j=2}^{n} \frac{1}{j}\right]\right\}$$

$$g_T^n = \frac{1}{2}\left\{1 + \frac{\alpha_s}{2\pi} \ln Q^2 \left[-3/2 + \frac{1}{n} + \frac{1}{n+1} - 2\sum_{j=2}^{n} \frac{1}{j}\right]\right\}$$

Then $g_2^n = \frac{1}{2}\frac{\alpha_s}{2\pi} \ln Q^2 \left\{-1 + \frac{2}{n+1}\right\}$

Agrees with OPE calculations that include the 'mass operator'.

$g_2^0 = 0$ Burkhardt - Cottingham sum rule

$\boxed{\text{NEWS}}$

To $O(\alpha_s)$ g_T has contributions from

O_m, $O_{k\perp}$ and O_g.

O_m depends explicitly on the quark mass and its matrix element has a direct parton interpretation (Feynman, 1972).

$O_{k\perp}$ and O_g are termed 'Higher twist' operators in the context of OPE.

Traditional to ignore O_m.

What we find:

Matrix elements of $O_{k\perp}$ and O_g also comes out proportional to m but the final result is independent of m.

For certain observables (polarization) effects of mass not negligible even at very high energy!

Summary

Tried to convey the message that the study of chiral symmetry on the light front can be quite adventurous.

There are lots and lots of things to be done.