NON-PERTURBATIVE ASPECTS OF $\mathcal{N}=8$ QCD_{1+1} AND MATRIX STRINGS

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- MATRIX STRING THEORY

  $\Leftrightarrow$ $\mathcal{N}=8$, $D=1+1$, $U(N)$ SUSY YANG-MILLS THEOR

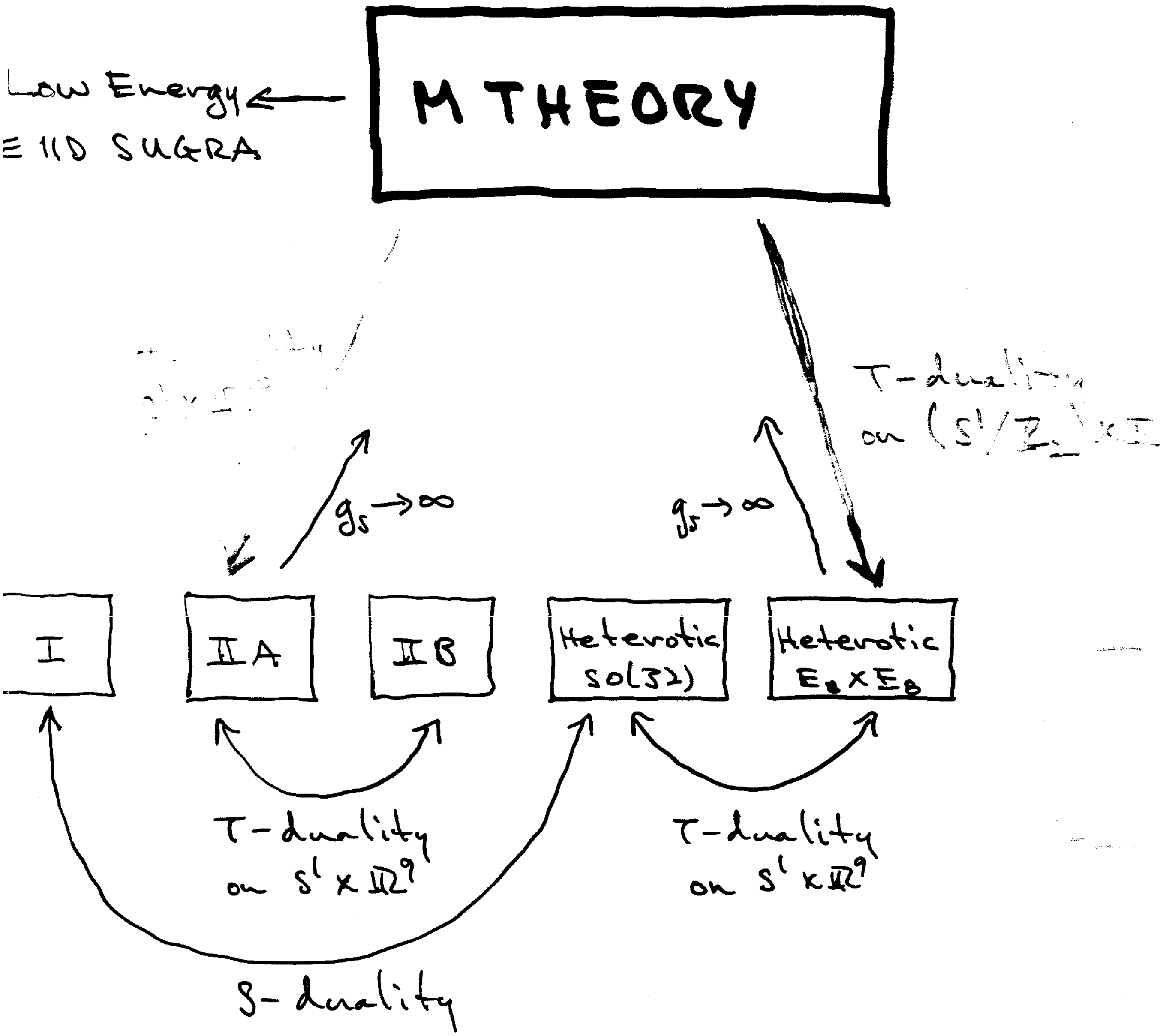
- PROPERTIES OF 2D $\mathcal{N}=8$ SYM

  $\Leftrightarrow$ NON-PERTURBATIVE DESCRIPTIONS OF STRING DYNAMICS, D-BRANES, M THEORY, -

- EXPLICIT NICOLAI MAP AT LARGE N

I. Kogan, G. Semenoff + R.J.S., Mod. Phys. Lett. **A12** (1997) 183.

I. Kogan + R.J.S., Phys. Lett. **B404** (1997) 286



S-DUALITY: $g_s \longrightarrow \dfrac{1}{g_s}$

T-DUALITY: $R \longrightarrow \dfrac{l_p^2}{R}$.

I. MATRIX STRING THEORY

$\mathcal{N}=8$ $u(N)$ SYM IN 1+1 DIMENSIONS ON A CYLINDER $(\tau,\sigma)\in\mathbb{R}^1\times S^1$:

$$S=\frac{R_9^2}{2\pi\alpha'}\int d\tau\oint d\sigma\ \mathrm{tr}\Big(\frac{1}{2}\sum_{i=1}^{8}(D_\mu X^i)^2+i\psi^T \not{D}\psi$$
$$-\frac{1}{2}F_{\mu\nu}^2+\frac{1}{2g_s^2}\sum_{i<j}[X^i,X^j]^2-\frac{1}{g_s}\sum_{i=1}^{8}\psi^T\gamma_i[X^i,\psi]\Big)$$

$g_{YM}=\frac{1}{g_s}$, $T\equiv\frac{1}{2\pi\alpha'}$ = STRING TENSION

R_9 - COMPACTIFICATION RADIUS IN 11-D

$X^i(\tau,\sigma)$ - $N\times N$ HERMITIAN MATRIX FIELD IN ADJOINT REP. OF COLOUR $u(N)$
$i=1,\dots,8$

$$\psi(\tau,\sigma)=\begin{pmatrix}\psi_L^\alpha(\tau+\sigma)\\ \psi_R^{\dot\alpha}(\tau-\sigma)\end{pmatrix}$$
- $N\times N$ MAJORANA-WEYL SPINOR FIELDS, IN ADJ. REP. OF $u(N)$

$\alpha,\dot\alpha=1,\dots,8$

$(X^i,\psi_L^\alpha,\psi_R^{\dot\alpha})$ TRANSFORM IN $(8_v,8_s,8_c)$ REPS. OF SYMMETRY GROUP $so(8)$ OF TARGET SPACE $\mathbb{R}^8$.

$$\gamma^i = \begin{pmatrix} 0 & \gamma^i_{\alpha\dot\alpha} \\ \gamma^i_{\dot\alpha\alpha} & 0 \end{pmatrix} \in 8_s \oplus 8_c \;, \quad \{\gamma_i, \gamma_j\} = 2\delta_{ij}$$

$A_\mu(\tau,\sigma)$ – NON-DYNAMICAL, RESIDUAL 2D $U(N)$ GAUGE FIELD, $\mu = 0, 1$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{i}{g_s}[A_\mu, A_\nu] \;;\; x^0 \equiv \tau, \; x^1 \equiv \sigma$$

$$\not{D} = \begin{pmatrix} D_+ & 0 \\ 0 & D_- \end{pmatrix}, \quad D_\pm = D_\tau \pm D_\sigma, \quad D_\mu = \partial_\mu + \frac{i}{g_s}[A_\mu, \cdot\,]$$

A M THEORY IN (10+1) DIMENSIONS:

- PLANCK LENGTH l_p, COORDS. $x^\mu = (t, x^i, x^{11})$

  $x^i = x^\perp, \quad i = 1, \ldots, 9$

  $x^{11} \sim x^{11} + 2\pi R_{11}$

- LIGHT-CONE: $x^\pm \equiv t \pm x^{11}$

- BOOST ALONG LONGITUDINAL AXIS x^{11}

  $\Rightarrow$ INFINITE MOMENTUM FRAME, $p_{11} \to \infty$

- ONLY STATES WITH $p_{11} > 0$ RELEVANT.

• COMPACT x^{11} $\Rightarrow$ $p_{11} = \frac{N}{R_{11}}$, N - A > 0 INTEGER

$$R_{11} \to \infty \, , \quad \frac{N}{R_{11}} \to \infty$$

• $R_{11} < \infty$ $\Rightarrow$ TYPE IIA SUPERSTRINGS IN $D = 9+1$:

$$R_{11} = g_s^{2/3} \, l_p \quad , \quad l_s \equiv \sqrt{\alpha'} = g_s^{-1/3} \, l_p$$

$$g_s \to \infty \quad \Rightarrow \quad R_{11} \to \infty \, .$$

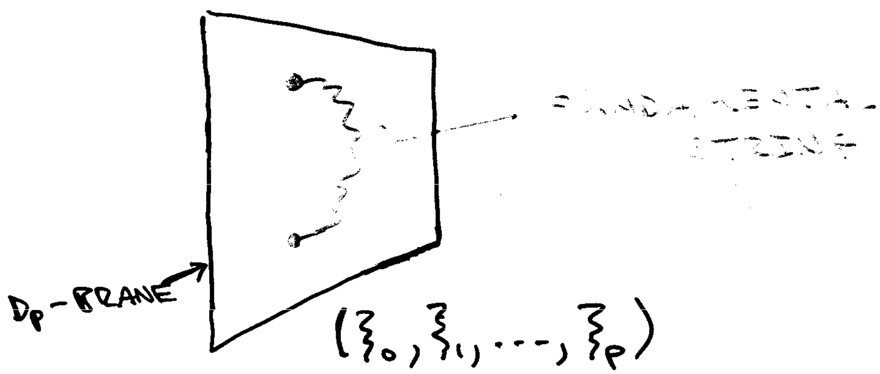
• <u>CONJECTURE</u>: ONLY DYNAMICAL DEGREES OF FREEDOM OF M THEORY IN THE LIGHT-CONE FRAME ARE D0-BRANES, WHICH ARE PARTONS OF THE TRANSVERSE SPACE $\mathbb{R}^9$.

5 D-BRANE FIELD THEORY:

• LOW-ENERGY DYNAMICS OF OPEN SUPERSTRINGS WITH <u>NEUMANN</u> BOUNDARY CONDS. $\equiv$ $D = 9+1$, $\mathcal{N} = 1$ $U(N)$ SYM THEORY:

$$S_{YM}^{(9)} = \int d^{10}x \, \mathrm{tr}\left(-\frac{1}{2} F_{\mu\nu}^2 + \frac{i}{2} \bar{\Psi} \overbrace{\Gamma^\mu}^{32 \times 32} D_\mu \Psi \right) .$$

- DIRICHLET p-BRANES:

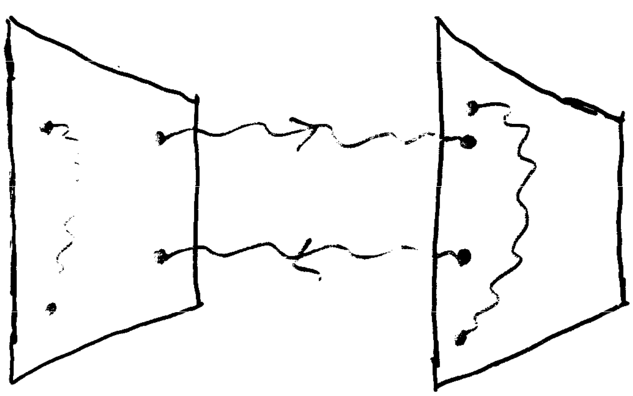


$A_\alpha(\xi)$, $\alpha = 0, \ldots, p$ $\Rightarrow$ tangential components to Dp-brane

$X_i(\xi) = 2\pi\alpha' A_i(\xi)$, $i = p+1, \ldots, 9$

$\Longrightarrow$ components orthogonal to D-brane, associated with coords. of Dp-brane

- U(N) GAUGE SYMMETRY:



$N = 2$ Dp-branes

$M \sim T|x^i - x^j|$

$P_{11} = \frac{N}{R_{11}}$ for $p = 0$

$U(N) \supset U(1)^N$

Massless vector states form a U(N) multiplet.

• T-DUALITY:

$$A_i \in \left[0, \frac{1}{R_i}\right] \xrightarrow{T} X_i = 2\pi\alpha' A_i \in \left[0, 2\pi\frac{\alpha'}{R_i}\right]$$

$$\text{NEUMANN} \xrightarrow{T} \text{DIRICHLET}$$

$$\text{winding \#'s } nR_i \text{ of } A_i \xrightarrow{T} \text{String momentum } p_i = \frac{nR_i}{\alpha'}$$

$$W[C] = \exp\left(i \int_C A_i \, dx^i\right) \xrightarrow{T} V_p = e^{i p_i X^i}.$$

• FOR D0-BRANES, COMPACTIFY $X^9 \in X^\perp$ TO RADIUS R_9 $\Rightarrow$ $X_9 \to R_9 D_\sigma$, $R_9 = g_s l_p$, $P_+ = \frac{N}{R}$

\begin{tikzcd}
\text{IIA STRING MOMENTUM } P_+ \arrow[rr, leftrightarrow, "9\text{-}11\text{ Flip}"] \arrow[d, leftrightarrow, "T\text{-duality}"] & & \text{D-PARTICLE} \equiv N \arrow[d, leftrightarrow, "T\text{-duality}"] \\
\text{IIB STRING WINDING} \equiv \arrow[rr, leftrightarrow, "S\text{-duality}"] & & \text{D-STRING WINDING}
\end{tikzcd}

• D0-BRANES PROBE SHORT-DISTANCES IN STRING THEORY

$\Rightarrow$ NONCOMMUTATIVE GEOMETRICAL SPACETIME STRUCTURE

$$\Delta X \gtrsim l_p.$$

II. PROPERTIES OF MATRIX STRINGS FROM $\mathcal{N}=8$ QCD_{1+1}

PROPOSAL: IIA STRING THEORY IN LIGHT-CONE FRAME $\Longleftrightarrow$ 2D, $\mathcal{N}=8$ $U(N)$ SYM IN THE $N\to\infty$ LIMIT.

FREE STRING LIMIT:

$g_s \to 0$ $(\Leftrightarrow g_{YM} \to \infty)$

$$S \sim -\int d\tau \oint d\sigma \, \mathrm{tr}\left(\frac{1}{2g_s^2}\sum_{i<j}[X^i,X^j]^2 - \frac{1}{g_s}\sum_{i=1}^{8}\psi^T\gamma_i[X^i,\psi]\right)$$

$[X^i, X^j] = 0$, $[X^i, \psi] = 0$ (COMMUTATIVE SPACETIME).

$$\Rightarrow X^i = U\begin{pmatrix} x_1^i & & 0 \\ & \ddots & \\ 0 & & x_N^i \end{pmatrix}U^{-1} \quad , \quad U \in U(N)$$

$\Rightarrow$ GREEN-SCHWARZ SUPERSTRING LIGHT-CONE COORDS. x_a^i , $\psi_{L,a}^{\alpha}$, $\psi_{R,a}^{\dot\alpha}$, $a = 1, \ldots, N$.

- FREELY-PROPAGATING STRINGS.

- DESCRIBED BY AN ORBIFOLD SUPERCONFORMAL FIELD THEORY:

$$S_0 = \frac{R_g^2}{2\pi\alpha'} \int d\tau \oint d\sigma \sum_{a=1}^{N} \left(\sum_{i=1}^{8} (\partial_\mu X_a^i)^2 + \sum_{\alpha=1}^{8} i \psi_{L,a}^{\alpha} \partial_+ \psi_{L,a}^{\alpha} + \sum_{\dot\alpha=1}^{8} i \psi_{R,a}^{\dot\alpha} \partial_- \psi_{R,a}^{\dot\alpha} \right)$$

$\equiv \mathcal{N}=8$ SUSY σ-MODEL IN THE ORBIFOLD TARGET SPACE $S^N \mathbb{R}^8 = (\mathbb{R}^8)^N / S_N$.

- PERMUTATION SYMMETRY OF N IDENTICAL PARTICLES:

$$S_N \subset U(N) \longrightarrow U(1)^N.$$

- $U(N) \sim U(1) \times SU(N) \Rightarrow U(1)$ FACTOR DESCRIBES CENTER OF MASS MOTION:

$$X_{cm}^i = \frac{1}{N} \sum_{a=1}^{N} X_a^i$$

$$P_i^{cm} = \frac{N}{R_{11}} \dot{X}_i^{cm}, \quad P_{11} = \frac{N}{R_{11}} \Rightarrow \frac{1}{P_{11}} P_i^{cm} = \dot{X}_i^{cm}.$$

IDENTIFIED AS A FUNDAMENTAL STRING

- LIMIT $N \to \infty \Rightarrow$ FREE 2^{nd} QUANTIZED HILBER SPACE.

"LONG" STRING STATES FROM ORBIFOLD SCFT.

- FREE LIGHT-CONE QUANTIZATION OF IIA STRINGS.

3 WEAKLY-COUPLED YANG-MILLS THEORY:

- STRONGLY-COUPLED STRINGS $g_s \to \infty$ $\left(\Leftrightarrow g_{YM} \to 0 \right)$:

$$S_\infty = \frac{R_9^2}{2\pi\alpha'} \int d\tau \oint d\sigma \, \mathrm{tr} \left(-\frac{1}{2} F_{\mu\nu}^2 + i \psi^T \not{D} \psi + \frac{1}{2} \sum_{i=1}^{8} \left(D_\mu X^i \right)^2 \right)$$

$\Rightarrow$ (PARTLY) ADJOINT QCD_{1+1}.

- INTERACTION WITH EXTERNAL COLOUR PROBE:

$$W_C[A] = \frac{\mathrm{tr}_E}{N} P \exp \left(i \oint_C A_\mu(x) \, dx^\mu \right)$$

- SCREENING RADIUS: $R_S \sim \frac{1}{g_{YM}^2} = g_s^2 \longrightarrow \infty$

$\Rightarrow$ MATRIX STRING THEORY AT $g_s \to \infty$ IS ESSENTIALLY CONFINING.

- SAME IS TRUE AT $g_s \to 0$, SO SCHWINGER-DYSON EQN

$\Rightarrow$ MATRIX STRING THEORY IS CONFINING $\forall g$.

- <u>PICTURE</u>: HADRONS BUILT FROM CONFINING QCD_{1+1} STRINGS, FORM $so(8)$ FLAVOUR MULTIPLETS, $u(N)$ COLOUR SINGLETS.

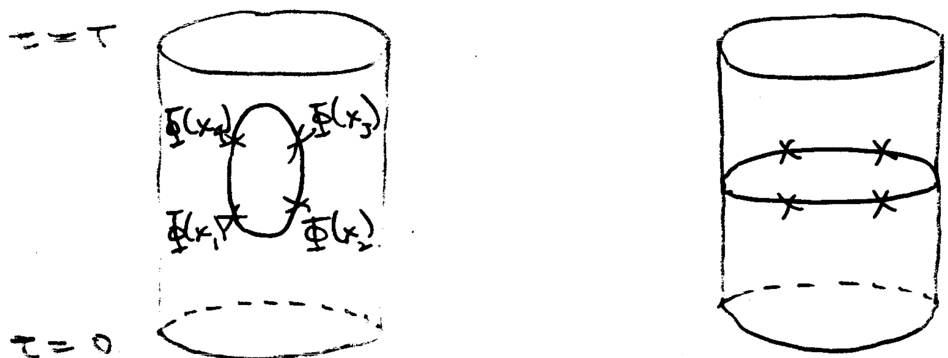
θ - VACUA: $\pi_1(\mathbb{R}^1 \times \mathbb{Z}) = \mathbb{Z}$ labels the windings of A_1 around S^1.

IN ADDITION TO PURE GLUON WINDINGS $W_C[A]$, WE ALSO HAVE:

$$W_C^{\{x_1 \cdots x_m\}}[A, \Phi] = \frac{\mathrm{tr}_F}{N} P\, \Phi(x_1) \exp\left(i \int_{x_1}^{x_2} A_\mu(x)\, dx^\mu\right)$$

$$\times\, \Phi(x_2) \cdots \Phi(x_m) \exp\left(i \int_{x_m}^{x_1} A_\mu(x)\, dx^\mu\right)$$

$x_1, \ldots, x_m \in C$, $\Phi = \psi$ or X^i.



- IF A_1 WINDS $\sigma \to 2\pi R_g n$, THEN

$$W_C[A] \to e^{2\pi i n/N} W_C[A], \quad \theta_n \equiv e^{2\pi i n/N} \in \mathbb{Z}_N$$

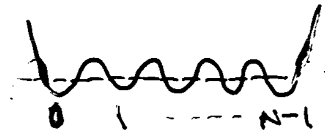
- EFFECTIVE GLOBAL GAUGE GROUP $SU(N)/\mathbb{Z}_N$ HAS

$$\pi_1(SU(N)/\mathbb{Z}_N) = \pi_0(\mathbb{Z}_N) = \mathbb{Z}_N$$

$\Rightarrow \exists$ N INEQUIVALENT VACUUM STATES $|0\rangle_n$
(θ-vacua) LABELLED BY $\theta_n \in \mathbb{Z}_N$

- CONNECTED BY INSTANTON FIELD CONFIGURATIONS

$W_c[A] \to \theta_n \cdot W_c[A]$, $n = 0, 1, \dots, N-1$.



SUMMARY OF SPECTRUM:

(1) Confining Strings

(b) Fundamental Strings $\equiv$ $U(1)$ Field in $U(N) \sim U(1) \times SU(N)$
D-strings $\equiv$ $SU(N)$ field

(c) Twisted long string states from orbifold CFT
($N \to \infty$ generates Fock space of the D-brane quantum field theory)

(d) Discretized light-cone quantization $\Rightarrow$ D-partons, wee-partons, etc. {Antonuccio, Pauli + Tsujimaru}

(e) Pure gluon + mixed hadronic winding modes
$\Rightarrow$ N inequivalent vacua which are connected by topological instantons.

III. INSTANTON CONFIGURATIONS AT LARGE-N

D=9+1, $\mathcal{N}=1$ SUSY

$$\delta_\varepsilon A_\mu = -2\varepsilon^T \Gamma_\mu \psi$$

$$\delta_\varepsilon \psi = \frac{1}{2} F_{\mu\nu} \Gamma^{\mu\nu} \varepsilon$$

$$\left(\Gamma^{\mu\nu} \equiv \frac{i}{2}[\Gamma^\mu, \Gamma^\nu]\right)$$

D=1+1, $\mathcal{N}=8$ SUSY

$$\delta_\varepsilon X^i = -2\varepsilon \gamma^i \psi$$

$$\delta_\varepsilon \psi = \frac{1}{2}\Big(\sum_{i=1}^{8} \not{\partial} X^i \gamma_i + \gamma_- - \frac{1}{2g_s} \sum_{i<j} [X^i, X^j] \gamma_{ij} \Big)\varepsilon + \varepsilon$$

$$\delta_\varepsilon A_\pm = -2\left(\varepsilon^\alpha (\gamma_i)_{\alpha\dot{\alpha}} \psi_R^{\dot{\alpha}} \pm \varepsilon^{\dot{\alpha}} (\gamma_i)_{\dot{\alpha}\alpha} \psi_L^\alpha \right)$$

Generate $16 \oplus 16$ rep. of super-Gallilean group in 11-D light-cone frame.

- $\mathcal{N}=8$ SUSY IS MAXIMAL IN $\mathbb{R}^8$ TARGET SPACE.

$\frac{1}{N}$ - EXPANSION OF THE PARTITION FUNCTION:

$$Z \equiv \int DA\, DX\, D\psi \, \exp\Big\{ iN \int d\tau \oint d\sigma \, \mathrm{tr}\Big(\frac{1}{2} \sum_{i=1}^{8} (D_\mu X^i)^2 + i\psi^T \not{D} \psi - \frac{1}{2} F_{\mu\nu}^2 + \frac{1}{2g_s^2} \sum_{i<j} [X^i, X^j]^2 - \frac{1}{g_s} \sum_{i=1}^{8} \psi^T \gamma_i [X^i, \psi] \Big) \Big\}$$

$$= \sum_{k \geq 0} N^{2-k} Z_k \quad \text{for large-}N.$$

{'t Hooft's topological large-N exp.}

$$Z_0 \equiv \lim_{N\to\infty} \frac{1}{N^2} Z = \int DA\, e^{iN\int d\tau \oint d\sigma\, \mathrm{tr}\, F_{\mu\nu}^2} \left(\mathcal{Z}[A]\right)^{1/2}$$

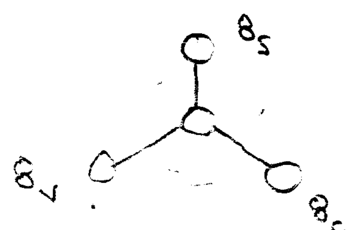
$$\mathcal{Z}[A] \equiv \int D\phi\, D\phi^\dagger\, D\chi\, D\bar\chi\, \exp\Big\{ iN \int d\tau \oint d\sigma$$
$$\times\, \mathrm{tr}\Big(\sum_{i=1}^{8} (D_+\phi_i^\dagger)(D_-\phi^i) + 2i\,\bar\chi \not{D} \chi$$
$$+ \frac{1}{g_s^2} \sum_{i<j} [\phi_i^\dagger, \phi^j][\phi^i, \phi_j^\dagger]$$
$$- \frac{2}{g_s} \sum_{i=1}^{8} \left(\bar\chi^\alpha \gamma^i_{\alpha\dot\alpha} [\phi^i, \chi^{\dot\alpha}] + \bar\chi^{\dot\alpha} \gamma^i_{\dot\alpha\alpha} [\phi_i^\dagger, \chi^\alpha] \right) \Big) \Big\}$$

$\phi^i(\tau,\sigma)$ – an $N\times N$ COMPLEX MATRIX FIELD, IN ADJ. REP. OF $u(N)$, $8_v \otimes \mathbb{C}$ REP. OF $so(8)$

$$\chi(\tau,\sigma) = \begin{pmatrix} \chi_L^\alpha(\tau+\sigma) \\ \chi_R^{\dot\alpha}(\tau-\sigma) \end{pmatrix}$$

– $N\times N$ COMPLEX FERMION MATRI[X] FIELD, IN ADJ. REP. OF $u(N)$, χ_L^α, $\chi_R^{\dot\alpha}$ IN $8_s \otimes \mathbb{C}$, $8_c \otimes \mathbb{C}$ REP. OF $so(8)$.

• SO(8) TRIALITY:



$\Rightarrow$ "LABEL" VECTOR AND SPINOR REPS. OF SO(8[)] IN THE SAME WAY.

- INTEGRATE OUT FERMIONS:

$$Z[A] = \int D\phi\, D\phi^\dagger \ \operatorname{Det} \Delta^{ab}[\phi, \phi^\dagger]$$
$$\times \exp\Big\{ iN \int d\tau \oint d\sigma \ \mathrm{tr}\Big(\sum_{i=1}^{8} (D_+ \phi_i^\dagger)(D_- \phi^i) + \frac{1}{g_s^2} \sum_{i<j} [\phi_i^\dagger, \phi^j][\phi^i, \phi_j^\dagger] \Big) \Big\}$$

$$\Delta^{ab}[\phi, \phi^\dagger] \equiv \begin{pmatrix} D_+^{ab}\, \delta_{\alpha\beta} & \frac{i}{g_s} f^{abc} \sum_{i=1}^{8} \gamma^i_{\alpha\dot\alpha}\, \phi^*_{i,c} \\ \frac{i}{g_s} f^{abc} \sum_{i=1}^{8} \gamma^i_{\dot\alpha\alpha}\, \phi^i_c & D_-^{ab}\, \delta_{\dot\alpha\dot\beta} \end{pmatrix}$$

$$\phi^i \equiv \phi^i_a \tau^a \ , \quad [\tau^a, \tau^b] = i f^{abc} \tau^c \ , \quad \mathrm{tr}\, \tau^a \tau^b = \tfrac{1}{2} \delta^{ab}$$

- <u>NICOLAI MAP:</u>

$$\xi^i = D_+ \phi^i + \frac{1}{2g_s} \gamma^i_{\alpha\dot\alpha} [\phi^{\dagger\alpha}, \phi^{\dagger\dot\alpha}]$$
$$\xi_i^\dagger = D_- \phi_i^\dagger + \frac{1}{2g_s} \gamma^i_{\dot\alpha\alpha} [\phi^{\dot\alpha}, \phi^{\alpha}]$$

$$\frac{\delta(\xi_a, \xi_a^*)}{\delta(\phi_b, \phi_b^*)} = \Delta^{ab}$$

- USE $u(N)$ JACOBI IDENTITY, $so(8)$ FIERZ IDENTITY, AND SYMMETRY PROPERTIES OF $spin(8)$ γ-MATRICES:

$$Z[A] = \int D\xi\, D\xi^\dagger \ \operatorname{Det} \Delta^{ab} \cdot |\operatorname{Det} \Delta^{ab}|^{-1}$$
$$\times \exp\Big\{ iN \int d\tau \oint d\sigma \ \mathrm{tr} \sum_{i=1}^{8} \xi_i^\dagger \xi^i \Big\}.$$

• ZEROES OF $\xi^i, \xi_i^\dagger \Rightarrow$ INSTANTON, ANTI-INSTANTON EQUATIONS:

$$D_+ \phi^i_{(I)} + \frac{1}{g_s} \frac{\delta}{\delta \phi^i_{(I)}} F[\phi_{(I)}, \phi^\dagger_{(I)}] = 0$$

$$D_- \phi^\dagger_{(I),i} + \frac{1}{g_s} \frac{\delta}{\delta \phi^\dagger_{(I),i}} F[\phi_{(I)}, \phi^\dagger_{(I)}] = 0$$

$$F[\phi, \phi^\dagger] = \int d\tau \oint d\sigma \sum_{i=1}^{8} \gamma^i_{\alpha\dot\alpha} \, \mathrm{tr}\, \phi^i [\phi^{\dagger\alpha}, \phi^{\dagger\dot\alpha}]$$

$\equiv$ PRE-POTENTIAL FOR THE $\mathcal{N}=8$ SUSY MODEL.

• EVALUATE $Z[A]$ ABOUT $\xi^i = \xi_i^\dagger = 0$:

$$Z[A] = \oint_{\mathcal{M}_I} \mathrm{sgn}\, \mathrm{Det}\, \Delta[\phi_{(I)}, \phi^\dagger_{(I)}]$$

$$= \oint_{\mathcal{M}_I} \exp\left\{ \frac{\pi i}{2} \left[\zeta(\Delta[\phi_{(I)}, \phi^\dagger_{(I)}]) - \eta(\Delta[\phi_{(I)}, \phi^\dagger_{(I)}\right.\right.$$

$\mathcal{M}_I =$ INSTANTON MODULI SPACE

$$\zeta(\Delta) \equiv \lim_{s\to 0} \frac{1}{\Gamma(s)} \int_0^\infty dt\, t^{s-1}\, \mathrm{tr}\, e^{-t|\Delta|}$$

$\equiv$ total # of eigenvalues of Δ

$$\eta(\Delta) \equiv \lim_{s\to 0} \frac{1}{\Gamma(\frac{s+1}{2})} \int_0^\infty dt\, t^{(s-1)/2}\, \mathrm{tr}\, \Delta\, e^{-t\Delta^2}$$

$\equiv$ (# of >0 eigenvalue of Δ)
$-$ (# of <0 eigenvalues of Δ).

$$\therefore \boxed{Z_0 = \oint_{\mathcal{M}_I} \int \mathcal{D}A \, \exp\left\{ iN \int d\tau \oint d\sigma \, \mathrm{tr}\, F_{\mu\nu}^2 - \frac{\pi}{4}\left[\xi(\Delta) - \eta(\Delta)\right] \right\}}$$

• NOTE: FOR STRONGLY-COUPLED STRINGS,

$$g_s \to \infty \quad (\Leftrightarrow g_{YM} \to 0)$$

$$\Rightarrow \quad D_+ \phi_{(I)} = D_- \phi_{(I)}^\dagger = 0$$

$$\Rightarrow \quad \phi_{(I)} \sim W_c[A] \,, \qquad \xi(A) = F_{01}$$

Z_0 describes moduli space of flat gauge connections at $g_s \to \infty$.

• FOR GENERIC g_s, GET MODIFICATION OF GROSS-TAYLOR $\frac{1}{N}$ STRING EXPANSION OF ORDINARY 2D YANG-MILLS THEORY.