Gauge and gravitational dressing

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WZNW model

$$S = \frac{k}{8\pi} \int_{\Sigma} \mathrm{Tr}\, (g^{-1}\partial_\mu g)(g^{-1}\partial_\mu g) +$$

$$+ \frac{k}{16\pi} \int_{M} \mathrm{Tr}\, (g^{-1}dg) \wedge (g^{-1}dg) \wedge (g^{-1}dg)\, d^3x$$

CFT $\quad c = \frac{3k}{k+2}$ for $SU(2)_k$

general $\quad c = \frac{k \dim G}{k + c_v}$

$$\Delta_r = \frac{T(r)}{k+2}.$$

$r: \; j = 0, \frac{1}{2}, \ldots\ldots \frac{k}{2}.$

Coset models

G/H

$$c_{G/H} = c_G - c_H$$

$$\Delta^{(R,\tilde{R})}_{G/H} = \Delta^{(R)}_G - \Delta^{(\tilde{R})}_H$$

Example – minimal models M_p

$$c_k = 1 - \frac{6}{(k+2)(k+3)} \qquad \left(\text{usually } p = k+2 = 3, 4, \ldots\right)$$

$$M_{k+2} = \frac{SU(2)_k \times SU(2)_1}{SU(2)_{k+1}}$$

$$c_k = -\frac{3(k+1)}{(k+3)} + \frac{3k}{k+2} + 1 = 1 - \frac{6}{(k+2)(k+3)}$$

Primary fields in M_{k+2} $\quad \Phi_{p,q}$

$$\Delta_{p,q}(c) = \frac{[(k+3)p - (k+2)q]^2 - 1}{4(k+2)(k+3)}$$

$p = 2\ell + 1$
$q = 2\ell' + 1$

$$\Delta = n^2 + \frac{\ell(\ell+1)}{k+2} - \frac{\ell'(\ell'+1)}{k+3} =$$

$$= \frac{\left[(k+3)(2\ell+1) - (k+2)(2\ell'+1)\right]^2 - 1}{4(k+2)(k+3)}$$

$n = \ell' - \ell$

One can consider other models,

for example $\frac{SU(2)_k \times SU(2)_2}{SU(2)_{k+2}}$ – $\mathcal{N} = 1$ Superconformal models

$\frac{SL(2)}{U(1)}$ – 2d strings and B.H.

and so on.

KZ equation.

$$\langle \varphi_1(z_1,\bar{z}_1) \ldots \varphi_n(z_n,\bar{z}_n)\rangle$$

where φ_i are irep of current algebra

$$\frac{\partial}{\partial z_i}\langle \ldots \rangle * \sum_{j\neq i} \frac{\vec{t}^{(i)}\cdot\vec{t}^{(j)}}{z_i - z_j}\langle \ldots \rangle = 0$$

where $\vec{t}^{(i)}$ and $\vec{t}^{(j)}$ are $SU(2)$ generators acting on indices of i and j <u>fields</u>!

The question one may ask (and must) – ~~can on~~ is it possible to write analog of KZ equation for coset model?

G/H model can be written as a gauged WZNW model

WZNW $\int \mathcal{D}g \, e^{i S_{WZNW}}$

G/H $\int \mathcal{D}g \, \mathcal{D}A \, e^{i S_{WZNW} + \int g^{-1}\partial_+ g A_- + \partial_- g g^{-1} A_+ + A_+ A_- + A_+ g A_- g^{-1}}$

Gauge field A is H-valued
$g \in G$

So we simply gauge subgroup H of the full symmetry group G

without A theory had semilocal (chiral) symmetry $G_L(z) \times G_R(\bar{z})$ $\quad g \to U g V^{-1}$

To study what is going on in presence of a gauge field A we shall fix the gauge using light cone (!)

$A_+ = 0$ (or $A_- = 0$).

In this case there is only one term in action

$A_+ = A_1 - A_0$
$A_+^\epsilon = A_1 + iA_0$

$$S_{WZNW} + \int Tr\,(g^{-1}\partial_+ g)\, A_-$$

In this theory we'll get now the new symmetries

$$\delta A_+ = \partial_+ \epsilon + [A_+\, \epsilon]$$
$$\delta A_- = \partial_- \epsilon + [A_-\, \epsilon]$$
$$\delta g = [\epsilon, g]$$

It is clear that fixing the gauge

$A_+ = 0$ leaves the freedom to transform

A_- with $\epsilon = \epsilon(\bar{z})$ $\quad (\bar{z} = z^-)$

$$\partial_+ \epsilon(\bar{z}) \equiv 0$$

So we have the residual symmetry $\epsilon(\bar{z})$

Thus instead of $G_L(z) \times G_R(\bar{z})$ symmetry

of WZNW model we have now a

new symmetry $H(\bar{z}) \times G$

So now we can consider the

<u>dressed</u> correlation function

$$\langle\langle \Phi_1 \dots \Phi_N \rangle\rangle =$$

$$= \int \mathcal{D}A_- \int \mathcal{D}g \; e^{i S_{WZNW} + \int g^{-1} \partial_+ g \, A_-}$$

How one can get KZ equation

$$J^a(z)\,\Phi(z') = \frac{t^a}{z-z'}\Phi(z') + \text{reg. terms}$$

$$J^a(z) = \sum_n \frac{1}{z^n} J^a_n \qquad J^a_0\,\Phi = t^a\,\Phi$$

$$T(z) = \frac{1}{k+2} :J^a J^a:$$

$$L_{-1} = \partial_z$$

Acting by L_{-1} and rewriting it as $J_{-1} J_0$

one can get from ∂_z operation a

structure $\frac{t^a_i t^a_j}{z_i - z_j}$ acting on primary fields

in result

$$\partial_z \langle \dots \rangle + \sum_{i \neq j} \frac{t^a_i t^a_j}{z_i - z_j} \langle \dots \rangle = 0$$

So we used the current algebra

$\partial_+ J_- = \partial_- J_+ = 0$ and equation

of motion.

In presence of a gauge field equations of motion are going to be modified. Instead of $\partial_- \Phi_+ = 0$ we have $D_- \Phi_+ = \partial_- \Phi_+ + [A_-, \Phi_+] = 0$.

So using the same approach as in 1.7 here we can repeat all details of derivation, but changing $\partial_- \to D_-$

In result extra terms of the type

$\langle\langle A_- \Phi \ldots \Phi_N \rangle\rangle$ will appear.

So we have to know OPE

$$A_+(z) \Phi(z') = ?$$

the answer is that $A^a_+(z)\, \Phi(z')$

$$= \frac{T^a \Phi}{z - z'}$$

Φ ... [illegible]

This is a very important new element – we still have equation in holomorphic sector (equation for z), but subgroup H acts on anti-holomorphic indices!

In result one can get an equation

$$\frac{\partial}{\partial z_i} \langle\langle \Phi(z_1) \dots \Phi(z_N) \rangle\rangle + \sum_{i \neq j} \left(\frac{1}{k+2} \frac{t_i^a t_j^a}{z_i - z_j} - \frac{1}{k+2} \frac{\bar{\tau}_i^b \bar{\tau}_j^b}{z_i - z_j} \right) \langle\langle \Phi(z_1) \dots \Phi(z_N) \rangle\rangle = 0$$

This equation is written when G and H are single groups (like $SL(2)/U(1)$)

For M_p when $G = G_1 \times G_2$ and H is a diagonal subgroup one can write it like that (for $\frac{SU(2)_k \times SU(2)_1}{SU(2)_{k+1}}$)

$$\frac{\partial}{\partial z_i} \langle\langle \Phi_1 \ldots\ldots \Phi_N \rangle\rangle +$$

$$+ \sum \frac{1}{k+2} \frac{t_i^a t_j^a}{z_i - z_j} + \frac{1}{3} \frac{\tilde{t}_i^a \tilde{t}_j^a}{z_i - z_j} - \frac{1}{k+3} \frac{\bar{\tau}_i^a \bar{\tau}_j^a}{z_i - z_j}$$

$$\langle\langle \Phi_1 \ldots\ldots \Phi_N \rangle\rangle = 0.$$

t and $\tilde{t} \in SU(2)_L$

$\bar{\tau} \in SU(2)_R$

Using this equation one can immediately get the spectrum of anomalous dimensions

$$\langle\langle \Phi(z)\, \Phi(0) \rangle\rangle = \frac{1}{z^{2\Delta}}$$

$$\frac{\partial}{\partial z} \langle\langle \Phi(z)\, \Phi(0) \rangle\rangle = -\frac{2\Delta}{z} \langle\langle \Phi(z)\, \Phi(0) \rangle\rangle$$

So $-\frac{\Delta}{z} \langle\langle \Phi(z)\, \Phi(0) \rangle\rangle$

$$+ \frac{1}{k+2} \frac{\vec{t}_1 \vec{t}_2}{z} + \frac{1}{3} \frac{\vec{\tau}_1 \vec{\tau}_2}{z} - \frac{1}{k+3} \frac{\vec{\tau}\,\vec{\tau}}{z} \langle\langle \Phi\, \Phi \rangle\rangle = 0$$

$$\vec{t}_1 \vec{t}_2 = \frac{1}{2}(\vec{t}_1 + \vec{t}_2)^2 - \frac{1}{2}\vec{t}_1^{\,2} - \frac{1}{2}\vec{t}_2^{\,2}$$

Total isospin with respect to each group is zero so each $\vec{t}\,\vec{t}$ factor transforms into $\vec{t}^{\,2} = l(l+1)$

For $\vec{t}$ only spins 0 and 1/2 are possible, i.e. $l - l' = 0$ or $l - l' = 1/2$

In result we get

$$\Delta = h + \frac{l'(l'+1)}{k+2} - \frac{l'(l'+1)}{k+3}$$

Contrary to 'ZZ' case where equations for both z and $\bar{z}$ could be written, here either [illegible] z or $\bar{z}$ only can be [illegible] (depending on type choice $A_+ = 0$ or $A_- = 0$).

The full conformal block for z can be derived and then $\bar{z}$ dependence can be found using analyticity and crossing.

One can get using the gauged equation conformal blocks in minimal models (it's lengthy, so skip the details).

Using the obtained equations one can study infinite-dimensional representation of $SL(2)/U(1)$ in this case the new features of affine $SL(2)$ algebra emerges – the Jordan block for the current algebra.

We had before $J_0^a |\Phi\rangle = t^a |\Phi\rangle$

$$\text{or } J^a(z)\,\Phi(0) = \frac{t^a}{z}\,\Phi$$

If there are 2 operators with the same dimensions one will get

$$J^0 V_{j,m} = m V_{j,m} \qquad J_0^0 \tilde{V}_{j,m} = m \tilde{V}_{j,m} + V_{j,m}$$

$$J^+ V_{j,m} = (m-j) V_{j,m+1} \qquad J^+ \tilde{V}_{j,m} = (m-j) \tilde{V}_{j,m+1}$$

$$J^- V_{j,m} = (m+j) V_{j,m-1} \qquad J^- \tilde{V}_{j,m} = (m+j) \tilde{V}_{j,m-1} + 2V_{j,m-1}$$

For WZNW model nothing is going to happen:

$$T = \frac{1}{k+2} :J^a J^a: = \frac{1}{k+2}\left(J^0 J^0 - \tfrac{1}{2} J^+ J^- - \tfrac{1}{2} J^- J^+\right)$$

and V terms cancel in $T \cdot \tilde{V}$.

But in coset models like $SL(2)/U(1)$

or 2d gravity $SL(2)/\text{Borel}$

$$T_{SL(2)/U(1)} = \frac{1}{k+2} :J J: - \frac{1}{k} :J^0 J^0: \qquad a)$$

$$T_{2d\ gravity} = \frac{1}{k+2} :J J: + \frac{\partial}{\partial z} J^0 \qquad b)$$

one gets $T \tilde{V} = \frac{\Delta}{z^2}\tilde{V} - \frac{m}{z^2} V$ b)

or $T \tilde{V} = \frac{\Delta}{z^2}\tilde{V} - \frac{2m}{k}\frac{V}{z^2}$ a)

It leads to a new type of correlations:

$$\langle \tilde{V}\, \tilde{V} \rangle \sim \frac{\ln|z|}{|z|^{2\Delta}}$$

But the theory of these operators is a separate story.

Conclusion.

One and a very simple fact:
It is possible to get an equation describing dressed corr. function in conformal field theory.

$$\langle A\ \Phi \dots \Phi \rangle$$

$$= \frac{1}{4g^2} \mathrm{Tr}\, F_{\mu\nu}^2 + \bar{\Psi} \not{D} \Psi$$

[illegible]