Renormalization of Fermions on LF

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issues:

- what operators appear in renormalized LF Hamiltonians with fermions
- how are the coefficients determined

topics:

- Motivation
- 2-loop analysis $\longrightarrow$ finiteness condition
- specific cutoffs
  - DLCQ
  - "local" cutoff
- summary

Motivation:

why LF-QCD:

- many high-energy reactions in relativistic systems dominated by light-like correlations (example: DIS)
- manifest boost invariance in longitudinal direction

In practice: drastic approximations necessary!

examples: - $\perp$ lattice QCD with large $a_\perp$
- "dimensionally reduced" models

char. feature: $\perp$ momenta cut off at rather low scale

$\hookrightarrow$ Even though these models are approximations to (or models for) QCD_{3+1}, they are formally equivalent to 1+1 dimensional theories with gauge and "Yukawa" couplings!

$\hookrightarrow$ important issue in these models:

renormalization of longitudinal (small p^+) divergences arising from Yukawa couplings

note: small p^+ divergences from coupling to A^- seem less of a problem in these quasi–two-dimensional models, since it is easy to construct gauge invariant states where the "gauge singularities" cancel manifestly!

today: discussion of divergences, regularization and renormalization of 1+1 dimensional Yukawa interactions

Lagrangean for Yukawa$_{1+1}$:

$$\mathcal{L} = \bar{\psi}\left(i\,\partial\!\!\!/ - m_F - g\phi\right)\psi - \frac{1}{2}\phi\left(\Box + m_B^2\right)\phi$$

canonical LF-Hamiltonian:

$$P^- = \int dx^- \mathcal{H},$$

where

$$\begin{aligned}\mathcal{H} \;=\; & \frac{m_B^2}{2}\phi^2 + \psi^\dagger_{(+)}\frac{m_{kin}^2}{\sqrt{2}i\partial_-}\psi_{(+)} \\ & + \psi^\dagger_{(+)}\left(g\phi\frac{m_V}{\sqrt{2}i\partial_-} + \frac{m_V}{\sqrt{2}i\partial_-}g\phi\right)\psi_{(+)} \\ & + \psi^\dagger_{(+)}g\phi\frac{1}{\sqrt{2}i\partial_-}g\phi\psi_{(+)}.\end{aligned}$$

Canonically: $m_{kin} = m_V = m_F$

construction of the canonical LF-Hamiltonian for Yukawa$_{1+1}$

Dirac equation:

$$0 = (i\,\partial\!\!\!/ - m_F - g\phi)\,\psi = \left(i\gamma^+\partial_+ + i\gamma^-\partial_- - m_F - g\phi\right)\psi$$

Multiply by γ^+ (use $\gamma^+\gamma^+ = 0$):

$$\boxed{2i\partial_-\psi_{(-)} = \gamma^+\,(m_F + g\phi)\,\psi_{(+)},}$$

where $\psi_{(\pm)} = \frac{1}{2}\gamma^\mp\gamma^\pm\psi$.

Note: ∂_- is LF-space derivative

$\hookrightarrow$ $\psi_{(-)}$ satisfies <u>constraint equation</u>

$\psi_{(-)}$ <u>not</u> dynamical degree of freedom!

$\hookrightarrow$ eliminate $\psi_{(-)}$ from $\mathcal{L}$, using

$$\psi_{(-)} = \frac{\gamma^+}{2i\partial_-}\,(m_F + g\phi)\,\psi_{(+)}$$

prehistory:

- fermion self-energy diverges (small p^+) in 2^{nd} (and in general also in higher) order pert. theory
- no such divergence in higher order corrections to the fermion-boson vertex
- at one loop, counter-term provided by "self-induced inertias" (normal ordering piece)
- divergence fixes only infinite piece in m^2_{kin}; finite part (Δm^2_{kin}) seems arbitrary
- "self-induced inertias" (from normal ordering) cancel 1-loop divergence, but give wrong (parity!) results $\hookrightarrow$ non-trivial finite part needed
- story repeats itself at higher loops

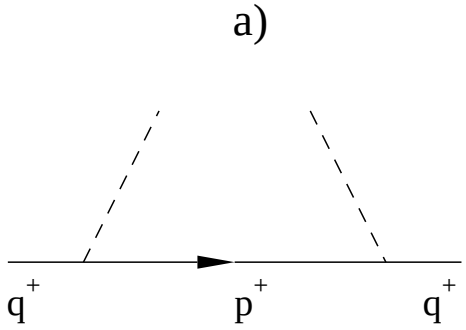
open issues

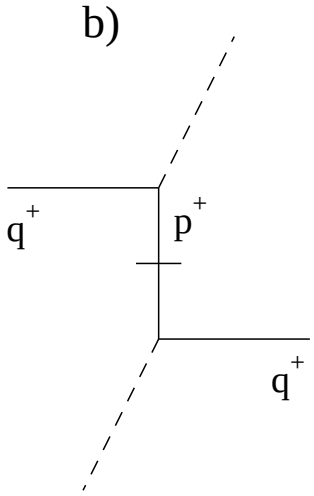
- general procedure to determine finite part (promising attempts: coupling coherence, parity conditions, ladder relations)
- is Δm^2_{kin} actually just a number or are "counter-term functions" required ?

today:

- when is Δm^2_{kin} just a number ?
- "finiteness" as a condition to determine Δm^2_{kin}

Compton scattering (tree level)





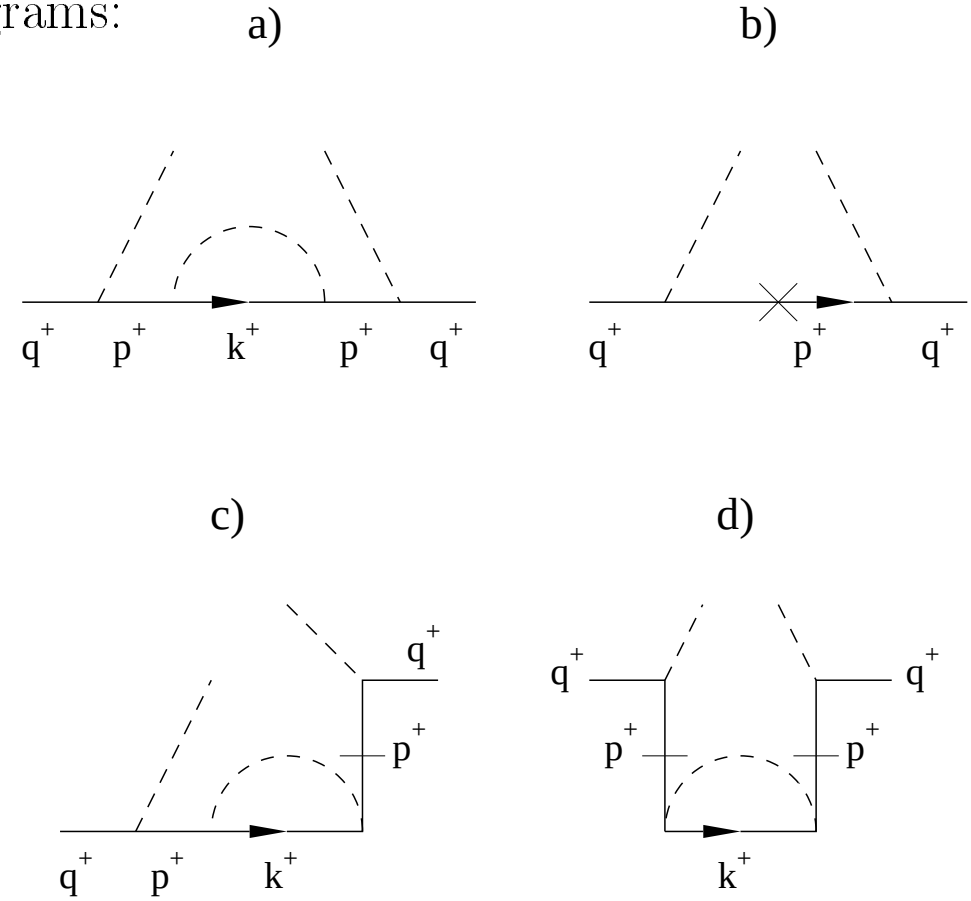
- two contributions at $\mathcal{O}(g^2)$: inst. fermion exchange and 2^{nd} order pert. term with physical fermion intermediate state
- both diagrams divergent as $p^+ \longrightarrow 0$
- $p^+ \longrightarrow 0$ singularities cancel iff $m_V = m_{kin}$

Implications for 1-loop self-energy:

- 2^{nd} order pert. theory term as well as "self-induced inertias" (obtained from contractions when normal-ordering instantaneous fermion exchange in LF-Hamiltonian both contain ($p^+ \to 0$ divergence)
- divergence cancels exactly iff $m_V = m_{kin}$ $\Rightarrow$ 2^{nd} order self-energy incl. self-induced inertia finite

one loop Compton scattering

relevant diagrams:



- all terms individually divergent as $p^+ \longrightarrow 0$.

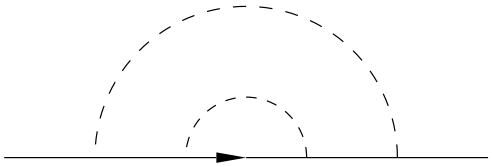
$$\lim_{p^+\to 0} p^+(T_a + T_b) \;\propto\; -\int_0^1 dx \frac{2+x}{m_F^2(1-x)+m_B^2 x} + \frac{1}{m_F^2}\ln\frac{m_B^2}{m_F^2} + \frac{\Delta m_{kin}^2}{m^4}$$

$$\lim_{p^+\to 0} p^+(T_c) \;\propto\; \int_0^1 dx \frac{2+2x}{m_F^2(1-x)+m_B^2 x}$$

$$\lim_{p^+\to 0} p^+(T_d) \;\propto\; -\int_0^1 dx \frac{x}{m_F^2(1-x)+m_B^2 x}$$

- divergence does not cancel when all diagrams are added — unless Δm^2_{kin} (i.e. the finite part of the $\mathcal{O}(g^2)$ kin. mass counter-term) is included and appropriately chosen!

Implications for 2-loop self-energy:



- If no or wrong finite kin. mass counter-term is added then 2-loop self-energy divergent!
  $\hookrightarrow$ need to add infinite $\mathcal{O}(g^4)$ counter-term, and so on (at higher orders).
- If correct finite $\mathcal{O}(g^2)$ counter-term, then no infinite $\mathcal{O}(g^4)$ counter-term needed, and so on
- The value for the finite piece of Δm^2_{kin}, which renders self-energy finite is identical to the one from the parity condition!
- since the 2-loop result diverges for all wrong values of the $\mathcal{O}(g^2)$ kin. mass and converges for the correct value, one can use "finiteness" as a condition to determine the correct kin. mass

- above 2-loop results true to all orders for 1+1 Yukawa interactions in planar approx.

Note: In order to derive this result, it was necessary to take the continuum limit (no cutoff) for the inner loop before doing the outer loop integration, so that there was no cutoff dependence of the inner loop when one studies the cutoff dependence of the outer loop!

This a potential source of trouble in nonpert. calcs., which are not done "loop by loop"!

2-loop analysis in DLCQ

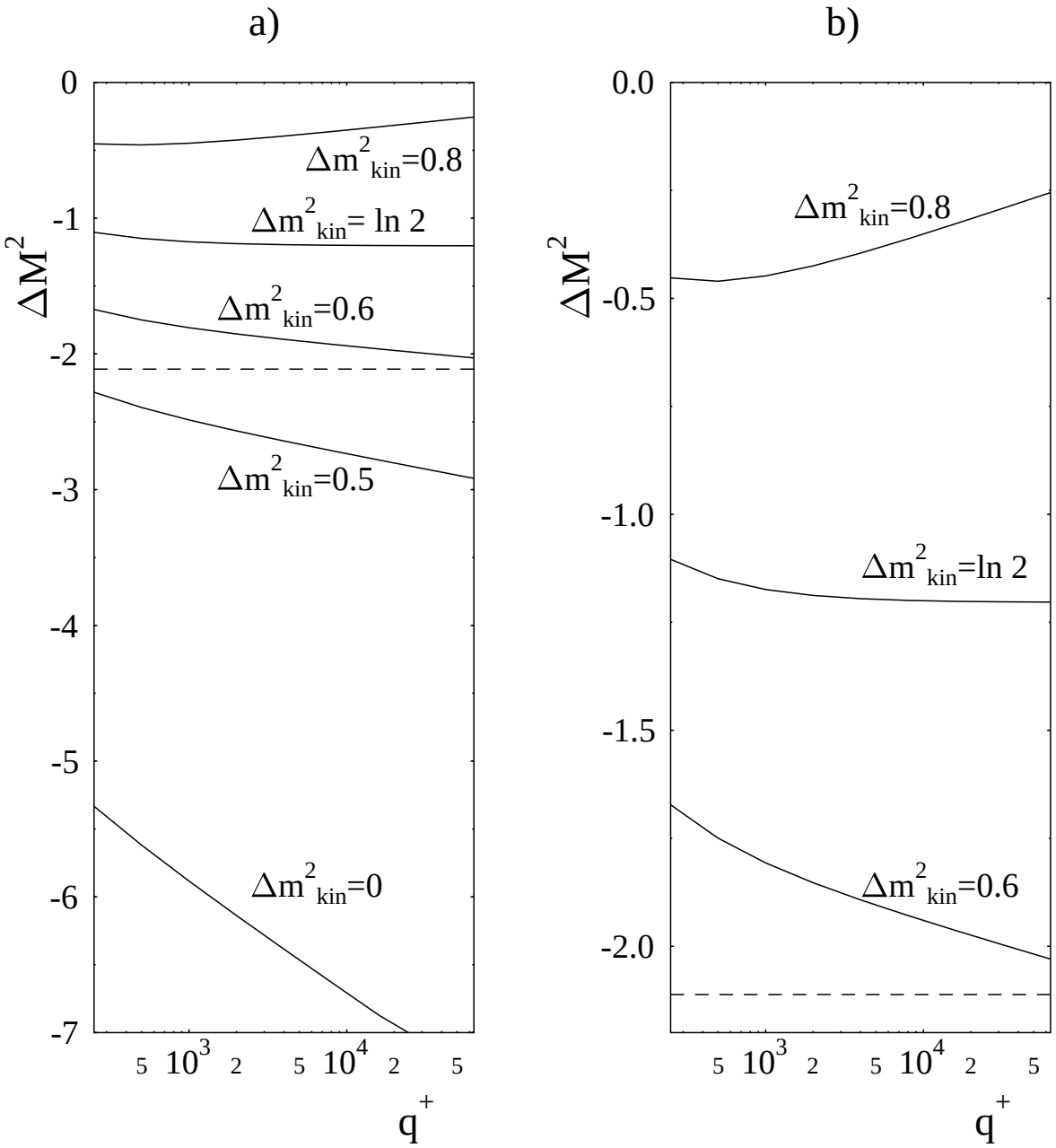
- calculate $\mathcal{O}(g^4)$ self-energy in DLCQ (i.e. with p^+ discretized momenta (anti-periodic b.c. for fermions and periodic b.c. without 0-modes for boson)
- keep standard $\mathcal{O}(g^2)$ "self-induced inertia" counter-term for fermion kin. energy and include additional finite $\mathcal{O}(g^2)$ mass counter-term Δm^2_{kin}
- calculate 2-loop self mass for various values of Δm^2_{kin} as a function of the fermion momentum (in integer units)

Results:

- unless one picks the correct value for Δm^2_{kin}, the DLCQ calculation diverges as the momentum (in integer units), i.e. the "DLCQ parameter K", is sent to infinity
- with the correct value for Δm^2_{kin}, the result is finite, but still disagrees with the covariant result :-(

2-loop self-mass in DLCQ (rainbow)

dependence on total momentum for various finite parts of the 1-loop counter-term:



dashed: cov. result

reason for this "disaster":

1-loop self-mass momentum dependent (even after adding self-induced inertias). Even though the momentum dependence of the 1-loop self-mass only matters for small momenta, it leads to a finite mistake in the 2-loop calculation, since it is multiplied by a $1/p^+$ factor in the 2-loop integral!

implications for renormalization:

- the problem repeats itself at higher loops
- need to introduce momentum dependent mass counter-term in order to compensate for the momentum dependence of the self-energy
  $\hookrightarrow$ if DLCQ is used as a cutoff, then "counter-term functions" are needed already in Yukawa$_{1+1}$!
- 0-modes are unlikely to "fix" the problem, since the problem arises from small fermion momenta and the fermions can be chosen to satisfy anti-periodic b.c.

Are counter-term functions unavoidable?

(In "quasi 2-dim" approximations to QCD)

- if DLCQ (or any other fixed p^+ cutoff) is used as a regulator, the answer seems to be yes!
- for regulators where the cutoff in sub-loops scales with the momenta flowing into sub-loops, the answer seems to be no!

example for such a "good" regulator: "local cutoff"

at each 3-point vertex, the quantity which is cut off is the "invariant mass transfer":

$$p^+|\delta P^-| \equiv p^+ \left| \sum_{i\in in} \frac{m_i^2}{p_i^+(in)} - \sum_{i\in out} \frac{m_i^2}{p_i^+(out)} \right|$$

where p^+ is the portion of plus momentum flowing into the vertex.

4-point vertices are cut off by first decomposing into two 3-point vertices and then using the cutoff rules for 3-point vertices.

main advantage: manifest longitudinal boost invariance!

Results for same analysis with "local cutoff"

- With "local cutoff", self-energy manifestly momentum independent and momentum dependent counter-terms avoided
- finite piece of mass counter-term can (in principle) be determined by demanding finiteness of physical mass

(checked to 2 loops in Yukawa$_{1+1}$ and nonperturbatively in a planar 1+1 dim. Yukawa-type model)

summary:

- dimensionally reduced or $\perp$ lattice models contain interactions that resemble Yukawa$_{1+1}$
- Yukawa interactions, which couple fermions and $\perp$ gauge degrees of freedom, can give rise to longitudinal (small p^+) divergences which are in general not cancelled by self-induced inertias
- divergence avoided if appropriate (finite) kinetic mass counter-term is added
- cancellation works only if cutoff such that self-energy momentum independent (e.g. "local cutoff")
- if DLCQ is used as a cutoff then one must introduce momentum-dependent kinetic mass, i.e. a "counter-term-function"
- prelim. results indicate: above results seem to generalize to models that include gauge interactions together with the Yukawa interactions (dimensionally reduced QCD, $\perp$ lattice QCD)