Rotational invariance in light-front perturbation theory

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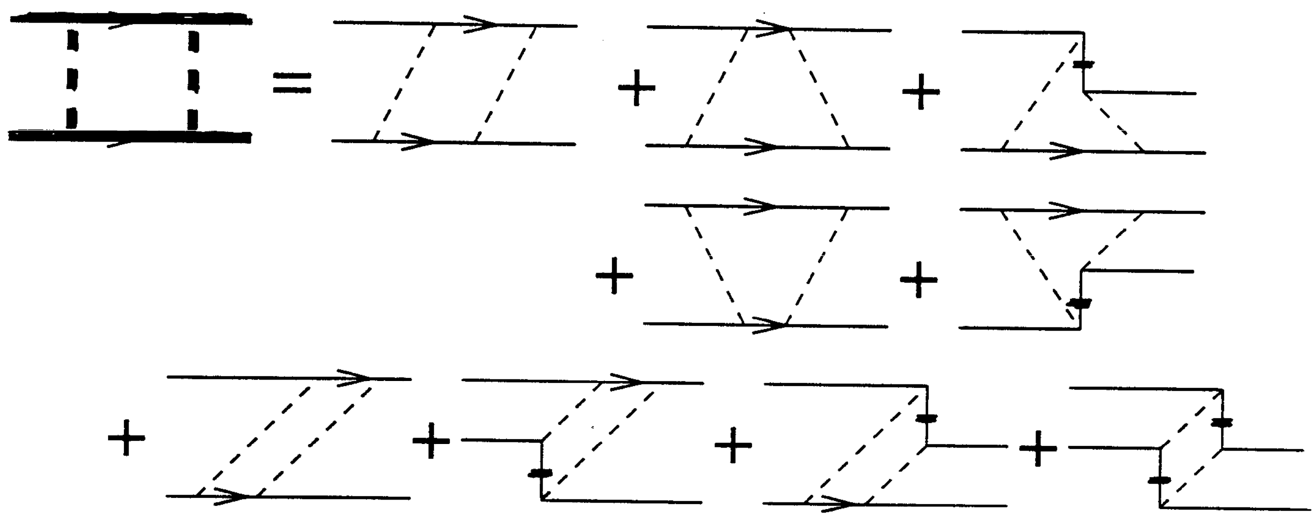
- Introduction
- Part I: LF diagrams & **Covariance** hep-ph/9702311
  - $\int dk^- \rightarrow$ LF time-ordered diagrams
  - Longitudinal divergences $\rightarrow$ **FIL**s
  - D=2 fermion self energy paradox
  - **Minus** regularization
  - Recipe for a **Covariant** LFPT
- Part II: LF bound state & **Covariance** unpublished
  - Choice of effective potential
  - OBEP: **error** $\propto$ the stretched box
  - Numerical examples
- Conclusions

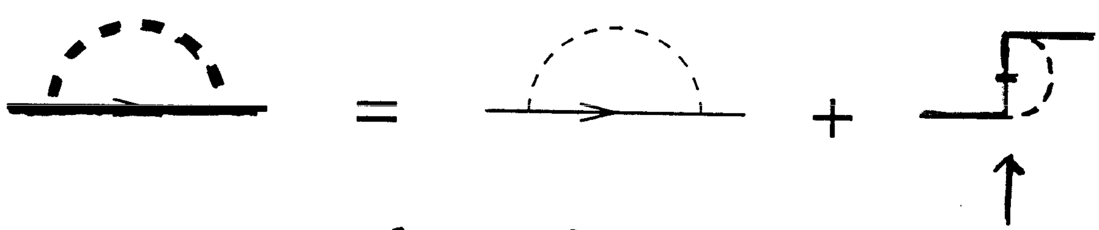
FOM

Part I
k^--integration $\rightarrow$ LF time-ordered diagrams

fermion —>— every fermion ⇒ instantaneous fermion

boson - - - - - spectrum condition ⇒ no vacuum creation/ annihilation contributions



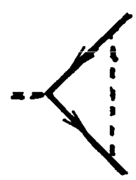
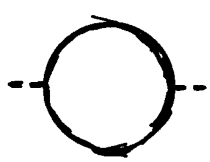
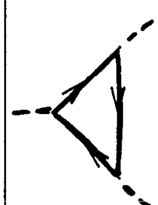


forced instantaneous loop (α)

Longitudinal divergences in Yukawa model

$D^- = 1 - \#\text{bosons}$

For scalar coupling reduced to D^-_{Y}.

$\#b = 1$	$D^-_{\mathrm{Y}} =$ ● 	$D^-_{\mathrm{Y}} = -1$ 	$D^-_{\mathrm{Y}} = -1$ 
$\#b = 0$	$D^-_{\mathrm{Y}} =$ ● 	$D^-_{\mathrm{Y}} = -1$ 	$D^-_{\mathrm{Y}} = -1$ 

$\Rightarrow$ The D=2 fermion self energy (FSE) paradox

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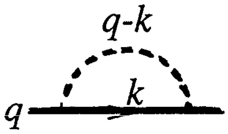
FSE paradox: Covariant calculation

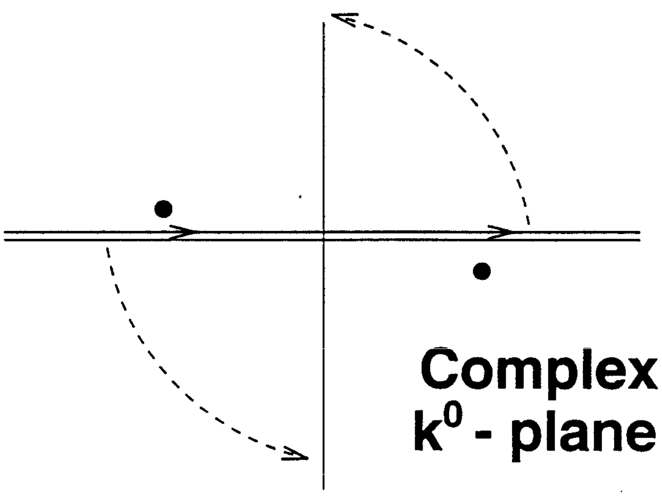


$$= \int_{\mathbf{M}} \mathrm{d}^2 k \frac{\not{k} + m}{(k^2 - m^2)((q-k)^2 - \mu^2)}$$

Introduce Feynman parameter

$$= \int_0^1 \mathrm{d}x \int_{\mathbf{M}} \mathrm{d}^2 k' \, \frac{\not{k}' + x\not{q} + m}{(k'^2 + C^2)^2}$$

Remove odd term $\not{k}'$ and Wick rotate



$$= \pi \int_0^1 \mathrm{d}x \frac{x\not{q} + m}{C^2} \quad \text{is finite}$$

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FSE paradox: Residue calculation

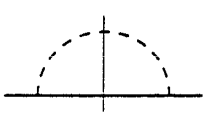
$$= \int \frac{\mathrm{d}k^+ \mathbf{d}\boldsymbol{k^-}}{4k^+(q^+ - k^+)} \frac{\boldsymbol{k^-}\gamma^+ + k^+\gamma^- + m}{(k^- - H_1^-)(k^- - H_2^-)}$$

Splitting the covariant diagram

$$= \int \frac{\mathrm{d}k^+ \mathbf{d}\boldsymbol{k^-}}{4k^+(q^+ - k^+)} \frac{\boldsymbol{H_1^-}\gamma^+ + k^+\gamma^- + m}{(k^- - H_1^-)(k^- - H_2^-)}$$

$$+ \int \frac{\mathrm{d}k^+ \mathbf{d}\boldsymbol{k^-}}{4k^+(q^+ - k^+)} \frac{\gamma^+ \cancel{(\boldsymbol{k^-} - \boldsymbol{H_1^-})}}{\cancel{(\boldsymbol{k^-} - \boldsymbol{H_1^-})}(k^- - H_2^-)}$$

Propagating diagram



$$= 2\pi i \int_0^{q^+} \frac{\mathrm{d}k^+}{4k^+(q^+ - k^+)} \frac{\frac{m^2}{2k^+}\gamma^+ + k^+\gamma^- + m}{H_2^- - H_1^-}$$

Forced instantaneous loop (**FIL**)

$$= \int \frac{\mathrm{d}k^+ \mathbf{d}\boldsymbol{k^-}}{4k^+(q^+ - k^+)} \frac{\gamma^+}{\boldsymbol{k^-} - H_2^-} \frac{\alpha(k^+)}{1 - i\delta_Q^+ k^-}$$

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Minus regularization

$$\left(\int dq^-\right)^n \int d^2k^\perp dk^+ \left(\frac{\partial}{\partial q^-}\right)^n F(q^-, k)$$

↑ external LF energy

Minus regularization	BPHZ
N.E. Ligterink	Bogoliubov Parasiuk Hepp Zimmerman
LF time-ordered and covariant diagrams	Covariant diagrams
Subtracts the lowest orders in q^- in the Taylor expansion	Subtracts the lowest orders in q^2 in the Taylor expansion
Instantaneous CT. **Nonlocal** CT **cancel** against **FIL**s	CT are local

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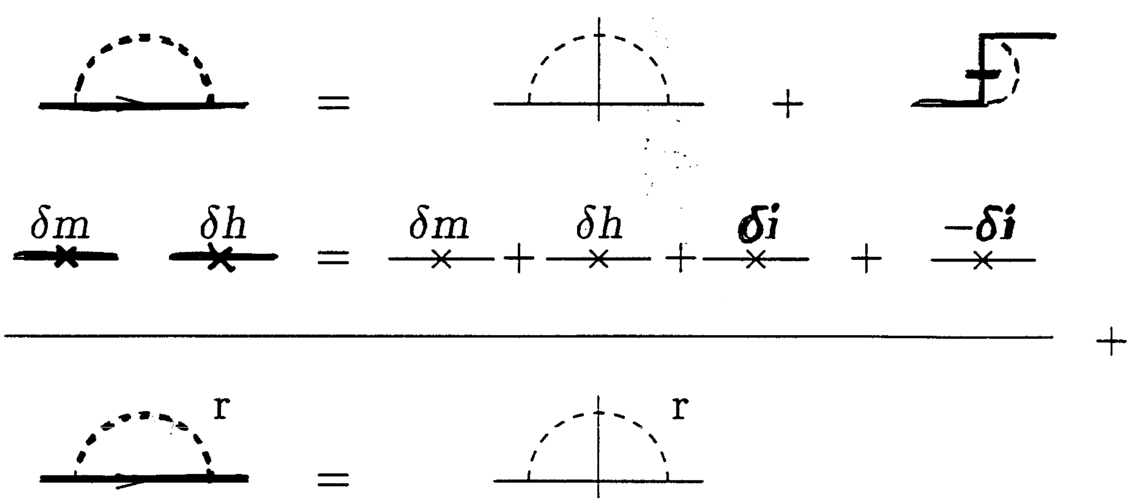
Minus regularization of the FSE:



δm: mass renormalization

δh: wave function renormalization

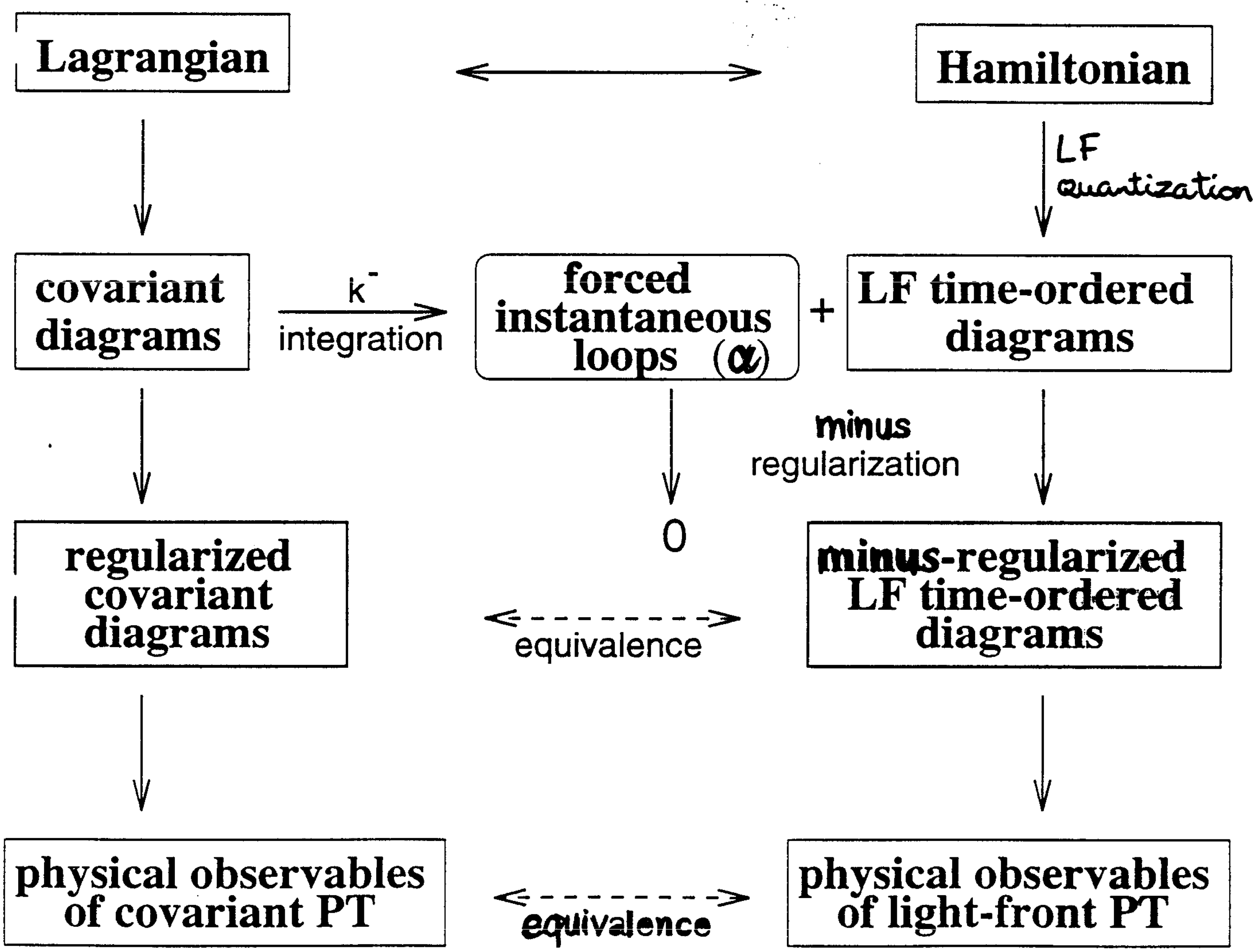
$\boldsymbol{\delta i}$: **non**covariant CT (instantaneous)
covariance $\Rightarrow$ cancelation against **FIL**

(Upon request I can give calculation)

Recipe for a covariant light-front PT

Ingredients for LF time-ordered diagrams:

- k^--integration
- **Minus** regularization



Enjoy a delicious, covariant and renormalized light-front PT!

Part II

LF bound state & covariance

Hamiltonian:

$$H = H_0 + V$$

with:

$$H_0 = \frac{{p_a^{\perp}}^2 + m^2}{2p_a^+} + \frac{{p_b^{\perp}}^2 + m^2}{2p_b^+}$$

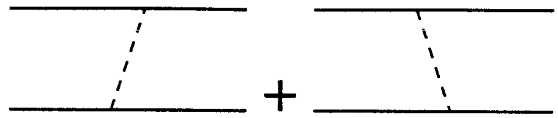
Lippmann-Schwinger solution:

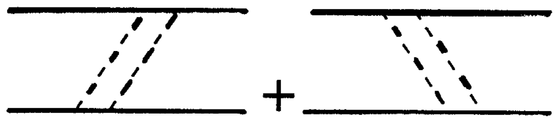
$$|\phi >= |\phi_0 > + \left(\frac{1}{E - H_0} V\right) |\phi >$$

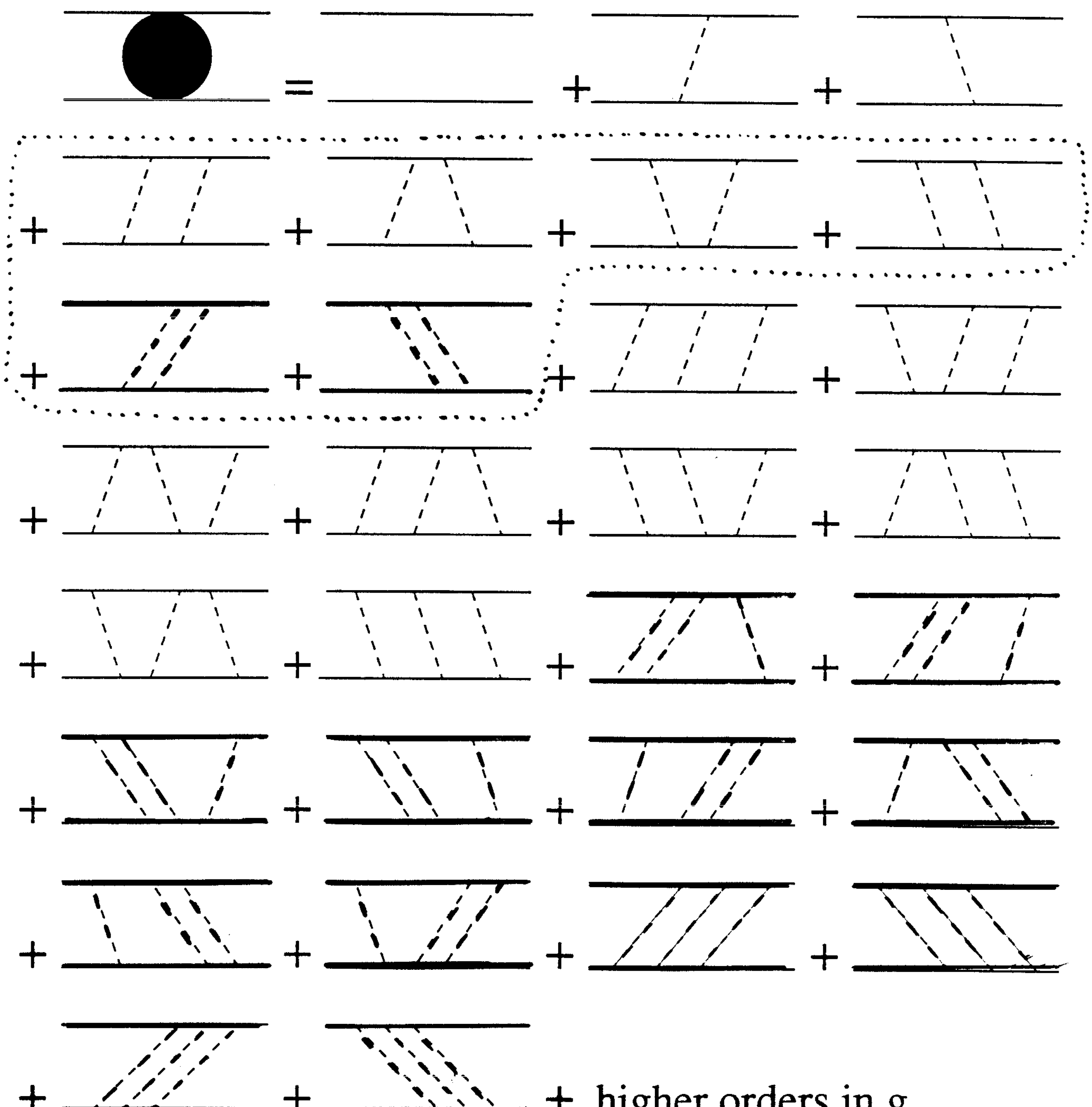
What do we take for effective potential V?

Choice of effective potential

$$V = V_1 + \mathbf{V_2} + \ldots..$$

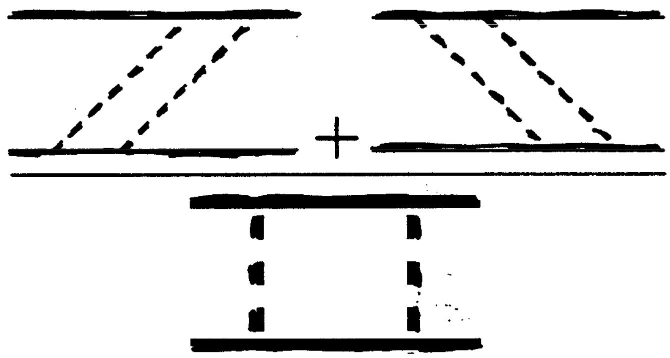
$V_1 =$ 

$\mathbf{V_2} =$ 

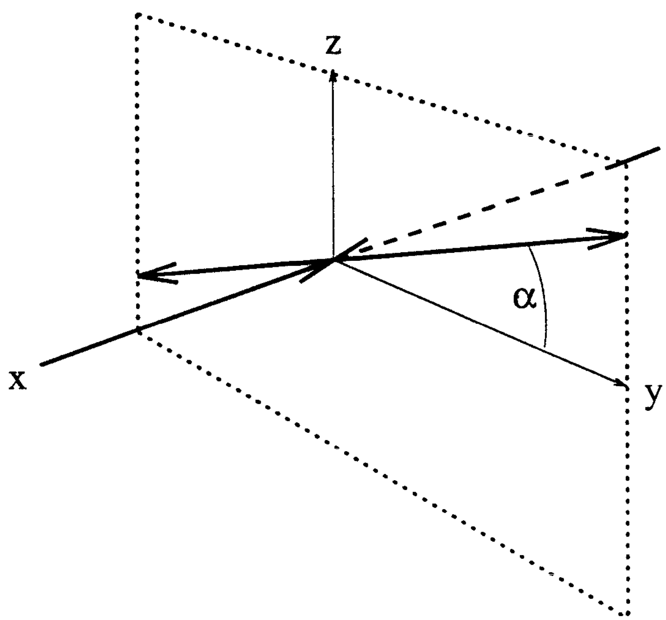


+ higher orders in g

Estimate error at g^4: The box

$\mathcal{R}^{\blacklozenge} =$ 

We calculate $\mathcal{R}^{\blacklozenge}$ in CM for process:

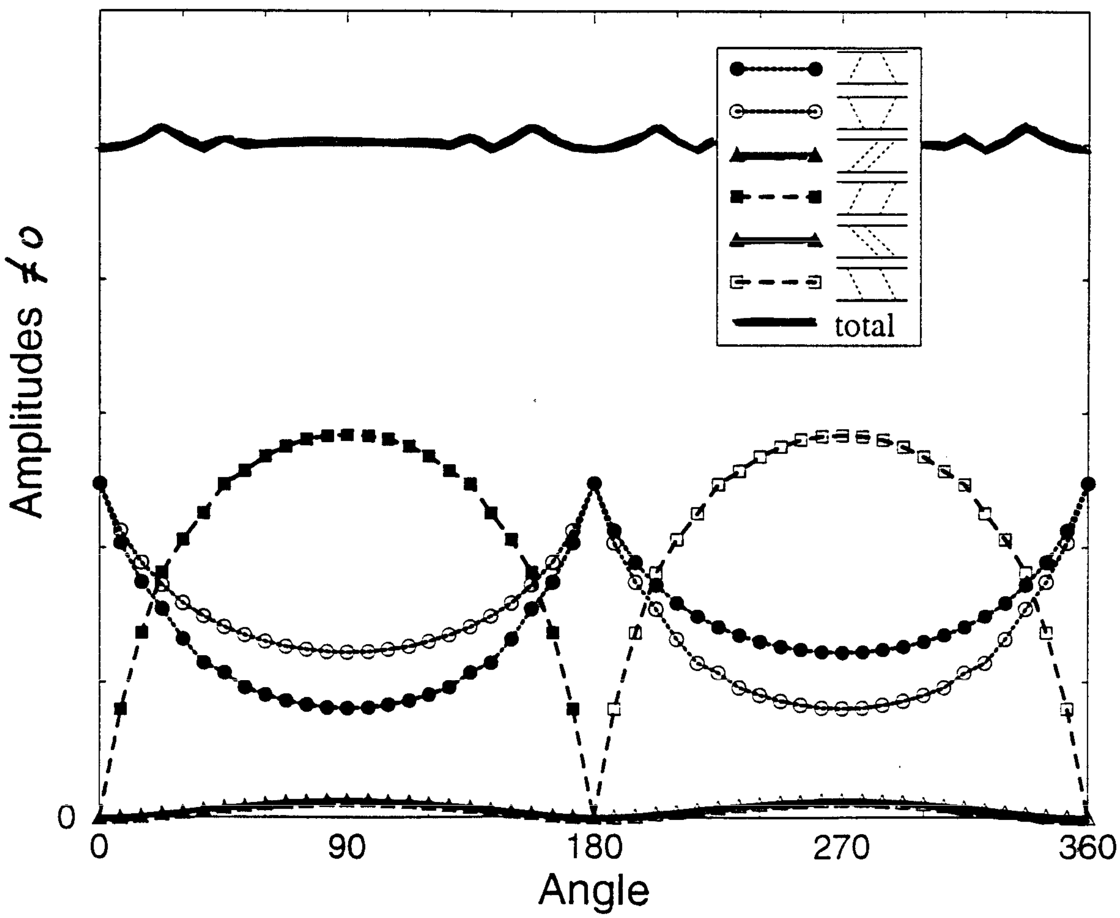


On-shell ext. particles $\Rightarrow$ rotational invariance
Parameters: α, μ/m

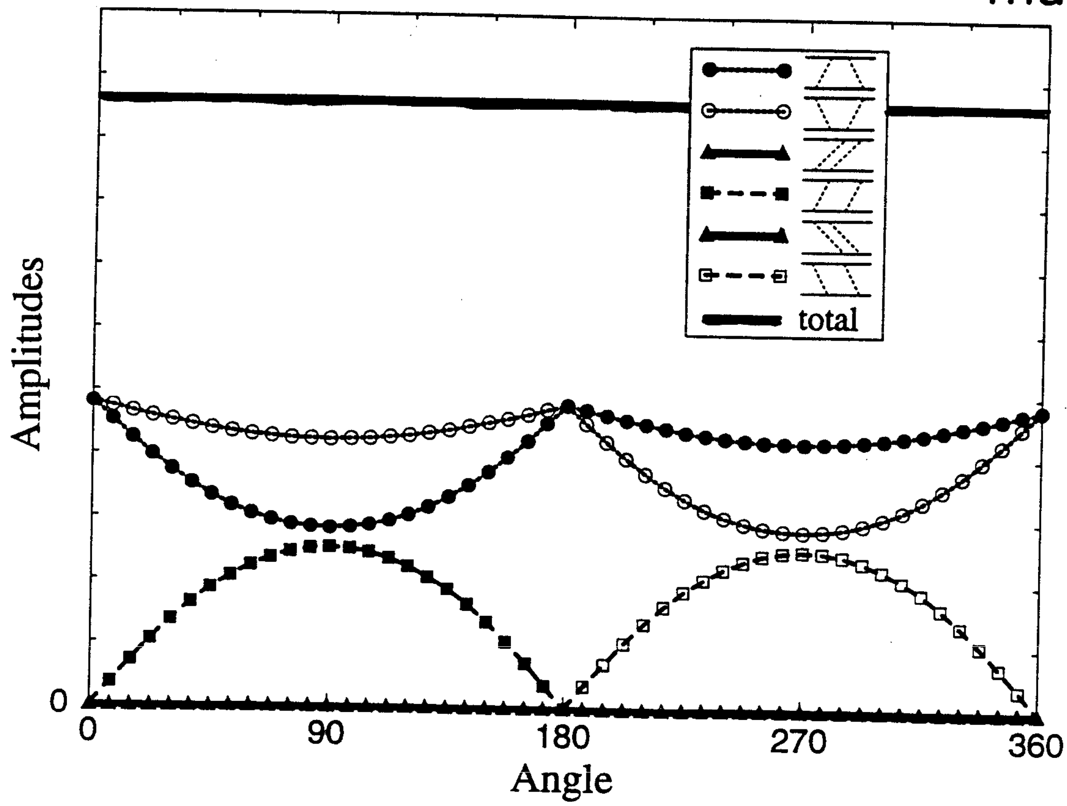
Wick-Cutkosky,

$|\vec{p}|=40$, $\mu=1$, $\mathcal{R}^{\blacklozenge}_{max}=2.5$ %

$m = 940$

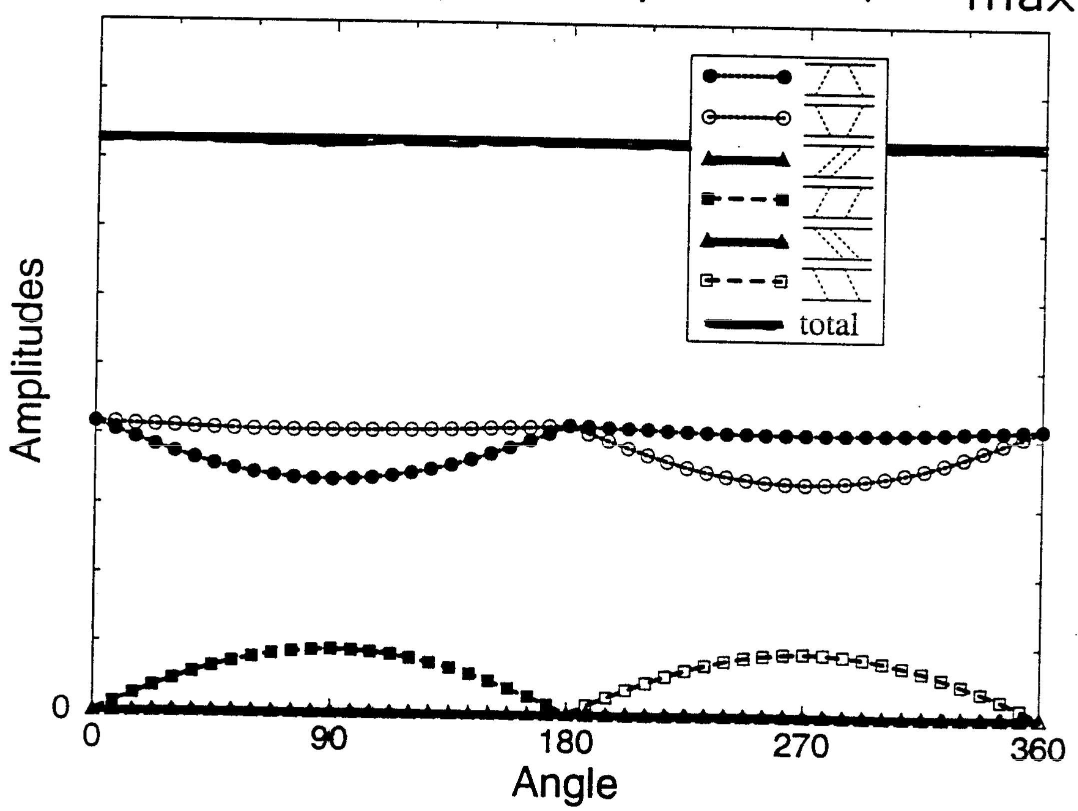


"Deuteron," $|\vec{p}|=40$, $\mu=140$, $\mathcal{R}^{\blacklozenge}_{max}=0.12$ %



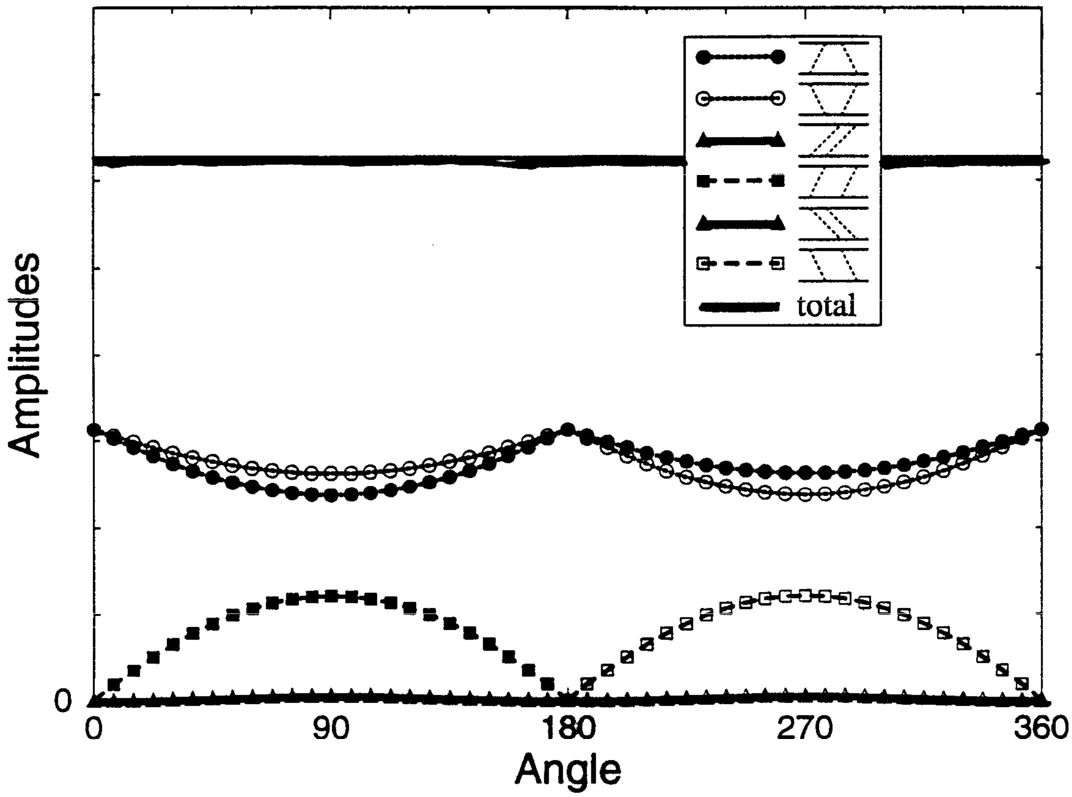
π

"Deuteron," $|\vec{p}|=40$, $\mu=763$, $\mathcal{R}^{\blacklozenge}_{max}=0.017$ %



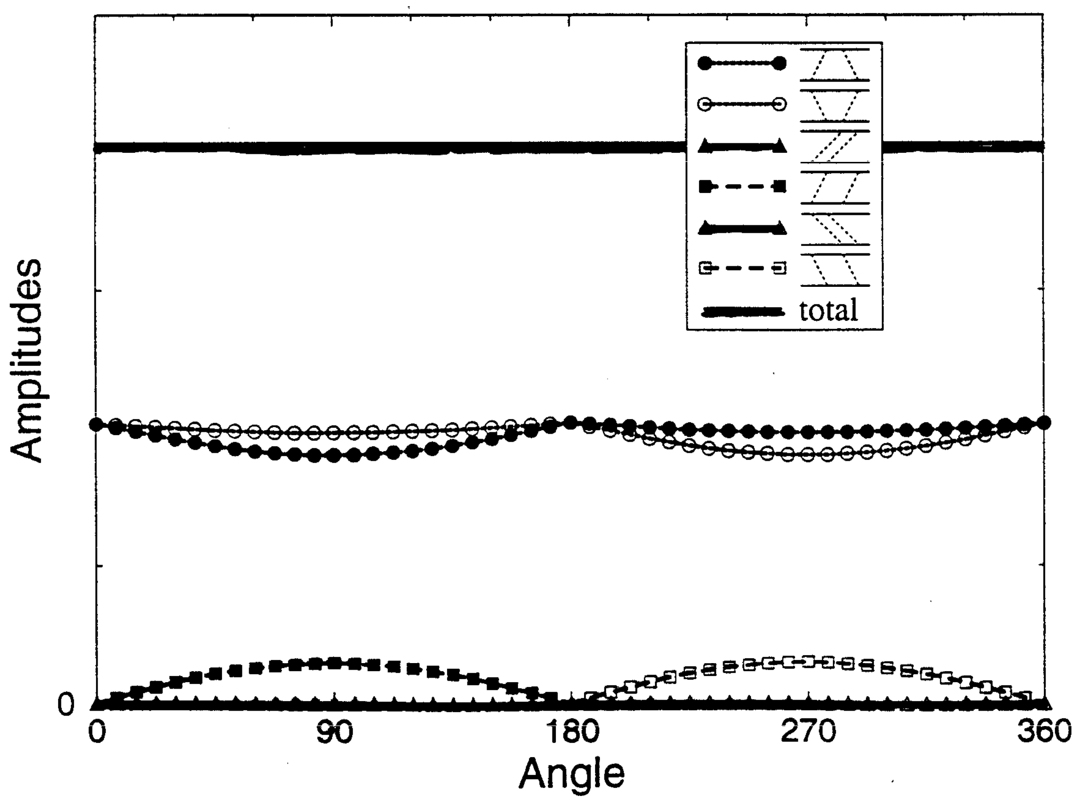
ρ

E =-100, $|\vec{p}|$=40, μ=140, $\mathcal{R}^{\blacklozenge}_{\max}$=0.88 %



π

E =-100, $|\vec{p}|$=40, μ=763, $\mathcal{R}^{\blacklozenge}_{\max}$=0.038 %



ρ

Conclusions

I

- Recipe for **covariant** LFPT includes k^--integration and **minus** regularization.

- The rise and fall of the **F**orced **I**nstantaneous **L**oops.

- Truncating Fock space $\Rightarrow$ error $\propto R^{\blacklozenge}$.

- One-boson exchange as effective potential $\Rightarrow R^{\blacklozenge} \propto$ stretched box.

(prelimanary results)

- Below threshold $R^{\blacklozenge}$ typically 0.02–2.5%.