The Meson Mass Spectra and Mixing Angles of (ω, ϕ) and (η, η') in the Light-Cone Quark Model

Ho-Meoyng Choi and Chueng-Ryong Ji

Department of Physics

North Carolina State University

Contents

I. Motivation

1. How to resolve the sign of the puzzle of $\omega - \phi$ mixing angle in LCQM,*
$\delta_V = -3.3°$, which is OPPOSITE to the usual convention that is based upon the quadratic SU(3) mass forumla(Gell-Mann-Okubo formula) ?

- Since the LCQM[1] fitting of the $\omega - \phi$ mixing angle has included the effect of SU(3) breaking, *i.e.*, $m_s/m_{u(d)} \sim 1.5$, it would be the right way to analyze the meson mass spectra based on the SU(3) breaking not on the Gell-Mann-Okubo(GMO) mass formula, which does not account for SU(3) breaking.

2. How to implement a dynamical mechanism on LCQM* ?

- Set up three different types of potentials to analyze the mass spectra of light pseudoscalar(π, K, η, η') and vector(ρ, K^*, ω, ϕ) mesons:

(1) Harmonic Oscillator(HO) + spin-spin(S-S)

(2) Coulomb + HO + S-S

(3) Coulomb + linear confining + S-S

* W. Jaus, Phys. Rev. D44, 2851 (1991)

→ Analyze the meson mass spectra!!

II. Quadratic mass formula and SU(3) breaking*, **

* De Rújula, Georgi, and Glashow ** N. Isgur

The generic two particle (f_1, f_2) mixing is given by

$$|f_1\rangle = -\sin\delta|n\bar{n}\rangle - \cos\delta|s\bar{s}\rangle$$

$$|f_2\rangle = \cos\delta|n\bar{n}\rangle - \sin\delta|s\bar{s}\rangle \tag{2.1}$$

where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and and $\delta = \theta_{SU(3)} - 35.26°$. We shall identify (f_1, f_2) with (ϕ, ω) and (η, η'). The quadratic mass eigenvalue equation is given by

$$\mathcal{M}^2|f_i> \; = M_i^2|f_i> \quad (i = 1, 2), \tag{2.2}$$

with

$$\mathcal{M}^2 = \begin{pmatrix} M_{n\bar{n}}^2 + 2\lambda & \sqrt{2}\lambda X \\ \sqrt{2}\lambda X & M_{s\bar{s}}^2 + \lambda X^2 \end{pmatrix}. \tag{2.3}$$

The parameter λ characterizes the strength of the quark-annihilation graph which couples the I=0 $u\bar{u}$ state to I=0 $u\bar{u}, d\bar{d}, s\bar{s}$ states with equal strength in the SU(3) limit. The parameter X pertains to SU(3) breaking, *i.e.*, $X \to 1$ in the SU(3) limit, such that the quark-annihilation graph *factors* into its flavor parts, with

$$\lambda : u\bar{u} \to u\bar{u}(d\bar{d})$$

$$\lambda X : u\bar{u} \to s\bar{s}(\text{or } s\bar{s} \to u\bar{u})$$

$$\lambda X^2 : s\bar{s} \to s\bar{s}$$

Solving Eqs.(2.1)-(2.3), we obtain

$$M_{f_1}^2 = (M_{n\bar{n}}^2 + 2\lambda)\sin^2\delta + 2\sqrt{2}\lambda X \sin\delta\cos\delta + (M_{s\bar{s}}^2 + \lambda X^2)\cos^2\delta, \tag{2.4}$$

$$M_{f_2}^2 = (M_{n\bar{n}}^2 + 2\lambda)\cos^2\delta - 2\sqrt{2}\lambda X \sin\delta\cos\delta + (M_{s\bar{s}}^2 + \lambda X^2)\sin^2\delta, \tag{2.5}$$

and

$$\tan^2\delta = \frac{M_{f_2}^2 - \epsilon M_{f_1}^2}{\epsilon M_{f_2}^2 - M_{f_1}^2}, \tag{2.6}$$

where $\epsilon = (M_{n\bar{n}}^2 + 2\lambda)/(M_{s\bar{s}}^2 + \lambda X^2)$. The constraint requires the invariance of the trace of quadratic mass matrix:

$$M_{n\bar{n}}^2 + M_{s\bar{s}}^2 + 2\lambda + \lambda X^2 = M_{f_1}^2 + M_{f_2}^2. \tag{2.7}$$

How to determine the sign of δ in quark model?

$$\tan 2\delta = \frac{2\sqrt{2}\lambda X}{M_{s\bar{s}}^2 - M_{n\bar{n}}^2 + \lambda X^2 - 2\lambda},$$

(A) $\omega - \phi$ mixing with $\delta_V = -3.3°$(or $\theta_{SU(3)} = 31.96°$)

$$\lambda_V < 0, \quad X_V > 0 \ (?) \ \text{ or}$$

$$\lambda_V > 0, \quad X_V < 0 \ (?)$$

while for $\delta_V = +3.3°$

$$\lambda_V X_V > 0$$

(B) $\eta - \eta'$ mixing with $\delta_P = -54.26°$(or $\theta_{SU(3)} = -19°$)

$$\lambda_P X_P > 0$$

III. Effective potential in the light-cone quark model

The effective Hamiltonian for the description of the meson mass spectra is defined by

$4m_q/p^+$

$$H_{q\bar{q}} = \sqrt{m_q^2 + k^2} + \sqrt{m_{\bar{q}}^2 + k^2} + V_{q\bar{q}}, \tag{3.1}$$

and the mass-squared operator is given by

$$M_{\rm op}^2 = 4(m^2 + k^2 + mV_{q\bar{q}}). \tag{3.2}$$

Three different types of potentials are as follows

$$\begin{aligned}
(A) \quad V_{q\bar{q}} &= \overbrace{V_{S.H.O}}^{V_0} + \overbrace{V_{S-S}}^{V_I} \\
&= A_1 + B_1 r^2 + \frac{\vec{S}_q \cdot \vec{S}_{\bar{q}}}{m_q m_{\bar{q}}} \nabla^2 V_{Coul}, && (3.3) \\
(B) \quad V_{q\bar{q}} &= V_{Coul} + V_{S.H.O} + V_{S-S} \\
&= A_2 - \frac{\kappa}{r} + B_2 r^2 + \frac{\vec{S}_q \cdot \vec{S}_{\bar{q}}}{m_q m_{\bar{q}}} \nabla^2 V_{Coul}, && (3.4) \\
(C) \quad V_{q\bar{q}} &= V_{Coul} + V_{lin} + V_{S-S} \\
&= A_3 - \frac{\kappa}{r} + B_3 r + \frac{\vec{S}_q \cdot \vec{S}_{\bar{q}}}{m_q m_{\bar{q}}} \nabla^2 V_{Coul}, && (3.5)
\end{aligned}$$

where $\nabla^2 V_{Coul} = 4\pi\kappa\delta^3(r)$.

Using the ground-state S-wave variational wavefunction,

$$\phi(k^2) = \Big(\frac{4}{\sqrt{\pi}\beta^3}\Big)^{1/2} \exp(-k^2/2\beta^2), \tag{3.6}$$

we obtain

$$< V_{q\bar{q}} > \; = A_1 + \frac{3}{2\beta_{q\bar{q}}^2} B_1 + 4\kappa \frac{\beta_{q\bar{q}}^3}{m_q^2 \sqrt{\pi}} \vec{S}_q \cdot \vec{S}_{\bar{q}}, \quad \text{for A} \tag{3.7}$$

$$= A_2 - \kappa \frac{2\beta_{q\bar{q}}}{\sqrt{\pi}} + \frac{3}{2\beta_{q\bar{q}}^2} B_2 + 4\kappa \frac{\beta_{q\bar{q}}^3}{m_q^2 \sqrt{\pi}} \vec{S}_q \cdot \vec{S}_{\bar{q}}, \quad \text{for B}, \tag{3.8}$$

$$= A_3 - \kappa \frac{2\beta_{q\bar{q}}}{\sqrt{\pi}} + \frac{2}{\beta_{q\bar{q}} \sqrt{\pi}} B_3 + 4\kappa \frac{\beta_{q\bar{q}}^3}{m_q^2 \sqrt{\pi}} \vec{S}_q \cdot \vec{S}_{\bar{q}}, \quad \text{for C}, \tag{3.9}$$

where

$$< \vec{S}_q \cdot \vec{S}_{\bar{q}} > \; = \begin{cases} 1/4 & \text{for vector} \\ -3/4 & \text{for pseudoscalar.} \end{cases} \tag{3.10}$$

and

$$< M_{q\bar{q}}^2 > \; = 4 \times \left\{ m_q^2 + \frac{3\beta_{q\bar{q}}^2}{2} + m_q < V_{q\bar{q}} > \right\}, \tag{3.11}$$

where we use $m_{u(d)}$= 0.25 GeV, m_s= 0.37 GeV, $\beta_{u\bar{u}}$= 0.3194 GeV and $\beta_{s\bar{s}}$= 0.3478 GeV.

•Methods of fitting three parmeters, $(A_i, B_i, \kappa)(i = 1, 2, 3)$:

(M1) Fitting the π, ρ and K^* meson masses;

$M_{Vn\bar{n}}^2 = m_\rho^2$, $M_{Pn\bar{n}}^2 = m_\pi^2$, and $M_{Vs\bar{s}}^2 = 2m_{K^*}^2 - m_\rho^2$.

(M2) Fitting π, ρ and K meson masses;

$M_{Vn\bar{n}}^2 = m_\rho^2$, $M_{Pn\bar{n}}^2 = m_\pi^2$, and $M_{Ps\bar{s}}^2 = 2m_K^2 - m_\pi^2$.

* Methods of fitting Parameters, (A_i, B_i, κ)

e.g.) $V = \underbrace{V_{S.H.O}}_{V_0} + \underbrace{V_{S-S}}_{V_I}$

step1) Variational Principle

$$\langle M^2 \rangle = \langle M_0^2 \rangle + \langle M_I^2 \rangle$$
$$= 4\{ m^2 + \frac{3\beta^2}{2} + mV_0 \} + 4mV_I$$

→ minimization

; $\frac{\partial \langle M_0^2 \rangle}{\partial \beta} = 0$

$\sim \beta - m\beta^{-3} B_1$

$\therefore$ $B_1 = \frac{\beta_{u\bar{u}}^4}{m_u} = \frac{\beta_{s\bar{s}}^4}{m_s}$

check with Jaus's Parameters

$$\frac{\beta_{u\bar{u}}^4}{m_u} = 0.042 \sim \frac{\beta_{s\bar{s}}^4}{m_s} = 0.040$$

→ model parameters seems O.K.

step2) Find A_1 and κ
using $\rho-\pi$ splitting

i.e.

$$\langle M^2_{n\bar{n}}\rangle_V = m^2_\rho \,,\ \langle M^2_{s\bar{s}}\rangle_P = m^2_\pi$$

* Numerical Results

$A_1 = -1.024$ GeV, $B_1 = 0.042$ GeV^3

$\kappa = 0.488$

and

$$\lambda_V = 0.6\, m^2_\pi \ \text{GeV}^2,\ X_V = -6.18$$
$$\lambda_P = 23.6\, m^2_\pi \ \text{GeV}^2,\ X_P = 0.68$$

Mass Spectra [Mev]

	Th.		Th.
π(135)	135	ρ(770)	770
K (494)	469	K^*(892)	848
η (547)	547	η' (958)	958
ω (782)	782	ϕ (1020)	1020