A NON-PERTURBATIVE EFFECT IN DEEP INELASTIC SCATTERING

E. A. Paschos

Phys. Lett. B 389 (96) 383

1. Introduction (data)
2. Space-time Picture in various frames
3. Proposed Solution / theoretical issues
4. Pair Creation in QCD à la Schwinger
5. Toy Models
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7. Summary - Outlook

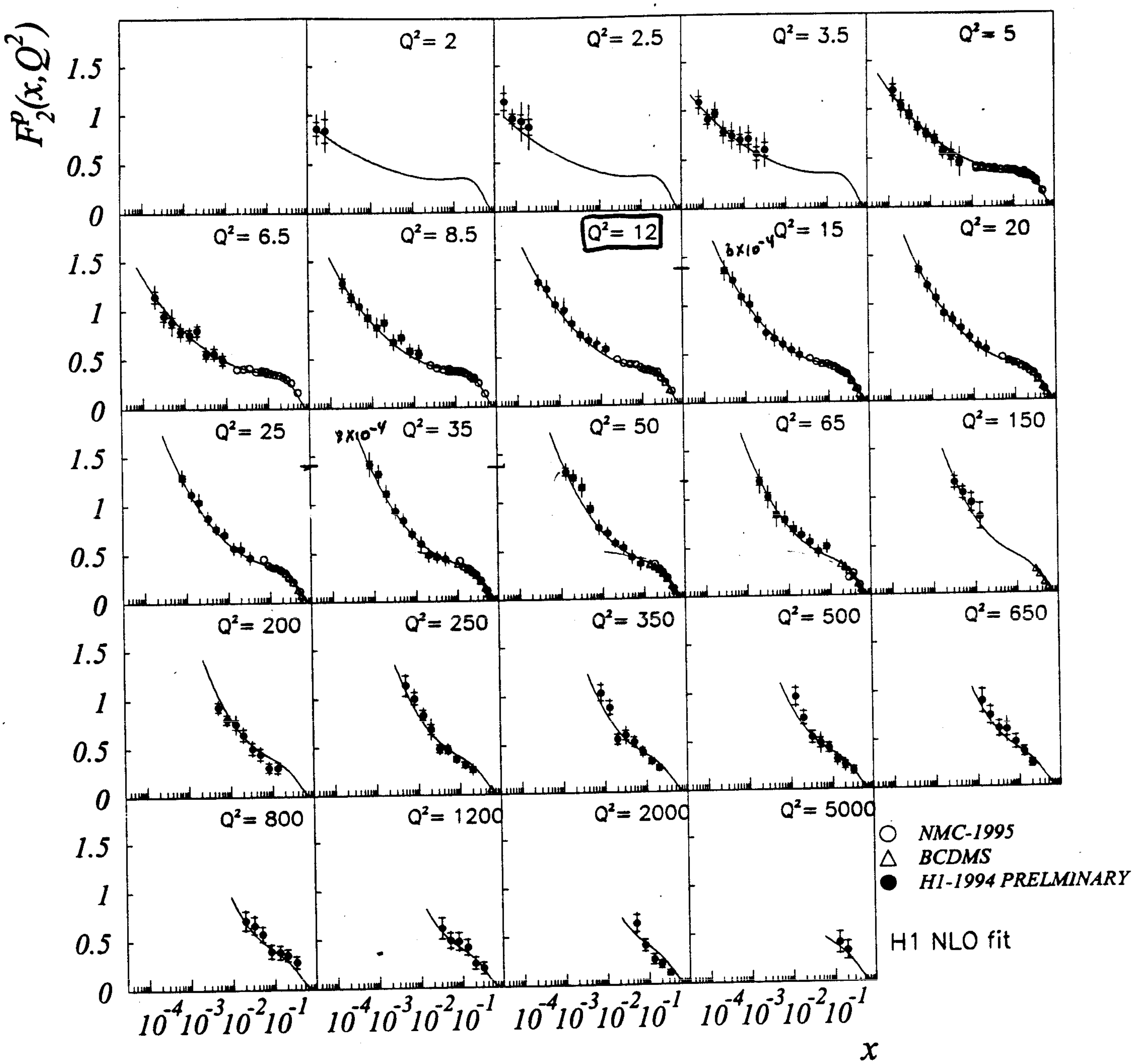
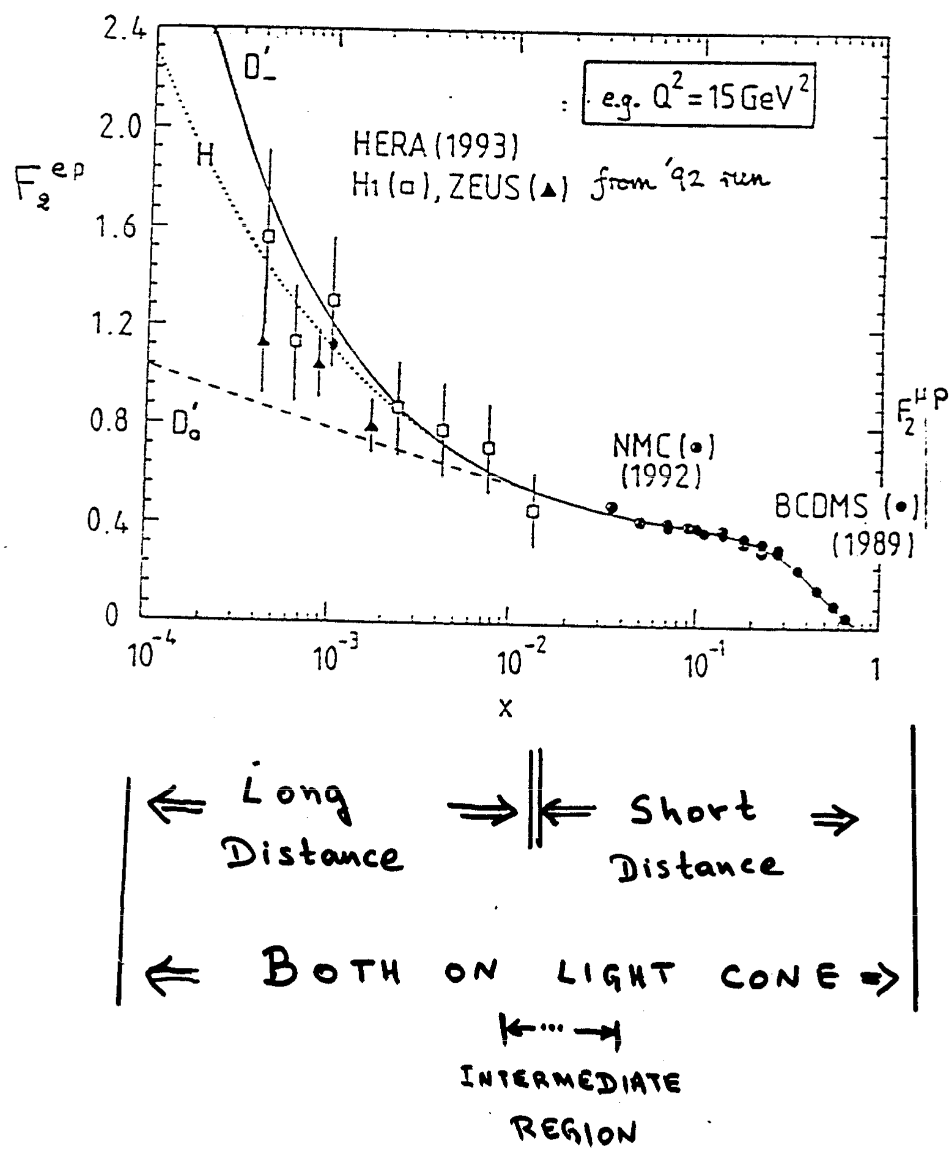
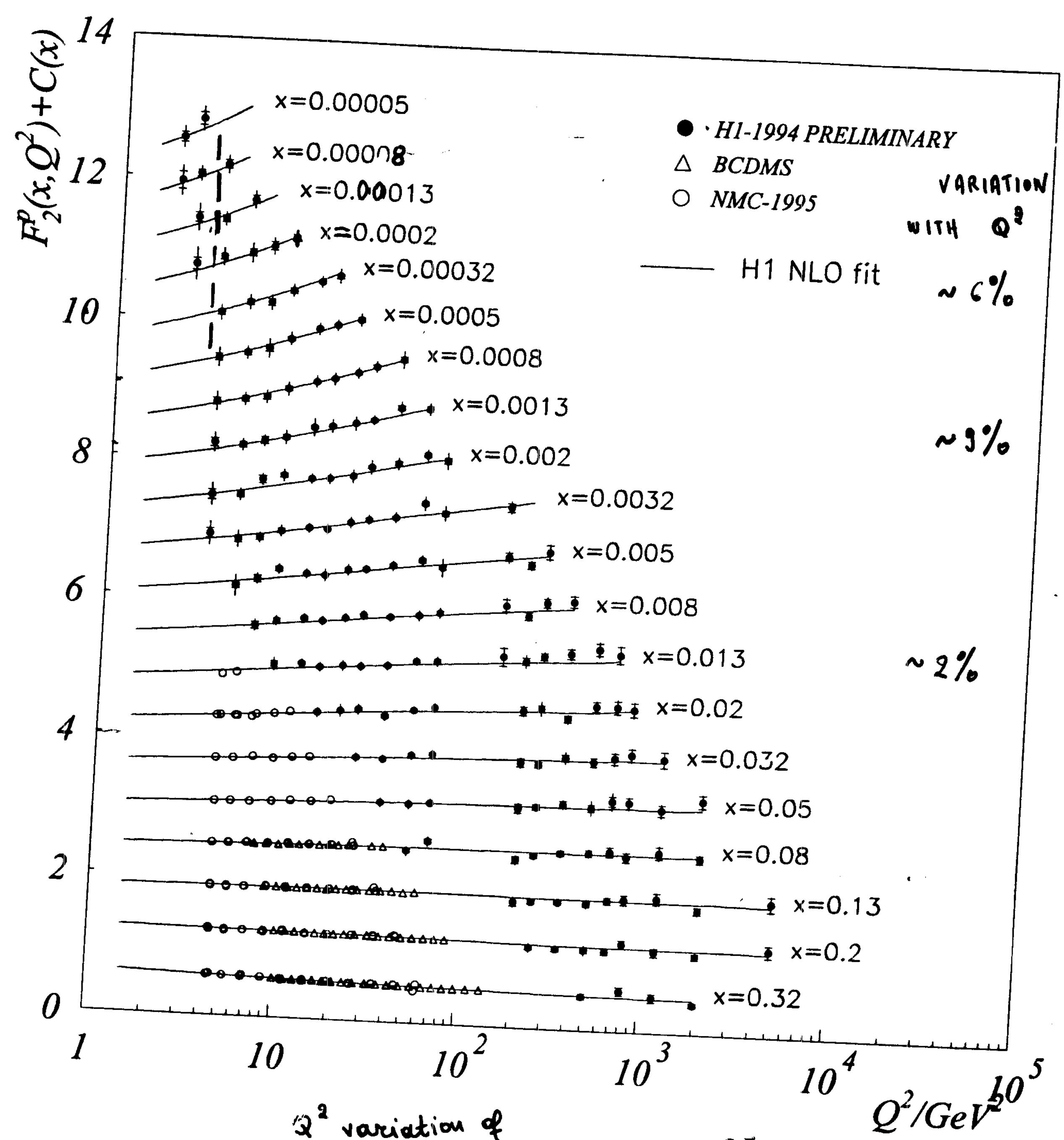


Figure 3: *Preliminary measurement of the proton structure function $F_2(x,Q^2)$ as function of x in different bins of Q^2. The inner error bar is the statistical error. The full error represents the statistical and systematic errors added in quadrature not taking into account the 1.5% systematic error on the luminosity measurement (4.5% for $Q^2 \leq 6.5$ GeV²). Open circles and triangles represent NMC and BCDMS measurements, respectively. A smooth transition becomes apparent from the NMC and BCDMS data (open circles and triangles, respectively) tc the H1 data. The curves represent a QCD fit to all data, see text.*

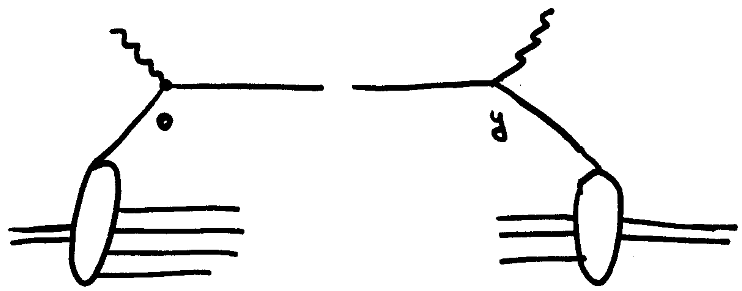


2.4
2.0
1.6
1.2
0.8
0.4
0
F_2^{ep}
D'_-
H
D'_0
e.g. $Q^2 = 15\,GeV^2$
HERA (1993)
H1 (□), ZEUS (▲) from '92 run
NMC (•)
(1992)
BCDMS (•)
(1989)
$F_2^{\mu p}$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
Long Distance
Short Distance
BOTH ON LIGHT CONE
INTERMEDIATE REGION



$F_2^p(x,Q^2)+C(x)$
x=0.00005
x=0.00008
x=0.00013
x=0.0002
x=0.00032
x=0.0005
x=0.0008
x=0.0013
x=0.002
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.02
x=0.032
x=0.05
x=0.08
x=0.13
x=0.2
x=0.32
H1-1994 PRELIMINARY
BCDMS
NMC-1995
H1 NLO fit
VARIATION WITH Q^2
~ 6%
~ 3%
~ 2%
Q^2/GeV^2
Q^2 variation of

SPACE-TIME STRUCTURE OF SCATTERING



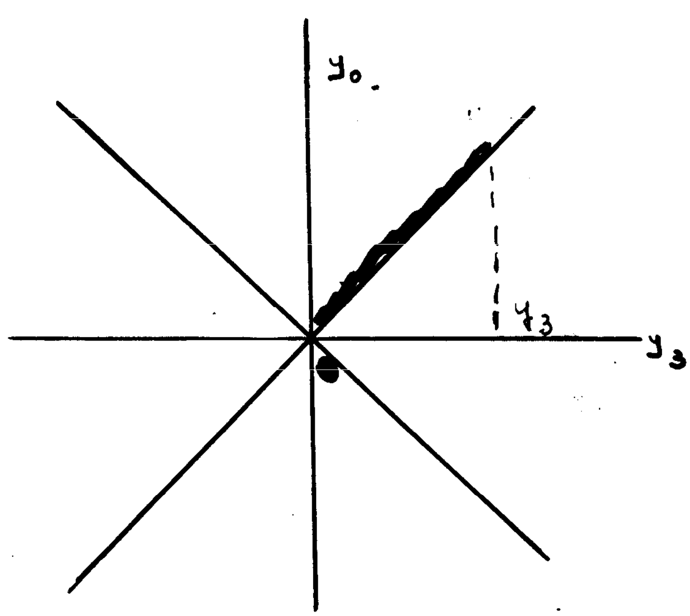
$$W_{\mu\nu}(q\cdot p, q^2) = \frac{1}{2\pi} \int d^4y \; e^{iq\cdot y} \langle p | [J_\mu(y), J_\nu(0)] | p \rangle$$

$$q\cdot y = q_0 y_0 - q_3 y_3 = \frac{1}{2}\left(q_+ y_- + q_- y_+ \right)$$

STATIONARY PHASE OCCURS : $y_- \approx \frac{\pm 1}{q_0 + q_3}$ and $y_+ = \frac{\pm 1}{q_0 - q_3} = y_0 + y_3$

FOR TIME-LIKE y

$$\therefore \quad y^2 = y_+ y_- - y_\perp^2 \leq y_+ y_- \approx \frac{-1}{q^2} = \frac{1}{Q^2} \rightarrow 0 \quad \text{LIGHTCONE}$$



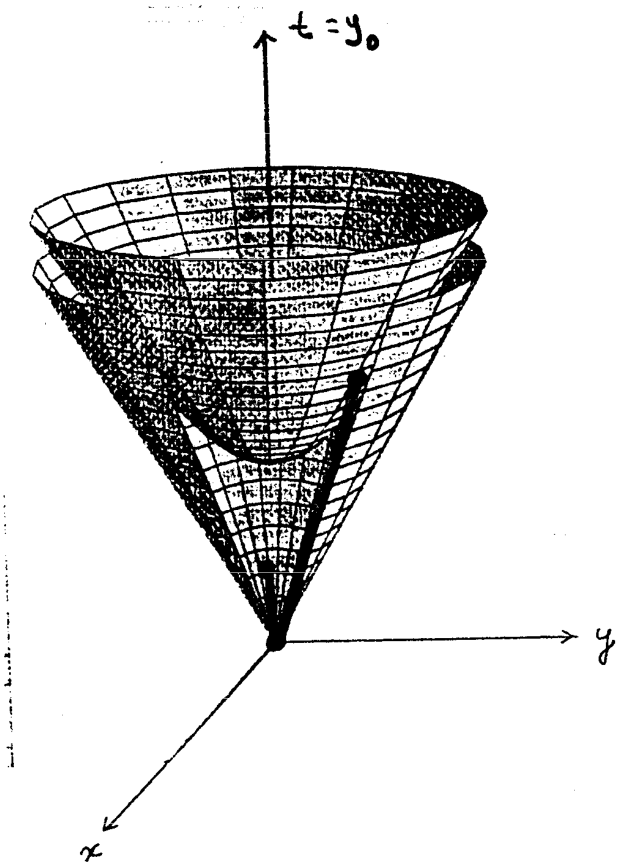
$$y_\perp^2 < y_+ y_- \sim \frac{1}{Q^2}$$

CONCLUSIONS: IT IS JUSTIFIED TO REPLACE COMMUTATOR BY LIGHTCONE SINGULARITY

$$\therefore \quad y_3 \approx \frac{1}{2} \frac{-1}{q_0 - q_3} = \frac{1}{2} \frac{-1}{q_0 - \sqrt{q_0^2 + Q^2}}$$

$$\approx \frac{1}{2} \frac{1}{+\frac{Q^2}{2q_0}} = \frac{1}{2} \frac{1}{M_p x_{Bj}}$$

1. SPACE-TIME STRUCTURE



blue : short distance } LIGHT-LIKE

red : long distance }

1 : $y^2 = y_\mu y^\mu = \frac{1}{Q^2}$ B. Ioffe : 1969

2 : $y_3 = \frac{1}{2 M_p} \frac{1}{x_{Bj}}$

VARIOUS FRAMES

REST FRAME OF PROTON

$$\mathcal{L}(x) \sim \frac{1}{2Mx}$$

$$\gamma_{ph} = \frac{\nu}{Q} = \frac{1}{2} Q \frac{2M\nu}{Q^2} = \frac{1}{2} \frac{Q}{x}$$

ELECTRON-PROTON CM-FRAME

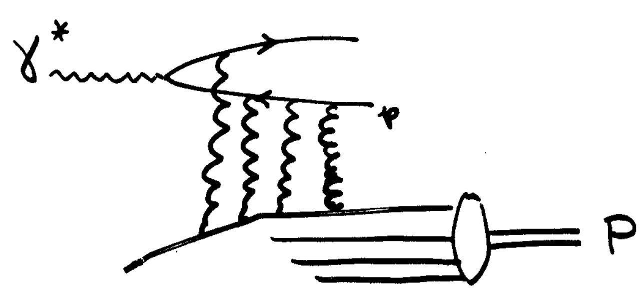
$$\mathcal{L} = \frac{1}{2} \frac{1}{q_0^* - q_3^*} \approx \frac{q_0^*}{Q^2} = \frac{1}{2} \frac{\sqrt{\tilde{y}}}{Q\sqrt{x}}$$

$$\tilde{y} = \frac{\nu}{E}$$

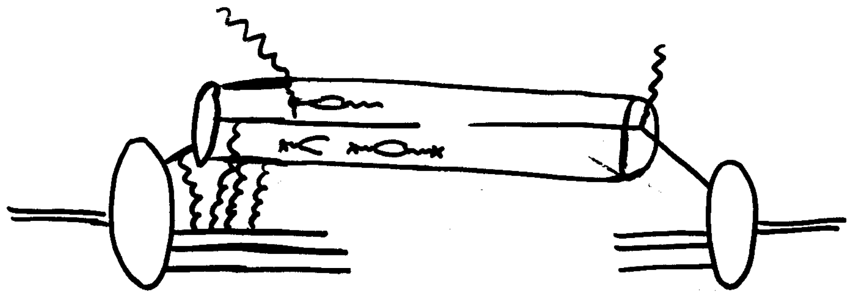
$$\gamma = \frac{q_0^*}{\sqrt{Q^2}} = \frac{\sqrt{\tilde{y}}}{2} \frac{1}{\sqrt{x}}$$

γ^*-Parton CM-FRAME

$$\gamma = \frac{1}{\sqrt{x}}$$



2. PROPOSAL. As the quark is hit by the current it tries to travel for long distances polarizing the QCD vacuum. The polarization stores energy in the vacuum creating a CHROMO-ELECTRIC and CHROMO-MAGMETIC tube, which breaks creating $(q-\bar{q})$-pairs (i.e. hadrons).



PROBLEM In a tube with E^α, B^α compute the creation of pairs

DIMENSIONS OF TUBE : $L(x) \simeq \frac{1}{4 M_p x}$ IN REST FRAME OF PROTON

$\lambda = 2.5 \frac{1}{\text{GeV}}$

Typical time for creation of pair $T \approx \frac{1}{\langle E_{pair} \rangle}$ Uncertainty principle.

NEED A MODEL OR A THEORY FOR THE CREATION OF PAIRS : ADOPT AN OLD CALCULATION FOR THE CREATION OF PAIRS (J. Schwinger PR 82 (1951) 664
93 (1954) 615

THE CALCULATION IS MODIFIED FOR QCD

CONCLUSIONS MORE GENERAL:

THEORETICAL ISSUES

FIRST : QED PERTURBATIVE

QCD NON-PERTURB.

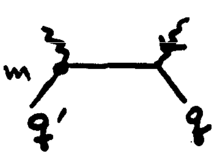
$$S = T\, e^{-i \int \left[g\, G_\nu^\alpha\, j^{\nu,\alpha} + e\, A_\mu j^\mu \right] d^4y}$$

$$\longrightarrow T\, e^{-ig \int G_\nu^\alpha\, j^{\nu,\alpha}\, d^4y} \left[1 - ie \int A_\mu j^\mu d^4y + \dots \right]$$

Rigorous

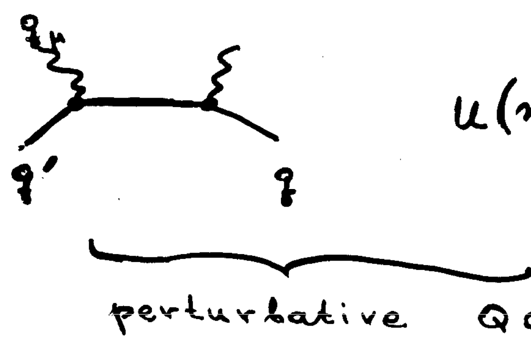
SECOND : MATRIX ELEMENTS : FACTORIZATION

$$\sum_X \left| \langle X | S | P \rangle \right|^2 =$$

$$\sum_X \left| \langle X | T\, e^{-ig \int G_\nu^\alpha\, j^{\nu,\alpha}\, d^4y} | q' \rangle \right|^2 \left\{ \mathrm{Im} \right.$$


$$\left. \right\} \langle q | P \rangle$$

$\Downarrow$

$$= \sum_n P_n \;\bullet$$


$$u(x, Q_0^2)$$

FACTORIZATION determines VARIABLE DEPENDENCE

S-MATRIX :

$$S = T e^{-i \int [e A_\mu(y) j^\mu(y) + g G^\alpha_\nu(y) j^{\alpha,\nu}(y)] dy^4}$$

$$= \sum_n \frac{(-i)^n}{n!} \iint T \left[e A_\mu(y) j^\mu(y) + g G^\alpha_\nu(y) j^\alpha(y) \right]^n \prod_{i=1}^n dy_i^4$$

$$= \ldots \frac{(-i)^n}{n!} \int \ldots \int T [\cdots] [e A(y_k) \cdot j(y_k) + g G^\alpha(y_k) \cdot j^\alpha(y_k)]$$

$$[e A(y_{k+1}) \cdot j(y_{k+1}) + g G^\alpha(y_{k+1}) \cdot j^\alpha(y_{k+1})] \cdots [\quad] dy_1 \ldots dy_k \, dy_{k+1} \cdots dy_n$$

For perturbation theory keep: $g G \cdot j(y_{k-1}) \; e A \cdot j(y_k)$

Then call

$e A(y_k) \cdot j(y_k) =: A(y_k)$ and
$g G^\alpha(y_{k-1}) \cdot j^\alpha(y_{k-1}) =: B(y_{k-1})$

Now :

$$T(A(y_1) B(y_2)) = \theta(y_1^0 - y_2^0) A(y_1) B(y_2) + \theta(y_2^0 - y_1^0) B(y_2) A(y_1)$$

because y_1 and y_2 dummy variables (integration) (two fermion fields in A and B)

$$\iint T(A(y_1) B(y_2)) dy_1 dy_2 = \iint T(B(y_1) A(y_2)) dy_1 dy_2$$

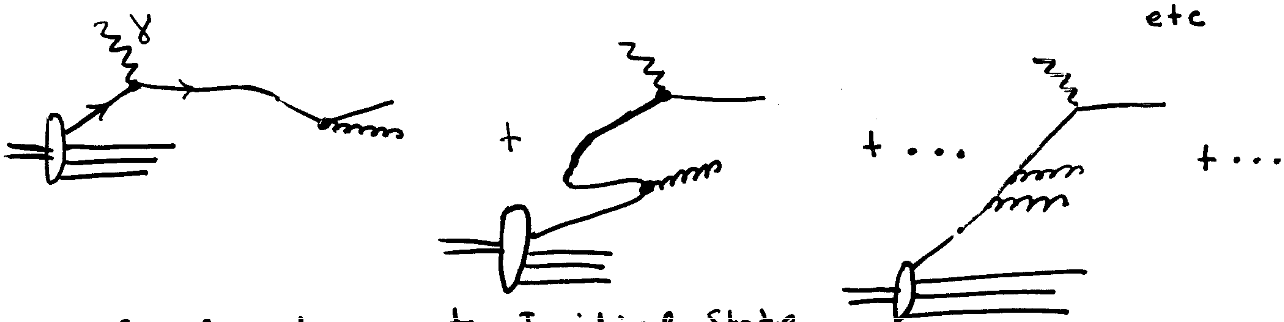
$$S = \sum \frac{(-i)^n}{n!} T \left\{ (A) B^{n-1} + B(A) B^{n-2} + \ldots + B^k (A) B^{n-k-1} + B^n \right\}$$

$$= \frac{(-i)^n}{n!} T \left\{ n A B^{n-1} + B^n \right\} = T [1 - i A] e^{-iB}$$

$$S \approx T\left\{ 1 - ie\int A_\mu(y)\, j^\mu(y)\, d^4y + \dots \right\} e^{-ig\int G^\alpha_\nu(y)\, j^{\alpha,\nu}(y)\, d^4y}$$

1. At <u>short distance</u> expand QCD and

$$j^\mu(y)\, j^{\alpha,\nu}(z) \dots j^{\beta,\kappa}(z_i) = \bar{q}\gamma^\mu \overbrace{q(y)\, \bar{q}}(z)\, \lambda^\alpha \gamma^\nu \underbrace{q(z) \dots}_{\text{etc}} q(z_i)$$

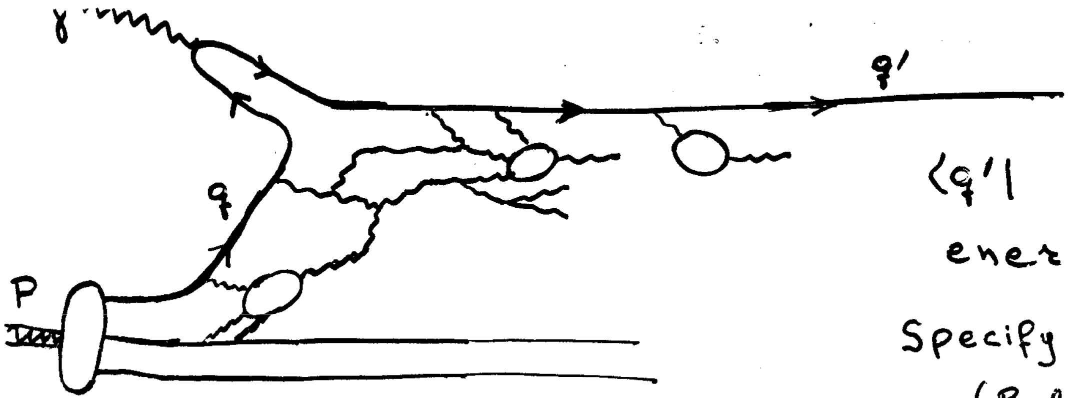


final rad. + Initial State rad.

Perturbative QCD

$$\Downarrow$$

$$q_i^{\text{pert.}}(x, Q^2)$$



$\langle q'|$ always very energetic

Specify QCD Vacuum

(Balatin, Matinyan + Savvidy '77)

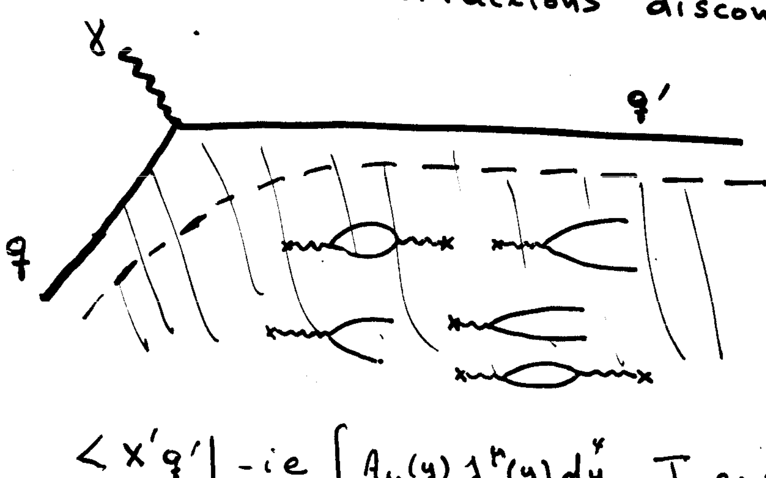
EMISSION OF GLUONS CREATING BACKGROUND FIELD

2. NON-PERTURBATIVE QCD EFFECT

$$-ie\int A_\mu(x)\, j^\mu(x)\, d^4x \quad \otimes \sum_n \int\cdots\int \frac{(-ig)^n}{n!}\cdot dy_1\ldots dy_n$$

$$G^\alpha_\nu(y_1)\,\bar q(y_1)\,\lambda^\alpha\gamma^\nu q(y_1)\;\; G^\beta_\rho(y_2)\,\bar q(y_2)\,\lambda^\beta\gamma^\rho q(y_2)\;\ldots\;\; G^\delta_\pi(y_n)\,\bar q(y_n)\,\lambda^\delta\gamma^\pi q(y_n)$$

All possible contractions disconnected from the EM-current



$$\langle X' q'|\, -ie\int A_\mu(y)\, j^\mu(y)\, d^4y \;\; T\exp\left\{-ig\int G^\alpha_\nu(z)\, j^{\alpha,\nu}(z)\, d^4z\right\}|q,\Omega\rangle$$

$$\approx \langle q'|\, -ie\int A_\mu(y)\, j^\mu(y)\, d^4y\,|q\rangle\,\langle X'|\, T\exp\left\{-ig\int G(z)\cdot j(z)\, d^4z\right\}|\Omega\rangle$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$q_i^0(x_P, Q^2)\cdot \qquad \left[1 - e^{-n(x)}\right]$$

3. PAIR CREATION

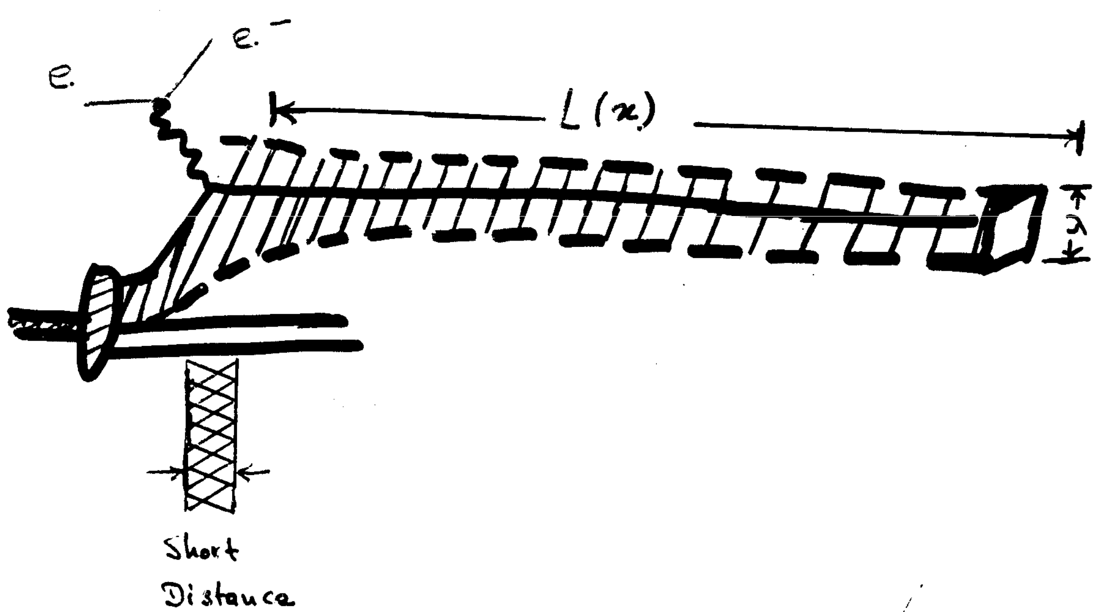
$$\sum_n P_n = \sum_{x'} \left| \langle x' | T \exp\left\{ -ig \int G_\nu^\alpha(z)\cdot j^{\nu,\alpha}(z)\, d^4z \right\} |\Omega\rangle \right|^2$$

$$S_0 = \langle \Omega | T \exp\left\{ -ig \int G(z)\cdot j(z)\, d^4z \right\} | \Omega \rangle$$

$$|S_0|^2 = \exp\left\{ -\int w(y)\, d^4y \right\}$$

Vacuum Persistence Probability.

W(y) is PROBABILITY FOR CREATING ONE PAIR PER UNIT VOLUME and UNIT TIME



SCHWINGER'S MECHANISM. ABELIAN FIELD:

After integration Fermion fields

$$\ln S_0(A) \doteq \mathrm{Tr}\ln\left\{[\not{p}-e\not{A}(x)-m+i\varepsilon]\frac{1}{\not{p}-m+i\varepsilon}\right\}$$

Doing Traces over γ's and using oper. identities

$$\mathrm{tr}\,\langle x|\, e^{is\left[(P-eA)^2+\sigma_{\mu\nu}\frac{F^{\mu\nu}}{2}\right]}\,|x\rangle$$

$$= \dots \int_{-\infty}^{\infty} ds\, \langle \omega|\, e^{is\left[P_0^2-e^2E^2x_0^2\right]}\,|\omega\rangle$$

$$w(y,m) = \frac{\alpha E^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\,e^{-\frac{m^2 n\pi}{eE}}$$

$A_\mu(x)=(0,0,0,teE)$

For $A_\mu(x) = (0, y^3 g B, 0, -teE)$

$$W(y,m) = \frac{\alpha_s}{\pi}\,E\,B\sum_{n=1}^{\infty}\frac{1}{n}\coth\left(n\frac{B}{E}\pi\right)e^{-\frac{nm^2\pi}{gE}}$$

$$\xrightarrow[\left|n\frac{B}{E}\pi\right|\ll 1]{\text{Lim.}} \frac{\alpha E^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\,e^{-\frac{n\pi m^2}{eE}}$$

Itzykson + Zuber
p. 196

NON-ABELIAN FIELDS

G. Cvetic
S. K. Bose
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SPACE + TIME HOMOGENEITY IS GAUGE COVARIANT PROPERTY

i.e., $G^{\ell}_{\mu\nu}$ is x- independent in particular gauges;

$$G^{\ell}_{\mu\nu}\left(\equiv \partial_\nu b^\ell_\mu - \partial_\mu b^\ell_\nu + i g f^{ij\ell} b^i_\mu b^j_\nu\right) = - F_{\mu\nu} n^\ell$$

n^ℓ is unit vector in color space

$F_{\mu\nu}$ a general field tensor (Abelian)

Define $W = \dfrac{dP}{d^4x} = -\dfrac{1}{\Omega} \ln |S_0(B)|^2 = -\dfrac{2}{\Omega} \mathrm{Re} \ln S_0(B)$

$$W = -\frac{1}{\Omega} \mathrm{Re} \left\{ \mathrm{Tr} \ln \left((\hat{\not p} - g_s \not B)^2 - (m - i\varepsilon)^2 \right) - \mathrm{Tr} \ln \left[\hat{\not p}^2 - (m - i\varepsilon)^2 \right] \right\}$$

Using:

i) Identity : $(\hat{\not p} - g \not B)^2 = (\hat{p} - g B)^2 - \frac{g}{2} \sigma_{\mu\nu} F^{\mu\nu}$

ii) Schwinger's integral repres. of logarithms

$$w = \mathrm{Re} \int_0^\infty \frac{ds}{s} \exp\left[-is(m^2 - i\varepsilon)\right]$$

$$\left\{ \mathrm{Tr} \langle x | \exp\left[is(\hat{p} - gB)^2\right] \exp\left[-is\frac{g}{2}\left(\frac{\vec{n}\cdot\vec{\lambda}}{2}\right)\sigma_{\mu\nu}F^{\mu\nu}\right] | x \rangle \right.$$

$$\left. - \mathrm{Tr} \langle x | \exp\left[isP^2\right] | x \rangle \right\}$$

We constructed an operator $\hat{U}$

$$\hat{U}\left[\hat{p} - gB(x)\right]^2 U^+ = |\alpha_1| \left[\hat{p}^0 \hat{p}^0 - \frac{g^2 A^2}{|\alpha_1|^2} \hat{X}^0 \hat{X}^0\right]$$

$$- |\alpha_2| \left[\hat{p}^1 \hat{p}^1 + \frac{gB^2}{|\alpha_2|^2} \hat{X}^1 \hat{X}^1\right]$$

Expressions in parentheses are Hamiltonians with mass $m = 1$ and 3×3-matrix frequencies

$$\omega = \frac{i g_s \tilde{A}}{|\alpha_1|} \quad \text{and} \quad \omega' = g\tilde{B}/|\alpha_2|$$

We can sum (trace) over the frequencies and integrate over $-s$.

(2.6

$$w = \frac{g_s^2}{4\pi} \frac{\tilde{a}\,\tilde{b}}{\pi} \sum_{k=1}^{\infty} \sum_{j=1}^{N_c} \frac{1}{k} \left\{ \left[\frac{\vec{n}\cdot\vec{\lambda}}{2} \right]_{jj} \right\}^2 \exp\left[-k \frac{2\pi m^2}{g_s \tilde{a} (n\cdot\lambda)_{jj}} \right]$$

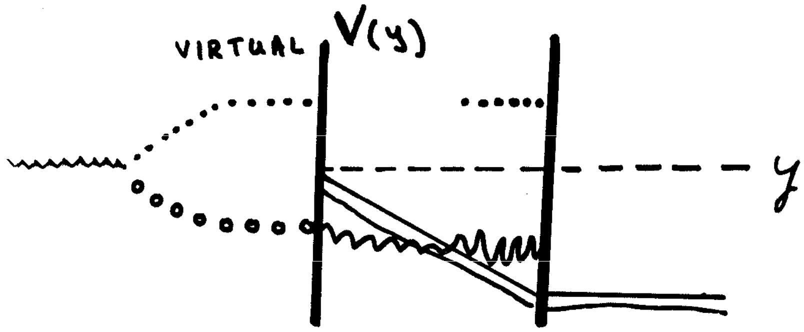
$$\cdot \coth\left[k \frac{\pi \tilde{b}}{\tilde{a}} \right]$$

with

$$\tilde{a} = \left[\vec{E}^2 - \vec{B}^2 + \sqrt{(\vec{E}^2 - \vec{B}^2)^2 + 4(\vec{E}\cdot\vec{B})^2} \right]^{1/2} / \sqrt{2}$$

$$\tilde{b} = \left[-\vec{E}^2 + \vec{B}^2 + \sqrt{(\vec{E}^2 - \vec{B}^2)^2 + 4(\vec{E}\cdot\vec{B})^2} \right]^{1/2} / \sqrt{2}$$

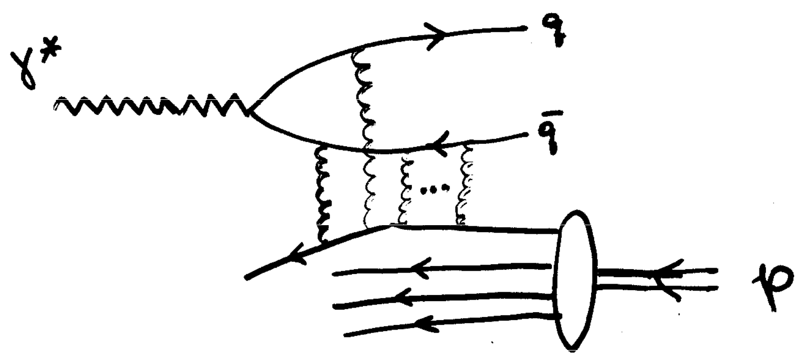
PHYSICAL MEANING



$$\frac{1}{\tau} = \exp\left\{ -2 \int_0^{y_0} \sqrt{\frac{2m}{\hbar}\left(|\varepsilon| - \tfrac{1}{2} g E y\right)}\, dy \right\}$$

$$= e^{-\frac{4}{3}\sqrt{\frac{2m}{\hbar}}\, \frac{|\varepsilon|^{3/2}}{gE}}$$

$$\varepsilon = m \qquad \text{and} \quad y_0 = \frac{2\varepsilon}{gE}$$



$$\frac{1}{\tau} = e^{-\frac{4}{3}\sqrt{\frac{2}{\hbar}}\, \frac{m^2}{gE_0}\sqrt{x}}$$

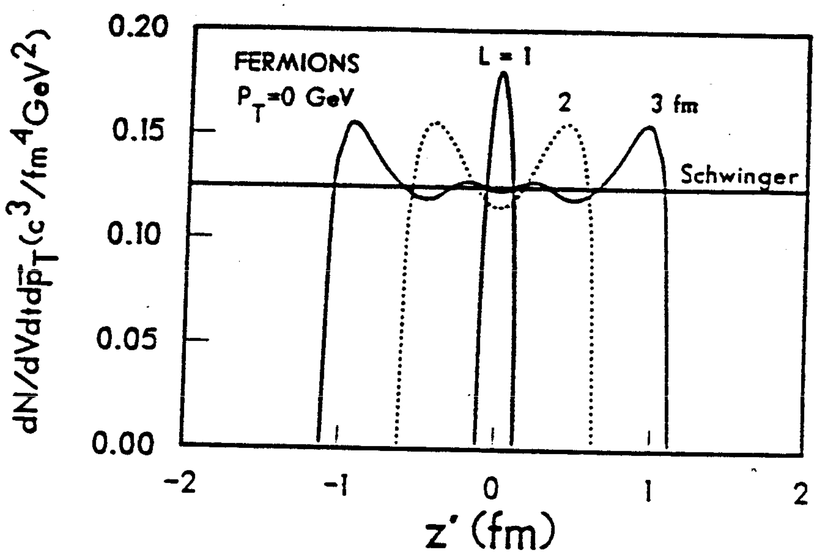
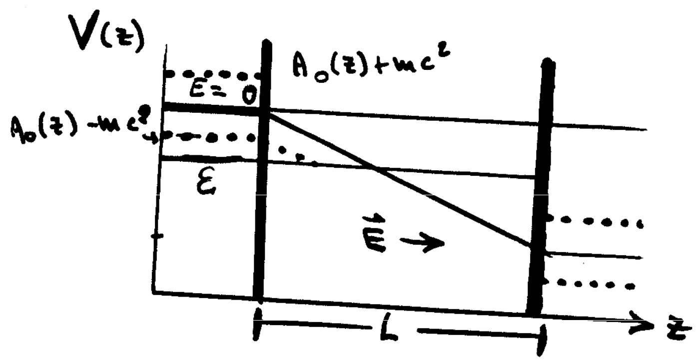


FIG. 3. We show here the particle-production rate $dN/dV\,dt\,d\mathbf{p}_T$ (per unit volume, per unit time, and unit transverse momentum) for fermions when the plates are separated by $L=1$, 2, and 3 fm. The particle production rate is plotted as a function of the location $z'=z-1/2$, which is measured relative to the midpoint between the plates.

R.C. Wang and C.Y Wong
Phys. Rev. D38 (1988) 348



Capacitor Plates

Brezin + Itzykson
Phys. Rev. D2 (1970) 1191

Itzykson + Zuber Field Theory
Chapter 4-3 (p. 185)

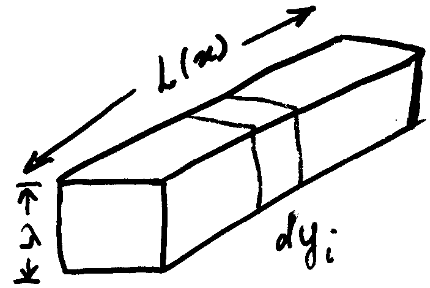
4. A TOY MODEL

1. HOW THE VARIOUS TERMS APPEAR IN THE STRUCTURE FUNCTIONS { multiplicative; coherent or incoherent; ...

2. MAGNITUDE OF THE EFFECTS

3. PREDICTIONS FOR DISTRIBUTION FUNCTIONS and COMPARISON WITH DATA

PARTITION OF TUBE



$$dP_i = \lambda^2 T\, w(y_i)\, dy_i \prod_{\substack{k=1 \\ k\neq i}}^{\infty} \left\{ 1 - \lambda^2 T\, w(y_k)\, dy_k \right\}$$

Probabil. that a pair is created per unit volume in time $T \sim \frac{1}{\Gamma}$

Probability that no-pair is created everywhere else

$$= \quad '' \quad e^{-\lambda^2 T \int_0^{L(x)} w(y)\, dy}$$

$$P_1 = \lambda^2 T \int_0^{L(x)} w(y)\, dy \; e^{-\lambda^2 T \int_0^{L(x)} w(y)\, dy}$$

Creation of n-pairs:

$$P_n = \frac{1}{n!} \left[\lambda^2 T \int_0^{L(x)} w(y)\, dy \right]^n e^{-\lambda^2 T \int_0^{L(x)} w(y)\, dy}$$

Poisson-distribution

$$\sum_{n=1}^{\infty} P_n = 1 - e^{-\lambda^2 T \int^{L(x)} w(y)\, dy}$$

Multipicity:

$$n(x) = \sum_{n=1}^{\infty} n P_n = \lambda^2 T \int_0^{L(x)} w(y)\, dy$$

Numerical Estimate.

$$w(y) = \frac{\alpha E^2}{\pi^2} \sum_n \frac{1}{n^2} e^{-\frac{n\pi m^2}{gE}}$$

$$n(x) = \lambda^2 T \int_0^{\frac{1}{4Mx}} w(y)\,dy = \lambda^2 T \frac{1}{\pi^3} \left(\frac{1}{2} g E\right)^2 \frac{1.0}{4Mx}$$

$$= 0.005 \frac{(T \text{ in GeV}^{-1})}{x}$$

Inputs: $\frac{1}{2} g_s E \approx 0.354 \ (\text{GeV})^2$

$$\lambda \approx 2.5 \frac{\sqrt{\pi}}{\text{GeV}}$$

$$T \approx \frac{1}{\langle E_{pair} \rangle}$$

Energy per unit length $\quad k = \frac{1}{2} \mathcal{E}^2 A = 0.177 \text{ GeV}^2$ String Tension

Gauß's Law $\quad \mathcal{E} A = \frac{1}{2} g$

$$\Rightarrow \frac{1}{2} \mathcal{E} \left(\frac{1}{2} g\right) = 0.177 \text{ GeV}^2$$

Summary

1. The kinematics of small x_{Bj} requires correlations between currents at long distances on the light-cone $L(x) \approx \frac{1}{2M x_{Bj}}$ (Rest frame of proton)

2. The scattered quark can travel long distances by polarizing the vacuum

3. A possible mechanism is provided by Schwinger's mechanism: Pair creation in an external QCD field. (Calculated as the vacuum persistence probability) Variable dependence explicit!!!

4. The rise of $F_2(x)$ is related to the rise in multiplicity $n(x) = f(x) + \frac{c}{x}$

   $f(x)$: Slow varying function of x

   c: a constant

5. Structure Function: $F_2^{tot}(x, Q^2) = F_2^{pert.}(x, Q^2) + F_2(x, Q_0^2)\, p(x)$

   $p(x) = 1 - e^{-n(x, Q^2)}$

IS DATA CONSISTENT?

6. More Correlations are desirable! Spin dependence, Isospin "

For: Professor E. A. Paschos

A NON-PERTURBATIVE EFFECT IN DEEP INELASTIC SCATTERING

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ABSTRACT

The HERA data at large Q^2 and small-x investigate *large distances on the light–cone.* At such large distances the scattered quarks can maintain their colour identity, i.e. they remain colour non–singlets and confined, by polarizing the vacuum. I calculate the probability for the creation of quark–antiquark pairs from the polarized QCD vacuum and their contribution to the structure function.

1. Introduction

The new experiments at HERA [1,2] extend the kinematic regions of deep inelastic scattering to much higher values of $Q^2 \lesssim 1000$ GeV^2 and smaller values of the Bjorken scattering variable $x \gtrsim 10^{-4}$. General arguments require that the scattering involves long distances on the light–cone[3-5].

Thus the struck quark moves with relativistic velocity to distances much larger than the radius of the proton, while at the same time remains confined. The fact that it remains confined means that it is still connected to the rest of the proton. For example, the struck quark can be one member of a virtual $q - \bar{q}$ pair. As the struck quark begins to move it transfers energy to virtual quark–antiquark pairs in the vacuum, thus polarizing the vacuum. When the deposited energy is large enough, the quark and antiquark separate and form hadrons.

In this article I describe the space–time structure of the process and assume that a chromoelectric flux tube is formed between the scattered quark and the rest of the proton. Quarks in the negative energy sea of the tube gain energy from the fields in the tube and tunnel into positive energy states leaving quark holes in the Dirac sea. The transition into positive energy states signals the breaking of the tube into two disjoint parts, which we identify with hadrons.

2. Space–time structure

Deep inelastic scattering investigates the tensor

$$W_{\mu\nu}(q \cdot p, q^2) = \frac{1}{2\pi} \int d^4y \, e^{iq \cdot y} \langle p | [J_\mu(y), \, J_\nu(0)] | p \rangle \, . \qquad (1)$$

We select the momentum of the current along the 3–axis, i.e., $q_\mu = (q_0, 0, 0, q_3)$. The phase of the Fourier transform becomes stationary when

$$y_- = y_0 - y_3 \sim \pm\frac{1}{q_0 + q_3} \quad \text{and} \quad y_+ = y_0 + y_3 \sim \pm\frac{1}{q_0 - q_3}\,. \tag{2}$$

For the time-like distances of y these equations imply

$$y^2 = y_+ y_- - y_1^2 - y_2^2 \le y_+ y_- \sim \frac{1}{Q^2}\,. \tag{3}$$

The large values of Q^2 investigated at HERA require that y^2 is very close to the light–cone. The dominant contribution of the commutator near the light–cone is singular, given by

$$W_{\mu\nu}(q \cdot p,\, q^2) = s_{\mu\nu\alpha\beta} \int d^4y\, e^{iq\cdot y} \frac{\partial}{\partial y^\alpha} \Delta_F(y)\, \langle p|\bar{q}(y)\gamma^\beta q(0)|p\rangle\,. \tag{4}$$

Keeping this contribution alone and following standard techniques, it can be shown that the scaling of the structure function follows. The data, however, show that scaling violations are large which means that QCD corrections are important. The new data also show that at fixed Q^2 the structure function $F_2(x, Q^2)$ increases by a factor of two as x decreases, indicating the creation of additional quanta. One method is to generate the structure function perturbatively starting with a boundary condition at $Q^2 = \mu^2$. A popular boundary condition[6] is to postulate a valence–like gluon distribution at $\mu = 0(\Lambda)$ and calculate its development perturbatively. This approach reproduces the data.

When we look at the distance occurring in the commutator of eq. (1), however, we find that

$$y_3 = \pm\frac{1}{2}\,\frac{1}{q_0 - q_3} \approx \pm\frac{1}{2M_p x} = 2L(x)\,. \tag{5}$$

The distance $L(x)$ depends on the scaling variable and for small x becomes very large in comparison to the radius of the proton. Considering eq. (1) as the correlator of two currents on a proton, we are forced to accept that the distance between the currents is many times the radius of the proton. Viewing the process in the electron–proton center–of–mass system it means that the struck quark is still connected to the remaining proton through a long tube of chromoelectric flux. The process is shown schematically in figure 1. The virtual $q - \bar{q}$ pairs in the tube experience the chromoelectric field from which they gain energy and tunnel to positive energy states. This is a different picture than perturbative QCD which I shall try to develop.

3. Pair creation

In addition to the perturbative effects, hadrons can be produced non–perturbatively by the gluonic field. This means that in the process we must treat QED perturbatively (single photon exchange) and QCD non–perturbatively. The S-matrix for the problem is given by

$$S = \mathrm{T}\, exp \left\{ -ie \int A_\mu(y)\, j^\mu(y)\, d^4y - ig \int G^\alpha_\nu(y)\, j^{\nu,\alpha}(y)\, d^4y \right\} \tag{6}$$

with $A_\mu(y)$ and $G^\alpha_\nu(y)$ the electromagnetic and gluonic fields coupled to the corresponding currents $j^\mu(y) = \bar{q}(y)\gamma^\mu q(y)$ and $j^{\nu,\alpha}(y) = \bar{q}(y)\frac{\lambda^\alpha}{2}\gamma^\nu q(x)$, respectively. The capital T indicates the time-ordered product. When we treat QED perturbatively, but QCD to all orders, it can be proofed that for the time–ordered product the following expansion holds

$$\langle X'|S|p\rangle = \langle X'|\,\mathrm{T}\,(-ie) \int A_\mu(y) j^\mu(y) d^4y \; exp \left\{ -ig \int G^\alpha_\nu(z) j^{\nu,\alpha}(z) d^4z \right\} |p\rangle. \tag{7}$$

We have written the matrix element of the action between a proton and a final hadronic state X', but the expansion also holds at the operator level. When we expand the QCD action in powers of g and contract the fields we obtain the perturbative QCD corrections which are frequently used in analyses of deep inelastic scattering [6]. A second contribution is obtained when we keep the QCD action intact and calculate the creation of quark–antiquark pairs. This is a non–perturbative phenomenon, which cannot be produced by a finite number of interactions of the quarks in $j^{\nu,\alpha}(y)$ with the gluonic field. The problem in the case of QED was solved by Schwinger [7,8] and we shall develop the mechanism for the creation of pairs in QCD.

For our case the scattering in the c-m system is shown in figure 1. Since the distance between the initial antiquark, $\bar{q}$, and the quark, q, is very large we will approximate the volume between them by a tube of chromoelectric flux.

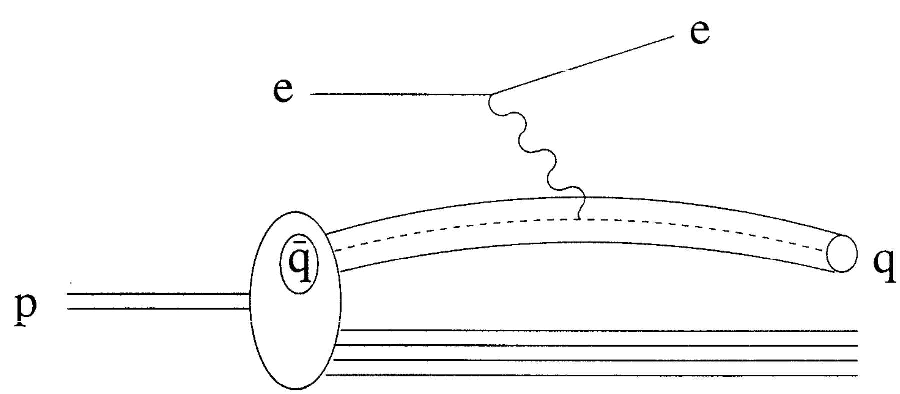

Fig. 1. The scattering of an electron on a proton with the development of the gluonic tube.

In addition to the perturbative expression for each quark distribution function, we shall show that there is now a term coming from the creation of pairs in the tube. We denote

$$q_i^{\rm tot}(x,Q^2) = q_i^{\rm pert}(x,Q^2) + q_i^0(x,Q_0^2)\, p(x,Q^2) \tag{8}$$

where $q_i^{\rm pert}(x,Q^2)$ is the distribution function obtained from perturbative QCD and $p(x,Q^2)$ is the probability of creating pairs from the vacuum. The second quark distribution, $q_i^0(x,Q^2)$, is generated by the electroweak current without the QCD corrections. We shall see, later on, that for large and intermediate values of x, $p(x,Q^2) \approx 0$ and for small x

$$lim_{x\to 0}\, p(x,Q^2) \to 1\,. \tag{9}$$

Thus the new effect manifests itself at small values of x.

We consider a potential which produces homogeneous gluoelectric and gluomagnetic fields

$$G_\mu^\alpha(y) = (0,\, y^3 gB,\, 0,\, -tgE)\,. \tag{10}$$

This is the field within the flux–tube with a chromoelectric field along the direction of the tube and a chromomagnetic field gB perpendicular to the tube. It is an Abelian field. The probability, per unit time and per unit volume that a pair is created by the fields is given by [10]

$$\omega(y,m^2) = \alpha_s \frac{|E|\,|B|}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \coth\left(n\frac{|B|}{|E|}\pi\right) e^{-\frac{nm^2\pi}{gE}} \tag{11}$$

which in the limit $n|\frac{B}{E}| \ll 1$ reduces to the Schwinger solution [7,8]

$$\omega(y,m^2) = \frac{\alpha_s E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n}\, e^{-\frac{nm^2\pi}{gE}}\,. \tag{12}$$

Modifications introduced by non–Abelian fields are discussed in ref. 10. The non–perturbative nature of these solutions is manifested in the exponential function, which has an essential singularity at $gE \to 0$. As we show below, these terms contribute to the function $p(x,Q^2)$ and brings in new dependence on the variables Q^2 and x, which originates from the length $L(x)$ and the variation of the gluonic fields with Q^2 and x.

The same problem was also solved for a finite size capacitor in order to study the finite size effects. In this case the authors solved [11] the one particle Dirac equation and obtained a solution which over most of the volume of the capacitor coincides with the Schwinger solution; deviations occur close to the plates.

We finally give a quantum mechanical interpretation of the result which demonstrates its generality. We consider a uniform electric field along the tube. The potential for the problem is shown in figure 2.

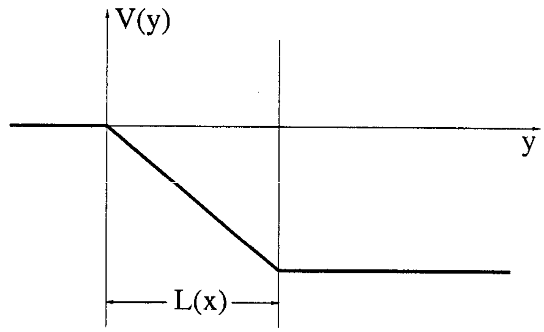

Fig. 2. The potential for a capacitator of constant electric field E and length $L(x)$.

We represent the potential

$$V(y) = \begin{cases} 0\,, & y < 0 \\ -gEy\,, & 0 < y < L \\ -gEL\,, & L < y \end{cases}$$

Consider a particle–hole pair with negative energy $-|\varepsilon|$. The probability for tunneling is given by

$$\begin{aligned} \frac{1}{\tau} &= exp\left\{-2\int_0^{y_0}\sqrt{\frac{2m}{\hbar}\left(|\varepsilon| - \frac{1}{2}gEy\right)}\,dy\right\} \\ &= e^{-\frac{4}{3}\sqrt{\frac{2m}{\hbar}}\frac{|\varepsilon|^{3/2}}{gE}} \end{aligned} \tag{13}$$

with $y_0 = \frac{2|\varepsilon|}{gE}$. We consider the emergence of a free quark as the creation of a pair.

4. Non–perturbative pair creation

In this section I attempt to develop a simple model for the creation of pairs in the tube. I consider a flux tube and partition it into small volume elements, as shown in figure 3. The probability of producing a pair in the time interval T at the volume element $\lambda^2 dy_i$ and no pairs elsewhere is

$$dP_1 = \lambda^2 T\omega(y_i)\,dy_i \prod_{K=1}^{\infty}\left(1 - \lambda^2 T\omega(y_K)\,dy_K\right)\,. \tag{14}$$

We denote the transverse dimension of the tube by λ and the time for the creation of a pair by T. These are two new parameters to be specified later on. The probability

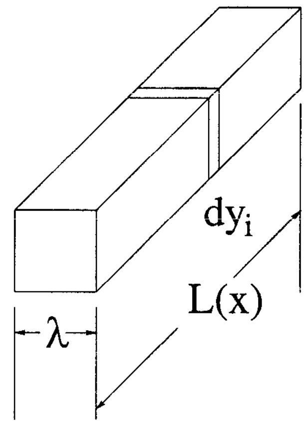

Fig. 3. A schematic drawing of the tube, where pairs are created by the gluonic field.

for producing one pair anywhere in the tube is

$$P_1 = \lambda^2 T \int_0^{L(x)} \omega(y) dy \; e^{-\lambda^2 T \int \omega(y) dy} \,. \tag{15}$$

We can generalize this result for n-pairs in the tube

$$P_n(x, Q^2) = \frac{1}{n!} \left[\lambda^2 T \int_0^{L(x)} \omega(y) dy \right]^n e^{-\lambda^2 T \int_0^{L(x)} \omega(y) dy} \,. \tag{16}$$

Finally, the sum over all possible pairs gives

$$p(x, Q^2) = \sum_{n=1}^{\infty} P_n(x, Q^2) = 1 - e^{-\lambda^2 T \int_0^{L(x)} \omega(y) dy} \,. \tag{17}$$

The functional form in eq. (16) is a Poisson distribution which follows from the property that the creation of pairs in each cell is independent of what happens in the other cells. One consequence is the multiplicity

$$n(x, Q^2) = \sum_{n=1}^{\infty} n P_n(x, Q^2) = \lambda^2 T \int_0^{L(x)} \omega(y) \, dy \tag{18}$$

which also determines the quark distribution function

$$q_i^{\text{tot}} = q_i^{\text{pert}}(x, Q^2) + q_i(x, Q_0^2) \cdot \left[(1 - e^{n(x,Q^2)}) \right] \,. \tag{19}$$

The last equation suggests a method for analysing the data. We can start with the distribution function at relatively low Q_0^2, then develop it through the Altarelli–Parisi equations to obtain $q_i^{\rm pert}(x,Q^2)$. To this we finally add the second term with $p(x,Q^2)$ calculated as described above and $q_i(x,Q_0^2)$ is the quark distribution function at a much lower value of Q^2.

5. Numerical estimates

In the previous publication [12] I considered the gluoelectric field in the tube being independent of the Q^2 and x and determined its value from the string tension determined in particle production [13]. The field created in the tube depends on the circumstances under which the tube was created. In the c–m system the energy transferred to the struck quark is

$$E_{\rm tot} = \frac{1}{\sqrt{2}}\sqrt{Q^2 x}\left(\frac{1}{\sqrt{\tilde{y}}} - \sqrt{\tilde{y}}\right) + \frac{1}{2}\sqrt{Q^2\frac{x}{\tilde{y}}} \tag{20}$$

with the inelasticity $\tilde{y} = \frac{\nu}{E_e}$ and $x = \frac{Q^2}{2M\nu}$. (Note that this $\tilde{y}$ is different from the y used in the previous part of this paper). The string tension is calculated as the energy per unit length

$$k \approx \frac{E_{\rm tot}}{L(x)} \tag{21}$$

with $L(x)$ defined in eq. (5).

Finally we obtain the product of coupling constant times of the chromoelectric field as $gE = 4k$ from

(i) the string tension $k = \frac{1}{2}E^2 A$, and

(ii) Gauss' Law : $EA = \frac{1}{2}g$

with $A = \lambda^2$ the cross–section of the tube. We see that in general the field has a complicated dependence on Q^2, x and $\tilde{y}$.

To sum up, the increase observed in $F_2(x,Q^2)$ may originate from the creation of pairs from the vacuum. Consequently the increase of $F_2^{\rm tot}(x,Q^2)$ from perturbative QCD can be relatively smaller. As a result a new analysis of the data is suggested in terms of two components: an increase from perturbative QCD and a faster increase from the creation of pairs. The limiting value of $F_2^{non-pert.}(x)$ at $x = 0$ is in the present theory finite. Summing the contributions from all the quarks we obtain

$$F_2^{total}(x,Q^2) = F_2^{pert.}(x,Q^2) + F_2(x,Q^2)(1 - e^{-n(x)}) \tag{22}$$

with $F_2^{pert.}(x,Q^2)$ the perturbative development of the structure function and $F_2(x,Q_0^2)$ the structure function at a lower value of $Q^2 \approx Q_0^2$ where perturbative corrections are not yet dominant.

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