Light-Cone Quark Model Studies of Radiative Meson Decays

Refs. (1) Nucl. Phys. **A618**, 291(1997)

(2) to be published in Phys. Rev. D

Chueng-Ryong Ji and Ho-Meoyng Choi

Department of Physics

North Carolina State University

Motivation

1. Extend the previous light-cone quark model(LCQM) predictions[1-4] of static properties for π, K, ρ, and axial-vector A_1 mesons to the various radiative meson decays.

[1] C.-R. Ji, P. L. Chung, and S. R. Cotanch, Phys. Rev. D **45**, 4214(1992).

[2] C.-R. Ji and S. R. Cotanch, Phys. Rev. D **41**, 2319(1990).

[3] Z. Dziembowski, Phys. Rev. D**37**, 778(1988).

[4] Z. Dziembowski and L. Mankiewicz, Phys. Rev. Lett.**58**, 2175(1987).

2. Recent lattice QCD results[5]: $\Delta M = M_{\eta'} - M_\eta$ increases(or decreases) as the quark mass m_q decreases(or increases), i.e., the effect of the topological charge contribution should be SMALL as m_q increases.

[5] Y. Kuramashi et.al., Phys. Rev. Lett.**72**, 3448(1994).

3. Understand the relations among the LCQM with the invarint mass(IM) scheme[6-9]:

[6] T. Huang et al., Phys. Rev. D**49**, 1490(1994).

[7] P. L. Chung et al., Phys. Lett. B**205**, 545(1988).

[8] W. Jaus, Phys. Rev. D**44**, 2851(1991).

[9] F. Schlumpf, Phys. Rev. D**50**, 6895(1994).

4. Analyze the meson mass spectra using $\delta_V = -3.3°$ for $\omega - \phi$ and $\theta_{SU(3)} = -19°$ for $\eta - \eta'$ mixing inferred from LCQM.

Outline

1. Lightcone Quark Model of Mesons.
2. What and how we calculated.
3. Old works
4. The Radiative Decays of $V(P_s) \to P_s(V)\gamma^*$, $A_1 \to \pi\gamma^*$ and $P_s \to \gamma^*\gamma$.
5. Relations among the IM Schemes
6. The Meson Mass Spectra in LCQM.
7. Conclusion and Discussion.

Lightcone Quark Model of Mesons

$$|M\rangle = \psi^M_{q\bar{q}'}|q\bar{q}'\rangle + \psi^M_{q\bar{q}'g}|q\bar{q}'g\rangle + \cdots$$

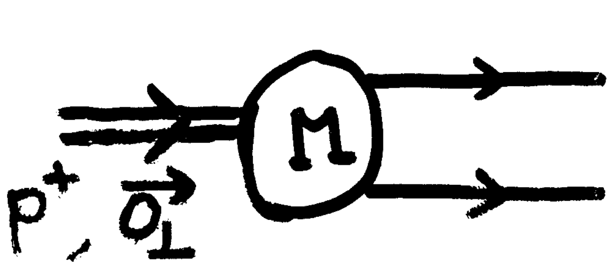
$$\approx \Psi^M_{Q\bar{Q}'}|Q\bar{Q}'\rangle$$

↑ key assumption (Mock Hadron)

where

$$|Q\rangle = \psi^Q_q|q\rangle + \psi^Q_{qg}|qg\rangle + \cdots$$

$$|\bar{Q}\rangle = \psi^{\bar{Q}}_{\bar{q}}|\bar{q}\rangle + \psi^{\bar{Q}}_{\bar{q}g}|\bar{q}g\rangle + \cdots$$



$$\left(\begin{array}{c} x_1 + x_2 = 1 \\ \vec{k}_{\perp 1} + \vec{k}_{\perp 2} = \vec{0}_\perp \end{array}\right)$$

$$\Psi^M_{Q\bar{Q}'} = \Psi_M(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$= \underbrace{\Phi(x_i, \vec{k}_{\perp i})}_{\text{Radial}} \; \underbrace{\chi_M(x_i, \vec{k}_{\perp i}, \lambda_i)}_{\text{Spin-Orbit}}$$

Radial (dependent on model potentials)

Spin-Orbit (interaction indep. Melosh transf.)

Ordinary equal-time static spin wavefunction:

$$\left(\tilde{\chi}_{M,\mu}\right)_{\mu_1,\mu_2} = \sum_{\mu_s,\mu_\ell} \sqrt{4\pi}\,|\vec{k}|^\ell\, Y_{\ell,\mu_\ell}(\hat{k}) \langle \tfrac{1}{2},\tfrac{1}{2},\mu_1,\mu_2 | s,\mu_s\rangle$$

$$\times \langle s,\ell,\mu_s,\mu_\ell | j,\mu\rangle$$

$$= \hat{\chi}^T_{\mu_1} \left(\tilde{\Gamma}_{M,\mu}\right) \hat{\chi}_{\mu_2}\,,$$

where $\hat{\chi}_{\frac{1}{2}} = \begin{pmatrix}1\\0\end{pmatrix}$ and $\hat{\chi}_{-\frac{1}{2}} = \begin{pmatrix}0\\1\end{pmatrix}$.

$$J^{PC} = 0^{-+}\ ; \quad \tilde{\Gamma}_{PS} = \frac{i}{\sqrt{2}}\sigma_2$$

$$1^{--}\ ; \quad \tilde{\Gamma}_{V,\mu} = \frac{i}{\sqrt{2}}\left[\hat{\varepsilon}(\mu)\cdot\vec{\sigma}\right]\sigma_1$$

$$1^{++}\ ; \quad \tilde{\Gamma}_{AV,\mu} = \frac{\sqrt{3}}{2}\left[\hat{\varepsilon}(\mu)\cdot(\vec{k}\times\vec{\sigma})\right]\sigma_2$$

$$\vdots$$

$$\left|\tilde{\chi}_{M,\mu}\right\rangle_t = \sum_{\mu_1,\mu_2}\left(\tilde{\chi}_{M,\mu}\right)_{\mu_1\mu_2} |\mu_1\rangle_t\, |\mu_2\rangle_t$$

$$= \left(|P_1,\uparrow\rangle_t,\ |P_1,\downarrow\rangle_t\right)\left(\tilde{\Gamma}_{M,\mu}\right)\begin{pmatrix}|P_2,\uparrow\rangle_t\\ |P_2,\downarrow\rangle_t\end{pmatrix}$$

$$|\tilde{\chi}_{M,\mu}\rangle_t = \sum_{\mu_1,\mu_2} (\tilde{\chi}_{M,\mu})_{\mu_1,\mu_2} |\mu_1\rangle_t |\mu_2\rangle_t$$

$$= \left(|p_1,\uparrow\rangle_t, |p_2,\downarrow\rangle_t \right) \left(\tilde{\Gamma}_{M,\mu} \right) \begin{pmatrix} |p_2,\uparrow\rangle_t \\ |p_2,\downarrow\rangle_t \end{pmatrix}$$

$$= \left(|p_1,\uparrow\rangle_c, |p_1,\downarrow\rangle_c \right) \underbrace{U^T \tilde{\Gamma}_{M,\mu} U}_{\equiv \, \Gamma_{M,\mu}} \begin{pmatrix} |p_2,\uparrow\rangle_c \\ |p_2,\downarrow\rangle_c \end{pmatrix},$$

where the melosh transf. U is given by

$$\begin{pmatrix} |p_i,\uparrow\rangle_t \\ |p_i,\downarrow\rangle_t \end{pmatrix} = \left(U \right) \begin{pmatrix} |p_i,\uparrow\rangle_c \\ |p_i,\downarrow\rangle_c \end{pmatrix}$$

$$\boxed{U = \begin{pmatrix} \bar{u}_c^{\uparrow} u_t^{\uparrow} & \bar{u}_c^{\downarrow} u_t^{\uparrow} \\ \bar{u}_c^{\uparrow} u_t^{\downarrow} & \bar{u}_c^{\downarrow} u_t^{\downarrow} \end{pmatrix} = \frac{1}{\sqrt{2p_i^+(p_i^0+m_i)}} \begin{pmatrix} p_i^+ + m_i & -p_i^1 - i p_i^2 \\ p_i^1 - i p_i^2 & p_i^+ + m_i \end{pmatrix}}$$

$$\chi_M(x_i, \vec{k}_{\perp i}, \lambda_i) = \bar{u}_{\lambda_1} \Gamma_M v_{\lambda_2}$$

$$J^{PC} = 0^{++} \; ; \; \Gamma_S = \frac{1}{2\sqrt{2} M_0}$$

$$0^{-+} \; ; \; \Gamma_{PS} = \frac{1}{\sqrt{2} M_0} \gamma_5$$

$$1^{-+} \; ; \; \Gamma_V = \frac{-1}{\sqrt{2}[M_0^2 - (m_1 - m_2)^2]^{1/2}} \left[\not{\epsilon} - \frac{\epsilon \cdot (k_1 - k_2)}{M_0 + m_1 + m_2} \right.$$

$$\vdots$$

$$M_0^2 = \sum_{i=1}^{2} \frac{m_i^2 + \vec{k}_{\perp i}^2}{x_i}$$

$$\Phi(x_i, k_{\perp i})$$

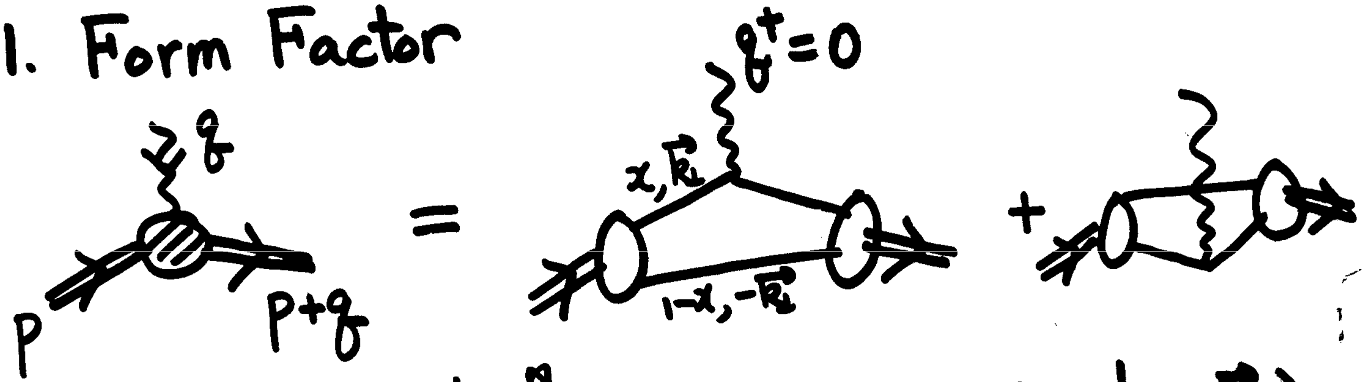
$$= \begin{cases} \sim \exp\left[-\frac{M_0^2}{8\beta^2}\right] & \text{Brodsky, Huang, Lepage; Huang, Ma, Shen} \\ \sim \sqrt{\frac{M_0}{x_1 x_2}} \exp\left[-\frac{M_0^2}{8\beta^2}\right] & \text{Chung, Coester, Polyzou; Jaus} \\ \sim \left(\frac{\beta^2}{M_0^2}\right)^n & \text{Schlumpf; Cardarelli, Salmè, ..., Pace} \\ \vdots & \vdots \end{cases}$$

Not too many parameters to fiddle around

$m_q\,(q = u, d)$, m_s, $\beta_{q\bar{q}}$, $\beta_{s\bar{s}}$ | Spin-Averaged Meson Masses, Dziembowski, Weber, ...

What and How we calculated

1. Form Factor

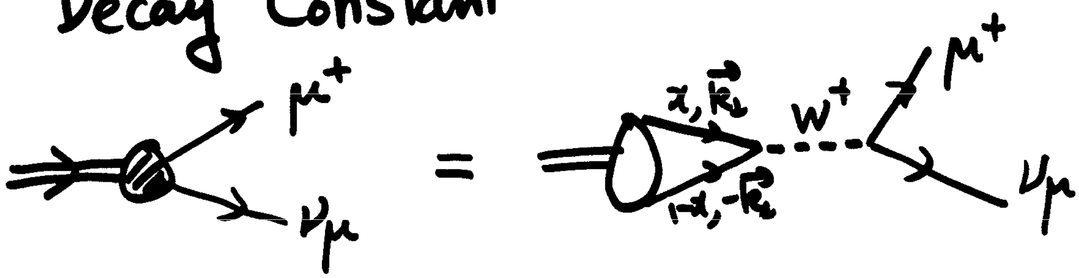


$$F_M(Q^2) = e_1 \int_0^1 dx \int_{-\infty}^{\infty} \frac{d^2\vec{k}_\perp}{16\pi^3} \psi^*(x, \vec{k}_\perp + (1-x)\vec{q}_\perp)\, \psi(x, \vec{k}_\perp)$$
$$+ \; (1 \leftrightarrow 2, \; x \leftrightarrow 1-x)$$

2. Charge Radius

$$\langle r^2 \rangle_M = -6 \left. \frac{d F_M(Q^2)}{d Q^2} \right|_{Q^2=0}$$

3. Decay Constant



$$f_M = 2\sqrt{3} \int_0^1 dx \int_{-\infty}^{\infty} \frac{d^2\vec{k}_\perp}{16\pi^3} \psi(x, \vec{k}_\perp)$$

4. Radiative Decay Width

$$\Gamma(A \to B\gamma) = \frac{\alpha}{2S_A+1} |G_{AB}(0)|^2 \left(\frac{M_A^2 - M_B^2}{2M_A} \right)^3 .$$

Old Works

CRJ & S. R. Cotanch, Phys. Rev. D41, 2319 ('90).

$\langle r^2 \rangle_{\pi^+}$, f_π, $\langle r^2 \rangle_{K^+}$, $\langle r^2 \rangle_{K^0}$, f_K, $\langle r^2 \rangle_{D^+}$, f_D, F_K

CRJ, P. L. Chung & S. R. Cotanch, Phys. Rev. D45, 4214 ('92).

A_1, $\langle \xi^2 \rangle$, $\langle \xi^4 \rangle$, $\langle \xi^6 \rangle$, $\cdots$

Numerical Results

Table I. Summary of numerical results for the static properties of various mesons

β(MeV)	300	320	340	360	Data
$\langle r^2\rangle_{\pi^+}$ (fm^2)	0.44	0.41	0.38	0.36	0.44 ± 0.05
f_π(MeV)	97	93	88	82	93
$\langle r^2\rangle_{K^+}$	0.41	0.38	0.35	0.33	0.34 ± 0.05
$\langle r^2\rangle_{K^0}$	−0.055	−0.050	−0.046	−0.042	−0.054 ± 0.026
f_K	121	122	121	120	113
$\langle r^2\rangle_{D^+}$	0.32	0.29	0.26	0.24	
f_D	112	122	131	141	< 183

Input: Constituent-quark masses;

$m_u = m_d = 330$ MeV

$m_s = 450$ MeV

$m_c = 1.5$ GeV

Spin-averaged Meson masses;

$M_\pi = \left(\frac{1}{4} m_\pi + \frac{3}{4} m_\rho\right)_{expt} = 612.4$ MeV

$M_K = 792.5$ MeV

$M_D = 1.9749$ GeV.

	$\langle r^2\rangle_\pi^{1/2}$ (fm)	f_π (MeV)	μ_p	μ_n	$\langle r^2\rangle_p^{1/2}$	$\langle r^2\rangle_n$	g_A/g_V
Model	0.64	93	2.80	-1.73	0.83	-0.15	1.20
Expt.	0.66	93	2.793	-1.913	0.84	-0.34	1.23

Dziembowski

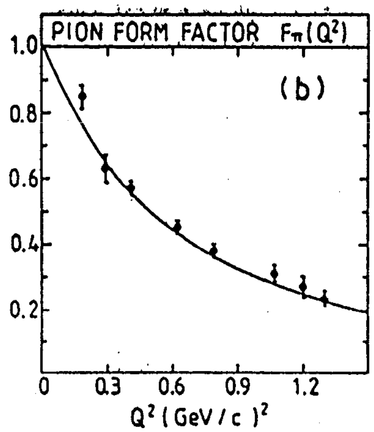


FIG. 1. (a) Pion form factor calculated in the present work with the pion wave function of Eq. (5); m_{const}=330 MeV, β=320 MeV. The data are from Ref. 22. (b) Pion form factor calculated in the present work with the pion wave function of Eq. (5); m_{const}=330 MeV, β=320 MeV. The data are from Ref. 28.

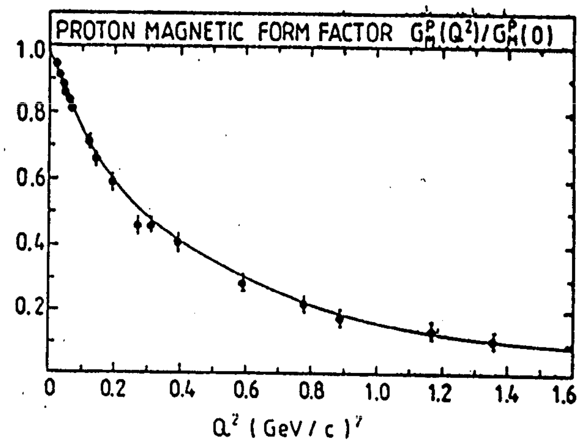
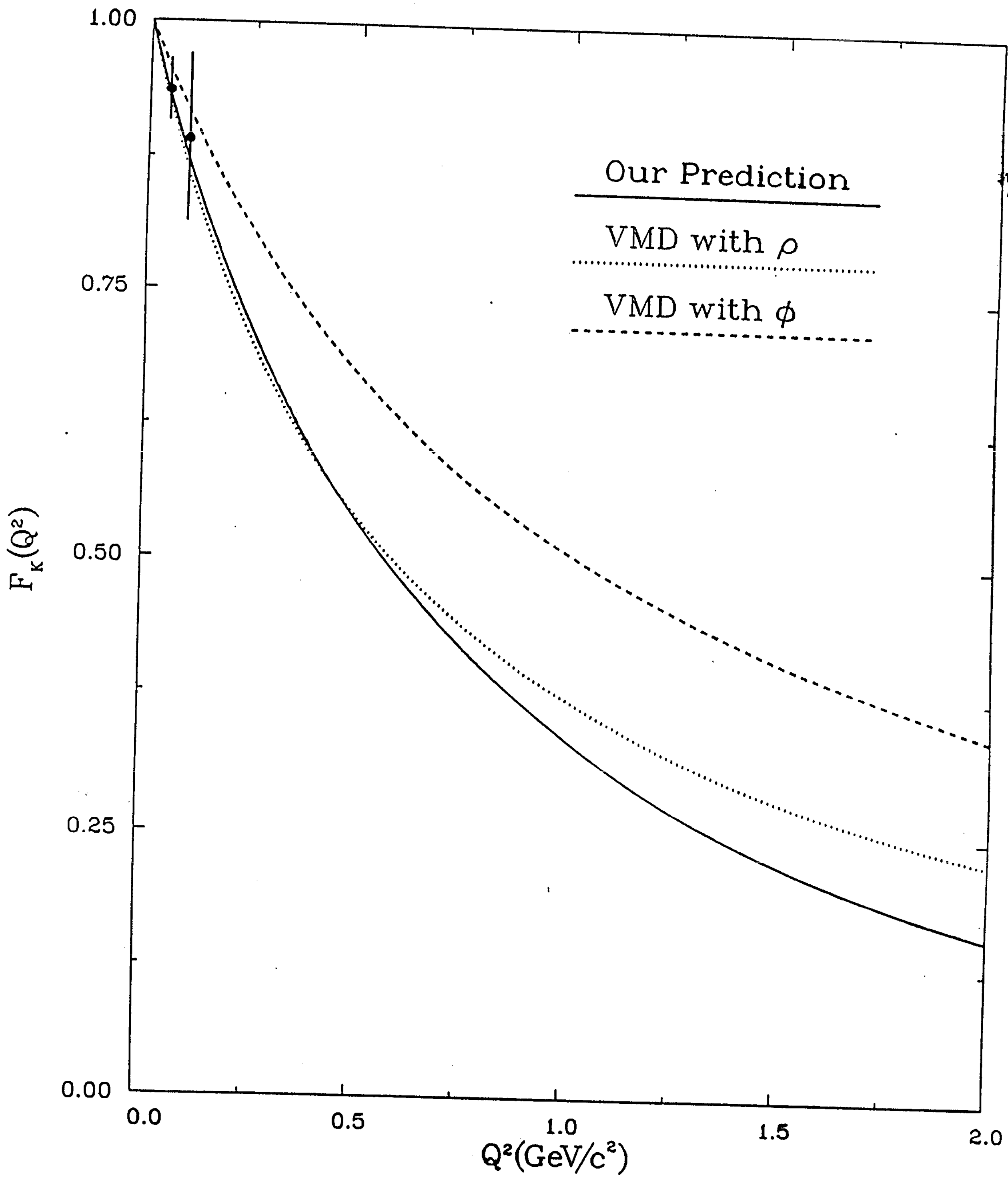


FIG. 2. (a) Proton magnetic form factor calculated in the present work with the nucleon wave function of Eq. (5); m_{const}=330 MeV, α=320 MeV. The data are from Ref. 29.

In this talk, we apply this model to flavored pseudoscalar meson in which the constituent quarks have unequal masses, i.e. $Q\bar{q}$, where Q is s or c and q is u or d.

We present numerical predictions for static properties of K and D mesons.

FIGURE 1


1.00
0.75
0.50
0.25
0.00
0.0
0.5
1.0
1.5
2.0
$F_K(Q^2)$
$Q^2(GeV/c^2)$
Our Prediction
VMD with ρ
VMD with ϕ

IV. The Radiative Decays of $V(P_S) \to P_S(V)\gamma^*$, $A_1 \to \pi\gamma^*$ and $P_S \to \gamma^*\gamma$

The flavor assignment of η and η' mesons in the $q\bar{q}$ basis are as follows:

$$\eta = -\sin\delta_{P_S}\frac{(u\bar{u}+d\bar{d})}{\sqrt{2}} - \cos\delta_{P_S}s\bar{s},$$
$$\eta' = \cos\delta_{P_S}\frac{(u\bar{u}+d\bar{d})}{\sqrt{2}} - \sin\delta_{P_S}s\bar{s}, \tag{4.1}$$

where $\delta_{P_S} = \theta_{SU(3)} - \theta_{ideal} \approx \theta_{SU(3)} - 35.3°$.

TABLE II. Mixing angles for η and η' and the corresponding spin averaged masses.

$\theta_{SU(3)}$	-sin δ_{P_S}	cos δ_{P_S}	m_η[MeV]	$m_{\eta'}$[MeV]
-10°	$\sqrt{1/2}$	$\sqrt{1/2}$	843	884
-23°	0.85	0.53	834	873

The transition form factors of $A \to B\gamma^*$, $A_1 \to \pi\gamma^*$, and $P_S \to \gamma^*\gamma$

$$< B(P')|J^\mu|A(P,\lambda) > \; = eG_{AB}(Q^2)\varepsilon^{\mu\nu\alpha\beta}\epsilon_\nu(P,\lambda)P'_\alpha P_\beta, \tag{4.2}$$
$$< \pi(P')|J^\mu|A_1(P,\lambda) > \; = \frac{e}{m_{A_1}}\Big[(\mathcal{P}\cdot q g^{\mu\nu} - \mathcal{P}^\mu q^\nu)G_1(Q^2) + \frac{1}{m_{A_1}^2}(\mathcal{P}\cdot q q^\mu - q^2\mathcal{P}^\mu)q^\nu G_2(Q^2)\Big]\,\epsilon_\nu(P,\lambda), \tag{4.3}$$
$$< \gamma(P+q)|J_\mu|P_S(P) > \; = ie^2 G_{P_S\gamma}(Q^2)\epsilon_{\mu\nu\rho\sigma}P^\nu\epsilon^\rho q^\sigma, \tag{4.4}$$

and the corresponding decay widths are given by

$$\Gamma(A \to B\gamma) = \frac{\alpha}{2S_A+1}|G_{AB}(0)|^2(\frac{M_A^2 - M_B^2}{2M_A})^3, \tag{4.5}$$
$$\Gamma(A_1 \to \pi\gamma) = \frac{4\alpha}{3}|\frac{G_1(0)}{M_{A_1}}|^2(\frac{M_{A_1}^2 - M_\pi^2}{2M_{A_1}})^3, \tag{4.6}$$
$$\Gamma(P_S \to \gamma\gamma) = \frac{\pi}{4}\alpha^2|G_{P_S\gamma\gamma}(0)|^2 M_{P_S}^3, \tag{4.7}$$

where α is the fine structure constant, S_A is the spin of the initial particle.

TABLE II. Decay widths for $V \to P_S\gamma$, $P_S \to V\gamma$, $A_1 \to \pi\gamma$ and $P_S \to 2\gamma$ with $P_S = \pi, K, \eta, \eta'$ and $V = \rho, K^*, \omega, \phi$ for various model parameters β

β[GeV]	0.32	0.34	0.36	0.38	ST[a]	Experiment[b]
$\Gamma(\rho^\pm \to \pi^\pm\gamma)$	78	73	69	64		68 ± 8[keV][c]
$\Gamma(\omega \to \pi\gamma)$	775	742	708	674		717 ± 51
$\Gamma(K^{*\pm} \to K^\pm\gamma)$	60	57	53	50		50 ± 5
$\Gamma(K^{*0} \to K^0\gamma)$	134	128	122	116		117 ± 10
$\Gamma(\rho \to \eta\gamma)^d$	66[77]	60[70]	56[65]	51[60]	40	58 ± 10
$\Gamma(\omega \to \eta\gamma)$	7.4[8.5]	6.9[7.9]	6.4[7.4]	6.0[6.8]	4.6	7.0 ± 1.8
$\Gamma(\eta' \to \rho\gamma)$	137[89]	126[80]	117[72]	108[66]	144	61 ± 8
$\Gamma(\eta' \to \omega\gamma)$	11.2[7.3]	10.4[6.6]	9.7[6.0]	9.1[5.6]	12.0	5.9 ± 0.9
$\Gamma(\phi \to \eta\gamma)$	54[40]	58[42]	61[45]	65[47]	71	56.9 ± 2.9
$\Gamma(\phi \to \eta'\gamma)$	0.26[0.43]	0.27[0.44]	0.28[0.45]	0.29[0.46]	0.23	< 1.8
$\Gamma(A_1 \to \pi\gamma)$	620	664	705	742		640 ± 246 [46,47]
$\Gamma(\pi^0 \to 2\gamma)$	7.58	7.06	6.50	5.91		7.8 ± 0.5[eV]
$\Gamma(\eta \to 2\gamma)$	0.61[0.78]	0.53[0.71]	0.47[0.65]	0.42[0.58]	0.44	0.47 ± 0.05
$\Gamma(\eta' \to 2\gamma)$	8.8[6.5]	8.3[6.1]	7.9[5.6]	7.3[5.1]	9.0	4.3 ± 0.6

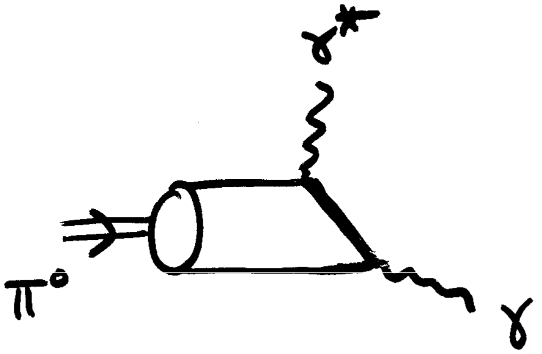
[a] ST = standard mixing($\theta_{SU(3)} = 0^o$) for $\beta = 0.36$ GeV.

[b] From Ref. [48], unless otherwise noted.

[c] The unit of decay width is [keV], unless otherwise noted.

[d] The values are the result from $\theta_{SU(3)} = -10^o[-23^o]$ mixing scheme.

H.M. Choi & CRJ, Nucl. Phys. A618, 291 ('97).



large Q^2. $\sim \frac{1}{Q^2}$

and no angular condition.

Fig.6

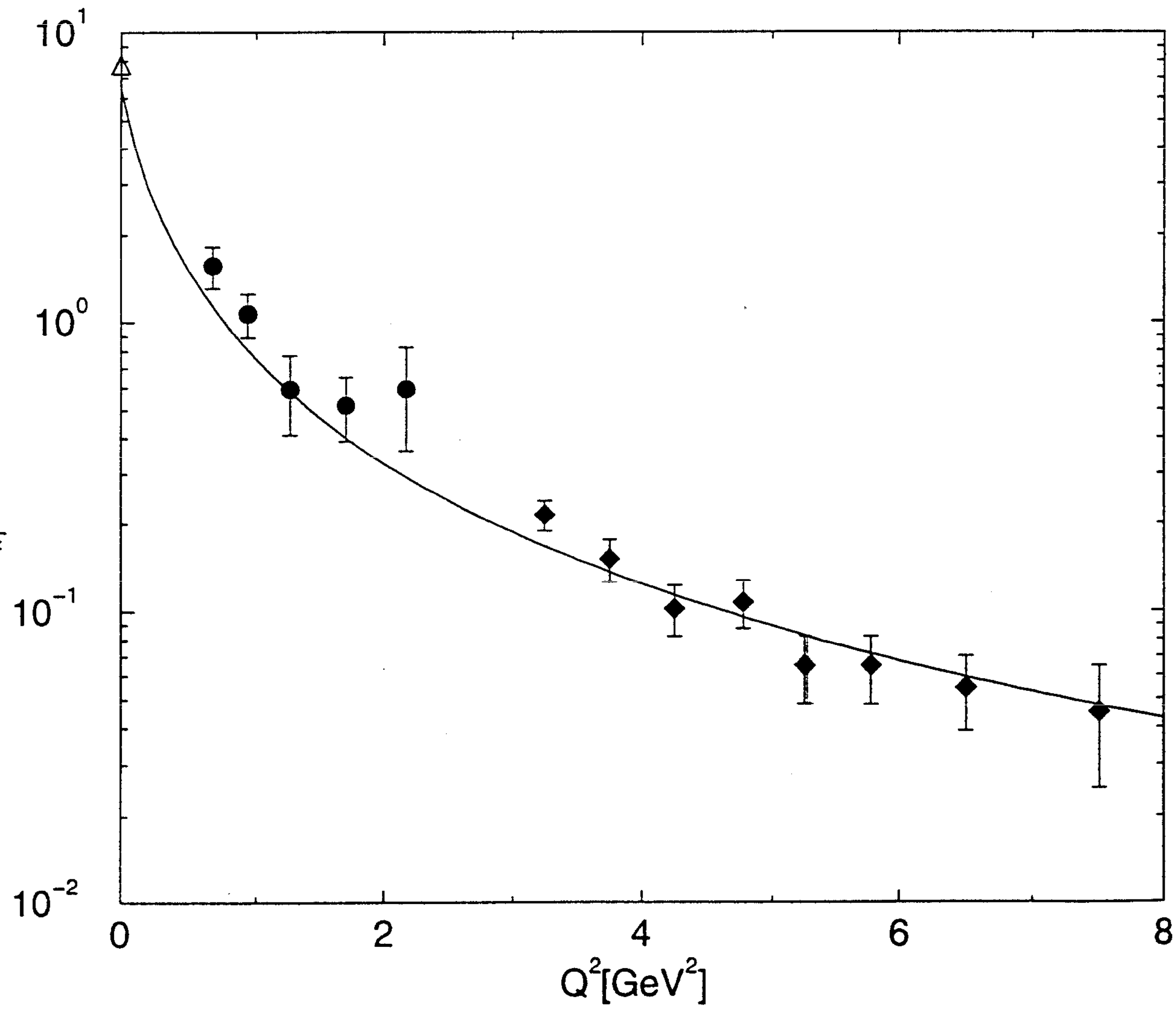
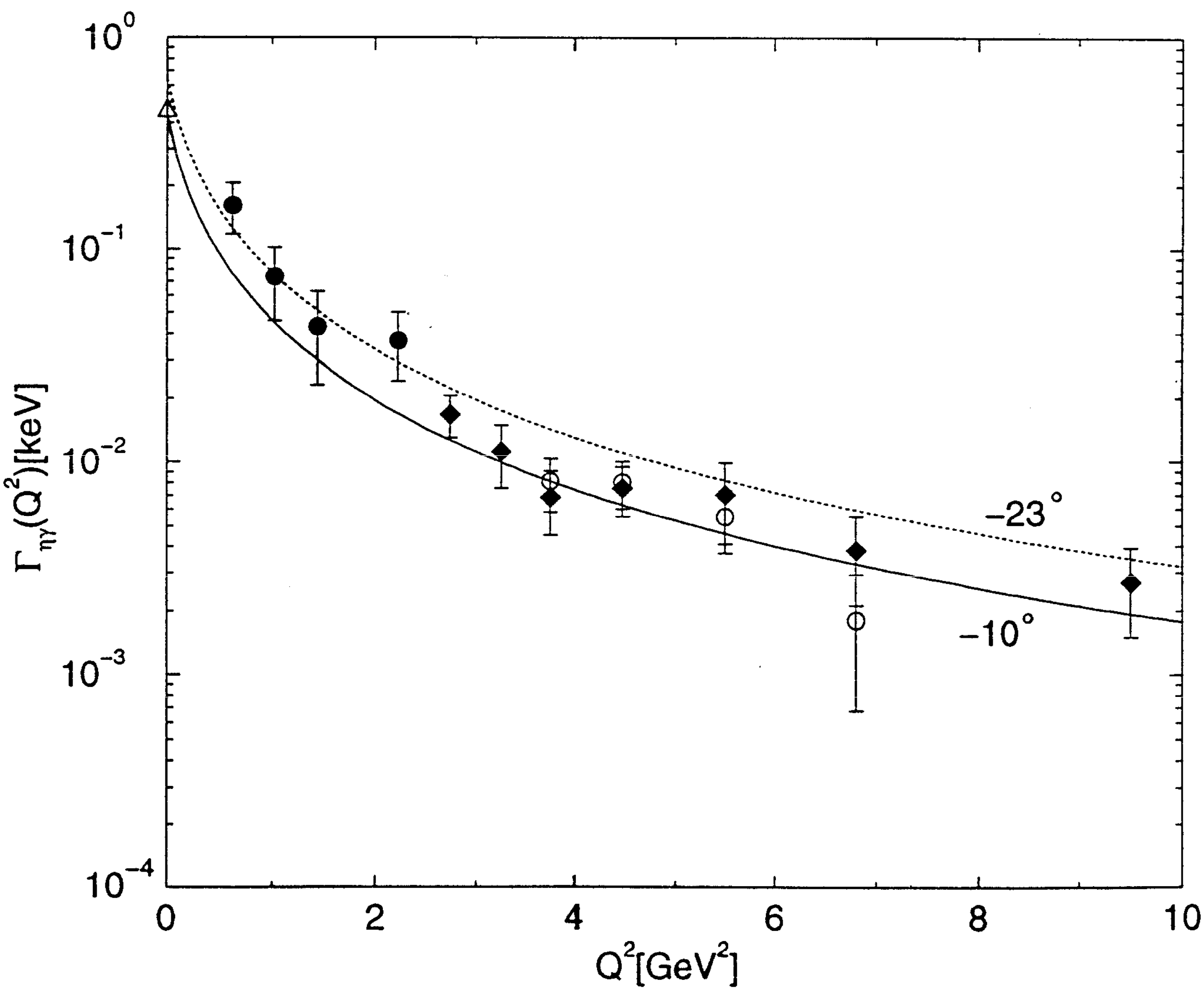
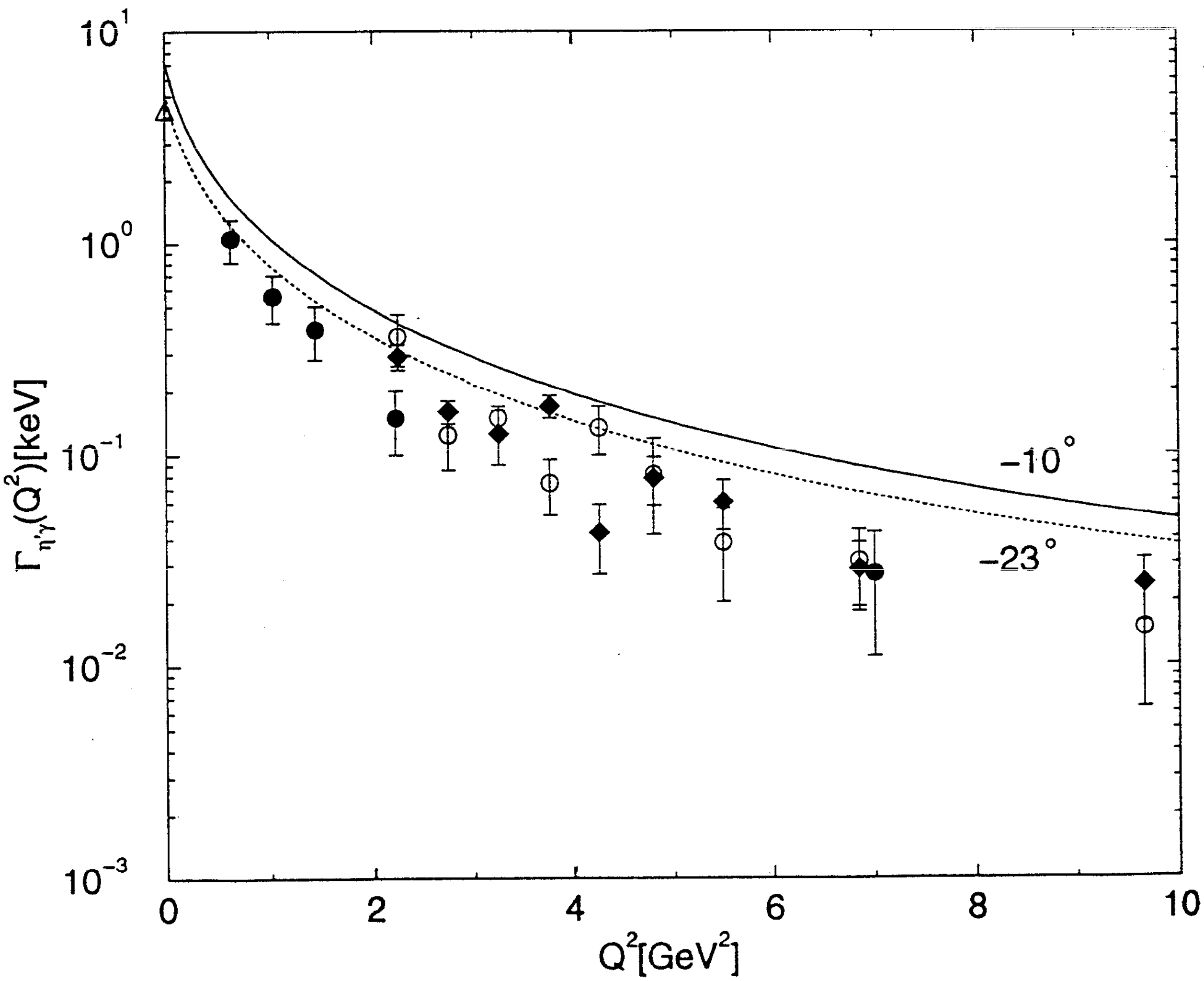


Fig.7



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\Gamma_{\eta\gamma}(Q^2)$[keV]
$-23°$
$-10°$
0
2
4
6
8
10
Q^2[GeV^2]

Fig.8



10^1
10^0
10^{-1}
10^{-2}
10^{-3}
$\Gamma_{\eta'\gamma}(Q^2)$[keV]
0
2
4
6
8
10
Q^2[GeV^2]
$-10°$
$-23°$

V. Relations among the IM Schemes

- **Model Description**

(A) Harmonic Oscillator(HO) wavefunction[6-8]

$$\Phi^H(k^2) = N_H \exp\Big[-\sum_{i=1}^{2} \frac{\mathbf{k}_{i\perp}^2 + m_i^2}{x_i}/8\beta^2\Big] \quad : \text{model H} \tag{5.1}$$

$$\phi^{C(J)}(k^2) = N_{C(J)} \exp(-\mathbf{k}^2/2\beta^2) : \text{model C(J)} \tag{5.2}$$

where $\mathbf{k} = (k_n, \mathbf{k}_\perp)$ is the three momentum and $N_C = (4/\sqrt{\pi}\beta^3)^{1/2}$ and $N_J = \pi\sqrt{2/3}N_C$, respectively. The model C and model J are related by

$$\phi^J(k^2) = \pi\sqrt{\frac{2}{3}}\phi^C(k^2), \tag{5.3}$$

(B) Power-law(PL) wavefunction[9]

$$\phi^{PL}(k^2) = N_{PL}(1 + \mathbf{k}^2/\beta^2)^{-2} \tag{5.4}$$

- **What makes DIFFERENCE between H and C(J) ?**

Definition of the normalization of the wave functions $\Phi^H(k^2)$ and $\phi^C(k^2)$:

$$\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}|\Phi^H(k^2)|^2 = 1,$$
$$\frac{1}{4\pi}\int d^3k|\phi^C(k^2)|^2 = 1 \tag{5.5}$$

Using the jacobian $\mathcal{J}$ of the variable transformation $\{x, \mathbf{k}_\perp\} \to \mathbf{k} = (k_3, \mathbf{k}_\perp)$,

$$\mathcal{J} = \frac{\partial k_3}{\partial x} = \frac{M_0}{4x(1-x)} \tag{5.6}$$

$x = \frac{1}{2}\left(1 + \frac{k_3}{\sqrt{k^2+m^2}}\right)$

The phase space $[d^3k]$ is transformed into

$$d^3k = dk_3 d^2\mathbf{k}_\perp = dx d^2\mathbf{k}_\perp \frac{M_0}{4x(1-x)}. \tag{5.7}$$

then

$$\frac{1}{4\pi}\int d^3k|\phi^C(k^2)|^2 = \frac{1}{4\pi}\int_0^1 dx \int d^2\mathbf{k}_\perp \frac{M_0}{4x(1-x)}|\phi^C(k^2)|^2 = 1. \tag{5.8}$$

From Eqs.(5.3) and (5.8), we obtain the RELATION between the model H and C(J)

$$\Phi^H(k^2) = \pi\sqrt{\frac{M_0}{x(1-x)}}\phi^C(k^2)$$
$$= \sqrt{n_c}\sqrt{\frac{M_0}{2x(1-x)}}\phi^J(k^2). \tag{5.9}$$

ONLY IF

$$N_H = N_H(x, \mathbf{k}_\perp) = \pi\sqrt{\frac{M_0}{x(1-x)}}(\frac{4}{\sqrt{\pi}\beta^3})^{1/2}\exp(m^2/2\beta^2), \tag{5.10}$$

•The EFFECT of the PRESENCE-ABSENCE of $\mathcal{J}$

TABLE V. Best fit quark masses and the model parameters β. PLJ and PLH are the power law(PL) models with and without Jacobi factor, respectively. q=u and d.

	H	J	PLH	PLJ
m_q[GeV]	0.25	0.25	0.28	0.28
m_s[GeV]	0.37	0.37	0.37	0.37
$\beta_{q\bar{q}}$[GeV]	0.36	0.3194	0.40	0.307
$\beta_{s\bar{s}}$[GeV]	0.38	0.3478	0.415	0.328

TABLE VI. Decay constants for $\pi \to \mu\nu$ and $V \to e^+e^-$, where the mixing angle $\delta_V = -3.3°$ is used. The results without Jacobi factor are included in square bracket to see the effect of the presence-absence of the Jacobi factor.

$f_{P(V)}$	H	J	PLH	PLJ	Experiment[MeV]
f_π	92.1	92.4[86.5]	92.8	92.5[79.2]	92.4±0.25
f_ρ	156.1	151.9[138.5]	155.5	154.7[120.1]	152.8± 3.6
f_ω	47.5	46.1[42.0]	47.3	47.1[43.4]	45.9± 0.7
f_ϕ	80.2	79.7[73.0]	79.4	79.6[62.3]	79.1± 1.3

$$\Gamma_{P_s \to \gamma\gamma} = \frac{\pi}{4} \alpha^2 g^2_{P_s \gamma\gamma} m^3_{P_s}$$

Axial-vector anomaly + PCAC.

Adler; Bell & Jackiw; Gasser & Leutwyler; Donoghue, Holstein & Lin; Itzykson & Zuber;

$$g_{\pi\gamma\gamma} = \frac{1}{4\pi^2 f_\pi}$$

$$g_{\eta\gamma\gamma} = \frac{1}{4\pi^2\sqrt{3}} \left[\frac{1}{f_8} \cos\theta_{SU(3)} - \frac{2\sqrt{2}}{f_0} \sin\theta_{SU(3)} \right]$$

$$g_{\eta'\gamma\gamma} = \frac{1}{4\pi^2\sqrt{3}} \left[\frac{1}{f_8} \sin\theta_{SU(3)} + \frac{2\sqrt{2}}{f_0} \cos\theta_{SU(3)} \right]$$

where

$$f_8 = (f_\pi + 2 f_{s\bar{s}})/3$$

$$f_0 = (2 f_\pi + f_{s\bar{s}})/3$$

$f_{s\bar{s}} = 112.63$ MeV	χPT.
$f_8/f_\pi = 1.148$	1.25
$f_0/f_\pi = 1.074$	1.04 ± 0.04

$\Gamma(P_s \to \gamma\gamma)$	π (eV)	η (keV)	η' (keV)
$\theta_{SU(3)} = -10°$	7.79	0.30	5.53
$-19°$	7.79	0.49	4.51
$-23°$	7.79	0.58	4.02
Jaus $(-19°)$	7.73	0.49	4.45
Expt.	7.8 ± 0.5	0.47 ± 0.05	4.3 ± 0.6

TABLE VII. Radiative decay widths for $V(P_S) \to P_S(V)\gamma$ transitions.

Widths	H	J	PLH	PLJ	Experiment[keV]
$\Gamma(\rho^\pm \to \pi^\pm\gamma)$	75	76	89	97	68 ± 8
$\Gamma(\omega \to \pi\gamma)$	712	730	855	938	717 ± 51
$\Gamma(\phi \to \pi\gamma)$	5.5	5.6	6.6	7.2	5.8 ± 0.6
$\Gamma(\rho \to \eta\gamma)$	59	59	69	76	58 ± 10
$\Gamma(\omega \to \eta\gamma)$	8.6	8.7	10.2	11.1	7.0 ± 1.8
$\Gamma(\phi \to \eta\gamma)$	55.9	55.3	72.4	74.2	56.9 ± 2.9
$\Gamma(\eta' \to \rho\gamma)$	66.1	67.5	79.1	86.8	61 ± 8
$\Gamma(\eta' \to \omega\gamma)$	4.7	4.8	5.5	6.1	5.9 ± 0.9
$\Gamma(\phi \to \eta'\gamma)$	0.56	0.57	0.71	0.76	< 1.8

TABLE VIII. Radiative decay widths $\Gamma(P_S \to \gamma\gamma)$ for $\eta - \eta'$ mixing angle $\theta_{SU(3)}$=-19°. The results are obtained from the axial-vector anomaly plus PCAC.

Widths	H	J	PLH	PLJ	Experiment
$\Gamma(\pi \to \gamma\gamma)$	7.79	7.73	7.67	7.72	7.8 ± 0.5[eV]
$\Gamma(\eta \to \gamma\gamma)$	0.49	0.485	0.52	0.52	0.47 ± 0.05[keV]
$\Gamma(\eta' \to \gamma\gamma)$	4.51	4.45	4.64	4.68	4.3 ± 0.6[keV]

Fig.1a

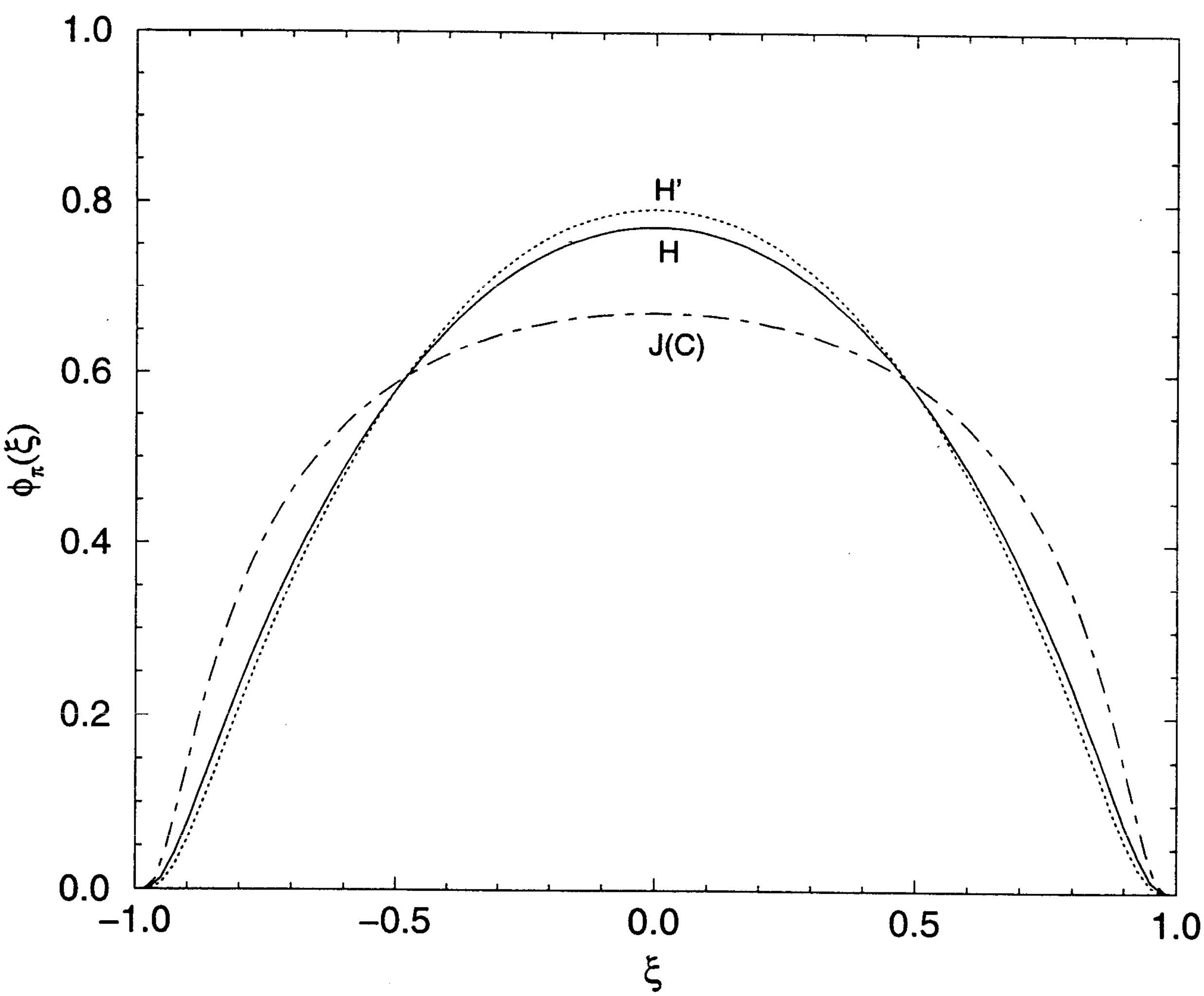


Fig.1b

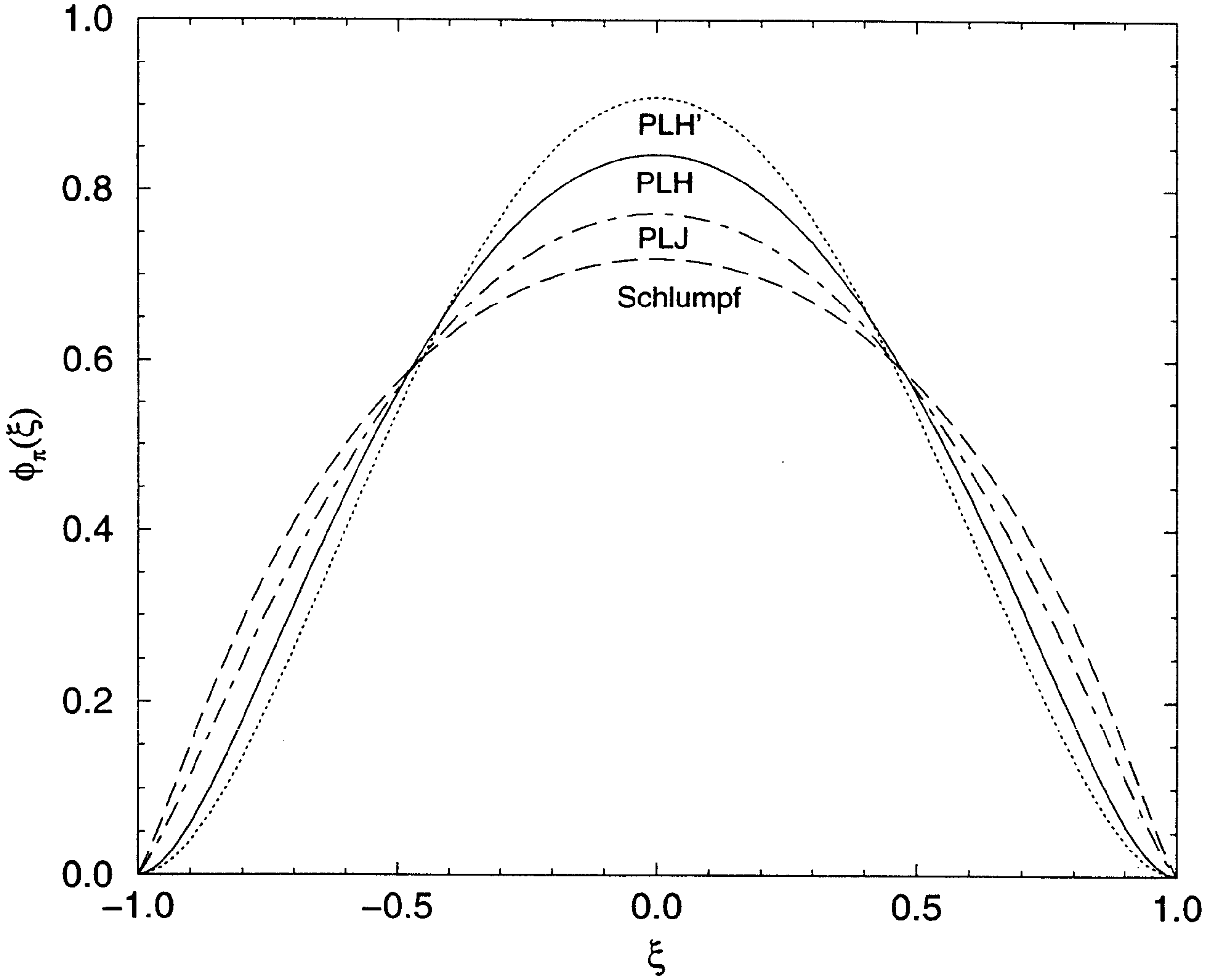
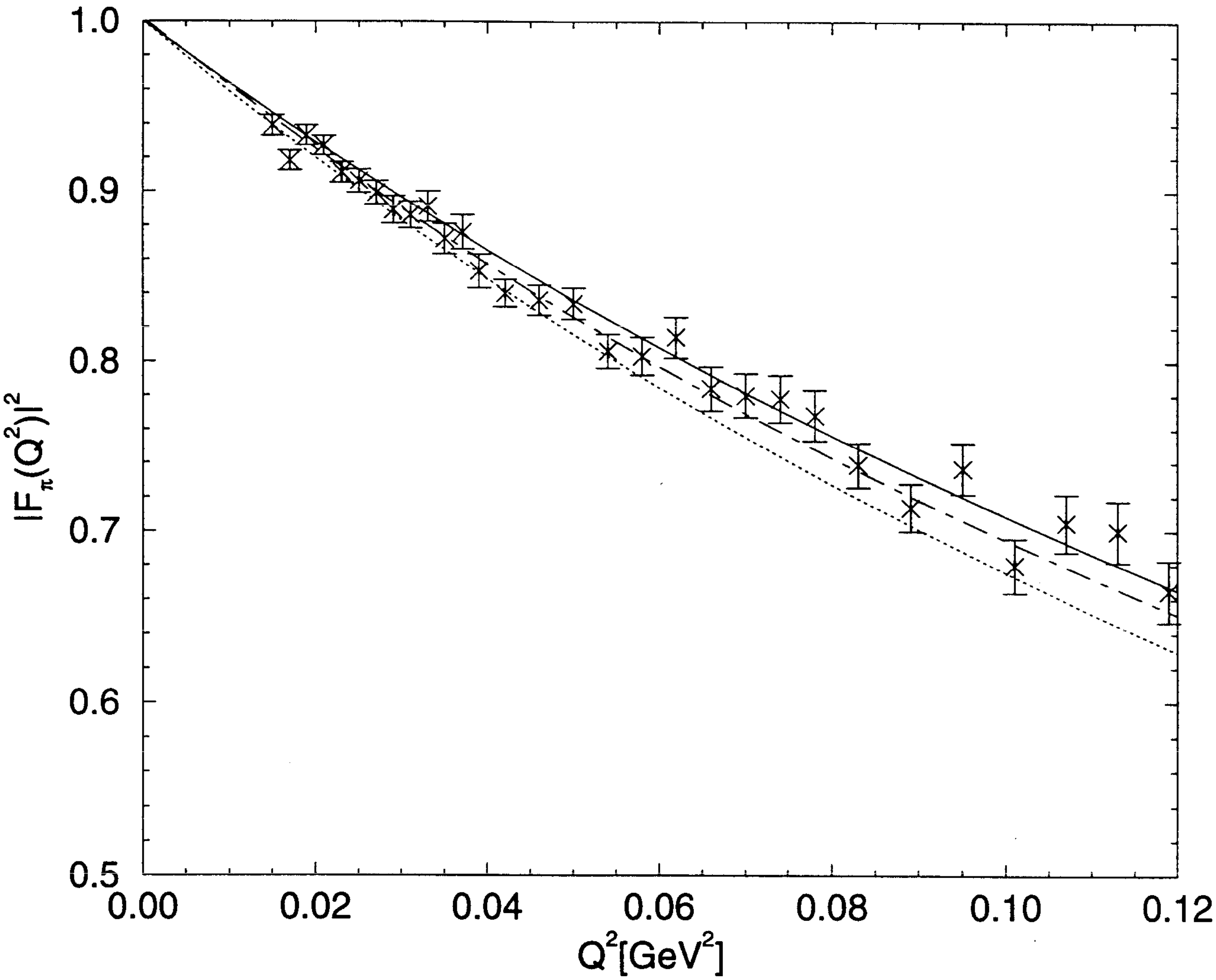
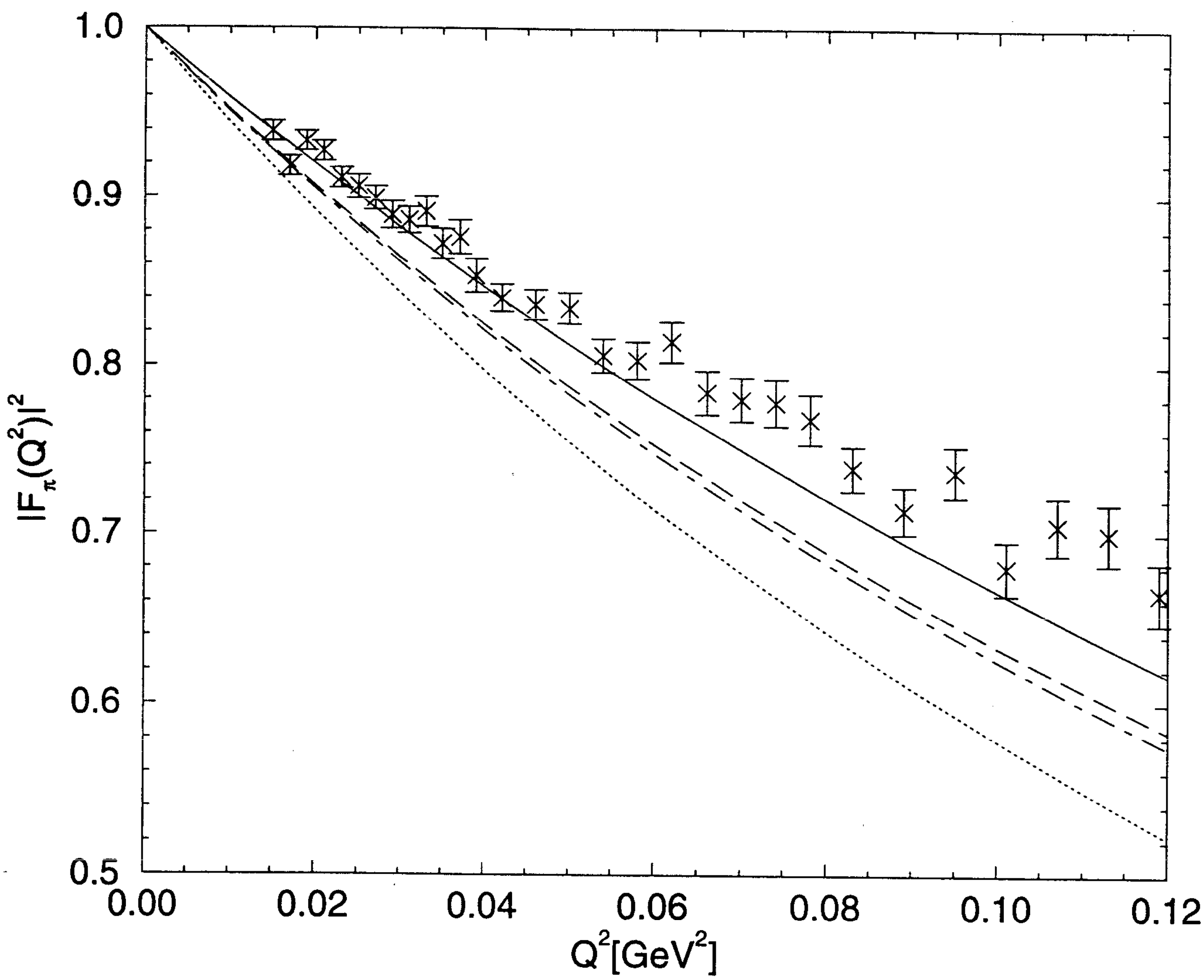


Fig.2a



1.0
0.9
0.8
0.7
0.6
0.5
$|F_\pi(Q^2)|^2$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
$Q^2[\mathrm{GeV}^2]$

Fig.2b



VI. The Meson Mass Spectra in LCQM

Our goal: Obtain meson mass spectra from (1) quark-annihilation diagrams accounting for the SU(3) breaking + (2) empirical mixing angles $\delta_V = -3.3°$)(or $\theta_{SU(3)} = 31.96°$) for $\omega-\phi$ and $\delta_P = -54.26°$(or $\theta_{SU(3)} = -19°$) for $\eta - \eta'$ mixing inferred from LCQM.

(1) Quadratic mass formula accounting for SU(3) breaking.

The generic two particle (f_1, f_2) mixing is given by

$$
\begin{aligned}
|f_1\rangle &= -\sin\delta|n\bar{n}\rangle - \cos\delta|s\bar{s}\rangle \\
|f_2\rangle &= \cos\delta|n\bar{n}\rangle - \sin\delta|s\bar{s}\rangle,
\end{aligned}
\tag{6.1}
$$

where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and and $\delta = \theta_{SU(3)} - 35.26°$. We shall identify (f_1, f_2) with (ϕ, ω) and (η, η'). These combinations satisfy the quadratic mass eigenvalue equation:

$$\mathcal{M}^2|f_i> \; = M_i^2|f_i> \quad (i = 1, 2), \tag{6.2}$$

SU(3) breaking.

with

$$\mathcal{M}^2 = \begin{pmatrix} M_{n\bar{n}}^2 + 2\lambda & \sqrt{2}\lambda X \\ \sqrt{2}\lambda X & M_{s\bar{s}}^2 + \lambda X^2 \end{pmatrix}. \tag{6.3}$$

$M^2_{q\bar{q}} = M^2_{0\,q\bar{q}} + 4m_q V_{q\bar{q}}$

$V_{q\bar{q}} = V_{S.H.O.} + V_{S-S}$

where λ: $u\bar{u} \to u\bar{u}(d\bar{d})$, λX: $u\bar{u} \to s\bar{s}$(or $s\bar{s} \to u\bar{u}$) and λX^2: $s\bar{s} \to s\bar{s}$.

Solving Eqs.(6.1)-(6.3), we obtain

$$\tan^2\delta = \frac{M_{f_2}^2 - \epsilon M_{f_1}^2}{\epsilon M_{f_2}^2 - M_{f_1}^2}, \tag{6.4}$$

where $\epsilon = (M_{n\bar{n}}^2 + 2\lambda)/(M_{s\bar{s}}^2 + \lambda X^2)$. The invariance of the trace of quadratic mass matrix requires

$$M_{n\bar{n}}^2 + M_{s\bar{s}}^2 + 2\lambda + \lambda X^2 = M_{f_1}^2 + M_{f_2}^2. \tag{6.5}$$

$$M_{f_1}^2 = (M_{n\bar{n}}^2 + 2\lambda)\sin^2\delta + 2\sqrt{2}\lambda X \sin\delta\cos\delta + (M_{s\bar{s}}^2 + \lambda X^2)\cos^2\delta$$

$$M_{f_2}^2 = (M_{n\bar{n}}^2 + 2\lambda)\cos^2\delta - 2\sqrt{2}\lambda X \sin\delta\cos\delta + (M_{s\bar{s}}^2 + \lambda X^2)\sin^2\delta$$

(2) Effective potential in LCQM

The effective Hamiltonian for the description of the meson mass spectra is defined by

$$H_{q\bar{q}} = \sqrt{m_q^2 + k^2} + \sqrt{m_{\bar{q}}^2 + k^2} + V_{q\bar{q}}, \tag{6.6}$$

and the mass-squared operator is given by

$$M_{\rm op}^2 = 4(m^2 + k^2 + mV_{q\bar{q}}), \tag{6.7}$$

The effective potential is

$$\begin{aligned} V_{q\bar{q}} &= V_{S.H.O} + V_{S-S} \\ &= A + Br^2 + \frac{\vec{S}_q \cdot \vec{S}_{\bar{q}}}{m_q m_{\bar{q}}} \nabla^2 V_{Coul}, \end{aligned} \tag{6.8}$$

where $\nabla^2 V_{Coul} = \nabla^2(-\kappa/r) = 4\pi\kappa\delta^3(r)$.

$\frac{\partial}{\partial\beta}\langle M_0^2 + 4mV_{S.H.O}\rangle = 0.$

Using the ground-state S-wave variational wavefunction,

$$\phi(k^2) = \left(\frac{4}{\sqrt{\pi}\beta^3}\right)^{1/2} \exp(-k^2/2\beta^2), \tag{6.9}$$

we obtain

$$< M_{q\bar{q}}^2 > = 4 \times \left\{ m_q^2 + \frac{3\beta_{q\bar{q}}^2}{2} + m_q \times (A + \frac{3}{2\beta_{q\bar{q}}^2} B + \kappa \frac{4\beta_{q\bar{q}}^3}{m_q^2\sqrt{\pi}} \vec{S}_q \cdot \vec{S}_{\bar{q}}) \right\}, \tag{6.10}$$

with

$$< \vec{S}_q \cdot \vec{S}_{\bar{q}} > = \begin{cases} 1/4 & \text{for vector} \\ -3/4 & \text{for pseudoscalar.} \end{cases} \tag{6.11}$$

•Methods of fitting three parmeters, (A,B,C):

(M1) Fitting the π, ρ and K^* meson masses;
$M_{Vn\bar{n}}^2 = m_\rho^2$, $M_{Pn\bar{n}}^2 = m_\pi^2$, and $M_{Vs\bar{s}}^2 = 2m_{K^*}^2 - m_\rho^2$.

(M2) π, ρ and K meson masses;
$M_{Vn\bar{n}}^2 = m_\rho^2$, $M_{Pn\bar{n}}^2 = m_\pi^2$, and $M_{Ps\bar{s}}^2 = 2m_K^2 - m_\pi^2$.

•Numerical results

Using Eqs.(6.10) and (6.4-6.5), we find for $\delta_V = -3.3°$(or $\theta_{SU(3)} = 31.96°$) and $\delta_P = -54.26°$(or $\theta_{SU(3)} = -19°$)

$$\begin{aligned} A &= 0.496\,\text{GeV}, \quad B = -0.062\,\text{GeV}^3, \quad \kappa = 0.488, \\ M^2_{Ps\bar{s}} &= 1.06(2m_K^2 - m_\pi^2)\,\text{GeV}^2, \quad \lambda_V = 0.6m_\pi^2\,\text{GeV}^2, \\ \lambda_P &= 13.5m_\pi^2\,\text{GeV}^2, \quad X_V = -2.01, \quad X_P = 0.92 \qquad \text{for M1,} \end{aligned} \tag{6.12}$$

$$\begin{aligned} A &= 0.379\,\text{GeV}, \quad B = -0.054\,\text{GeV}^3, \quad \kappa = 0.488, \\ M^2_{Vs\bar{s}} &= 0.97(2m_{K^*}^2 - m_\rho^2)\,\text{GeV}^2, \quad \lambda_V = 0.6m_\pi^2\,\text{GeV}^2, \\ \lambda_P &= 13.5m_\pi^2\,\text{GeV}^2, \quad X_V = -2.60, \quad X_P = 0.98 \qquad \text{for M2.} \end{aligned} \tag{6.13}$$

The corresponding mass spectra for light pseudoscalar and vector mesons are given in Table IX.

TABLE IX. Fit of the ground state meson masses based on Eq.(6.10) with the parameters given in Eqs.(6.4) and (6.5). Here the mixing angles of $\omega - \phi$ and $\eta - \eta'$ are used as $\delta_V = -3.3°$(or $\theta_{SU(3)} = 31.96°$) and $\delta_P = -54.26°$(or $\theta_{SU(3)} = -19°$), respectively.

0^{-+}	Experiment[MeV]	M1[M2]	1^{--}	Experiment	M1[M2]
π	135±0.00035	135[135]	ρ	770± 0.8	770[770]
K	494± 0.016	508[494]	K^*	892± 0.24	892[884]
η	547± 0.19	522[504]	ω	782± 0.12	786[787]
η'	958±0.14	972[982]	ϕ	1020±0.008	1017[1016]

Conclusion and Discussion

1. L.C.Q.M. > "naive" Q.M.
   ∃ predictive power for the not-yet-measured or poorly measured expt'l data.

2. Connection of L.C.Q.M. to a dynamical eq. of motion can be made by the variational principle and LCQM model wavefunction can be regarded as a zeroth order solution of the dynamical equation.

3. First order perturbation in V_s-S and III with SU(3) breaking up to X^2 order seems consistent result both for mass spectra and other wf related observables.

4. Connection to QCD still remains as a challenge.