Constituent Gluons

in Hamiltonian Light-Front QCD

Sergio Szpigel

(The Ohio State University)

Robert J. Perry (OSU)

Martina Brisudova (LANL)

Outline

1) Motivation and General Strategy

2) Renormalized LF Hamiltonian with Constituent Gluons

3) Effective Quark-Antiquark Potential for Charmonium

4) Conclusion and Future Perspectives

Wilson et al --> Phys. Rev. D49 , 6720 (1994).

- inspired by the Constituent Quark Model (CQM)

- based on Hamiltonian Light-Front Quantization

Constituent Picture for Hadrons from QCD

Hadrons ⟶ dominant valence structure

↓

Coupled Few-body Equations

2 steps

I) Renormalization

Similarity Renormalization Group

Glazek,Wilson --> Phys. Rev. D 48, 5863 (1993); Phys. Rev. D 49, 4214 (1994).

Coupling-coherence

Perry --> Nuc. Phys. B 403, 587 (1993); Ann. Phys. 232, 116 (1994).

- obtain a Low-energy Effective Hamiltonian

$$H_{eff} = H_{free} + g V_1 + g^2 V_2 + \ldots$$

Perry --> Proc. of 'Hadrons 94', Gramado, Brasil (1994).

$$H_{eff}(g^2) \longrightarrow V_{conf}$$

Two-body Logarithmic Confining Interaction

- Incomplete cancelation between the instantaneous interaction and the effective high-energy one gluon exchange interaction.

- acts in every Fock sector and confines quarks and gluons

II) Bound-state Problem

Bound-State Perturbation Theory (BSPT)

- obtain the low-lying eigenstates

$$H_{eff} = H_0 + V$$

$$V = H_{eff} - H_0$$

H_0 : leading order --> non-perturbatively

V : corrections --> perturbatively

Constituent Picture

H_0 --> dominant interactions that conserve particle number

V --> interactions that change particle number

↓

emission and absorption of low-energy gluons

V_{conf} -> H_0 => BSPT converge
V_{conf} survives to higher orders

↓

non-perturbative mechanism that supresses the coupling with many-gluon states at low-energies

Nonlinear Gluon Interactions

↓

Gluon energy can be dynamically lifted up

Wilson --> The hadronic spectrum is not continuous
Massive low-energy gluons are better
to describe hadron phenomenology

↓

Gluons acquire an effective mass at low-energy

Perry --> V_{conf} confine gluons

↓

a mass gap develops

emission and absorption of gluons is supressed
as the cutoff goes below the gluon mass /mass gap

**Non-perturbative problem in
Fock-sectors containing gluons**

Brisudova, Perry --> Phys. Rev. D54, 1831 (1996).

- $Q\bar{q}$, $q\bar{Q}$

Brisudova, Perry, Wilson --> Phys. Rev. Lett. 78, 1227 (1997).

- Heavy Quarkonia

leading order -> Heavy Mesons

↓

color singlet 'valence' $q\bar{q}$ bound-states

- $O(g^2)$ effective one-body operators and dominant two-body potentials ($V_{coul} + V_{conf}$)

- constituent quark masses

corrections -> BSPT

How to improve :

-> higher orders in similarity

-> higher Fock-sectors (e.g. $q\bar{q}g$)

$$|\psi\rangle = |\psi_{q\bar{q}}\rangle + |\psi_{q\bar{q}g}\rangle$$

How to set up the bound-state calculation more effectively

$$H = H_0 + V$$

H_0 : better approximation

↓

V : smaller corrections

Phenomenology

H incorporates non-perturbative effects from intermediate and low-energy gluons

↓

Assign an Effective Mass to Gluons at Intermediate and Low-Energy Scales

Renormalized LF Hamiltonian with Constituent Gluons

I) Renormalization

- bare QCD Hamiltonian in LF gauge ($A^{+}{}_{a} = 0$) -> $H_B(\Lambda)$

$\Lambda^2/\mathcal{P}^+$: cutoff on free energy change at the interaction vertices

- Similarity Transformation : lower the cutoff $\Lambda \rightarrow \lambda$

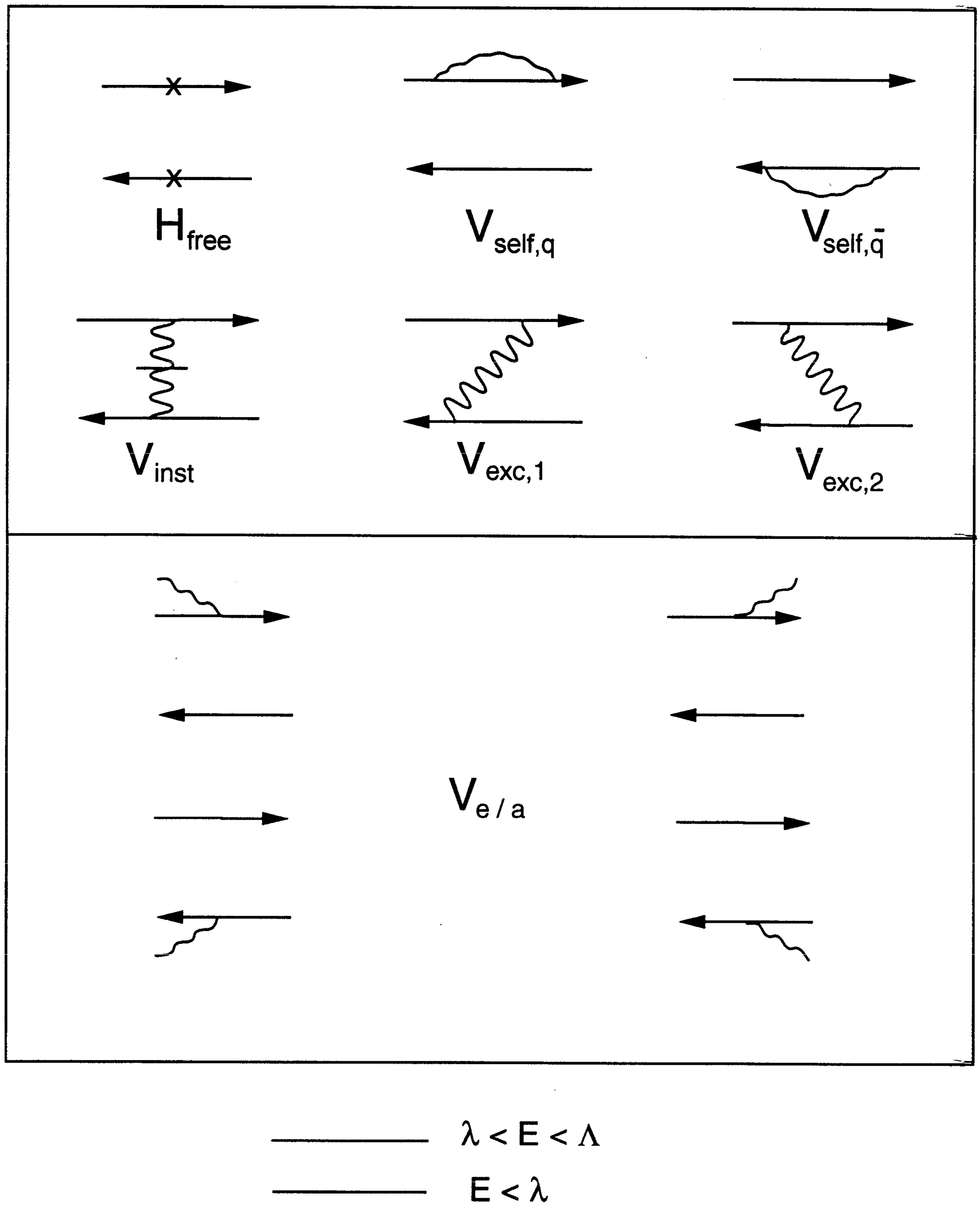
- Coupling-Coherence (fix counterterms)

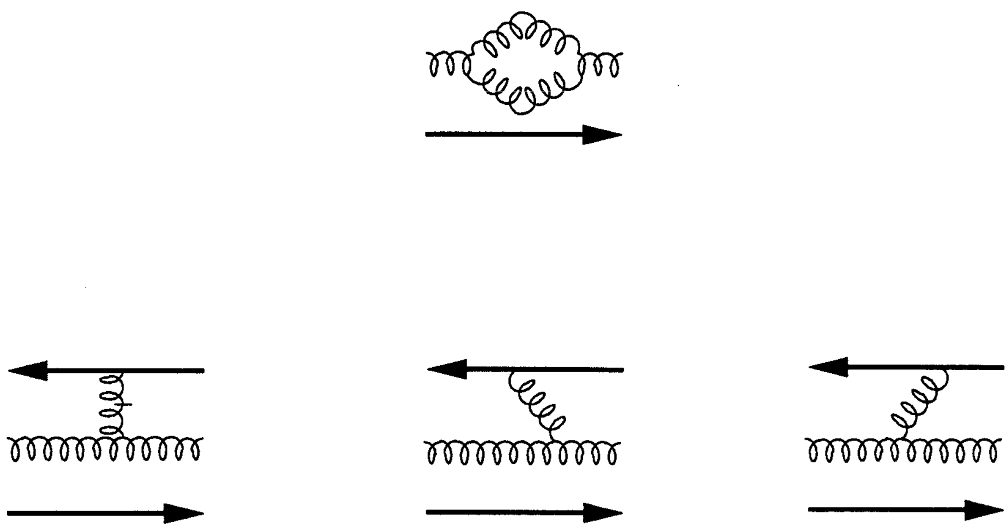
$$H^{(2)}{}_R(\lambda) = H_{free} + f_\lambda \left(V_{e/a} + V_{inst} + V^{(2)}{}_{eff}(\lambda) \right)$$

$\downarrow$ ($V^{(2)}{}_{eff}(\lambda)$): logarithmic confining potential

- effects of high-energy (massless) gluons 'integrated out' and replaced by effective interactions

- below λ gluons cease to couple perturbatively


H_{free}
$V_{self,q}$
$V_{self,\bar{q}}$
V_{inst}
$V_{exc,1}$
$V_{exc,2}$
$V_{e/a}$
$\lambda < E < \Lambda$
$E < \lambda$



II) Inclusion of the effective gluon mass operator

$$H_{free} \to H'_{free} = H_{free} + \int \frac{d^2q_\perp \, dq^+}{16\pi^3 q^+} \frac{M_g^2(q_\perp, q^+)}{q^+} a^\dagger_{i,q} \, a_{i,q}$$

Gluon Dispersion Relation (< λ)

$$q^- = \frac{q_\perp^2}{q^+} \longrightarrow \frac{q_\perp^2 + M_g^2(q_\perp, q^+)}{q^+}$$

Phenomenological input

- higher-order operators

- higher Fock-sectors

- $M_g(q_\perp, q^+) \to m_g \, F(x_g)$

xg : gluon longitudinal momentum fraction in a given state

III) Second Similarity Transformation

- lower the cutoff to a scale $\mu < \lambda$ where we expect mesons can be approximated by $q\bar{q}$-component.

$$H'^{(2)}_R(\mu) = H'_{free} + f'_\mu \, (V_{e/a} + V_{inst} + f_\lambda V^{(2)}_{eff}(\lambda))$$

$$+ f'_\mu \, V^{(2)}_{eff}(\lambda,\mu,m_g)$$

- effects of gluons with energy $\mu < E < \lambda$ 'integrated out' and replaced by effective interactions

- does not generate new divergences

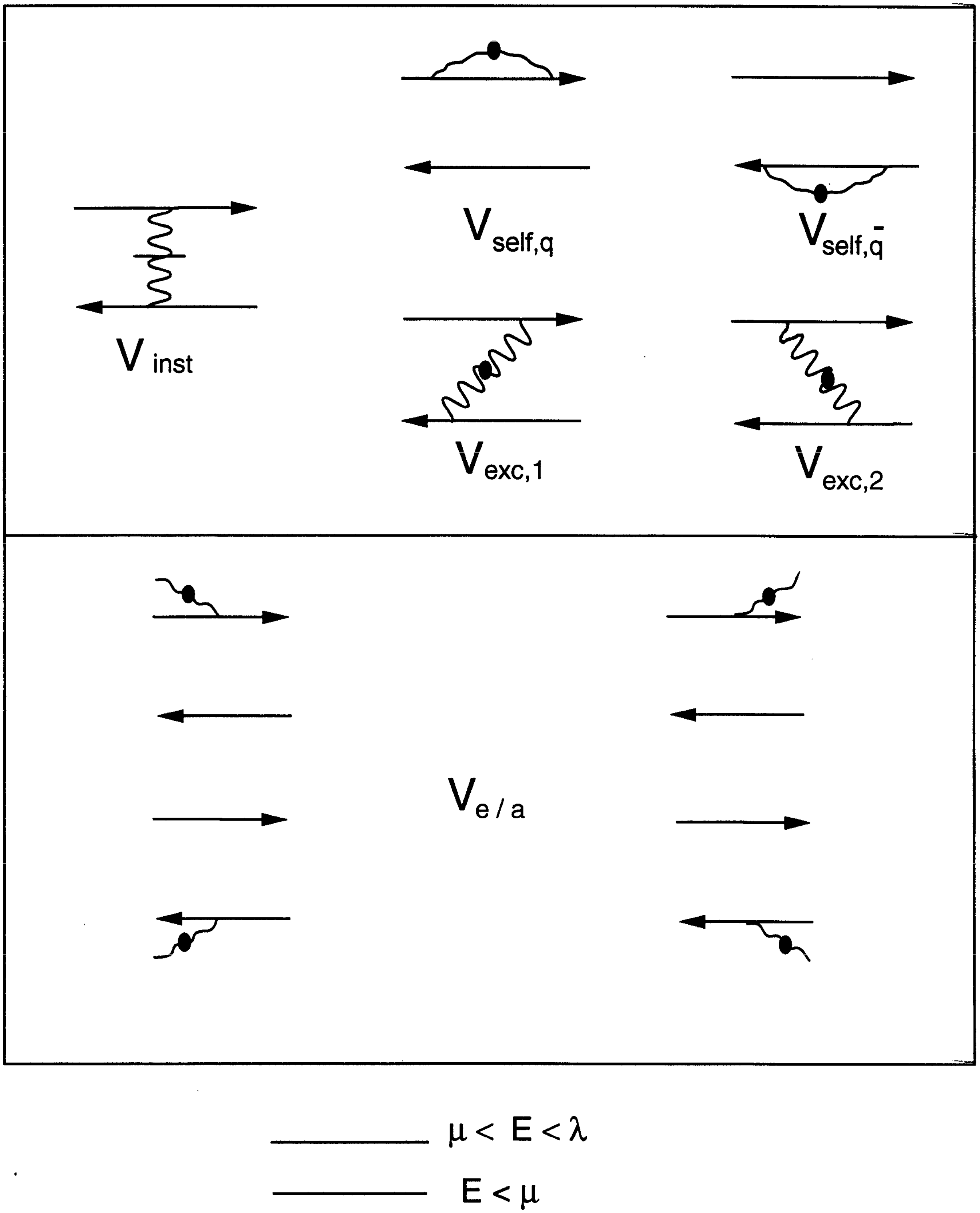
  no need for new counterterms

- also runs the cutoff perturbatively

  around a different free Hamiltonian

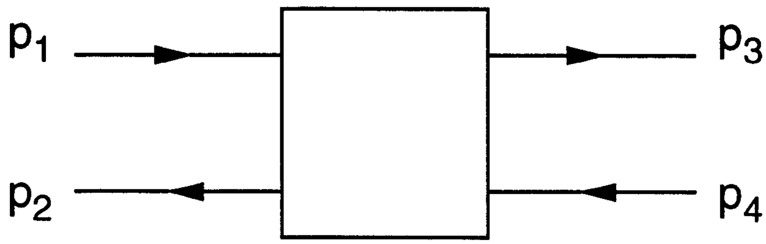
- two new parameters

  mg, λ



V inst
V self,q
V self,q̄
V exc,1
V exc,2
V e / a
μ < E < λ
E < μ

Effective Quark-Antiquark Potential for Charmonium



$$< q\bar{q} \mid H'^{(2)}{}_R(\mu) \mid q\bar{q} > = <H'_{free}> + \Sigma' + V_{q\bar{q}}$$

$$<H'_{free}> = \frac{p_{1\perp}{}^2 + m^2}{p_1{}^+} + \frac{p_{2\perp}{}^2 + m^2}{p_2{}^+}$$

$$\Sigma = <V^{(2)}{}_{self}(\lambda)> + <V^{(2)}{}_{self}(\lambda, \mu, m_g)>$$

$$V_{q\bar{q}} = <V_{inst}> + <V^{(2)}{}_{exc}(\lambda)> + <V^{(2)}{}_{exc}(\lambda, \mu, m_g)>$$

--> most infrared divergent part (spin independent) $q^+ \to 0$

--> coordinate change : $x \to p_z \to \vec{p} = (p_z, p_\perp)$

--> non-relativistic reduction : $|\vec{p}| << m \to O(|\vec{p}|^2)$

--> Fourier Transform : $V_{q\bar{q}}(r_z, r_\perp)$

Brisudova, Perry, Wilson --> Phys. Rev. Lett. 78, 1227 (1997).

$$V_{conf}(r_z, r_\perp) = V_0(r) + \sum_{k=1} V_{2k}(r)\, P_{2k}(\cos\theta)$$

$V_0(r)$ → angular average

$\sum_{k=1} V_{2k}(r)\, P_{2k}(\cos\theta)$ → rotationally non-invariant

leading order :

$$H_0 = < H_{free} > + \Sigma + V_0(r) + V_{coul}(r)$$

corrections in BSPT :

$$V = V_{e/a} + \sum_{k=1} V_{2k}(r)\, P_{2k}(\cos\theta) + V_{spin}$$

↓

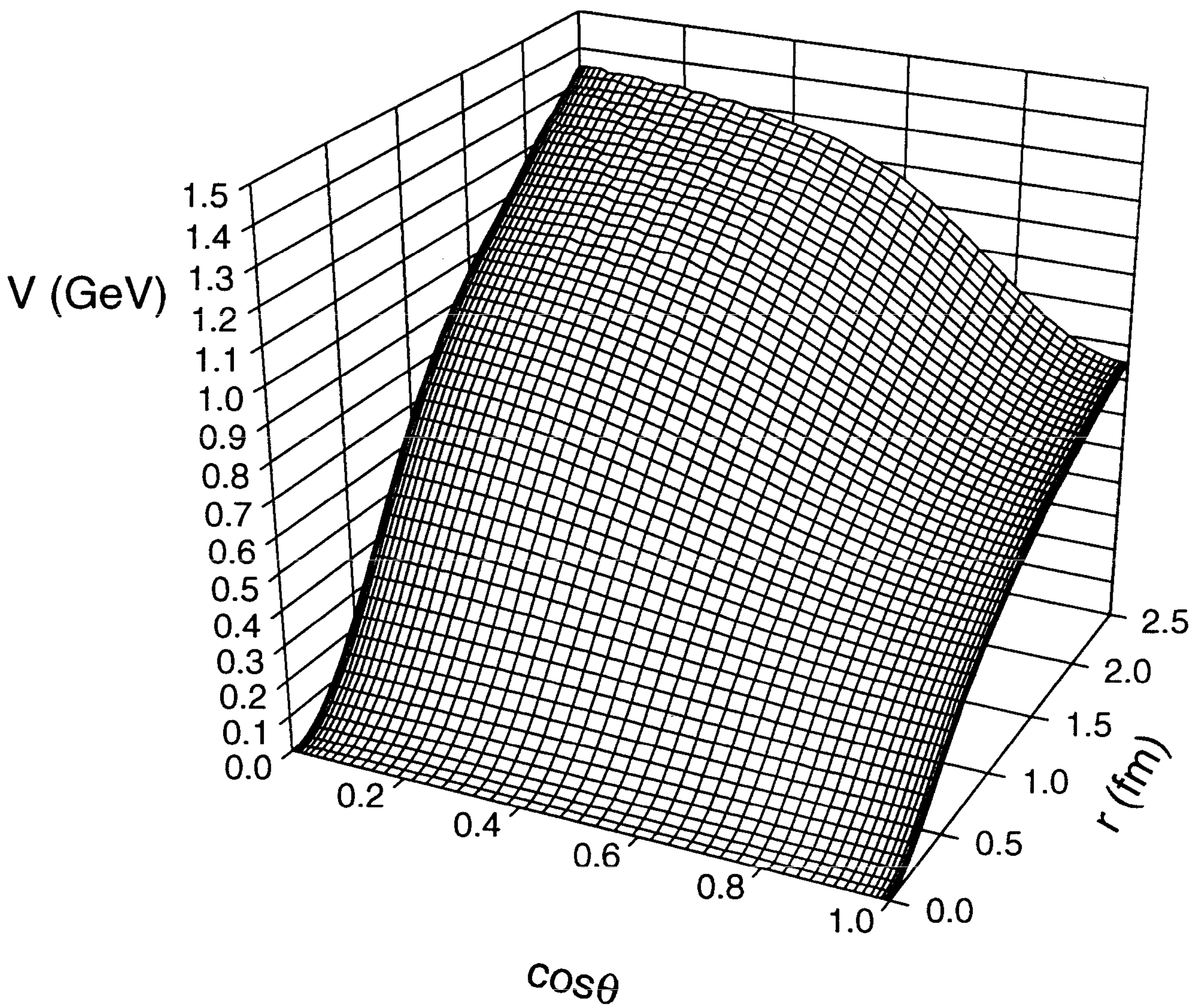
ground state (1S) --> very small

lowest excited states (1P, 2S) --> 30 %

↓

An important piece of the Physics is missing in H_0

Charmonium (massless gluons)



Rotational symmetry is dynamical in the LF and is broken by the cutoff

↓

Exact restoration --> states with arbitrarily large number of gluons

Approximated restoration for the low-lying states

↓

including the corrections from $V_{e/a}$

$H'^{(2)}_R(\mu)$ ->effective interactions that replace the effects of emission/absorption of gluons with $\mu < E < \lambda$

↓

reduce the corrections due to the violation of rotational symmetry → $m_g F(x_g)$

↓

$F(x_g) = | x_g |$

$$V_{q\bar{q}} \to \theta\left(\frac{\mu^2}{\mathcal{P}^+} - \left|\frac{\mathcal{M}_0^2 - \mathcal{M}_0'^2}{P^+}\right|\right) 4g^2 C_F (2m)^2$$

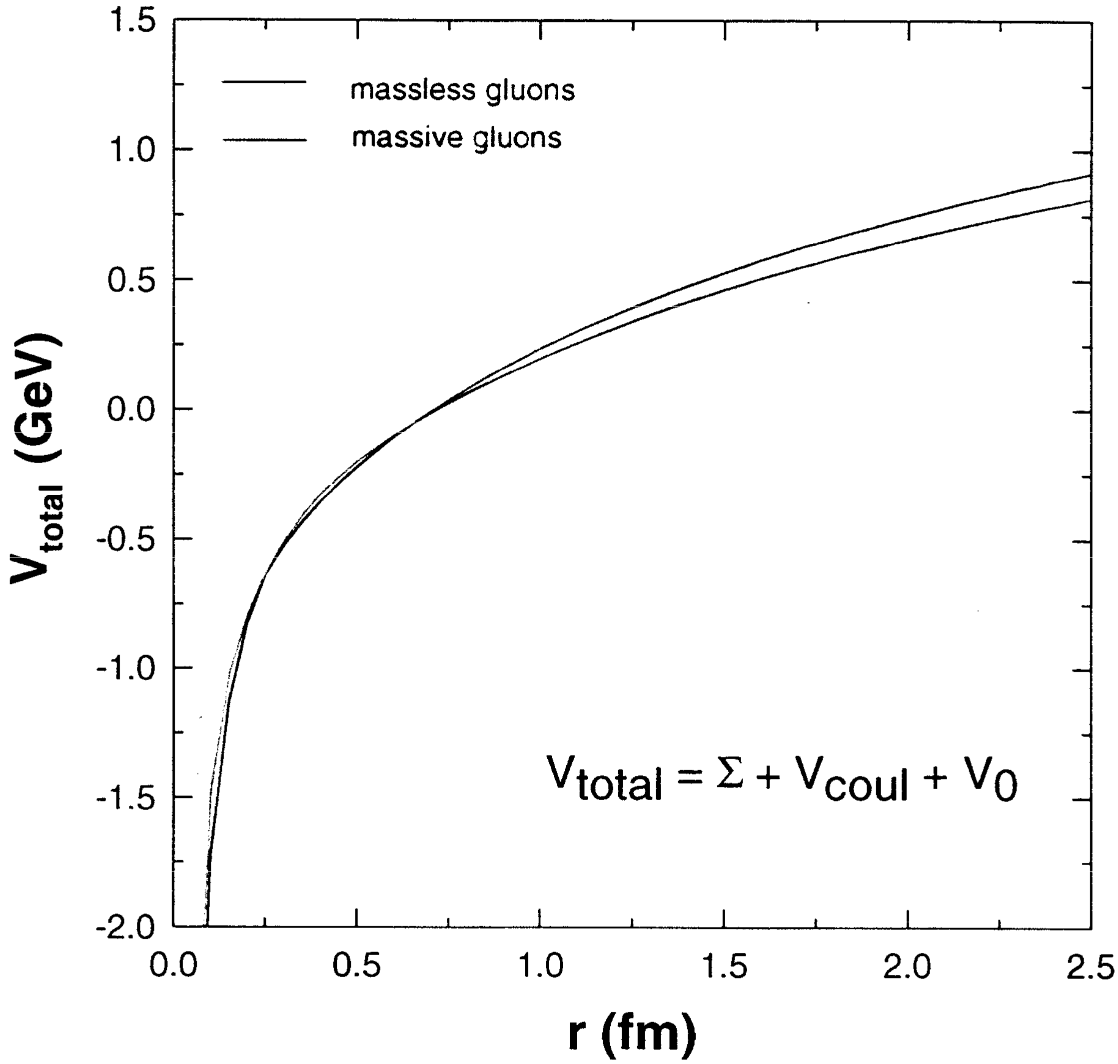
$$\times \left[-\frac{1}{\mathbf{q}^2} - \frac{1}{q_z^2}\left(\frac{q_\perp^2}{\mathbf{q}^2}\right)\theta\left(\frac{\lambda^2}{\mathcal{P}^+} - \frac{\mathbf{q}^2(2m)}{|q_z|P^+}\right)\right.$$

$$+ \frac{1}{q_z^2}\left(\frac{q_\perp^2}{\mathbf{q}^2 + M_g{}^2(q_z, q_\perp)}\right)\theta\left(\frac{\lambda^2}{\mathcal{P}^+} - \frac{\mathbf{q}^2(2m)}{|q_z|P^+}\right)$$

$$\left. - \frac{1}{q_z^2}\left(\frac{q_\perp^2}{\mathbf{q}^2 + M_g{}^2(q_z, q_\perp)}\right)\theta\left(\frac{\mu^2}{\mathcal{P}^+} - \frac{(\mathbf{q}^2 + M_g{}^2(q_z, q_\perp))2m}{|q_z|P^+}\right)\right]$$

$\mathbf{q} \equiv \mathbf{p} - \mathbf{p}' \equiv (q_z, q_\perp)$: exchanged (gluon) momentum.

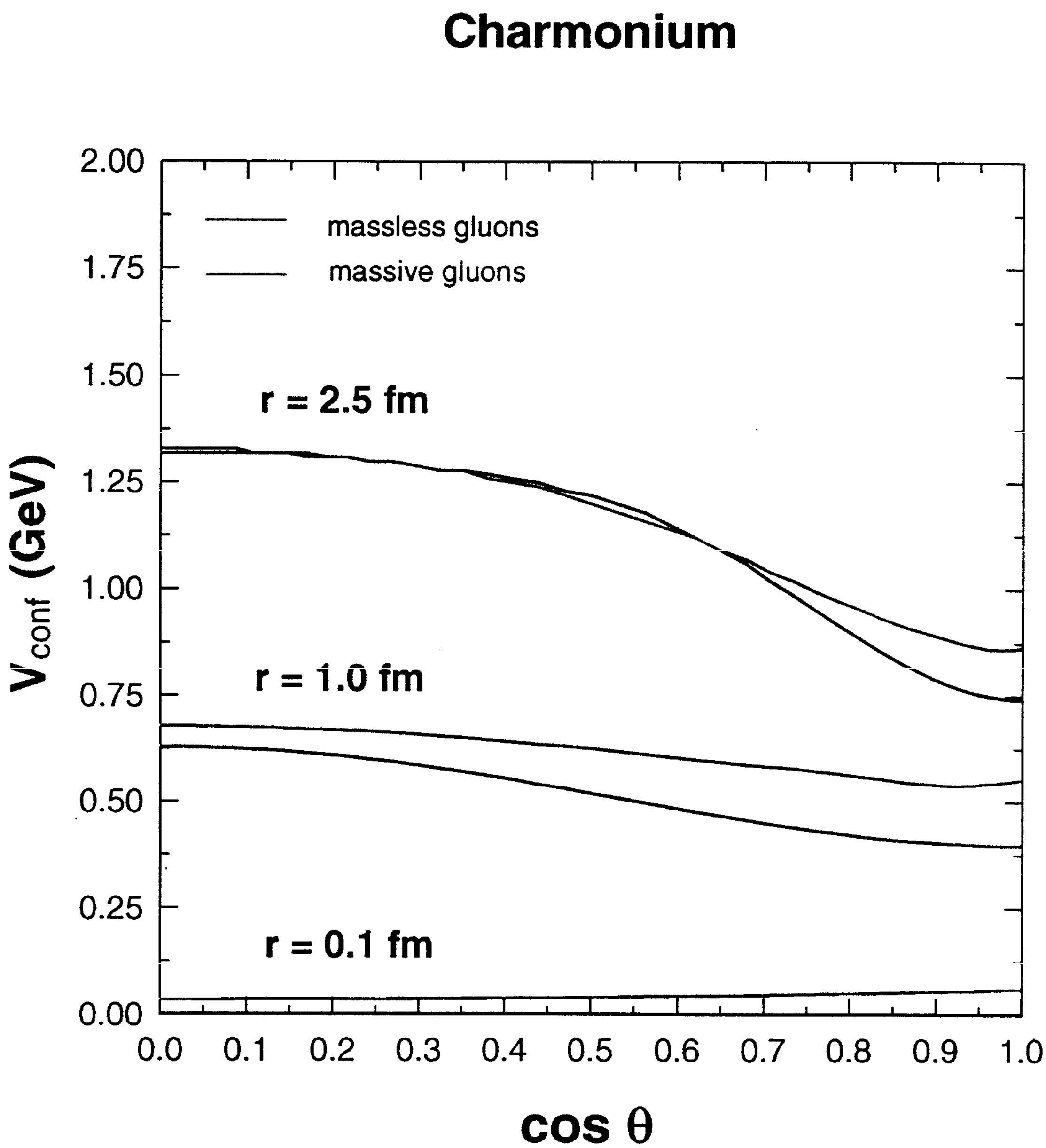
Charmonium



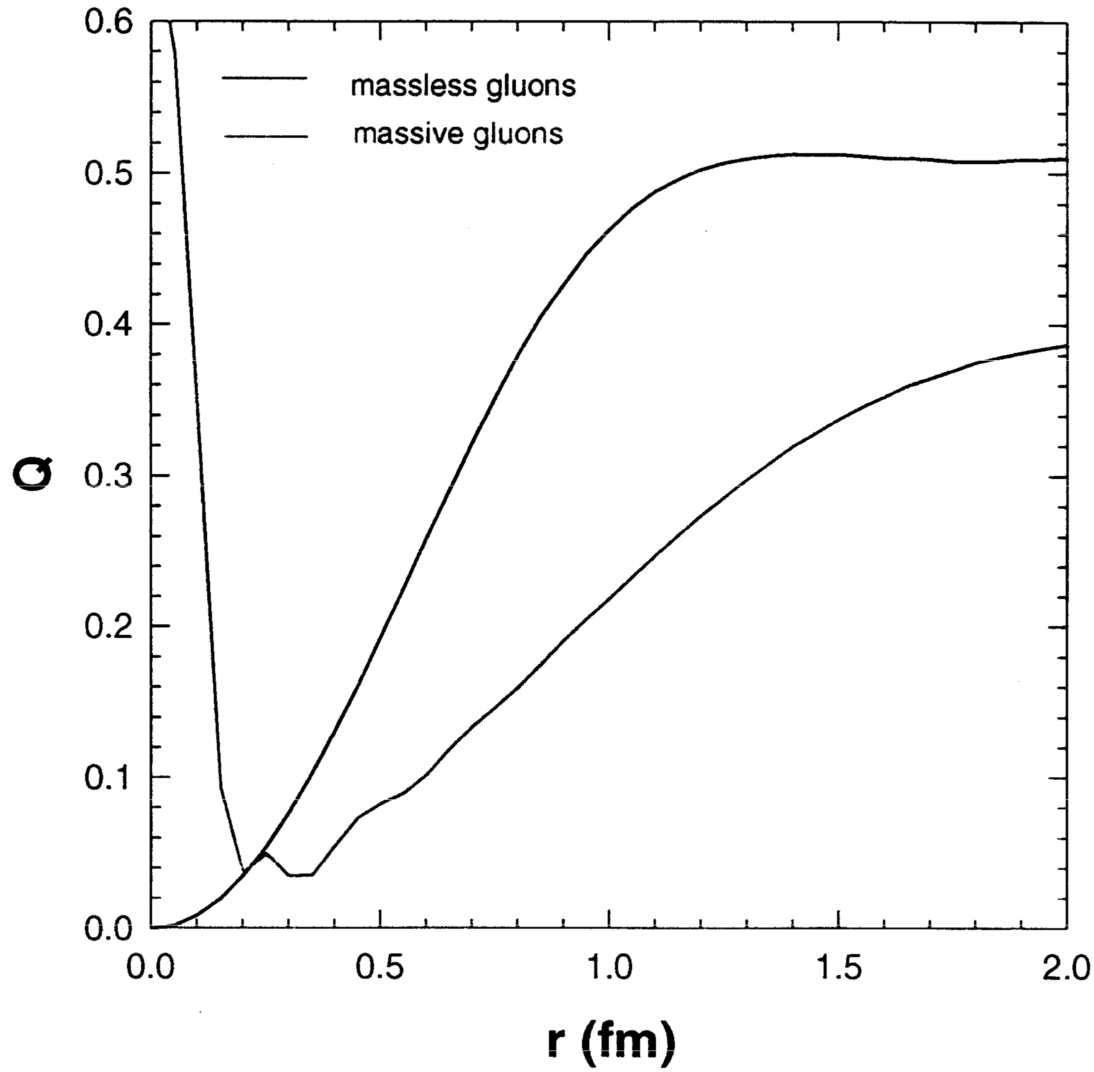
m = 1.5 GeV
λ = 1.7 GeV
α = 0.5

m = 1.5 GeV
λ = 4.0 GeV
α = 0.6

μ = 1.5 GeV
mg = 1.6 GeV


Charmonium
massless gluons
massive gluons
r = 2.5 fm
r = 1.0 fm
r = 0.1 fm
V_{conf} (GeV)
cos θ
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

$$Q = |V_{conf}(r_\perp, r_z=0) - V_{conf}(r_\perp=0, r_z)| / V_0(r)$$



Conclusion and Future Perspectives

- Renormalized Light-Front QCD Hamiltonian $O(g^2)$
    - Similarity Transformation
    - Coupling-Coherence
    - Effective Gluon Mass One-Body Operator

- Effective Quark-Antiquark Potential for Charmonium
    - V_{conf} less rotationally non-invariant at intermediate range -> $m_g \mid x_g \mid$
    - useful to gain intuition about the $O(g^2)$ results
    - hint to analyse higher order terms

Next step:

- Solve for the low-lying bound-states
- evaluate the corrections from rotationally non-invariant part
- spin dependent interactions
- choice of $M(q^+, q_\perp)$