Insights from Light-Front Quantization on the Unruh Effect

A. Deur

Thomas Jefferson National Accelerator Facility

In collaboration with:

S. J. Brodsky, C. D. Roberts and B. Terzić

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•The Unruh effect: Accelerated observers can detect particles which are unobservable to inertial observers. Occurs even if the accelerated observer is in vacuum.

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•Interpretation: If a vacuum loop's pair of virtual particles occurs near an event horizon, one of the particles may fall beyond the event horizon and the pair annihilation is prevented: one particle becomes real (taking energy from the Black Hole (Hawking radiation) or from the frame acceleration (Unruh effect)).





LF quantization: $p^+ \ge 0 \Rightarrow$ trivial vacuum (aside from possible zero-modes). No loops of virtual particle with non-zero momenta.

What about the Unhru effect in LF quantization?



Field decomposition

In frame x_{μ} (with vacuum $|0\rangle$):

Field decomposition in positive and negative frequency modes:

$$\phi = \int dp \left(\hat{a}_p f_p + \hat{a}_p^{\dagger} f_p^* \right), \text{ with } f_p \propto e^{i p^{\mu} x_{\mu}}$$

For IF: $f_p \propto e^{-i(\omega t - pz)}$, annihilation creation operators For LF: $f_{p^+} \propto e^{-\frac{i}{2}(p^- x^+ + p^+ x^-)}$.



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In another frame x'_{μ} (with vacuum $|0'\rangle$):

$$\phi = \int dp' \left(\hat{b}_{p'} f'_{p'} + \hat{b}^{\dagger}_{p'} f'_{p'} \right)$$

If **no un**ambiguous separation positive and negative modes in different frames, then $\hat{b}_{p'} = \alpha \hat{a}_p + \beta \hat{a}_p^{\dagger}$ with $\beta \neq 0$

$$\Rightarrow \hat{b}_{p'} |0'\rangle = 0 \quad \text{but } \hat{b}_{p'} |0\rangle = (\alpha \hat{a}_p + \beta \hat{a}_p^{\dagger}) |0\rangle = \hat{a}_p^{\dagger} |0\rangle \propto |p\rangle$$

i.e., $\hat{b}_{p'} |0\rangle \propto |p\rangle \neq 0$
Unruh effect



Inertial frames



If the frames are related by a Lorentz boost (viz inertial frames): $f_p \propto e^{ip^{\mu}x_{\mu}} = e^{i(\omega t - pz)} = e^{i(\omega't' - p'z')} \implies \hat{a}_p = \hat{b}_{p'}$. $\beta = 0$: No Unruh effect.



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If the frames are related by a Lorentz boost (viz merial frames): $f_p \propto e^{ip^{\mu}x_{\mu}} = e^{i(\omega t - pz)} = e^{i(\omega't' - p'z')} \qquad \Rightarrow \hat{a}_p = \hat{b}_{p'}. \quad \beta = 0: \text{ No Unruh effect.}$ $\frac{\partial f_p}{\partial t'} = \left[\frac{\partial t}{\partial t'}\partial_t + \frac{\partial z}{\partial t'}\partial_z\right]f_p = i\left[\omega\cosh\theta - p\sinh\theta\right]f_p = -i\omega'f_p$ Frequencies in the boosted frame are boosted frequencies. $\Rightarrow \text{ unambiguous separation of positive and negative modes.}$ No Unruh effect in inertial frames, as it should be.



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Inertial frames



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Accelerating frames





Accelerating frames



Canonical quantization done at constant proper time.

Heisenberg uncertainty principle originates from commutation relations

- \Rightarrow Uncertainty principle operates at equal time.
- \Rightarrow complex IF vacuum.





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If events \mathscr{C}_1 and \mathscr{C}_2 at times t_1 and t_2 are spacelike-separated, time-ordering of \mathscr{C}_1 and \mathscr{C}_2 is frame-dependent.

In some frames, $t_1 < t_2$ and Z-graphs appear. They contribute negative probabilities.

 \Rightarrow Need disconnected vacuum loops to balance the negative probabilities.



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Accelerated frames: virtual particles may borrow 4-momentum from acceleration process and become observable: Unruh effect.



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Boost or acceleration: rescales axes without reorienting them \Rightarrow uncertainty principle always operates along x^- .

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This persists in an accelerated LF frame: no Unruh effect.



Direct propagation

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This persists in an accelerated LF frame: no Unruh effect.

Also: Momentum conservation \Rightarrow no vacuum loops: LF particles must have $p^+ \ge 0$. Vacuum $p^+ = 0 \Rightarrow$ one of the particles of the vacuum loop would have $p^+ < 0$, which is forbidden.



Direct propagation

Standard derivation (Unruh 1976)

S. M. Carroll, Spacetime and Geometry: An Introduction to General Relativity, Cambridge University Press



Acceleration: • Hyperboloid in IF: $t^2(\rho) = z^2(\rho) - \alpha^2$ • Hyperbola in LF: $\tau(\rho) = \frac{1}{\alpha^2 x^{-}(\rho)}$ (ρ is a parameter)



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ξ: Rindler space; η: Rindler time Acceleration: "Orbit" in Rindler spacetime (ξ=const., η evolves)



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The coordinates above cover only Region I.

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The coordinates above cover only Region I. Other coordinates needed for region IV:

$$z = -\frac{1}{a}e^{a\xi}\cosh(a\eta); \ t = -\frac{1}{a}e^{a\xi}\sinh(a\eta)$$
$$x^{-} = -\frac{1}{a}e^{a(-\eta+\xi)}; \ x^{+} = -\frac{1}{a}e^{a(\eta+\xi)}$$



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In Rindler space-time, the field is decomposed using regions I and IV Rindler modes:

$$\phi = \int dp \left(\hat{b}_{p}^{(\mathrm{I})} g_{p}^{(\mathrm{I})} + \hat{b}_{p}^{(\mathrm{I})\dagger} g_{p}^{(\mathrm{I})*} + \hat{b}_{p}^{(\mathrm{IV})} g_{p}^{(\mathrm{IV})} + \hat{b}_{p}^{(\mathrm{IV})\dagger} g_{p}^{(\mathrm{IV})*} \right)$$



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In Minkowski (=inertial) frame (t, z) (IF) or (x^+, x^-) (LF), no spacetime partition \Rightarrow the field is decomposed as usual:

$$\phi = \int dp \left(\hat{a}_p f_p + \hat{a}_p^{\dagger} f_p^* \right)$$
(IF)

$$\phi = dp^+ (\hat{a}_{p^+} f_{p^+} + \hat{a}_{p^+}^{\dagger} f_{p^+}^*)$$
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$$\phi = \int dp (a_p f_p + a_p^* f_p^*) \quad (\text{IF})$$

$$\phi = \int dp^+ (\hat{a}_{p^+} f_{p^+} + \hat{a}_{p^+}^\dagger f_{p^+}^*) \quad (\text{LF})$$

Then we have the definition of the vacua: $\hat{b}_p^{(I)}|0\rangle_R = 0 = \hat{b}_p^{(IV)}|0\rangle_R$; $\hat{a}_p|0\rangle_{M,IF} = 0$; $\hat{a}_{p^+}|0\rangle_{M,LF} = 0$



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To derive his effect, Unruh combined Rindler modes so that they match the spacetime dependence of Minkowski modes.

 $\Rightarrow \text{Rindler mode combinations } h_p^{(I)}, h_p^{(IV)} (IF) \text{ or } h_p^{(I)}, h_p^{(IV)} (LF) \text{ whose annihilation operators } \hat{c}_p^{(I,IV)} (\text{ or } \hat{c}_p^{(I,IV)}) \text{ annihilate } |0\rangle_{M,IF} (\text{ or } |0\rangle_{M,LF}).$



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$$h_p^{(I)} = \frac{1}{\sqrt{2\sinh(\frac{\pi\omega}{a})}} \left(e^{\pi\omega/2a} g_p^{(I)} + e^{-\pi\omega/2a} g_{-p}^{(IV)*} \right) \quad (\omega \text{ is the field frequency and } a \text{ the acceleration.})$$

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$$h_{p}^{(I)} = g_{p}^{(I)} \qquad \text{No } g_{p}^{(VI)*} \text{ because it would require } p^{+} < 0 \quad (\text{same reason why there is no LF})$$

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Jef

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<u>Preliminary</u>: Rindler time translation \Leftrightarrow Minkowski boost.

Recall the definition of Rindler coordinates:

$$z = \frac{1}{a}e^{a\xi}\cosh(a\eta); \quad t = \frac{1}{a}e^{a\xi}\sinh(a\eta);$$
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Rindler time translation
$$\eta \to \eta + \Delta$$

 $\Rightarrow \qquad t' = t \cosh(a\Delta) + x \sinh(a\Delta)$
 $z' = z \cosh(a\Delta) + t \sinh(a\Delta)$ } IF
 $x^{+'} = e^{a\Delta}x^{+}$
 $x^{-'} = e^{-a\Delta}x^{-}$ } LF

Usual boost formulae with rapidity $a\Delta$ (*a* is the acceleration).

This is because acceleration = succession of boosts with changing rapidity.



This derivation uses the periodicity in imaginary time ($\tilde{t} \equiv it$) of a QFT at finite temperature $T \equiv /1\beta$.

1.) Green function of $\phi(\tilde{t}, z)$: $G_{\beta}(\tilde{t}, z) = -\frac{1}{Z} \text{Tr } e^{-\beta H} \int [\phi(\tilde{t}, z)\phi(0, 0)]$

Partition function

imaginary-time ordering operator



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If *H* is also the time-evolution operator, then:

 $e^{-\beta H}\phi(\tilde{t},z)e^{\beta H} = \phi(\tilde{t}-\beta,z)$

This and cyclicity property of trace $\Rightarrow G_{\beta}(\tilde{t}, z) = G_{\beta}(\tilde{t} - \beta, z)$: QFT periodicity with period β



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2.) On the other hand: Rindler line element, $ds^2 = -\xi^2 d\eta^2 + d\xi^2$. With imaginary Rindler time $\tilde{\eta} \equiv i\eta$: $ds^2 = \xi^2 d\eta^2 + d\xi^2$: polar coordinates. For non-singular coordinates, we must have $\tilde{\eta} = \tilde{\eta} + 2\pi i$.



This derivation uses the periodicity in imaginary time ($\tilde{t} \equiv it$) of a QFT at finite temperature $T \equiv /1\beta$.

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If *H* is also the time-evolution operator, then: $e^{-\beta H}\phi(\tilde{t},z)e^{\beta H} = \phi(\tilde{t}-\beta,z)$

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In Euclidean spacetime the dilation operator serves as LF Hamiltonian.

Fubini, Hanson & Jackiw, PRD 7 1732 (1973)

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No LF Unruh effect.

Jefferson Lab

X

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Simple thermometer: deuterium gas. *T* given by Maxwell-Boltzmann law + distribution between D ground state $|S = 0\rangle$ and excited state $|S = 1\rangle$.



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Dynamical boost induces spin-orbit force. \Rightarrow State $|S = 1\rangle$ depopulates to $|S = 0\rangle$

McGee, Phys. Rev. 158 1500 (1967); Brodsky, Primack, Phys. Rev. 174 2071 (1968)

\Rightarrow The detector cools.

LF: boosts are kinematical (no cooling) & vacuum is trivial (no Unruh temperature)

 \Rightarrow Suggests Unruh heating + IF-boost cooling = 0.



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The IF cooling effect has been overlooked \Rightarrow No compensation mechanism for the Unruh effect, which seems objectively observable.



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- Apparently contradicts General Relativity's basic principle that for vanishing distances, curved spacetime → flat spacetime (=inertial frame, without Unruh effect).
- However IF vacuum is complex, with characteristic distance scale (size of the vacuum loops). \Rightarrow cannot take the small distance limit \Rightarrow definition of positive and negative modes ambiguous: Unruh effect.
- Classical physics (with trivial vacuum): no Unruh effect. Indeed, $T_{\text{Unruh}} \propto \hbar$: quantum effect.
- LF vacuum is trivial. No distance scale prevents reaching the flat spacetime limit, with well-defined positive and negative modes ⇒ no Unruh effect



Common claim: No need to test or question the Unruh effect since it is a logical consequence of QFT.

- "the Unruh effect itself does not need experimental confirmation any more than free quantum field theory does" (Standard review on the Unruh effect: arXiv:0710.5373)
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Then, origin of this grave problem is the same as the origin cosmological constant problem, with same resolution: trivial LF vacuum \Rightarrow no $\sim 10^{120}$ discrepancy with observed cosmological constant. Brodsky, Shrock, PNAS. 108, 45 (2011)

Brodsky, et al, PRC 82, 022201 (2010); 85, 065202 (2012).



Supplementary slides



LF vacuum

•Firmly established: no virtual particle loops in LF vacuum.

- •Rôle of possible zero-momentum LF modes in the LF vacuum is less clear. But, irrelevant to the Unruh effect regardless of their possible existence:
 - 1. $p^+ = 0$ modes do not transfer momentum/kinetic energy to the Unruh detector: zero-modes cannot heat thermometers.
 - 2. Vacuum structure is not invoked in IF demonstration of Unruh effect. Demonstration is based on coordinate definitions of the forms of dynamics + consequent quantization conditions + generic properties of quantum field theory. Only the *interpretation* of the effect invokes the vacuum structure to provides an intuitive picture.
 - 3. Discussions of the Unruh effect are often set for simplicity in (1+1)D. There the triviality of the LF vacuum (perturbative & non-perturbative) is established.
 - 4. New perspective in "Implication to Black Hole Evaporation": By definition, zero-point energy occurs at a single point in space (their only possible physical contribution being from the infinite momentum loop, viz with conjugate distance →0) ⇒ zero-modes do not provide the distance scale necessary to prevent reaching the flat spacetime limit.

Possible nontrivial nonperturbative vacuum? Also irrelevant because no field coupling nor other expansion parameter enters in the Unruh effect.

