Consistency of the pion form factor and unpolarized TMDs beyond leading twist in the light-front quark model

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(in collaboration with Prof. Chueng-Ryong Ji)

3D Structure of the Nucleon via GPDs, June 25-28, 2024, Incheon, Korea

Outline

1. Motivation

2. Form Factors on the Light-Front
   - LF zero mode issue

3. Light-Front Quark Model(LFQM)
   - New Development of self-consistent LFQM
   - Pion Form Factor

4. Unpolarized TMDs of pion

5. QCD evolution of Pion PDFs

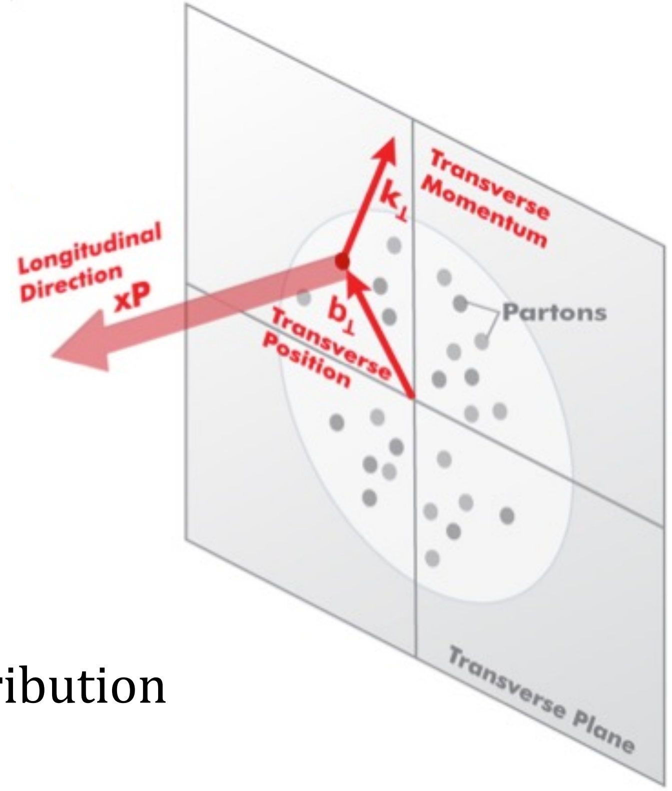
6. Conclusions

1. Motivation

• Understanding the internal structure of hadron is an important objective in modern nuclear and particle physics.

- Experimental studies (e.g. JLab, COMPASS, EIC, J-PARC, etc.) are aimed at probing the 3D structure of hadrons, particularly focused on Generalized Parton Distributions (GPDs) and Transverse Momentum Dependent Distributions (TMDs).

For precision 3D imaging of hadrons,
it is essential to measure positions
and momenta of the partons
transverse to the hadron's direction of motion.



$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$: Wigner distribution

3D hadron structure from 5D tomography

$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$: Wigner distribution

$\int d\boldsymbol{b}_\perp$ $\qquad$ $\int d\boldsymbol{k}_\perp$

$f(x, \boldsymbol{k}_\perp)$

TMDs :semi-inclusive processes

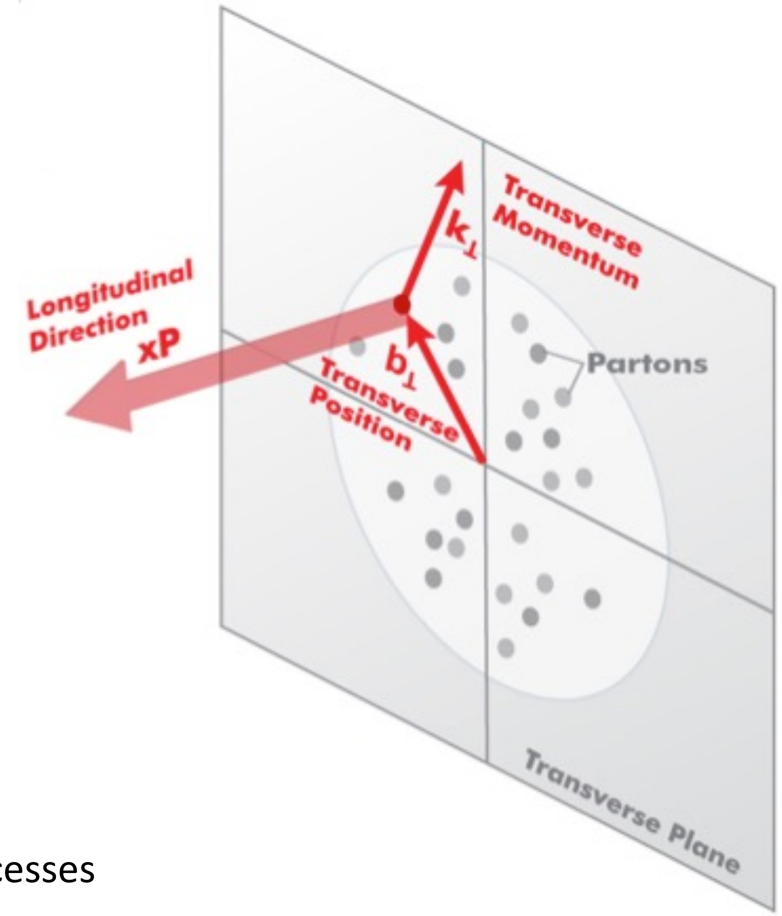
$f(x, \boldsymbol{b}_\perp)$

Impact parameter distributions

$\int d\boldsymbol{k}_\perp$ $\qquad$ $\int d\boldsymbol{b}_\perp$

$f(x)$

PDFs: inclusive and semi-inclusive processes



3D hadron structure from 5D tomography

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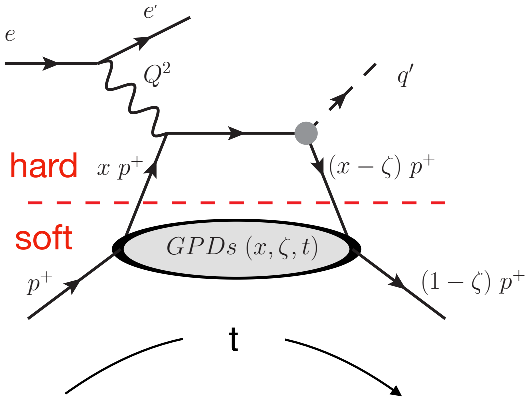
Impact parameter distributions

FT $\boldsymbol{b}_\perp \leftrightarrow \Delta$



$H(x, 0, t)$ $\xleftarrow{\zeta = 0}$ $H(x, \zeta, t = -\Delta^2)$

$$\zeta = \frac{(P - P')^+}{P^+}$$

GPDs :exclusive processes

$\int d\boldsymbol{k}_\perp$ $\int d\boldsymbol{b}_\perp$

$f(x)$

PDFs: inclusive and semi-inclusive processes

3D hadron structure from 5D tomography

$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$: Wigner distribution

$\int d\boldsymbol{b}_\perp$ → $f(x, \boldsymbol{k}_\perp)$

TMDs :semi-inclusive processes

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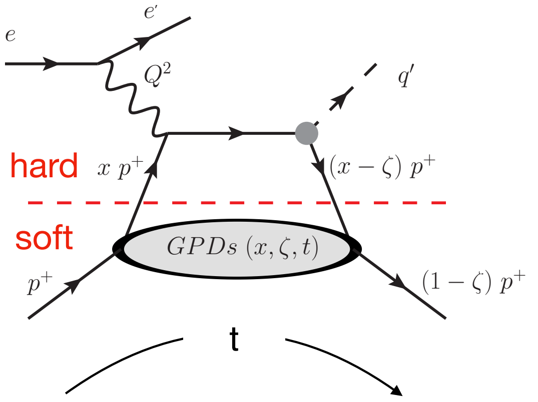

$H(x, 0, t)$ ⇐ $\zeta = 0$ ⇐ $H(x, \zeta, t = -\Delta^2)$

$$\zeta = \frac{(P - P')^+}{P^+}$$

GPDs :exclusive processes

$\int dx$ → $F(t)$

Form factors: elastic scattering

$\int d\boldsymbol{k}_\perp$, $\int d\boldsymbol{b}_\perp$, $H(x, 0, 0)$ → $f(x)$

PDFs: inclusive and semi-inclusive processes

3D hadron structure from 5D tomography

$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$: Wigner distribution

$\int d\boldsymbol{b}_\perp$ → $f(x, \boldsymbol{k}_\perp)$

TMDs :semi-inclusive processes

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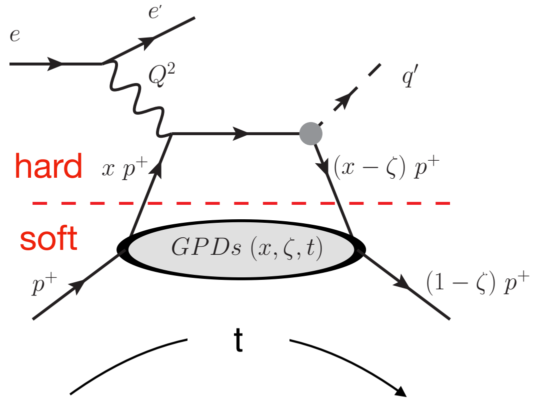

$H(x, 0, t)$ ⇐ $\zeta = 0$ ⇐ $H(x, \zeta, t = -\Delta^2)$

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GPDs :exclusive processes

$\int dx$ → $F(t)$

Form factors: elastic scattering

$H(x, 0, 0)$ → $f(x)$

$\int d\boldsymbol{k}_\perp$, $\int d\boldsymbol{b}_\perp$ → $f(x)$

PDFs: inclusive and semi-inclusive processes

⇒ **Study the interplay among the pion's Form Factor, TMDs, and PDFs in the LFQM.**

Schematic descriptions of form factors, TMDs, and PDFs of the Pion in the LFQM

$$\langle P'|\bar{q}\ (0)\gamma^{\mu}\ q(0)|\mathrm{P}\rangle = \wp^{\mu}\ F_{\pi}(q^2) \qquad \wp\cdot q = 0$$

FF $$F_{\pi}^{(\mu)}(Q^2) = \iint dx\, d^2\boldsymbol{k}_{\perp}\ f^{(\mu)}(x, \boldsymbol{k}_{\perp}, Q^2), \qquad (\mu = +, \perp, -)$$

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↓ as $Q^2 \to 0$

$$F_{\pi}^{(\mu)}(0) = 1 = \iint dx\ d^2\boldsymbol{k}_{\perp}\ f^{(\mu)}(x, \boldsymbol{k}_{\perp}) \quad \text{TMDs} \quad (\mu = +, -)$$

Schematic descriptions of form factors, TMDs, and PDFs of the Pion in the LFQM

$$\langle P'|\bar{q}\ (0)\gamma^{\mu}\ q(0)|\mathrm{P}\rangle = \wp^{\mu}\ F_{\pi}(q^2) \qquad \wp\cdot q = 0$$

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as $Q^2 \to 0$

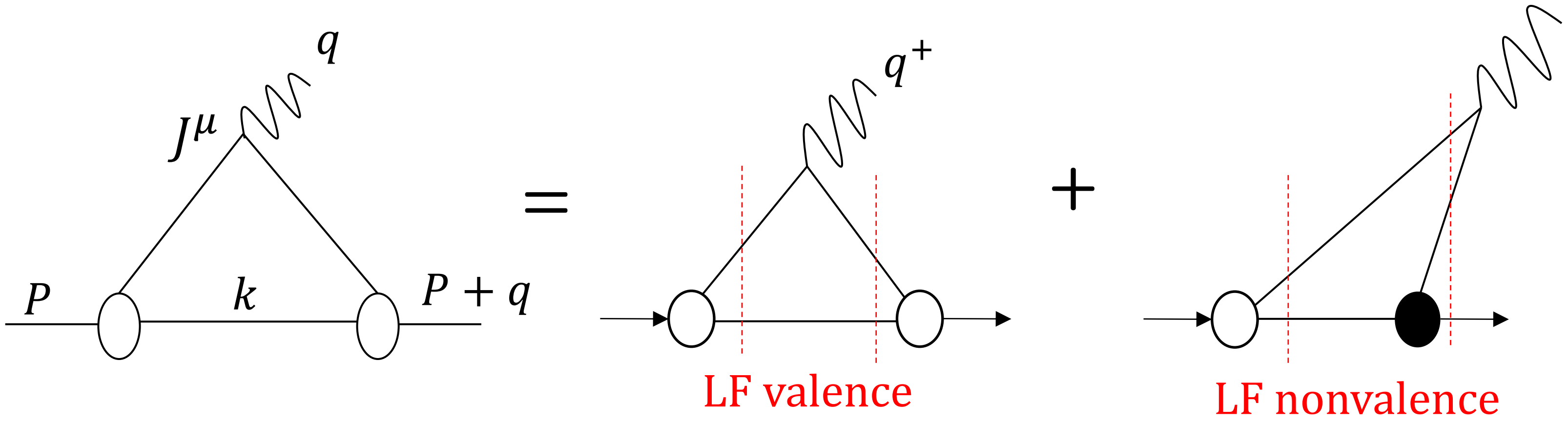
$$F_{\pi}^{(\mu)}(0) = 1 = \iint dx\, d^2\boldsymbol{k}_{\perp}\ f^{(\mu)}(x,\boldsymbol{k}_{\perp})$$ TMDs $(\mu = +,-)$

PDFs $$f^{(\mu)}(x) = \int d^2\boldsymbol{k}_{\perp}\ f^{(\mu)}(x,\boldsymbol{k}_{\perp})$$

$(\mu = +,-)$

$c.f.)\ \mu = \perp$ case later...

2. Form Factors on the Light-Front



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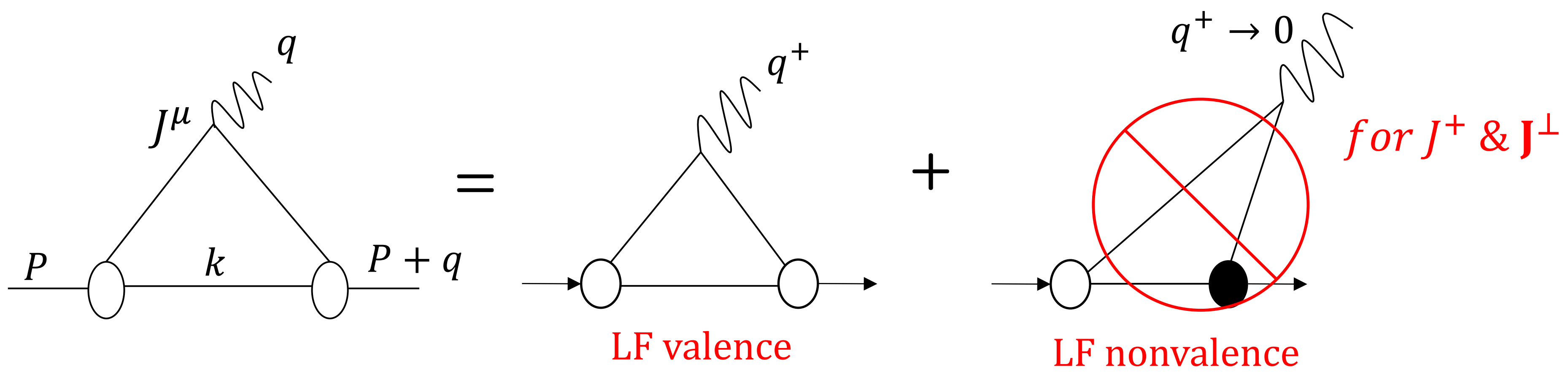

$\Longrightarrow$ facilitates the partonic interpretation of the amplitude!

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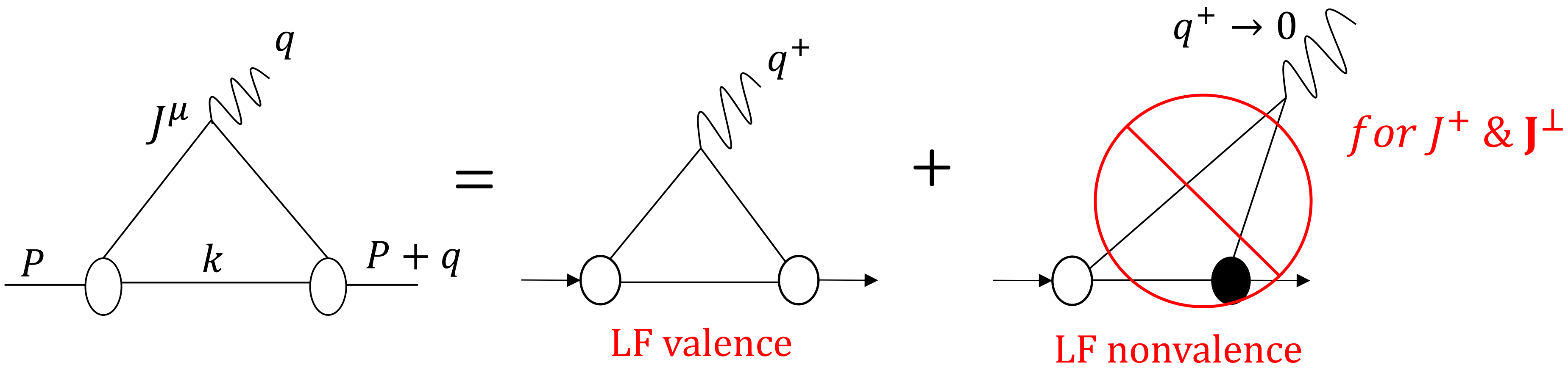

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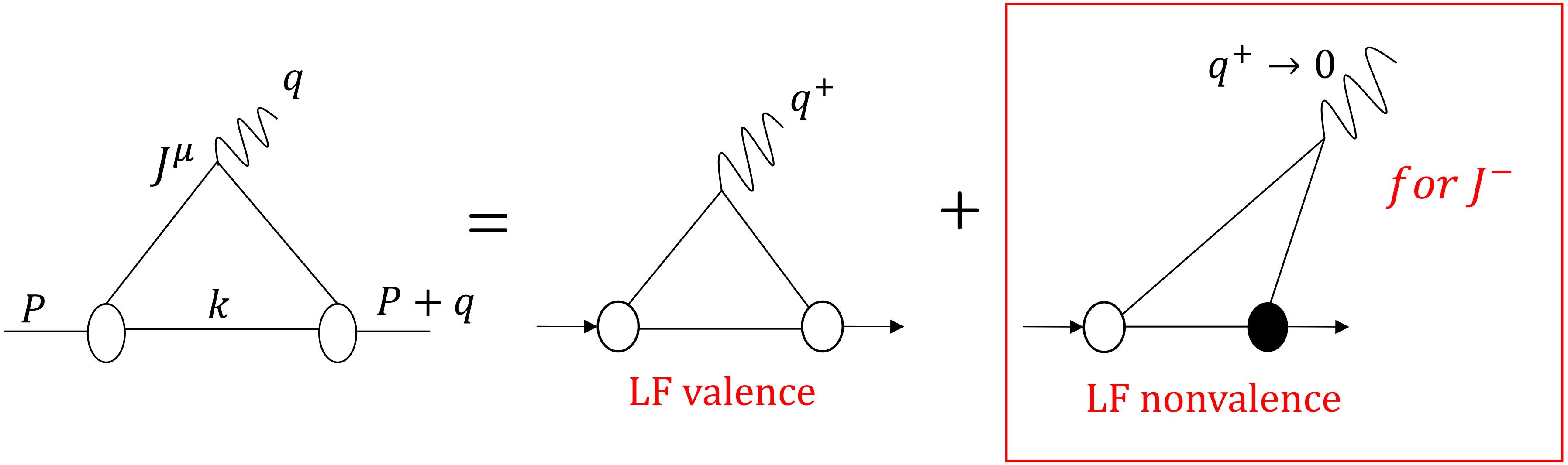
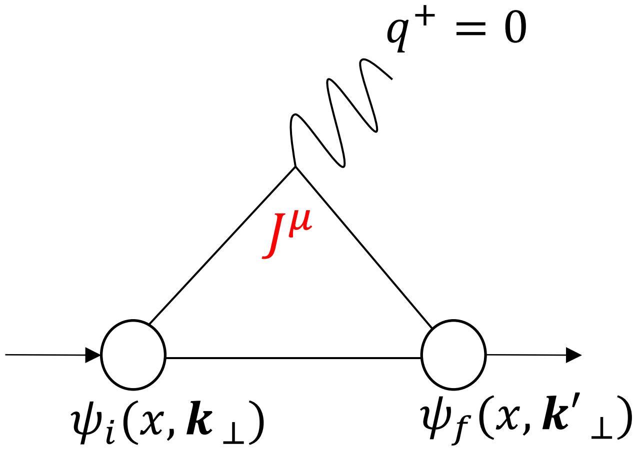


Nonvanishing : LF Zero-Mode !

We developed a "new method" to obtain the form factor

$$F_\pi^{(+)}(Q^2) = F_\pi^{(\perp)}(Q^2) = F_\pi^{(-)}(Q^2)$$

within the valence picture of the LFQM.



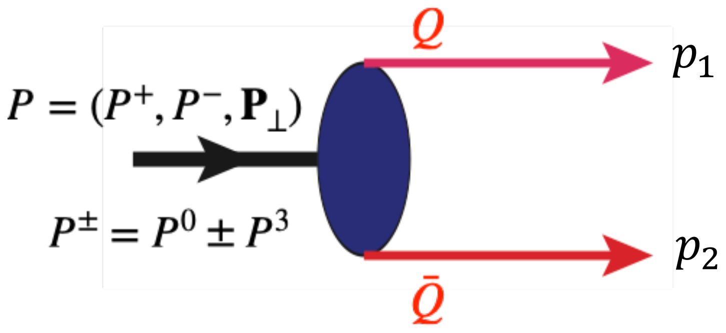
$$\langle P'|\bar{q}\,(0)\gamma^\mu\, q(0)|\mathrm{P}\rangle = \wp^\mu\, F_\pi(q^2)$$

$$F(Q^2) = \int [dx][d^2\boldsymbol{k}_\perp]\,\psi_f^*\,(x, \boldsymbol{k}'_\perp)\psi_i(x, \boldsymbol{k}_\perp)$$

$for\ any\ J^\mu = (J^+, \mathbf{J}^\perp, J^-)$

☞ We resolved the LF zero mode problems from J^-.

3. Light-Front Quark Model(LFQM)



$M \rightarrow M_0$

$$P^- = p_1^- + p_2^- \quad \rightarrow \quad \frac{M_0^2 + \boldsymbol{P}_\perp^2}{P^+} = \frac{m_1^2 + \boldsymbol{p}_{1\perp}^2}{p_1^+} + \frac{m_2^2 + \boldsymbol{p}_{2\perp}^2}{p_2^+}$$

$$M_0^2 = \frac{m_1^2 + \boldsymbol{k}_\perp^2}{x} + \frac{m_2^2 + \boldsymbol{k}_\perp^2}{1 - x}$$

Invariant mass

Meson state: Noninteracting "on-mass" shell Q & $\bar{Q}$ representation.

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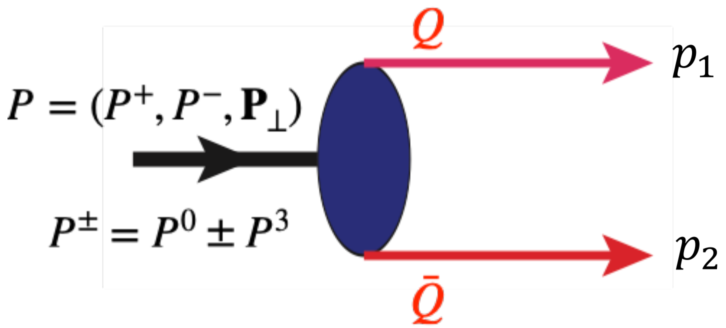

$$P^- = p_1^- + p_2^- \rightarrow \frac{M_0^2 + \boldsymbol{P}_\perp^2}{P^+} = \frac{m_1^2 + \boldsymbol{p}_{1\perp}^2}{p_1^+} + \frac{m_2^2 + \boldsymbol{p}_{2\perp}^2}{p_2^+}$$

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$$\Psi_{\lambda\bar{\lambda}}(x, \boldsymbol{k}_\perp) = \phi(x, \boldsymbol{k}_\perp)\mathcal{R}_{\lambda\bar{\lambda}}(x, \boldsymbol{k}_\perp)$$

Spin-Orbit for PS meson

$$\phi(x, \boldsymbol{k}_\perp) = \frac{4\pi^{3/4}}{\beta^{3/2}}\sqrt{\frac{\partial k_z}{\partial x}}\exp\left(-\frac{\boldsymbol{k}^2}{2\beta^2}\right)$$

$$\mathcal{R}_{\lambda_q\lambda_{\bar{q}}} = \frac{\bar{u}_{\lambda_q}(p_q)\gamma_5 v_{\lambda_{\bar{q}}}(p_{\bar{q}})}{\sqrt{2}M_0}$$

H.J. Melosh: PRD 9, 1095(1974).

$$\int_0^1 dx \int \frac{d^2\boldsymbol{k}_\perp}{16\pi^3} |\phi(x, \boldsymbol{k}_\perp)|^2 = 1$$

$$\sum_{\lambda' s} \mathcal{R}^\dagger \mathcal{R} = 1.$$

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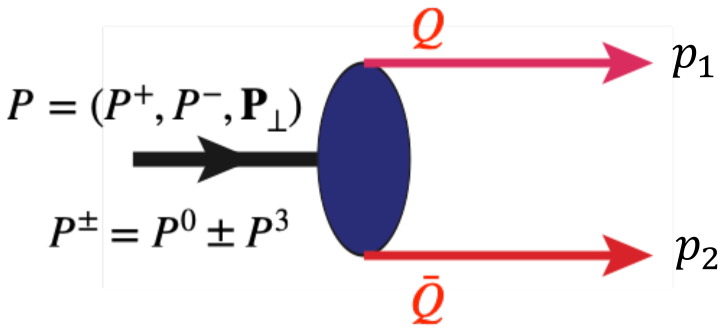

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The interaction between $Q\bar{Q}$ is incorporated into the mass operator via $M := M_0 + V_{Q\bar{Q}}$

$$H_{Q\bar{Q}} = \sqrt{m_Q^2 + \vec{k}^2} + \sqrt{m_{\bar{Q}}^2 + \vec{k}^2} + V_{Q\bar{Q}} \qquad V_{Q\bar{Q}} = a + br(br^2) - \frac{4\kappa}{3r} + \frac{2\vec{S}_Q \cdot \vec{S}_{\bar{Q}}}{3m_Q m_{\bar{Q}}} \nabla^2 V_{\text{Coul}}$$

$$H_{Q\bar{Q}} |\Psi\rangle = M_{Q\bar{Q}} |\Psi\rangle$$

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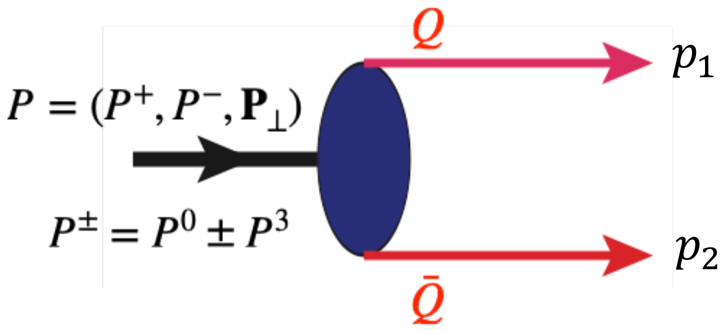

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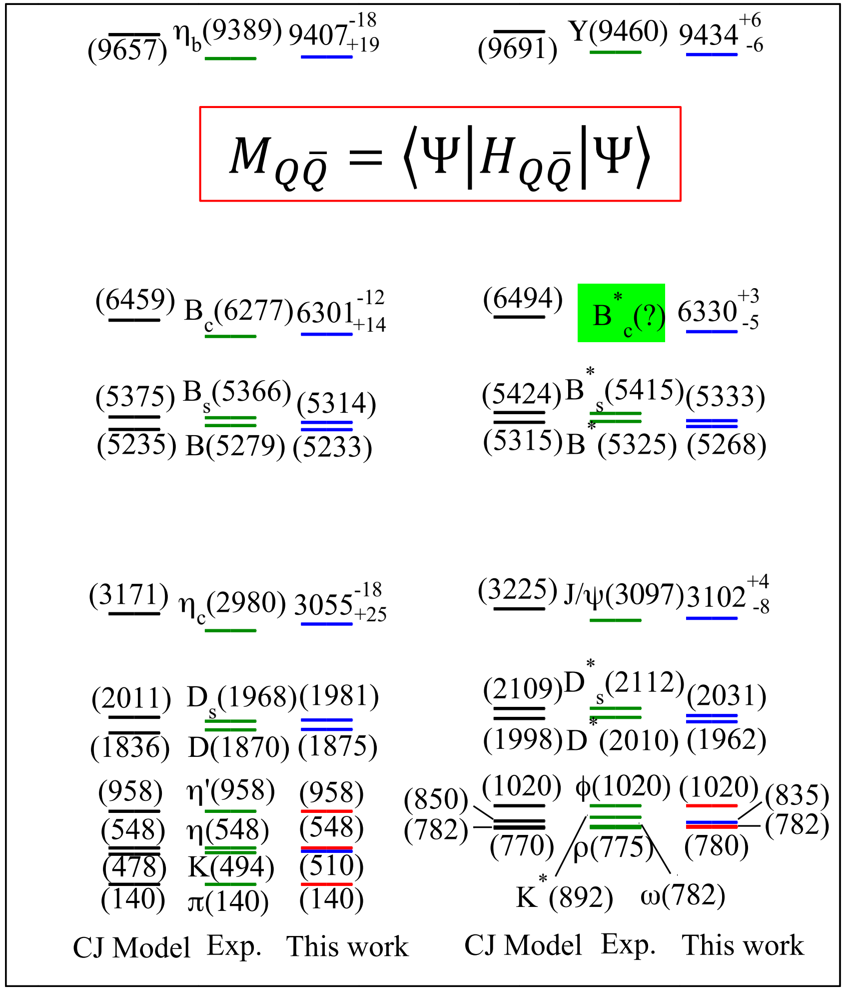
$$H_{Q\bar{Q}} |\Psi\rangle = M_{Q\bar{Q}} |\Psi\rangle$$

Bakamjian-Thomas(BT) constuction!

Optimized model parameters(in unit of GeV) and 1S state meson mass spectra

Model	m_q	m_s	m_c	m_b	β_{qq}	β_{sq}	β_{ss}	β_{qc}	β_{sc}	β_{cc}	β_{qb}	β_{sb}	β_{cb}	β_{bb}
Linear	0.22	0.45	1.8	5.2	0.366	0.389	0.413	0.468	0.502	0.651	0.527	0.571	0.807	1.145
HO	0.25	0.48	1.8	5.2	0.319	0.342	0.368	0.422	0.469	0.699	0.496	0.574	1.035	1.803



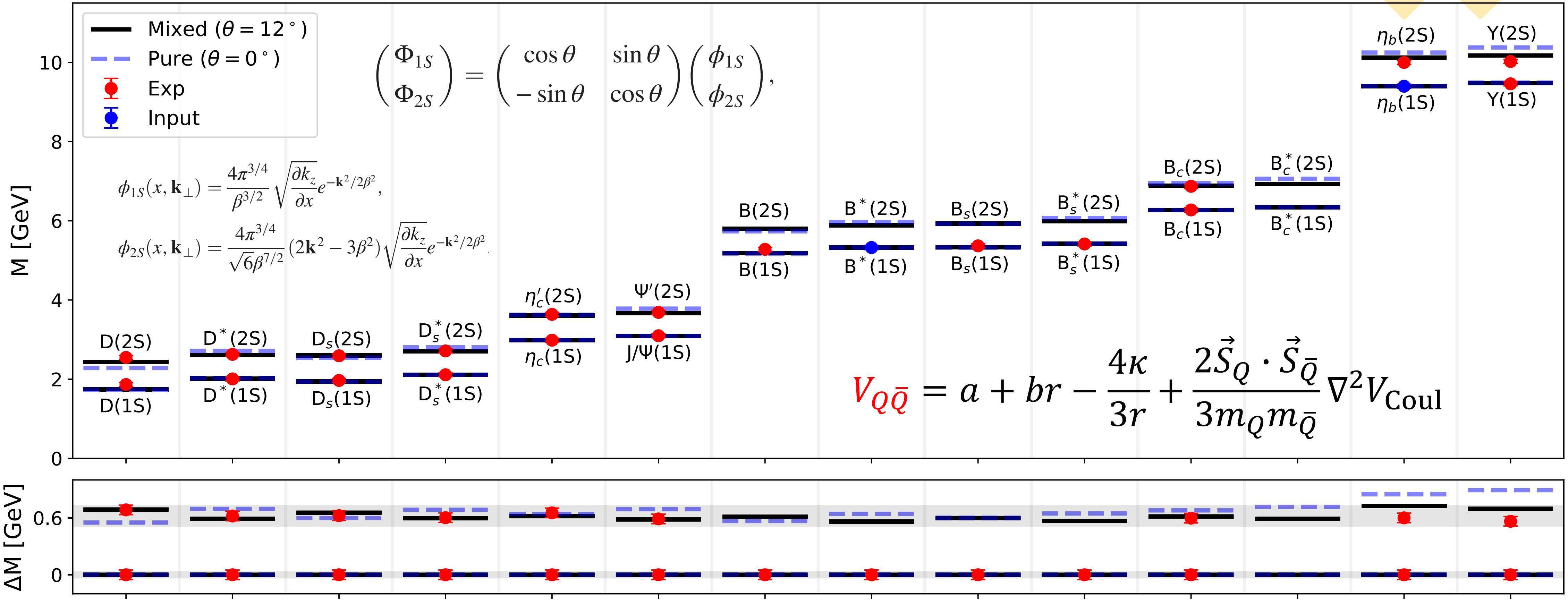
PRC92, 055203(2015) by HMC, CRJ, Z. Li, and H. Ryu

Mass spectroscopy analysis

- PRD59, 074015(99); PLB460, 461(99) by HMC and CRJ
- PRC92, 055203(2015) by HMC, CRJ, Z. Li, and H. Ryu
- PRD 100, 014026(2019) by N. Dhiman, H. Dahiya, HMC, CRJ
- PRD106, 014009(2022) by A. J. Arifi, HMC, CRJ, YO

Analysis of $(1S, 2S)$ state heavy meson spectroscopy

PRD106, 014009(2022) by A. J. Arifi, HMC, CRJ, YO



$$\begin{pmatrix}\Phi_{1S}\\ \Phi_{2S}\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\phi_{1S}\\ \phi_{2S}\end{pmatrix},$$

$$\phi_{1S}(x,\mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\beta^{3/2}}\sqrt{\frac{\partial k_z}{\partial x}}e^{-\mathbf{k}^2/2\beta^2},$$

$$\phi_{2S}(x,\mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\sqrt{6}\beta^{7/2}}(2\mathbf{k}^2 - 3\beta^2)\sqrt{\frac{\partial k_z}{\partial x}}e^{-\mathbf{k}^2/2\beta^2}.$$

$$V_{Q\bar{Q}} = a + br - \frac{4\kappa}{3r} + \frac{2\vec{S}_Q \cdot \vec{S}_{\bar{Q}}}{3m_Q m_{\bar{Q}}}\nabla^2 V_{\text{Coul}}$$

$\frac{1}{2}\cot^{-1}(2\sqrt{6}) < \theta < \frac{\pi}{4}$ constrained by empirical mass gap: $\Delta M_P > \Delta M_V$

BT Construction on Hadronic Matrix Element Calculations

PRD 89, 033011(14); PRD91, 014018(15); PRD95, 056002(17) by HMC and C.-R. Ji PRD 103, 073004(21); Adv. High Energy Phys., 4277321(21) by HMC

PRD 107, 053003(23); PRD108, 013006(23) by A. J. Arifi, HMC and C.-R. Ji

$\langle P'|\bar{q}\ \Gamma^{\mu}\ q|\mathrm{P}\rangle = \wp^{\mu}\ \mathcal{F}$

$\mathcal{F}$: Physical observables

$\wp^{\mu}$: Lorentz factors

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$\mathcal{F}$: Physical observables $\wp^{\mu}$: Lorentz factors

- Most Previous LFQM: Apply BT ($M \rightarrow M_0$) only to LHS.

$$M \rightarrow M_0$$

$$\langle P'|\bar{q}\ \Gamma^{\mu}\ q|\mathrm{P}\rangle = \wp^{\mu}\ \mathcal{F}$$

$$\mathcal{F} = \frac{1}{\wp^{\mu}}\langle P'|\bar{q}\ \Gamma^{\mu}\ q|\mathrm{P}\rangle$$

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$$\langle P'|\bar{q}\ \Gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\ \mathcal{F}$$

$\mathcal{F}$: Physical observables $\wp^\mu$: Lorentz factors

- Most Previous LFQM: Apply BT ($M \rightarrow M_0$) only to the matrix element.

$$M \rightarrow M_0$$

$$\langle P'|\bar{q}\ \Gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\ \mathcal{F}$$

$$\mathcal{F} = \frac{1}{\wp^\mu}\langle P'|\bar{q}\ \Gamma^\mu\ q|\mathrm{P}\rangle$$

Fails to show the independency of "μ"
due to the LF zero-modes from the "bad" current.

New development of a "self-consistent" LFQM based on the BT construction.

- In our LFQM: Apply BT ($M \rightarrow M_0$) equally to both sides

$$M \rightarrow M_0$$

$$\langle P'|\bar{q}\ \Gamma^{\mu}\ q|\mathrm{P}\rangle = \wp^{\mu}\ \mathcal{F}$$

$$\mathcal{F} = \left\langle P'\left|\frac{\bar{q}\ \Gamma^{\mu} q}{\wp^{\mu}}\right|\mathrm{P}\right\rangle$$

becomes independent of the current components!

Pion Form Factor

General structure for $P(P) \rightarrow P(P')$ transition: PRD 103, 073004(21); Adv. High Energy Phys., 4277321(21) by HMC

$$\langle P'|\bar{q}\ \gamma^{\mu}\ q|\mathrm{P}\rangle = \wp^{\mu}\ F(q^2)\ + q^{\mu}\ \frac{(M^2 - M'^2)}{q^2} H(q^2), \qquad \wp^{\mu} = (P + P')^{\mu} - q^{\mu}\ \frac{(M^2 - M'^2)}{q^2}$$

$$q^{\mu} = (P - P')^{\mu}$$

Pion Form Factor

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$$q^\mu = (P - P')^\mu$$

For elastic process,
only gauge invariant form factor $F(q^2)$ survives!

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\ F_P(q^2) \qquad \wp \cdot q = 0$$

Pion Form Factor

$$\langle P'|\bar{q}\ \gamma^{\mu}\ q|\mathrm{P}\rangle = \wp^{\mu}\ F_P(q^2), \qquad \wp^{\mu} = (P+P')^{\mu} - q^{\mu}\ \frac{(M^2 - M'^2)}{q^2}$$

In $q^+ = 0$ frame,

$$\langle P'|\bar{q}\ \gamma^{\mu}\ q|\mathrm{P}\rangle = \int_0^1 \mathrm{d}p_1^+ \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\ \phi'(x,\mathbf{k}'_\perp)\phi(x,\mathbf{k}_\perp)\ \sum_{\lambda's} \mathcal{R}^\dagger_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}}\gamma^{\mu}\frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right]\mathcal{R}_{\lambda_1\bar{\lambda}},$$

$$F_P^{(\mu)}(Q^2) = \int_0^1 \mathrm{d}p_1^+ \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\ \phi'(x,\mathbf{k}'_\perp)\phi(x,\mathbf{k}_\perp)\ \frac{1}{\wp^{\mu}}\ \sum_{\lambda's} \mathcal{R}^\dagger_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}}\gamma^{\mu}\frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right]\mathcal{R}_{\lambda_1\bar{\lambda}},$$

Pion Form Factor

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\ F_P(q^2), \qquad \wp^\mu = (P+P')^\mu - q^\mu\ \frac{(M^2 - M'^2)}{q^2}$$

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Previous LFQM method, which couldn't resolve the zero-mode.

Apply $M \rightarrow M_0$ only to

$$F_P^{(\mu)}(Q^2) = \int_0^1 \mathrm{d}p_1^+ \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\ \phi'(x,\mathbf{k}'_\perp)\phi(x,\mathbf{k}_\perp)\ \frac{1}{\wp^\mu} \sum_{\lambda' s} \mathcal{R}^\dagger_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}}\gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right] \mathcal{R}_{\lambda_1\bar{\lambda}},$$

This leads to $F_\pi^{(+)}(Q^2) = F_\pi^{(\perp)}(Q^2) \neq F_\pi^{(-)}(Q^2)$

Pion Form Factor

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\ F_P(q^2), \qquad \wp^\mu = (P+P')^\mu - q^\mu\ \frac{(M^2 - M'^2)}{q^2}$$

In $q^+ = 0$ frame,

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \int_0^1 \mathrm{d}p_1^+ \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\ \phi'(x,\mathbf{k}'_\perp)\phi(x,\mathbf{k}_\perp) \sum_{\lambda' s} \mathcal{R}^\dagger_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}}\gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right] \mathcal{R}_{\lambda_1\bar{\lambda}},$$

Apply $M \rightarrow M_0$ both to

Our New method, which now resolves the zero-mode.

$$F_P^{(\mu)}(Q^2) = \int_0^1 \mathrm{d}p_1^+ \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\ \phi'(x,\mathbf{k}'_\perp)\phi(x,\mathbf{k}_\perp)\ \frac{1}{\wp^\mu} \sum_{\lambda' s} \mathcal{R}^\dagger_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}}\gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right] \mathcal{R}_{\lambda_1\bar{\lambda}},$$

Then we get $F_\pi^{(+)}(Q^2) = F_\pi^{(\perp)}(Q^2) = F_\pi^{(-)}(Q^2)$

$$F_\pi^{\mathrm{SLF}(\mu)}(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi(x,\mathbf{k}_\perp)\phi'(x,\mathbf{k}'_\perp)}{\sqrt{\mathbf{k}_\perp^2+m^2}\sqrt{\mathbf{k}'^2_\perp+m^2}} \mathcal{O}^{(\mu)}_{\mathrm{LFQM}}$$

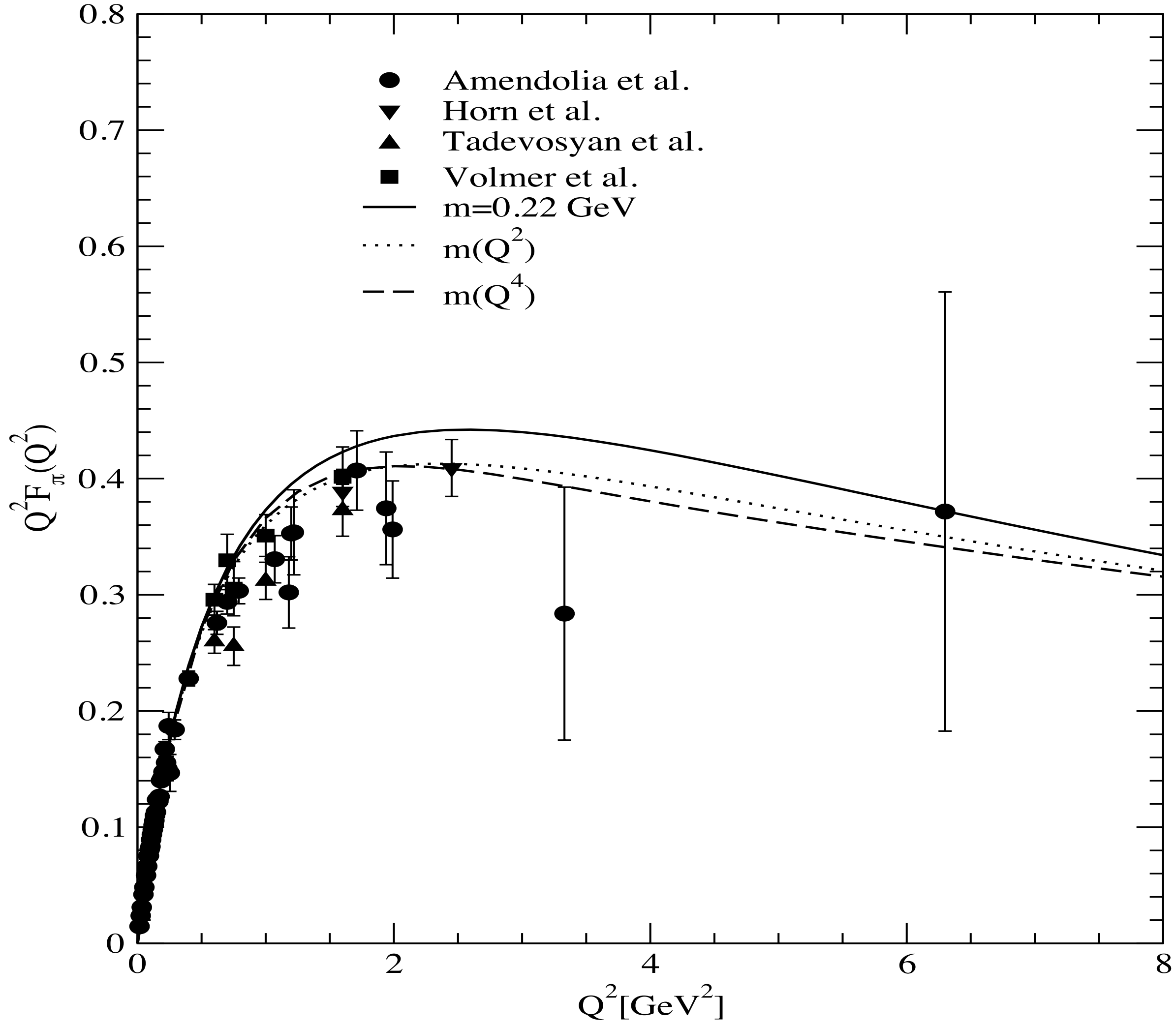
TABLE II: The operators $\mathcal{O}^{(\mu)}_{\mathrm{LFQM}}$ and their helicity contributions to the pion form factor in the standard LFQM.

$F_\pi^{(\mu)}$	$\mathcal{O}^{(\mu)}_{\mathrm{LFQM}}$	$\mathcal{H}^{(\mu)}_{(\uparrow\to\uparrow)+(\downarrow\to\downarrow)}$	$\mathcal{H}^{(\mu)}_{(\uparrow\to\downarrow)+(\downarrow\to\uparrow)}$
$F_\pi^{(+)}$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2$	0
$F_\pi^{(\perp)}$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2$	0
$F_\pi^{(-)}$	$\frac{2(1-x)\mathbf{q}_\perp^2 M_0^2(\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2+\mathbf{q}_\perp\cdot\mathbf{k}'_\perp)}{x[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$	$\frac{2\mathbf{q}_\perp^2\{(\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2)(\mathbf{k}_\perp^2+\mathbf{k}_\perp\cdot\mathbf{q}_\perp+m^2)+(1-x)(\mathbf{k}_\perp\times\mathbf{q}_\perp)^2\}}{x^2[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$	$\frac{2\mathbf{q}_\perp^2\{(1-x)m^2\mathbf{q}_\perp^2\}}{x^2[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$

$$\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$$

$$F_\pi^{(+)}(Q^2) = F_\pi^{(\perp)}(Q^2) = F_\pi^{(-)}(Q^2)$$

"The first proof of the pion form factor's independence from current components in the LFQM!"



$f_\pi^{LFQM} = 130$ MeV

(Exp.=131 MeV)

$r_\pi^{LFQM} = 0.654$ fm

(Exp.=0.659(4)fm)

$$F_\pi^{(+)}(Q^2) = F_\pi^{(\perp)}(Q^2) = F_\pi^{(-)}(Q^2)$$

4. Unpolarized TMDs of pion

C. Lorce, B. Pasquini, P. Schweitzer EPJC76,415(2016)

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma^+\psi(z)|P\rangle|_{z^+=0} = f_1^q(x, p_T),$$

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma_T^j\psi(z)|P\rangle|_{z^+=0} = \frac{p_T^j}{P^+} f_3^q(x, p_T),$$

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma^-\psi(z)|P\rangle|_{z^+=0} = \left(\frac{m_\pi}{P^+}\right)^2 f_4^q(x, p_T),$$

which are related with the forward matrix elements $\langle P|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle$ as

$$2P^+ \int dx\, f_1^q(x) = \langle P|\bar{\psi}(0)\gamma^+\psi(0)|P\rangle,$$

$$2p_T \int dx\, f_3^q(x) = \langle P|\bar{\psi}(0)\gamma^\perp\psi(0)|P\rangle,$$

$$4P^- \int dx\, f_4^q(x) = \langle P|\bar{\psi}(0)\gamma^-\psi(0)|P\rangle,$$

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$$4P^- \int dx\, f_4^q(x) = \langle P|\bar{\psi}(0)\gamma^-\psi(0)|P\rangle,$$

PDF TMD

$$f(x) = \int d^2p_T\, f(x, p_T).$$

4. Unpolarized TMDs of pion

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$$4P^- \int dx\, f_4^q(x) = \langle P|\bar{\psi}(0)\gamma^-\psi(0)|P\rangle,$$

Sum rules :

$$\int dx\ f_1^q(x) = 1$$

$$2\int dx\ f_4^q(x) = 1$$

In free quark model:

$$x f_3^q(x, p_T) = f_1^q(x, p_T)$$

4. Unpolarized TMDs of pion

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$$2p_T \int dx\, f_3^q(x) = \langle P|\bar\psi(0)\gamma^\perp\psi(0)|P\rangle,$$

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Sum rules :

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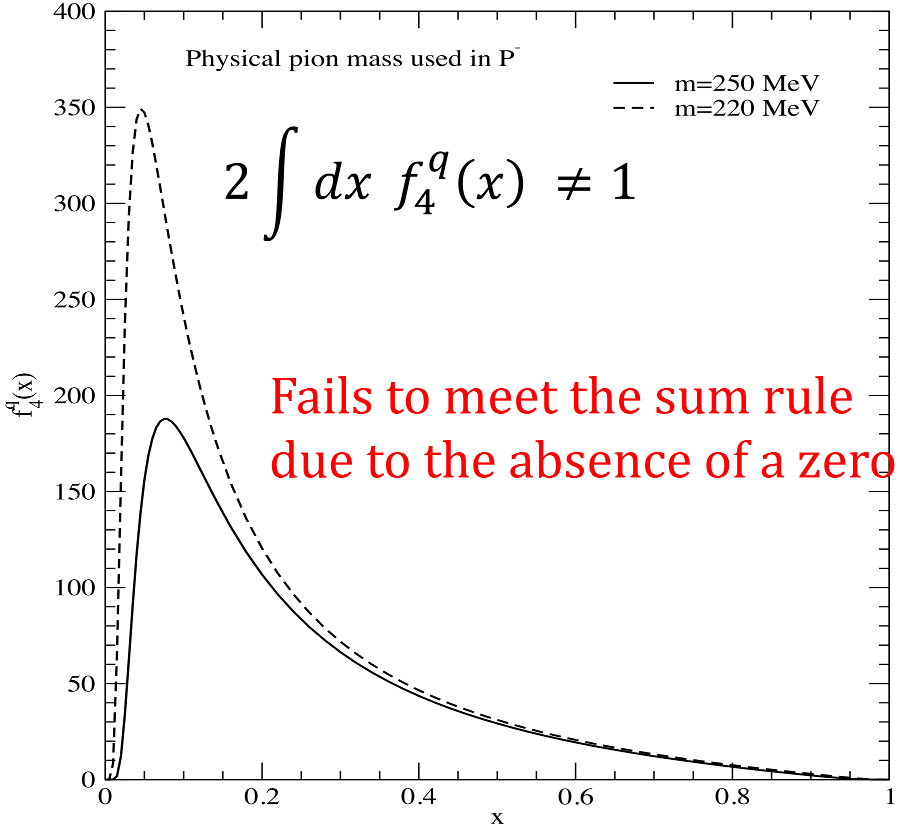
$$2\int dx\ f_4^q(x) = 1$$

In free quark model:

$$x f_3^q(x, p_T) = f_1^q(x, p_T)$$

However...



$$2\int dx\ f_4^q(x) \neq 1$$

Fails to meet the sum rule due to the absence of a zero mode!

Derivation of Sum rules

C. Lorce, B. Pasquini, P. Schweitzer

EPJC76,415(2016)

$$\langle P'|\ J^{\mu}\ |\mathrm{P}\rangle = \bar{\wp}^{\mu}\ F_{\pi}(q^2)$$

$$\bar{\wp}^{\mu} = (P + P')^{\mu}$$

$$1 = F_{\pi}(Q^2 = 0) = \lim_{Q \to 0} \frac{\langle P'|\ J^{\mu}\ |\mathrm{P}\rangle}{\bar{\wp}^{\mu}}$$

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$$\Downarrow$$

$$\lim_{Q\to 0} \frac{\langle P'|\ J^{+}\ |\mathrm{P}\rangle}{\bar{\wp}^{+}} = \int dx\ f_1^q(x)$$

$$\lim_{Q\to 0} \frac{\langle P'|\ J^{-}\ |\mathrm{P}\rangle}{\bar{\wp}^{-}} = 2\int dx\ f_4^q(x)$$

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$$\langle P'|\, J^\mu \,|\mathrm{P}\rangle = \bar{\wp}^\mu\, F_\pi(q^2)$$

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$$\lim_{Q\to 0} \frac{\langle P'|\, J^+ \,|\mathrm{P}\rangle}{\bar{\wp}^+} = \int dx\;\, f_1^q(x) \;= 1$$

$$\lim_{Q\to 0} \frac{\langle P'|\, J^- \,|\mathrm{P}\rangle}{\bar{\wp}^-} = 2\int dx\;\, f_4^q(x) \;\neq 1$$

This Work

$$\langle P'|\, J^\mu \,|\mathrm{P}\rangle = \wp^\mu\, F_\pi(q^2)$$

$$\wp^\mu = (P + P')^\mu - q^\mu \frac{(M^2 - M'^2)}{q^2}$$

$$1 = F_\pi(Q^2 = 0) = \lim_{Q\to 0} \left\langle P'|\, \frac{J^\mu}{\wp^\mu} \,|\mathrm{P} \right\rangle$$

Derivation of Sum rules

C. Lorce, B. Pasquini, P. Schweitzer

EPJC76,415(2016)

$$\langle P'|\ J^{\mu}\ |\mathrm{P}\rangle = \bar{\wp}^{\mu}\ F_{\pi}(q^2)$$

$$\bar{\wp}^{\mu} = (P + P')^{\mu}$$

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$$\lim_{Q\to 0} \frac{\langle P'|\ J^{+}\ |\mathrm{P}\rangle}{\bar{\wp}^{+}} = \int dx\ f_1^q(x)\ = 1$$

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This Work

$$\langle P'|\ J^{\mu}\ |\mathrm{P}\rangle = \wp^{\mu}\ F_{\pi}(q^2)$$

$$\wp^{\mu} = (P + P')^{\mu} - q^{\mu}\,\frac{(M^2 - M'^2)}{q^2}$$

$$1 = F_{\pi}(Q^2 = 0) = \lim_{Q\to 0} \left\langle P'|\ \frac{J^{\mu}}{\wp^{\mu}}\ |\mathrm{P}\right\rangle$$

$M \ \rightarrow\ M_0$

$$\lim_{Q\to 0} \left\langle P'|\ \frac{J^{+}}{\wp^{+}}\ |\mathrm{P}\right\rangle = \int dx\ f_1^q(x)$$

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$$\lim_{Q\to 0} \frac{\langle P'|\ J^{+}\ |\mathrm{P}\rangle}{\bar{\wp}^{+}} = \int dx\ f_1^q(x) = 1$$

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This Work

$$\langle P'|\ J^{\mu}\ |\mathrm{P}\rangle = \wp^{\mu}\ F_{\pi}(q^2)$$

$$\wp^{\mu} = (P + P')^{\mu} - q^{\mu}\ \frac{(M^2 - M'^2)}{q^2}$$

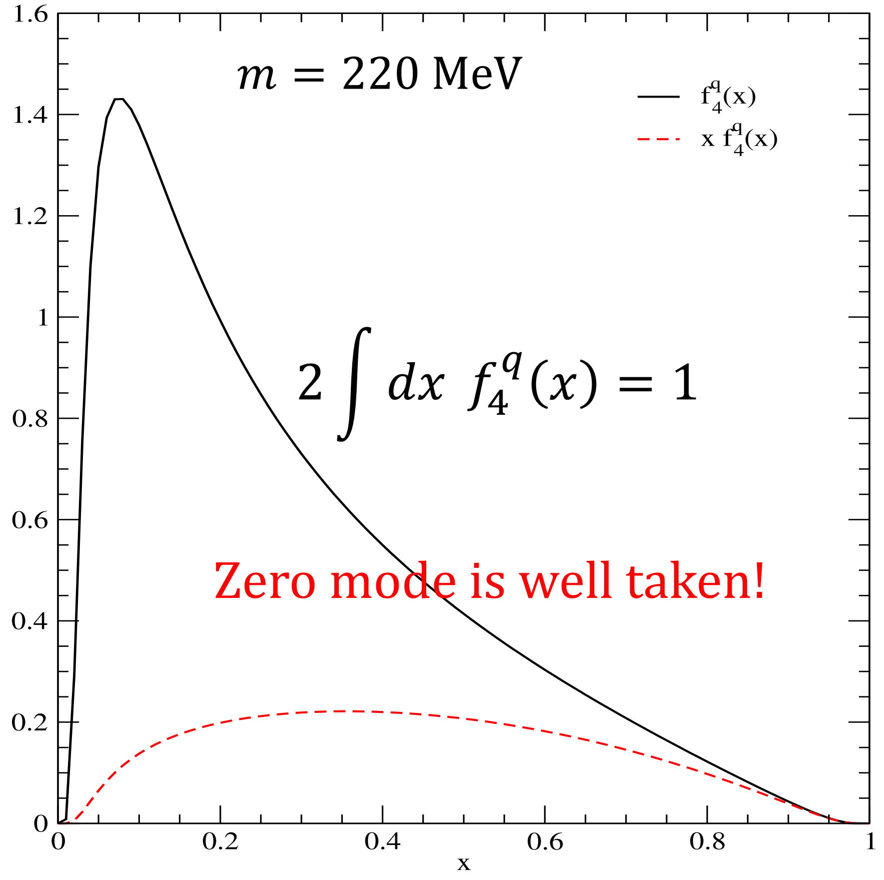
$$1 = F_{\pi}(Q^2 = 0) = \lim_{Q\to 0} \left\langle P'\middle|\ \frac{J^{\mu}}{\wp^{\mu}}\ \middle|\mathrm{P}\right\rangle$$

$$M \rightarrow M_0$$

$$\lim_{Q\to 0} \left\langle P'\middle|\ \frac{J^{+}}{\wp^{+}}\ \middle|\mathrm{P}\right\rangle = \int dx\ f_1^q(x) = 1$$

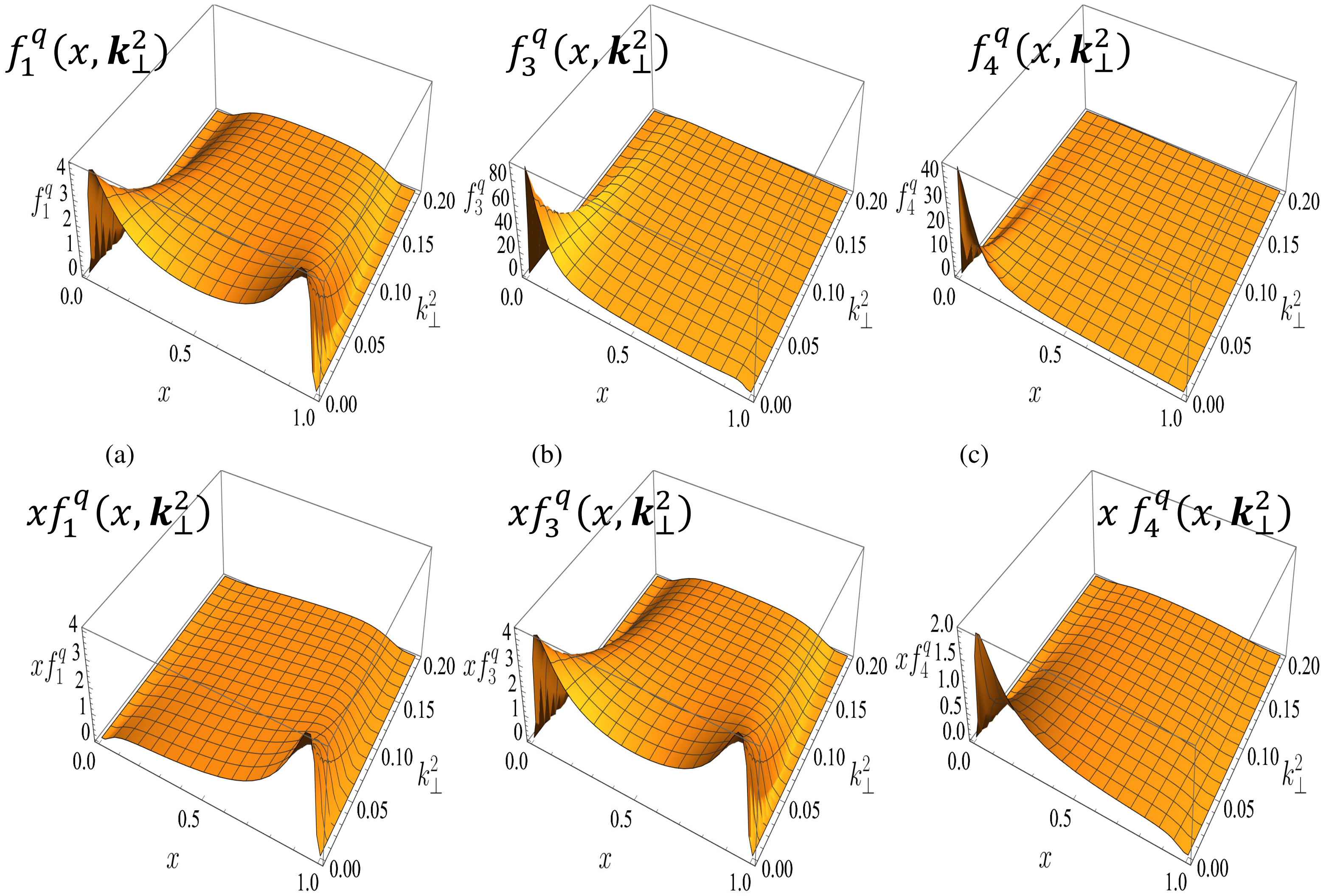
$$\lim_{Q\to 0} \left\langle P'\middle|\ \frac{J^{-}}{\wp^{-}}\ \middle|\mathrm{P}\right\rangle = 2\int dx\ f_4^q(x) = 1$$

LF Zero-Mode for twist-4 PDF and its Resolution



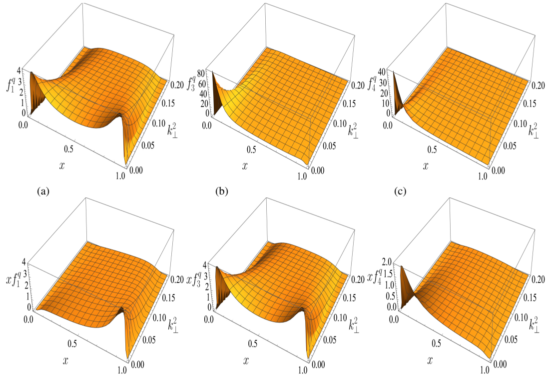
$$\lim_{Q \to 0} \left\langle P' \right| \frac{J^-}{\wp^-} \left| \mathrm{P} \right\rangle = 2 \int dx\ f_4^q(x) = 1$$

Unpolarized TMDs for Pion

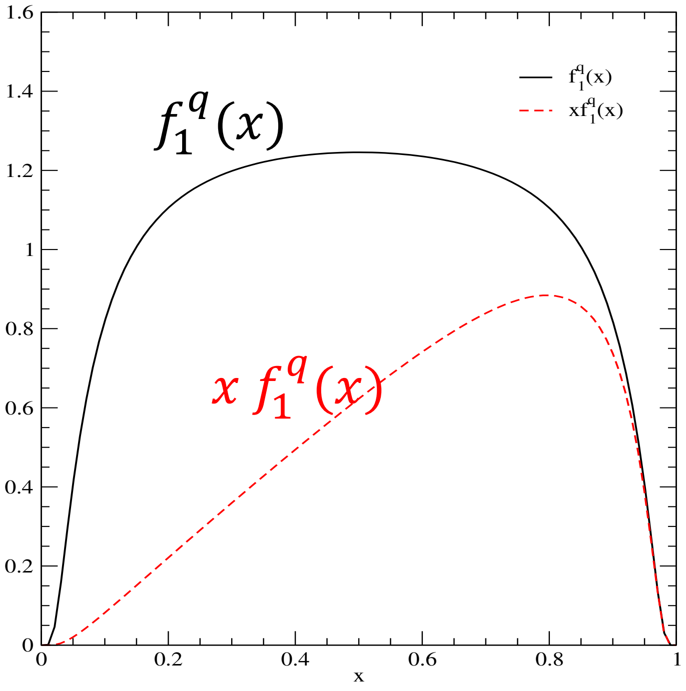


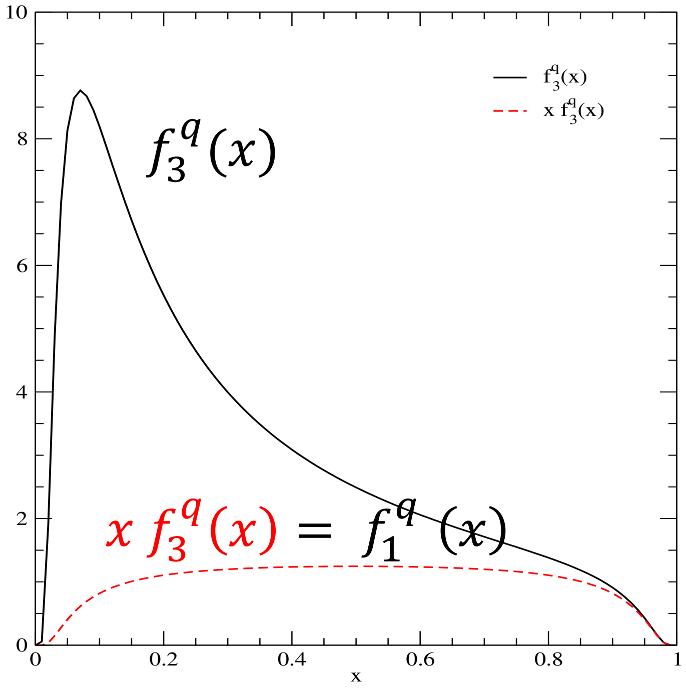
$$xf_3^q(x, \boldsymbol{k}_\perp^2) = f_1^q(x, \boldsymbol{k}_\perp^2)$$

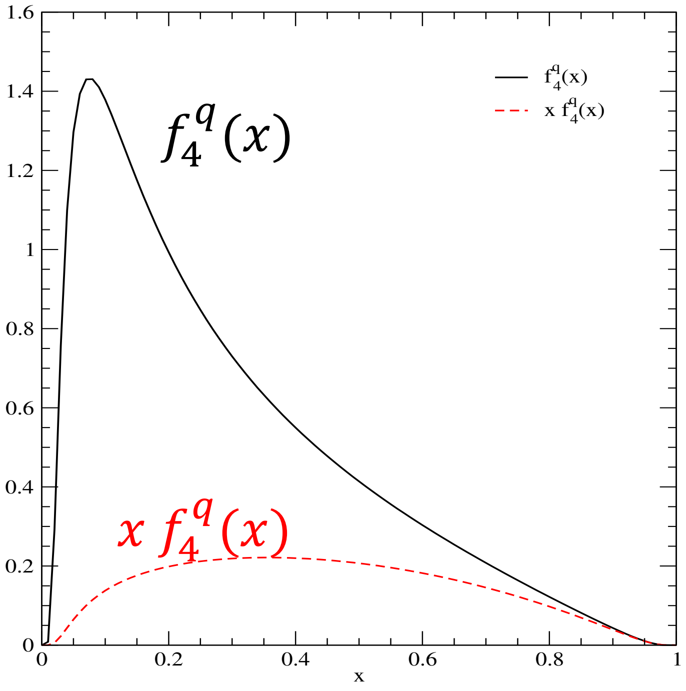
Unpolarized PDFs for Pion



$\int d^2\boldsymbol{k}_\perp$ at the initial scale $\mu_0 = 1$ GeV

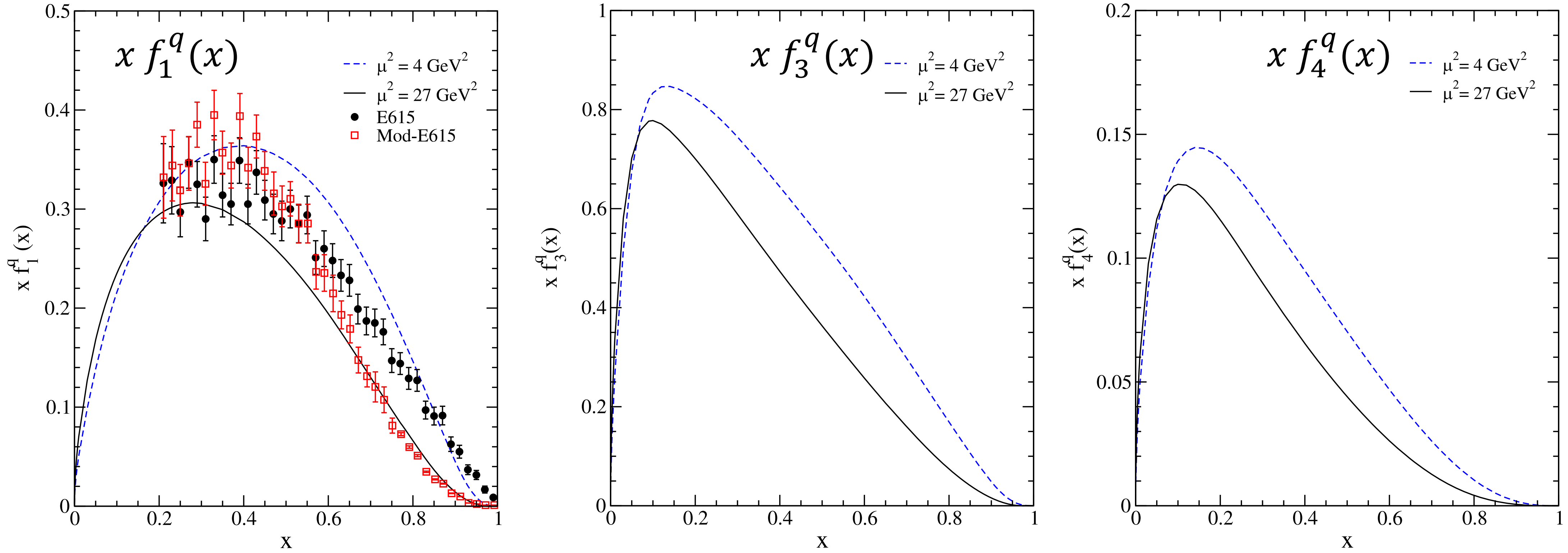






4. QCD Evolution of Pion PDFs

Evolved from $\mu_0^2 = 1\ \mathrm{GeV}^2$ to $\mu^2 = 4$ and $27\ \mathrm{GeV}^2$



We use the **Higher Order Perturbative Parton Evolution toolkit (HOPPET)** to solve the NNLO DGLAP equation.

Mellin moments: $\langle x^n \rangle = \int_0^1 dx\ x^n f(x)$

TABLE III. Mellin moments of the pion valence PDF, $f_1^q(x)$, evaluated at the scale $\mu^2 = 4\ \mathrm{GeV}^2$.

	$\langle x \rangle_{t2}^u$	$\langle x^2 \rangle_{t2}^u$	$\langle x^3 \rangle_{t2}^u$	$\langle x^4 \rangle_{t2}^u$
This work	0.236	0.101	0.055	0.033
[83]	0.2541(26)	0.094(12)	0.057(4)	0.015(12)
[84]	0.2075(106)	0.163(33)	...	...
[56]	0.24(2)	0.098(10)	0.049(7)	...
[57]	0.24(2)	0.094(13)	0.047(8)	...

[83] B. Joo et al. , PRD 100, 114512(2019)

[84] M. Oehm et al. , PRD 99, 014508(2019)

[56] M. Ding et al. PRD 101, 054014(2020)

[57] Z.-F. Cui et al. EPJC 80, 1064(2020)

TABLE IV. Mellin moments of the pion valence PDF, $f_1^q(x)$, evaluated at the scale $\mu^2 = 27\ \mathrm{GeV}^2$.

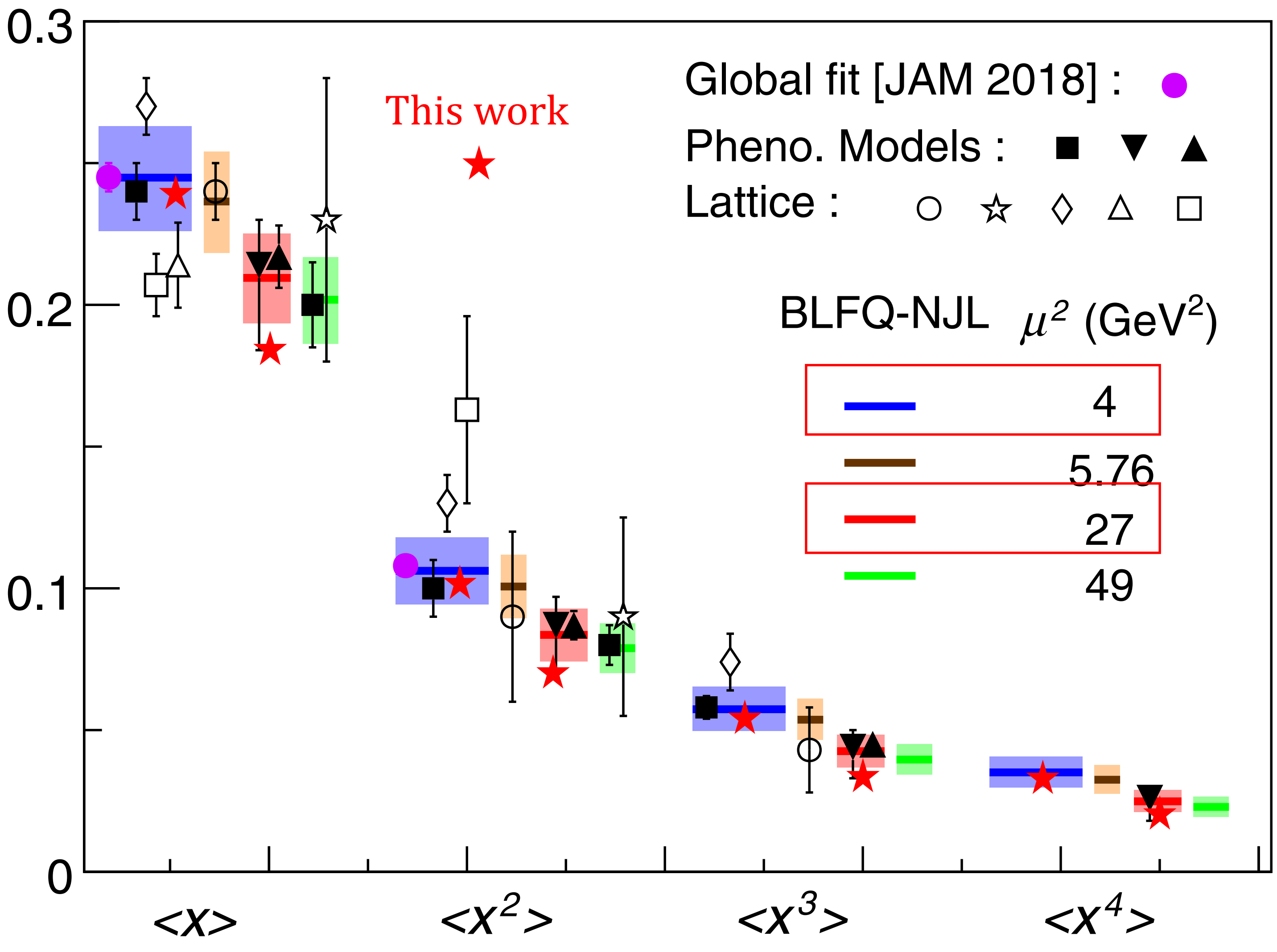
	$\langle x \rangle_{t2}^u$	$\langle x^2 \rangle_{t2}^u$	$\langle x^3 \rangle_{t2}^u$	$\langle x^4 \rangle_{t2}^u$
This work	0.182	0.069	0.034	0.019
[85]	0.18(3)	0.064(10)	0.030(5)	...
[57]	0.20(2)	0.074(10)	0.035(6)	...
[86]	0.184	0.068	0.033	0.018
[21]	0.217(11)	0.087(5)	0.045(3)	...

[85] R. S. Sufian et al. , PRD 99, 074507(2019)

[86] S.-I. Nam, PRD 86, 074005(2012)

[21] K. Wijesooriya, P. Reimer, R. J. Holt, PRC 72, 065203 (2005).

Lowest four moments of pion valence PDF



Adopted from J. Lan et al. (BLFQ Collab.), PRL 122, 172001 (2019)

Mellin moments: $\langle x^n \rangle = \int_0^1 dx\ x^n f(x)$

TABLE V. Mellin moments of the twist-3 pion PDF, $f_3^q(x)$, evaluated at the scales $\mu^2 = 4\ \mathrm{GeV}^2$ and $\mu^2 = 27\ \mathrm{GeV}^2$, respectively.

	$\langle x \rangle_{t3}^u$	$\langle x^2 \rangle_{t3}^u$	$\langle x^3 \rangle_{t3}^u$	$\langle x^4 \rangle_{t3}^u$
$\mu^2 = 4\ \mathrm{GeV}^2$	0.471	0.164	0.079	0.045
$\mu^2 = 27\ \mathrm{GeV}^2$	0.365	0.111	0.049	0.026

TABLE VI. Mellin moments of the twist-4 pion PDF, $f_4^q(x)$, evaluated at the scales $\mu^2 = 4\ \mathrm{GeV}^2$ and $\mu^2 = 27\ \mathrm{GeV}^2$, respectively.

	$\langle x \rangle_{t4}^u$	$\langle x^2 \rangle_{t4}^u$	$\langle x^3 \rangle_{t4}^u$	$\langle x^4 \rangle_{t4}^u$
$\mu^2 = 4\ \mathrm{GeV}^2$	0.069	0.021	0.009	0.005
$\mu^2 = 27\ \mathrm{GeV}^2$	0.053	0.014	0.006	0.003

5. Conclusions

- We developed a new method for ensuring self-consistency in the LFQM.

Our LFQM: Noninteracting Q & $\bar{Q}$ representation consistent with the Bakamjian-Thomas(BT) constuction!

$$P^- = p_q^- + p_{\bar{q}}^- \text{ , i.e. } M^2 \rightarrow M_0^2$$

$$\langle 0|\bar{q}\,\Gamma^\mu q|\mathrm{P}\rangle = \mathfrak{F}\,\wp^\mu$$

$\mathfrak{F}$: physical observables ($\mathfrak{F} = f_P, F \cdots$)

$\wp^\mu$: Lorentz factors ($\wp = P^\mu \cdots$)

$$\mathfrak{F} = \left\langle 0\left|\frac{\bar{q}\Gamma^\mu q}{\wp^\mu}\right|\mathrm{P}\right\rangle = \iint dx\, d^2\mathbf{k}_\perp \cdots \left(\frac{\Gamma^\mu}{\wp^\mu}\right)\cdots$$

Constrained by BT construction!

⇒ This allows one to obtain the physical observables independent of the current components !

Partial Extractions of TMD, PDF, GPD from Pion Form Factor

Form factor: $F^{(\mu)}(t) \equiv \iint dx\, d\boldsymbol{k}_\perp\, f^{(\mu)}(x, \boldsymbol{k}_\perp, t)$ Note) $Q^2 \to -t$

$f^{(\mu)}(x, \boldsymbol{k}_\perp, t \to 0)$

$\int d\boldsymbol{k}_\perp$

TMD $f(x, \boldsymbol{k}_\perp)$

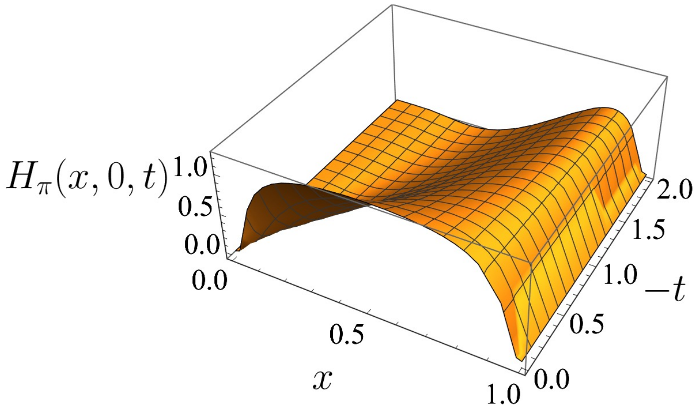
GPD $H(x, 0, t)$

$f_1^q(x, \boldsymbol{k}_\perp) \leftrightarrow f^{(+)}(x, \boldsymbol{k}_\perp, 0)$

$2f_4^q(x, \boldsymbol{k}_\perp) \leftrightarrow f^{(-)}(x, \boldsymbol{k}_\perp, 0)$

$H(x, 0, t) = \int d\boldsymbol{k}_\perp\, f^{(+)}(x, \boldsymbol{k}_\perp, t)$

GPD at $\zeta = 0$



$\int d\boldsymbol{k}_\perp$

$H(x, 0, 0)$

PDFs $f_1^q(x)$: twist-2 PDF

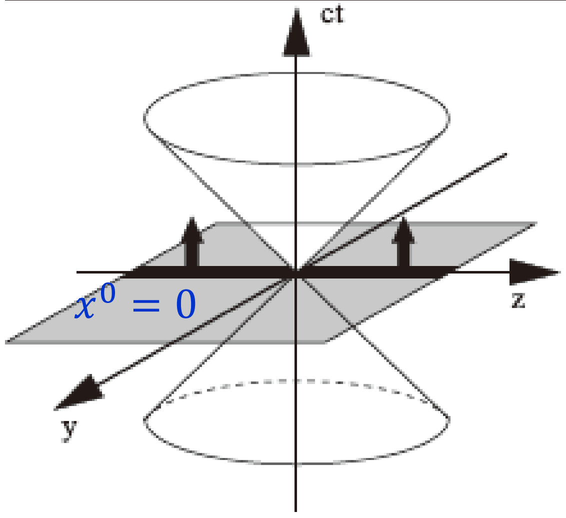
$f_4^q(x)$: twist-4 PDF

Backup Slides

2. Why Light-Front?

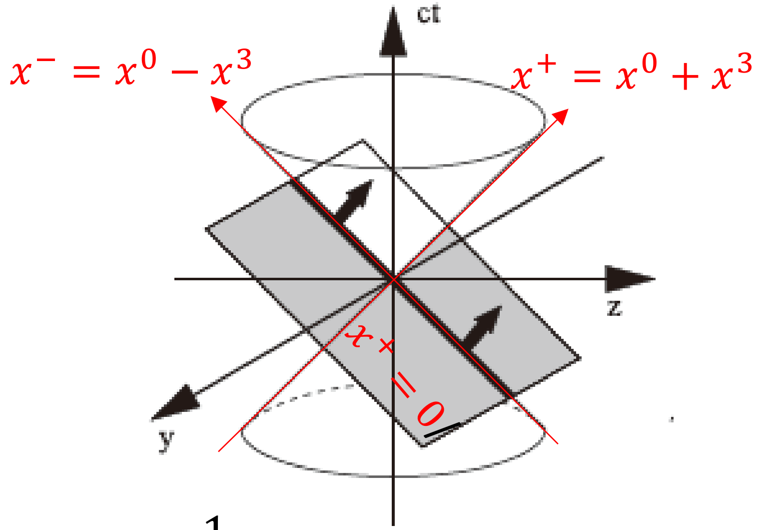
Light-Front Dynamics (LFD) (by Dirac in 1949)

Instant form $(x^0 = ct = 0)$



$$a \cdot b = a^0 b^0 - \vec{a} \cdot \vec{b}$$

Front form $(x^+ = x^0 + x^3 = 0)$

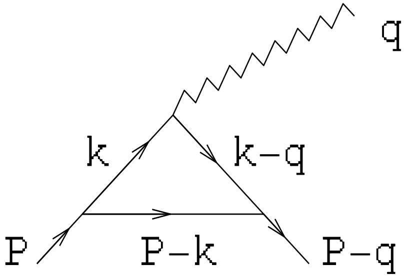


$$a \cdot b = \frac{1}{2}(a^+ b^- + a^- b^+) - \boldsymbol{a}_\perp \cdot \boldsymbol{b}_\perp$$

Hamiltonian	P^0	$P^- = P^0 - P^3$
Momentum	$\boldsymbol{P}_\perp = (P^1,\ P^2)$ P^3	$\boldsymbol{P}_\perp$ $P^+ = P^0 + P^3$
Energy-Momentum Dispersion Relation	$P^0 = \sqrt{M^2 + \vec{P}^2}$	$P^- = \dfrac{M^2 + \boldsymbol{P}_\perp^2}{P^+}$

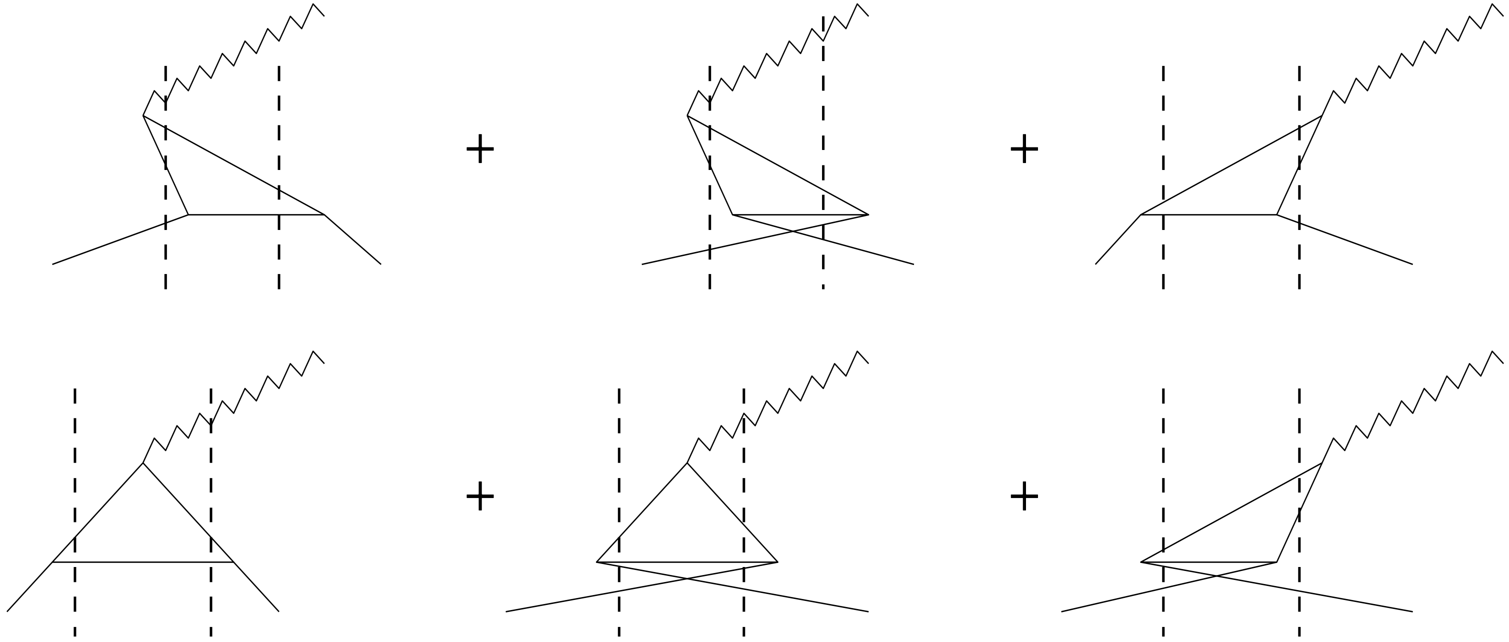
Irrational vs. Rational

- Advantage of LFD in the calculation of Form Factors :
  Equal-t vs Equal Light-front τ formulations



Equal t (Instant form)

=



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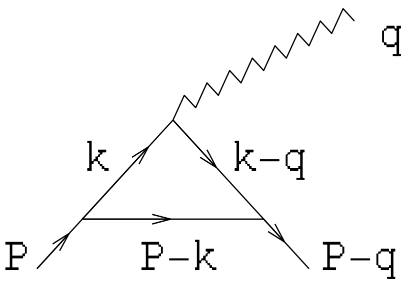

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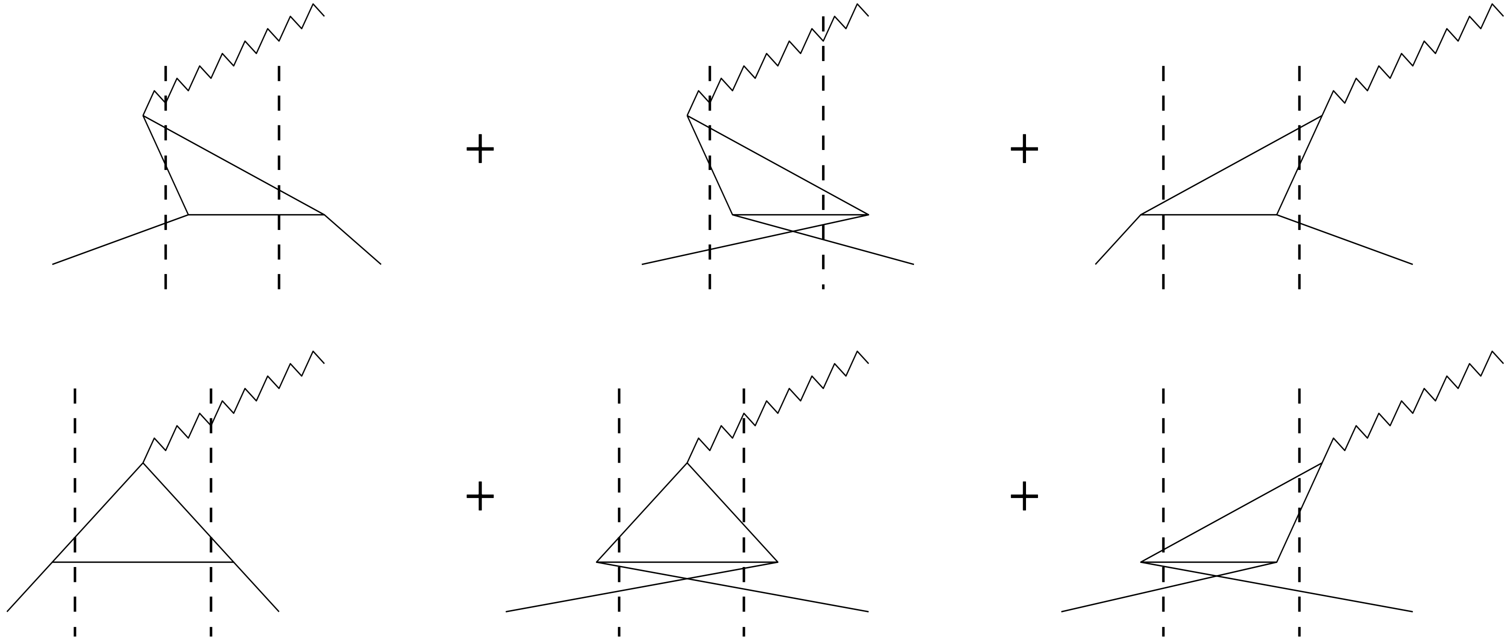
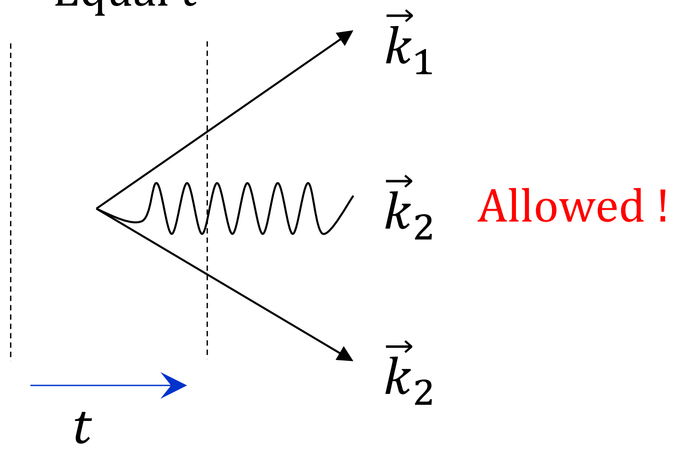
=



$$k^0 = \sqrt{m^2 + \vec{k}^2}$$

Equal t



$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$$

- Advantage of LFD in the calculation of Form Factors :
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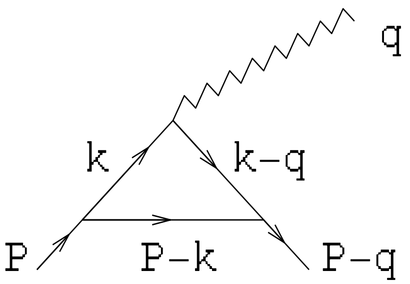
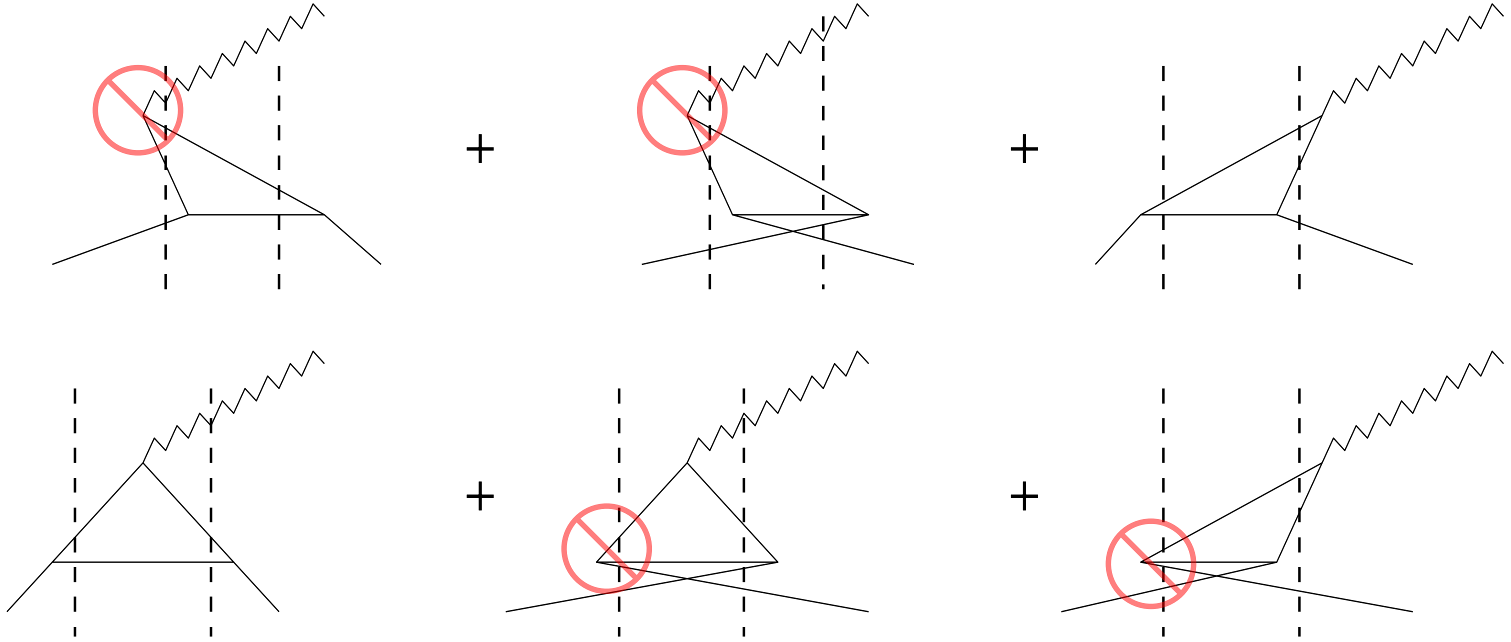


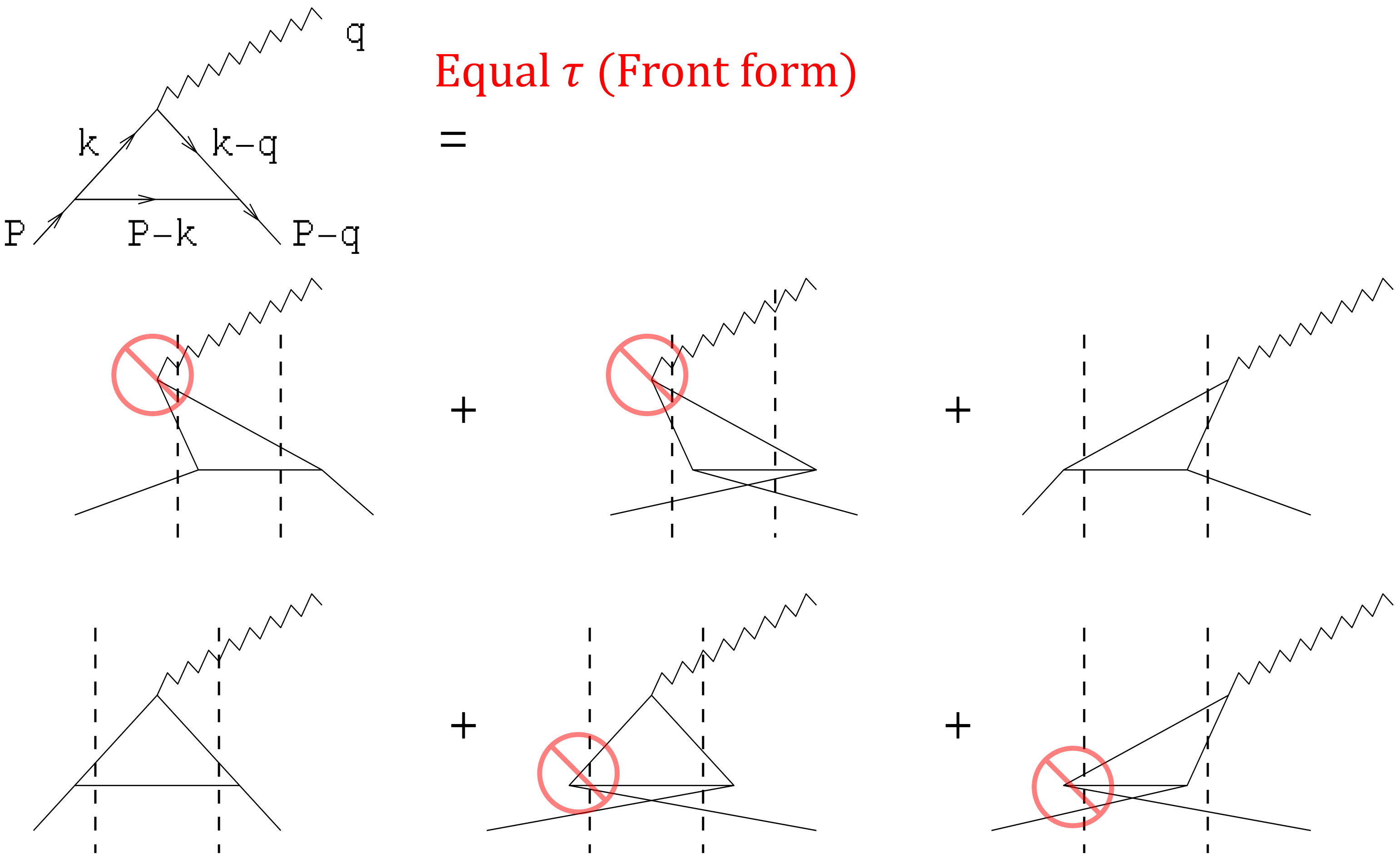
Equal τ (Front form)

=

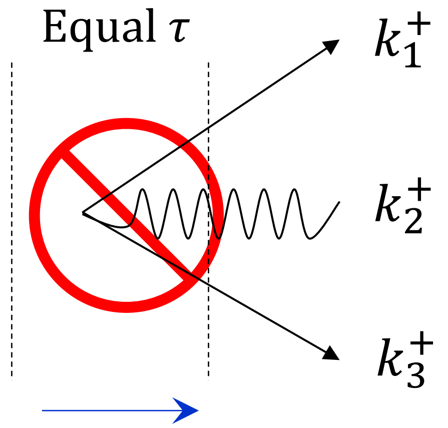


- Advantage of LFD in the calculation of Form Factors :

Equal-t vs Equal Light-front τ formulations



$$k^- = \frac{m^2 + k_\perp^2}{k^+}$$



$$\tau = t + z/c$$

$$k_1^+ + k_2^+ + k_3^+ = 0$$

- Advantage of LFD in the calculation of Form Factors :

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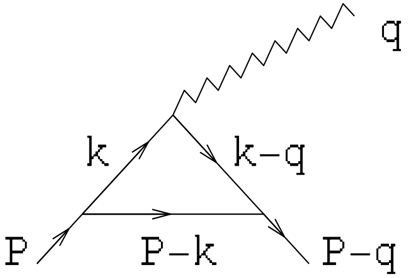
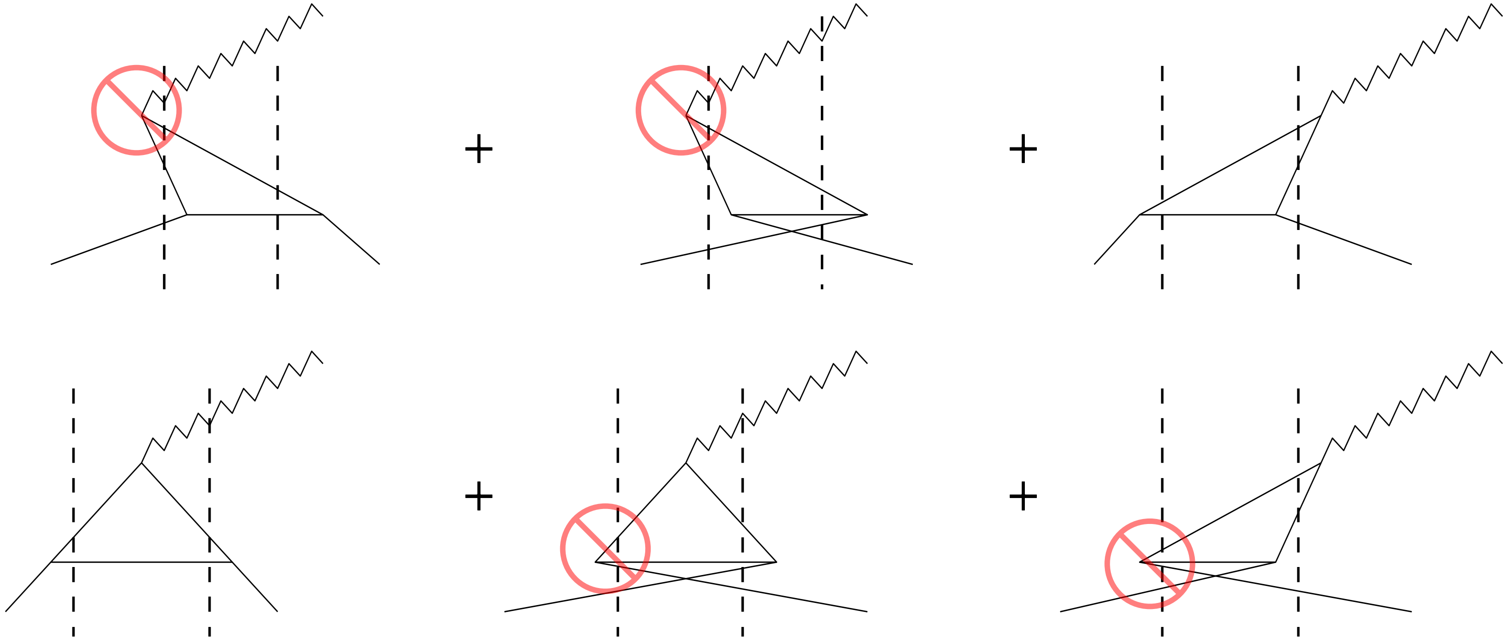


Equal τ (Front form)

=

LF nonvalence
(higher Fock state)



LF valence

Decay Constant and DAs

$$\langle 0|\bar{q}(0)\gamma^{\mu}\gamma_5 q(0)|P\rangle = i f_\pi P^{\mu}$$

BT ↓

$$f_\pi = \left\langle 0 \left| \frac{\bar{q}\gamma^{\mu}\gamma_5 q}{P^{\mu}} \right| \mathrm{P} \right\rangle \text{ with } M \to M_0$$

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$$f_\pi = \sqrt{2N_c}\int_0^1 dx \int \frac{d^2\boldsymbol{k}_\perp}{16\pi^3}\frac{\phi(x,\boldsymbol{k}_\perp)}{\sqrt{m^2+\boldsymbol{k}_\perp^2}}(2m)$$

$$\phi_\pi(x,\mu_0) = \frac{\sqrt{2N_c}}{f_\pi}\int^{\mu_0^2}\frac{d^2\boldsymbol{k}_\perp}{16\pi^3}\frac{\phi(x,\boldsymbol{k}_\perp)}{\sqrt{m^2+\boldsymbol{k}_\perp^2}}(2m)$$

independent of “μ”!

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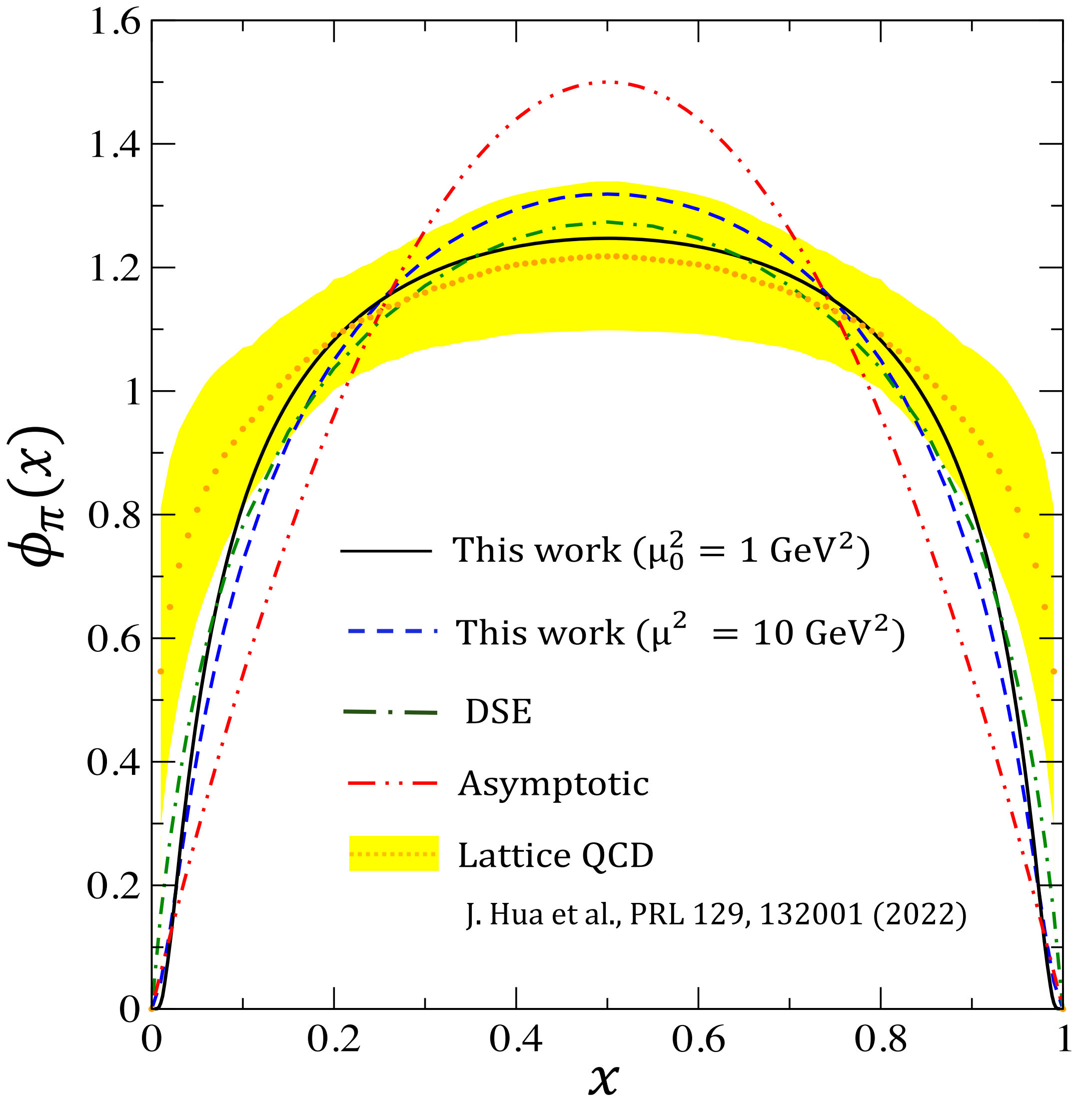

See PRD 107, 053003(23); PRD108, 013006(23) by A. Arifi, HMC and CRJ for the analysis of higher-twist DAs.