3D Structure of the Nucleon
via Generalized Parton Distributions

Howard Johnson Incheon Airport Hotel
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Beam Spin Asymmetry in DVMP on Helium-4

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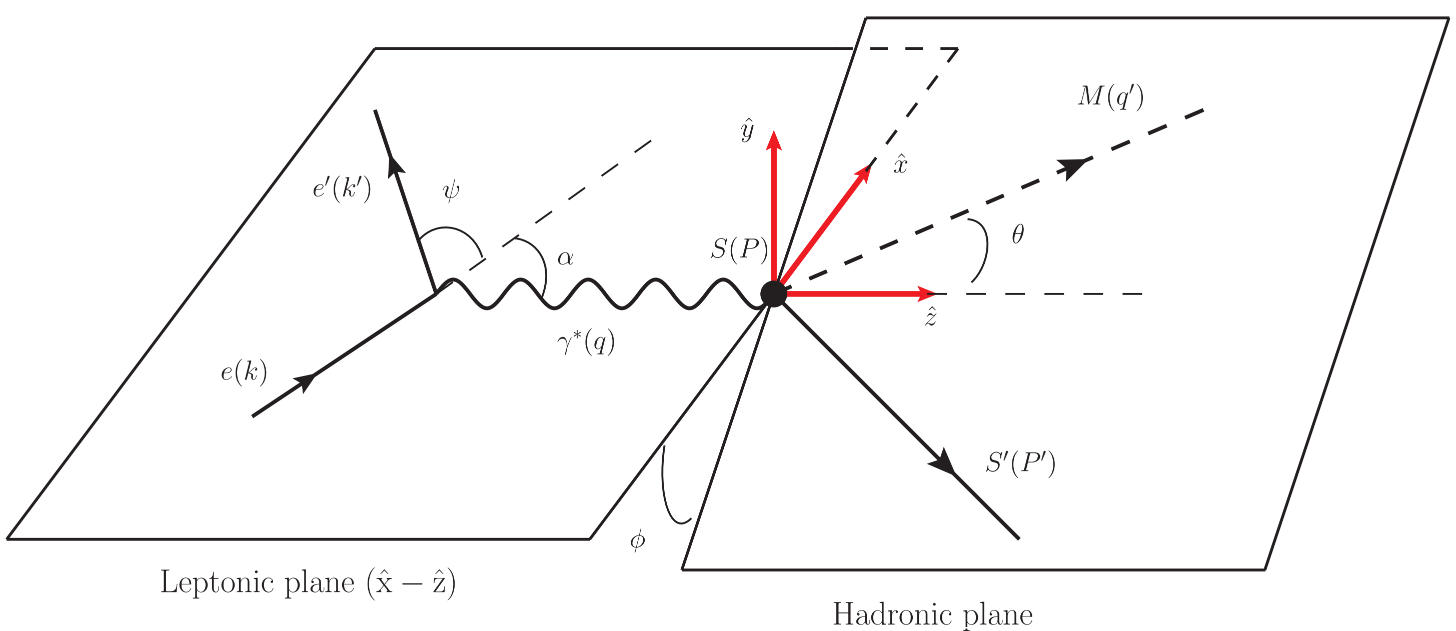
with Prof. Chueng-Ryong Ji and Andrew Lundeen



I. Motivation

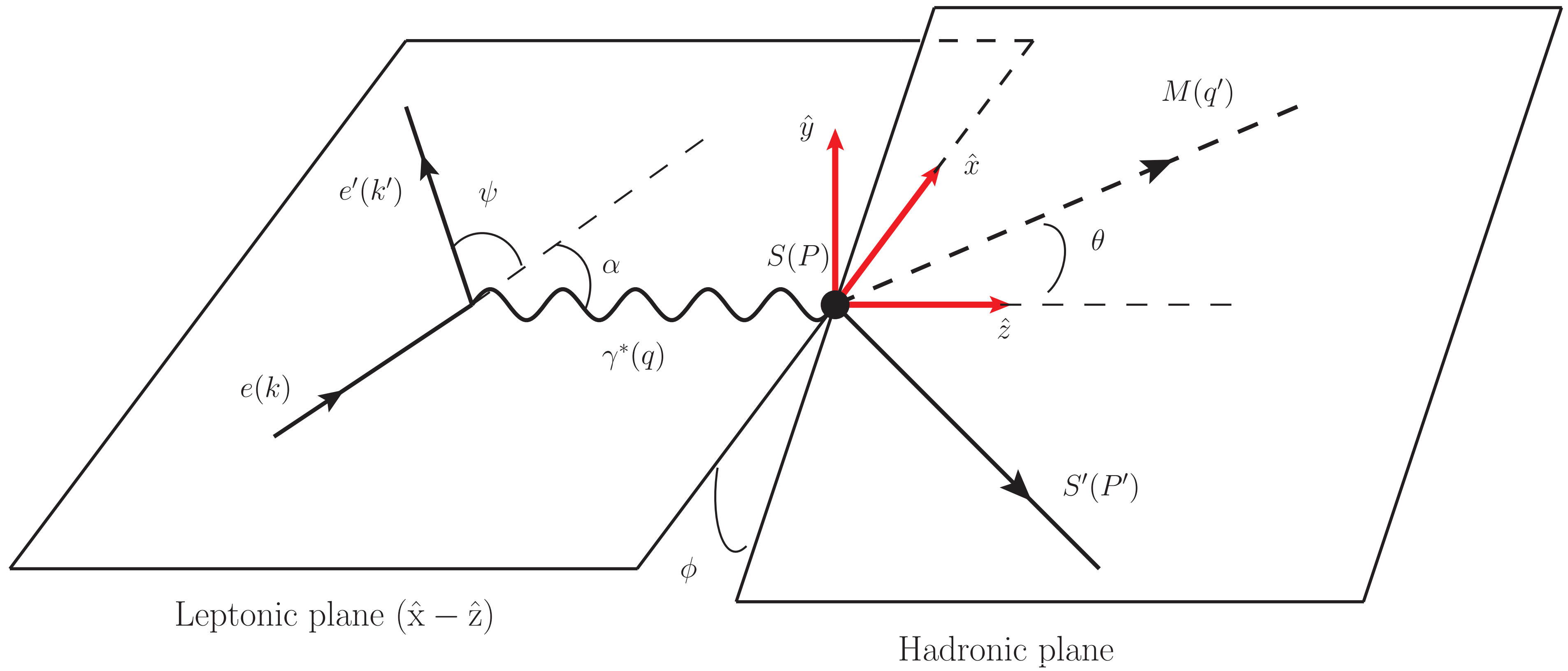
II. Two approaches

III. GPD results

IV. Results

I. Motivation

Beam Spin Asymmetry (BSA)



To probe the internal hadron structure, and understand the QCD dynamics,

Asymmetry in the cross section when the spin of the polarized electron beam is flipped.

$$A_{LU}(\phi) = \frac{d\sigma_{e\uparrow} - d\sigma_{e\downarrow}}{d\sigma_{e\uparrow} + d\sigma_{e\downarrow}} = \frac{d\sigma_{BSA}}{d\sigma_T\ (1 + \epsilon\ \cos(2\phi)) + d\sigma_L \epsilon_L + d\sigma_{LT}\ \cos(\phi)\sqrt{\epsilon_L(1+\epsilon)/2}}$$

Virtual meson production → **No interference with Bethe-Heitler process**

Non-vanishing BSA

For the **pseudoscalar meson (0^{-+})** production on He-4 target,

the zero-BSA is confirmed in the F. Cao's work.

Cao, Frank Thanh, *Doctoral Dissertations (2019)*

On the other hand, non-vanishing BSA is expected for **scalar meson (0^{++})** production.

Ji, Chueng-Ryong et al, Phys. Rev. D **99**, 11 116008 (2019)

$$A_{LU}(\phi) \sim \mathscr{F}_1 \, \mathscr{F}_2^* \; - \; \mathscr{F}_2 \, \mathscr{F}_1^*$$

To obtain the finite BSA, at least **two Compton Form Factors** are required.

But, GPD based model, a single leading-twist GPD $H(x, \zeta, t)$ exist for this process.

Since # of GPDs is the same as # of CFFs,

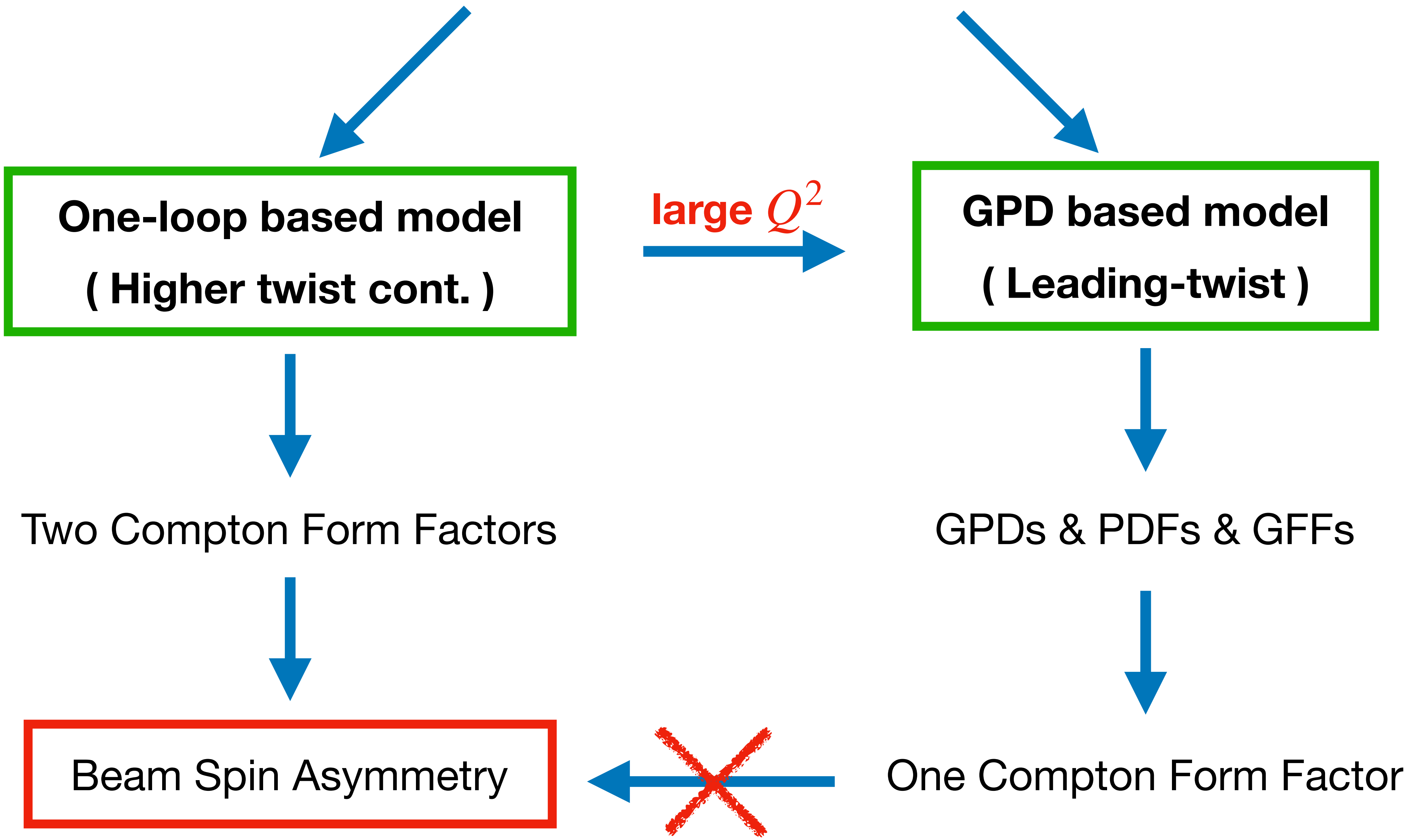
the leading-twist approximation is not sufficient! → simple theoretical test

Strategy

Assuming
coherent process
is dominant in small $|t|$

Virtual Meson Production

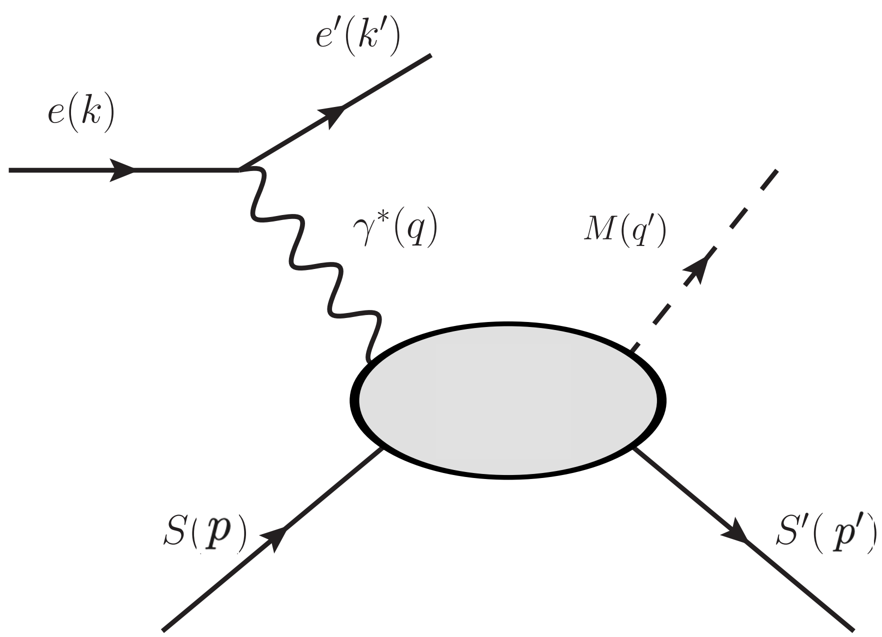
$$(\gamma^* + {}^4\mathrm{He} \rightarrow \mathrm{f}^0(0^{++}) + {}^4\mathrm{He})$$



II. Two approaches

Parameterization

Generalized structure functions (Compton Form Factors (CFFs))



Considering **symmetries** (gauge invariance, ...),

for the pseudoscalar meson ($J^{PC} = 0^{-+}$) : $\epsilon^{\mu\nu\alpha\beta}$

scalar meson (0^{++}) : $d^{\mu\nu\alpha\beta} = g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta}$

$$\mathcal{M}^{\mu}_{S} = (\mathcal{F}_1(Q^2, x_B, t)\ q_\alpha + \mathcal{F}_2(Q^2, x_B, t)\ \mathcal{P}_\alpha)\ d^{\mu\nu\alpha\beta} q_\beta \Delta_\nu$$

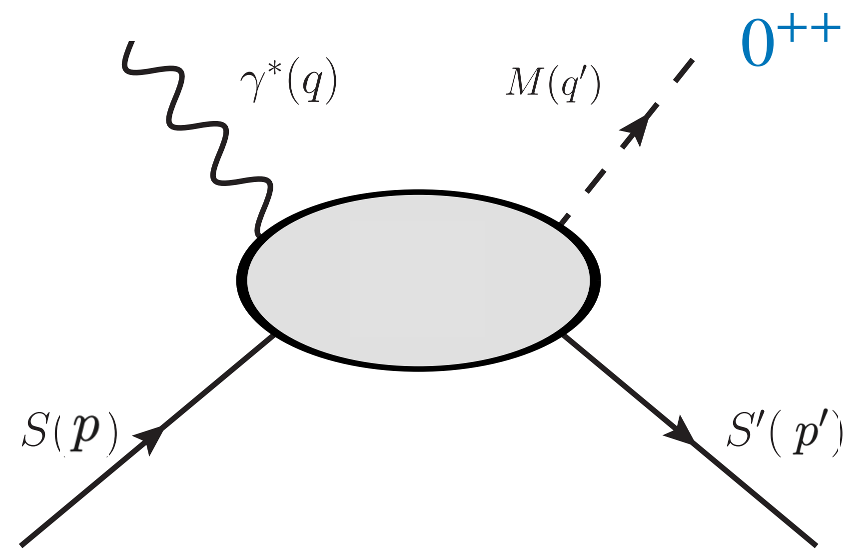
$$= \{q^2\ \Delta^\mu - (\Delta \cdot q)\ q^\mu\} \mathcal{F}_1(Q^2, x_B, t) + \{(\mathcal{P} \cdot q)\ \Delta^\mu - (\Delta \cdot q)\ \mathcal{P}^\mu\} \mathcal{F}_2(Q^2, x_B, t)$$

where $\Delta = p - p' = q' - q, \ \ \mathcal{P} = p + p'$

Ji, Chueng-Ryong et al., Phys. Rev. D **99**, 11 116008 (2019)

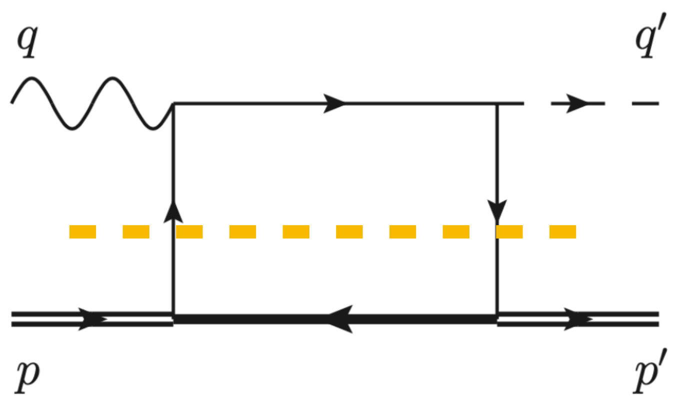
I. One-Loop Based Model

$$\mathscr{M}^\mu = A^\mu \mathscr{F}_1 + B^\mu \mathscr{F}_2 =$$



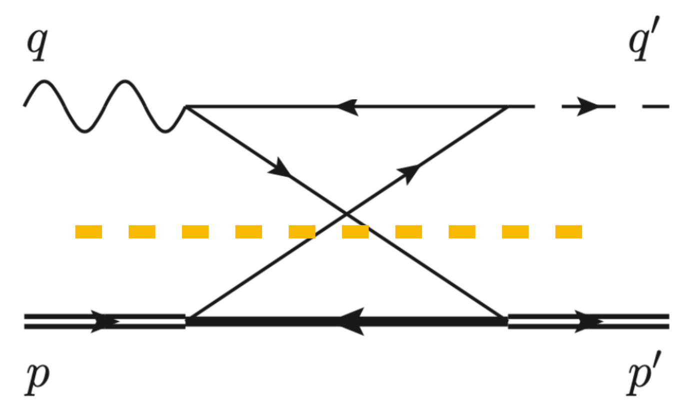
S-box

$=$



$+$

U-box



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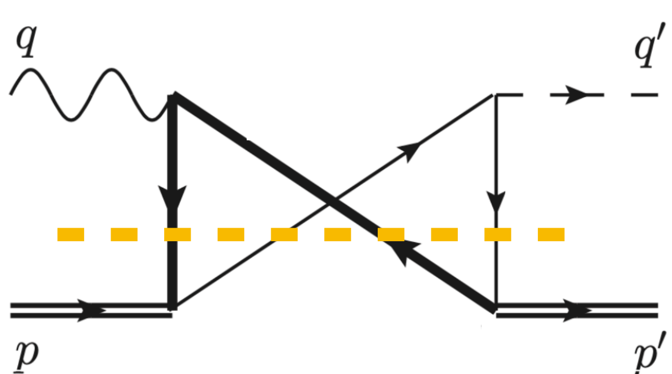


Cat's ears

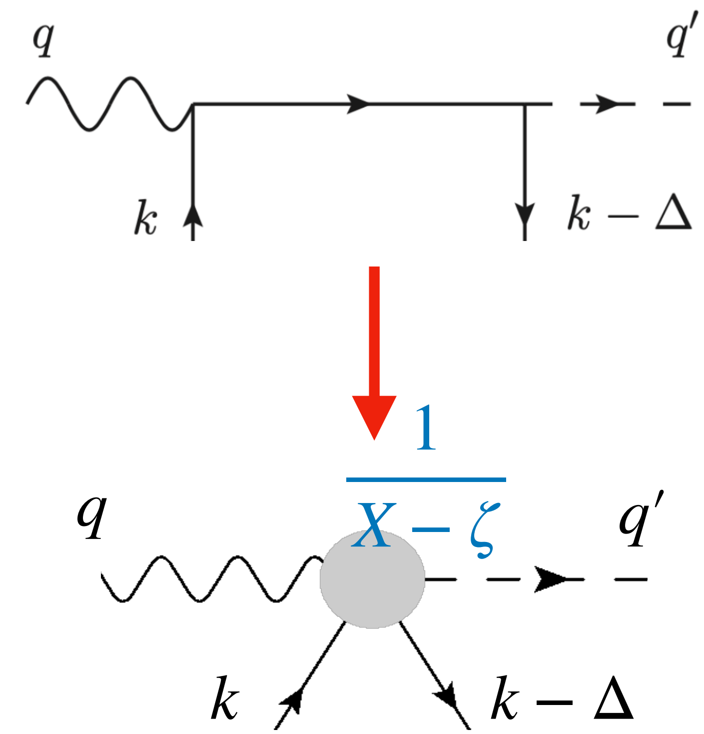
$+$

Photon-target coupled.

Large Q^2 Expansion (Factorization)

In the large Q^2, the upper propagators are

$$\frac{2k^\mu + q^\mu}{(k+q)^2 - m^2} \rightarrow \left(\frac{1}{X-\zeta}\right)\left(\frac{2k^\mu + q^\mu}{Q^2/\zeta}\right)$$



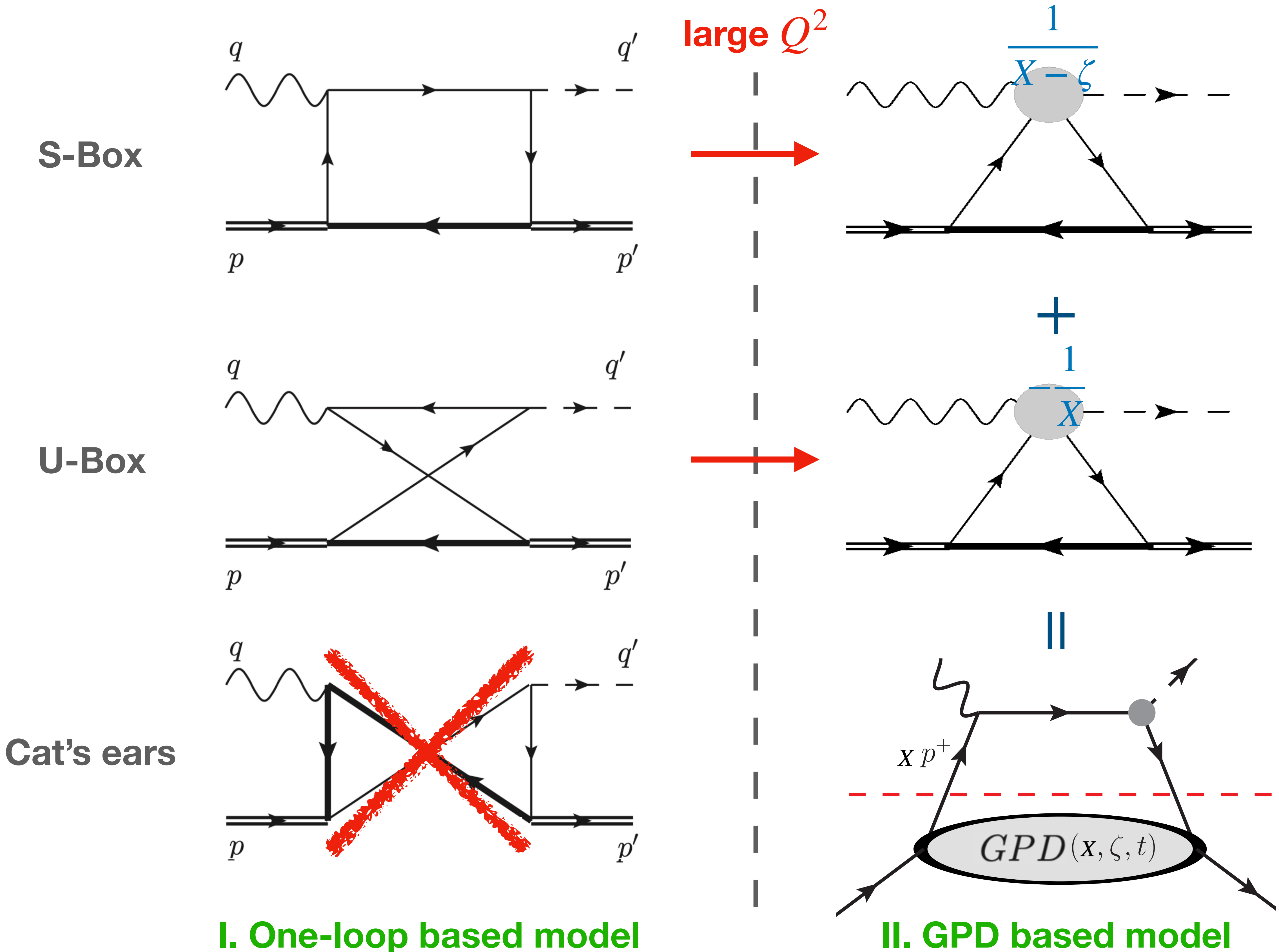
Then, the scattering amplitude in the large Q^2 limit can be

$$\mathcal{M}^\mu \sim \int dX \left(\frac{1}{X-\zeta} - \frac{1}{X}\right) \mathrm{H}(X,\zeta,t) \;\times \begin{cases} \zeta^1 \; Q^{-2} & \text{for } \mu = + \\ \zeta^0 \; Q^{-1} & \mu = - \\ \zeta^{-1} Q^0 & \mu = \perp \end{cases}$$

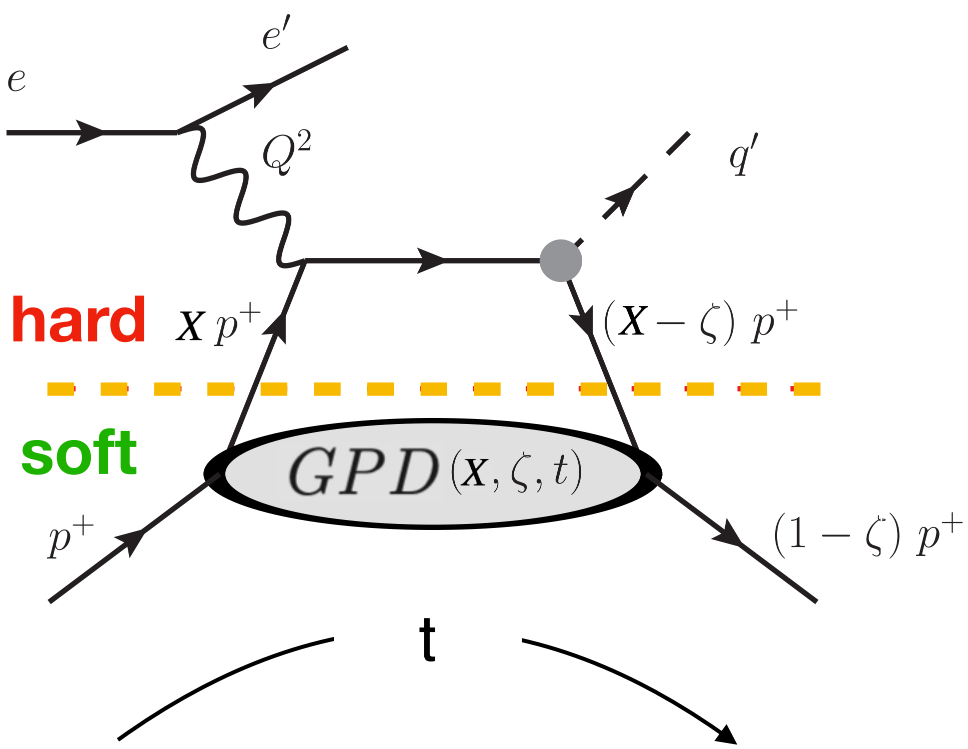
Remarks : Regardless of the choice μ,

we have **the same leading-twsit GPD** with different order of ζ and Q.

Model-I → Model-II



II. GPD Based Model



Skewed (off-forward) parton distribution :

hard part (OPE) $\otimes$ **soft part (GPD H)**

Relation with CFF :

$$\mathrm{Re}[\mathscr{F}(Q^2, x_B, t)] \sim \mathscr{P} \int dx \left(\frac{1}{X-\zeta} - \frac{1}{X} \right) \mathrm{H}(X, \zeta, t)$$

$$\mathrm{Im}[\mathscr{F}(Q^2, x_B, t)] \sim -i\pi\ \mathrm{H}(X = \zeta, \zeta, t)$$

p^+ : Light-front (LF) longitudinal momentum of incoming target,

Q^2 : Virtuality $\rightarrow$ factorizable in $Q^2 >> |t|,\ M_t^2,\ M_s^2,\ \cdots$,

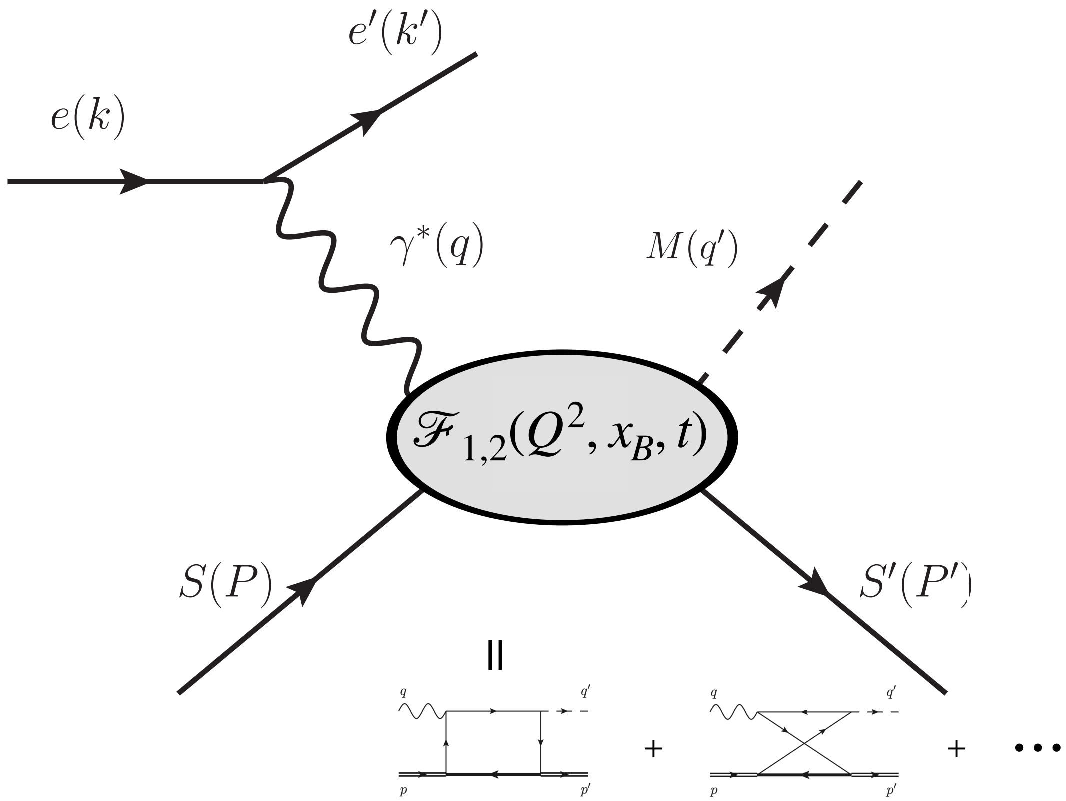
X : Longitudinal momentum fraction,

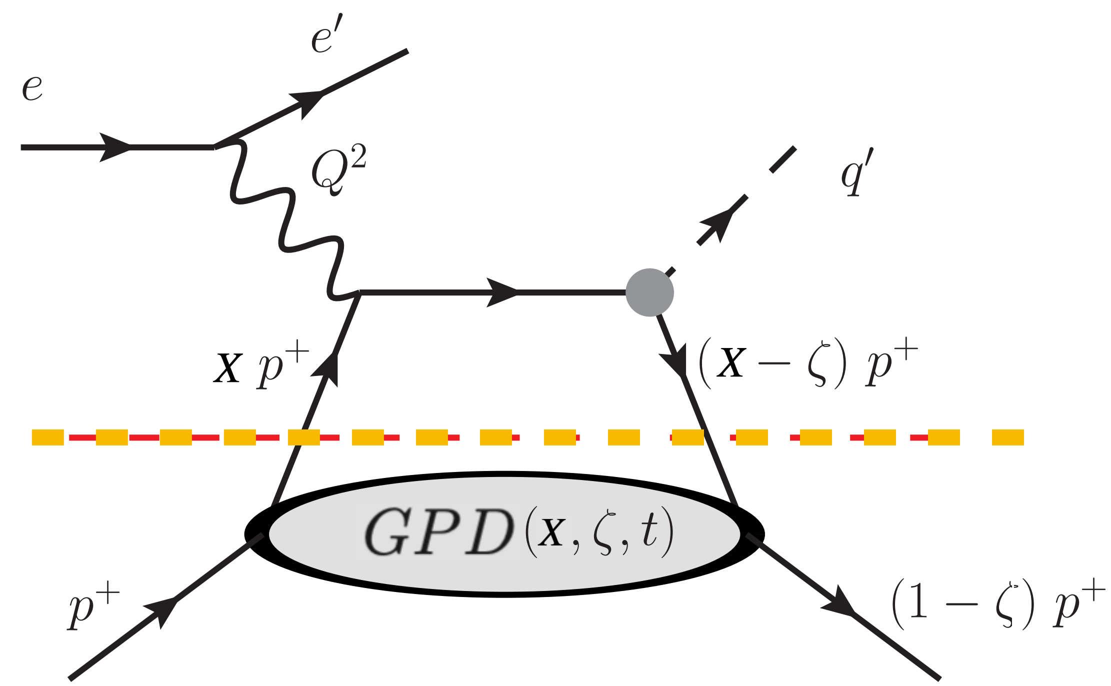
ζ : Skewness, $(p^+ - p'^+)/p^+$, asymmetry of LF longitudinal momentum of target,

t : Square momentum transfer, $\Delta^2 = (q' - q)^2 = (p - p')^2$,

"kick" transverse momentum depending on scattering angle.

Model-I vs Model-II

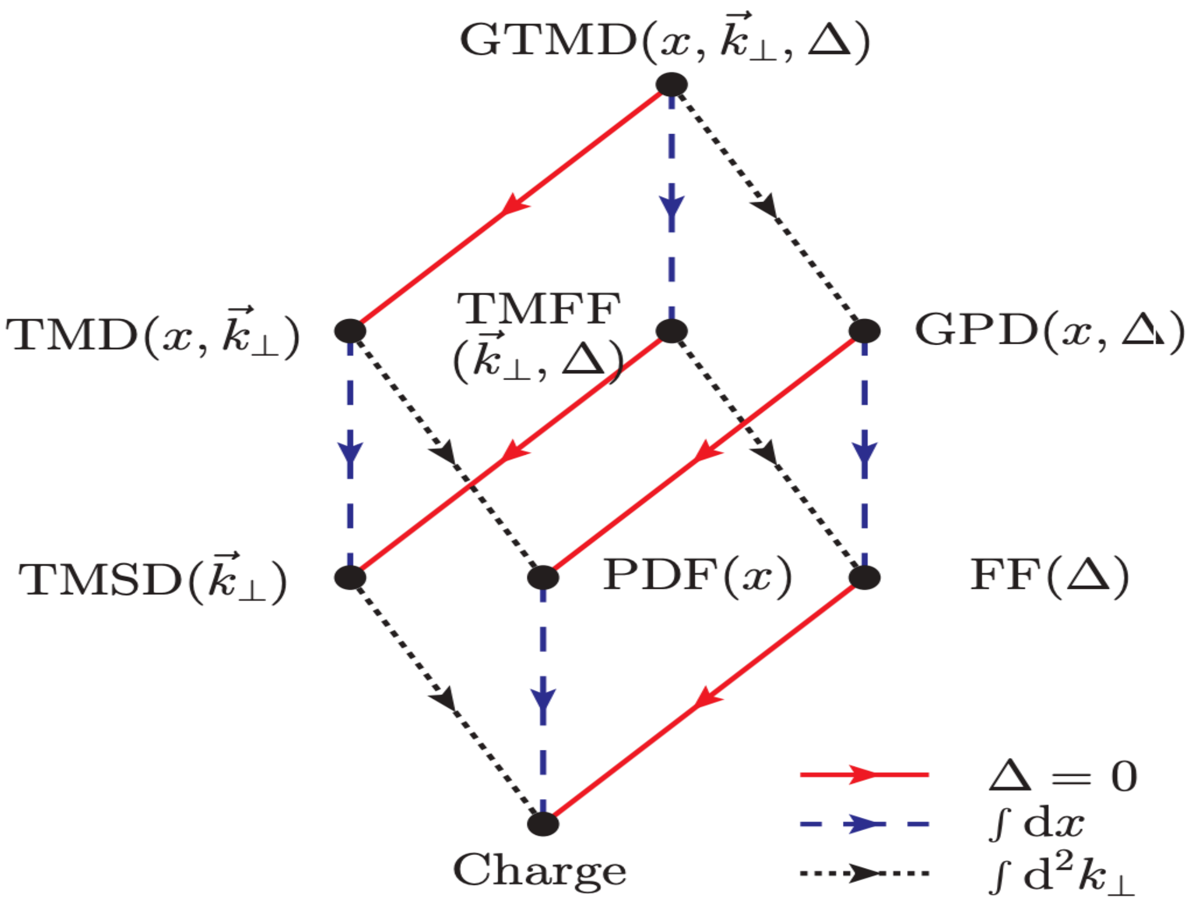




DV(f0)P	I. One-loop based	II. Leading-twist GPD
Factorizable	X	O
Diagrams	S-, U- box, and Cat's ears …	Handbags
Gauge Invariance	O	X
# of CFFs	2	1
Exp.	Compton Form Factors, Beam Spin Asymmetry, …	

III. GPD Result

GPD sum rule



B. Pasquini, C. Lorcé

II. GPD based model

Parton Distribution Function :

$$H^q(x, 0, 0) = f(x)$$

Form Factor (1st Mellin moment) :

$$\int dx\ H^q(x, \xi, t) = F^q(t),$$

Energy-Momentum Tensor (2nd Mellin moment) :

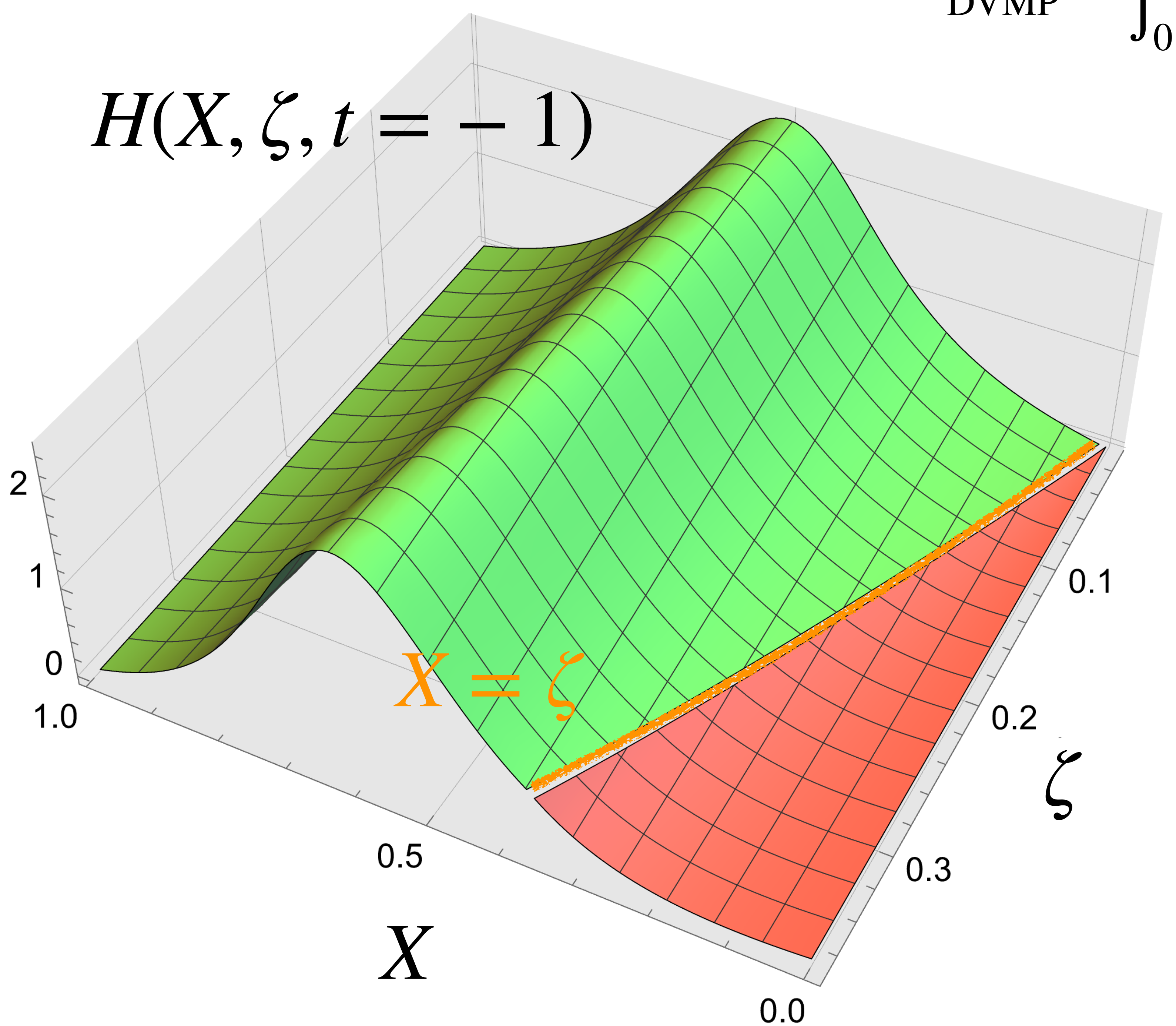
$$\int dx\ xH^q(x, \xi, t) = A^q(t) + 4\xi^2 C^q(t),$$

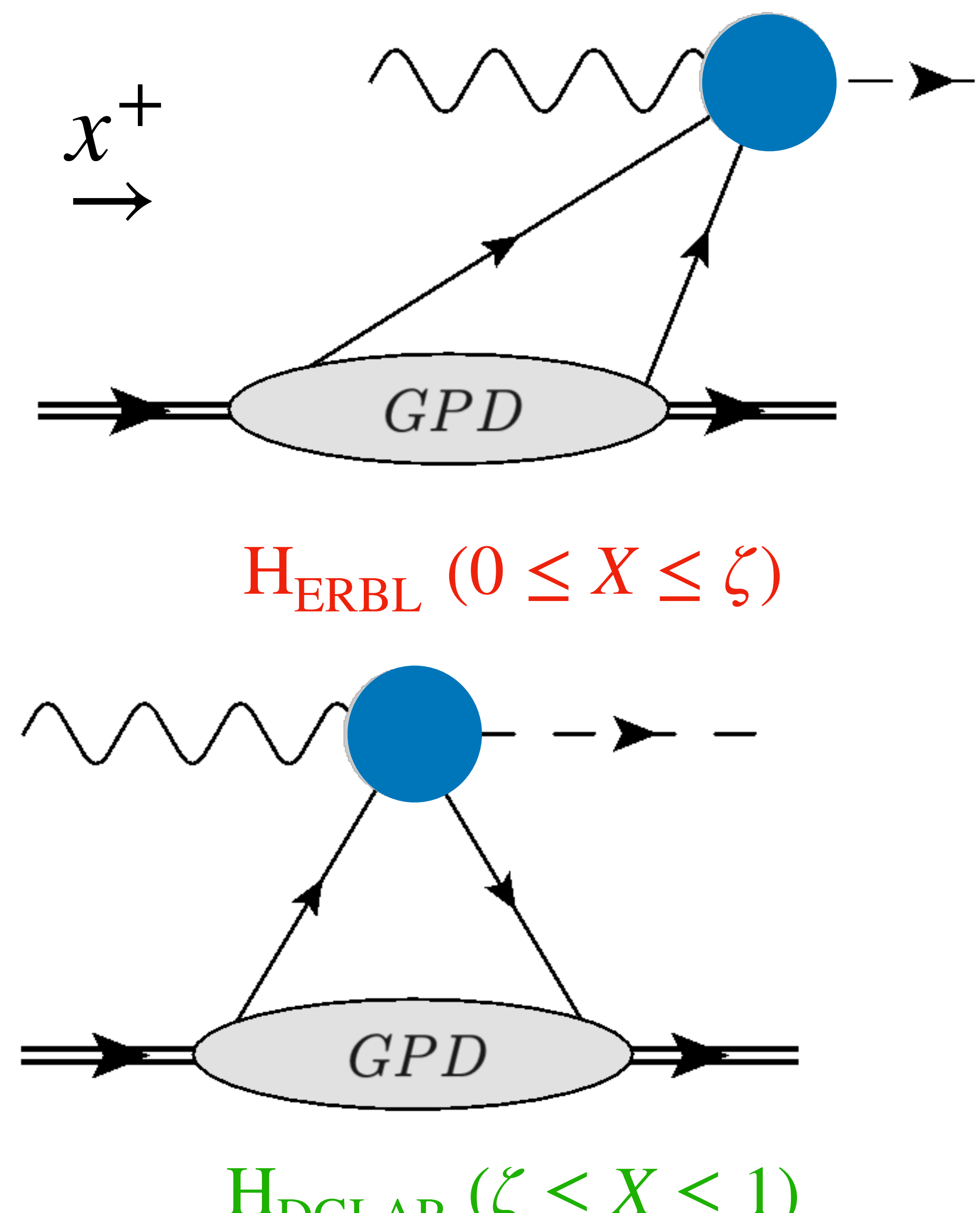
$$\lim_{\xi,t\to 0} \int \frac{dy^-}{2\pi}\, e^{ixP^+y^-} \langle P' | \bar{\psi}(0)\ \hat{\mathcal{O}}\ \psi(y) | P \rangle \Big|_{y^+=\mathbf{y}_\perp=0} = \int \frac{dy^-}{2\pi}\, e^{ixP^+y^-} \langle P | \bar{\psi}(0)\ \hat{\mathcal{O}}\ \psi(y) | P \rangle \Big|_{y^+=\mathbf{y}_\perp=0}$$

$$\int dx \int \frac{dy^-}{2\pi}\, e^{ixP^+y^-} \langle P' | \bar{\psi}(0)\ \hat{\mathcal{O}}\ \psi(y) | P \rangle \Big|_{y^+=\mathbf{y}_\perp=0} = \langle P' | \bar{\psi}(0)\ \hat{\mathcal{O}}\ \psi(0) | P \rangle \Big|_{y^+=\mathbf{y}_\perp=0}$$

Generalized Parton Distribution (GPD)

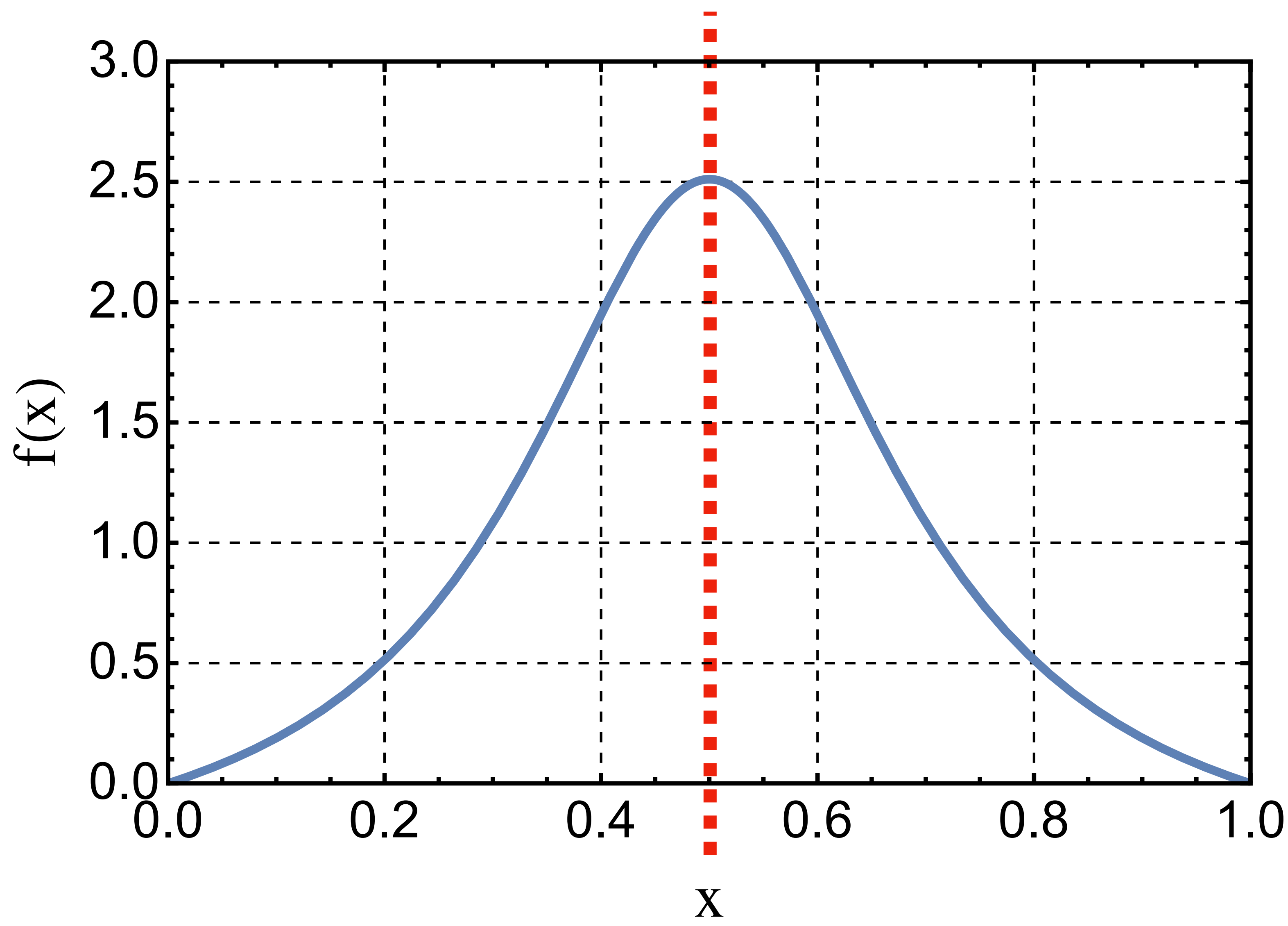
$$\mathscr{M}^{\mu}_{\mathrm{DVMP}} \sim \int_0^1 dx \left(\frac{1}{X-\zeta} - \frac{1}{X}\right) \mathrm{H}(X,\zeta,t)$$





Parton Distribution Function

$$\lim_{\zeta,t\to 0} H(X,\zeta,t) = H(X,0,0) = f(X)$$



Helium is consist of

two effective partons

in this model.

Expect that

for a single parton,

It has largest probability

at x=0.5.

Electromagnetic Form Factor

Using the polynomiality,

1st Melin moment is

$$\int dX \, \frac{H(X, \zeta, t)}{1 - \zeta/2} = F(t),$$

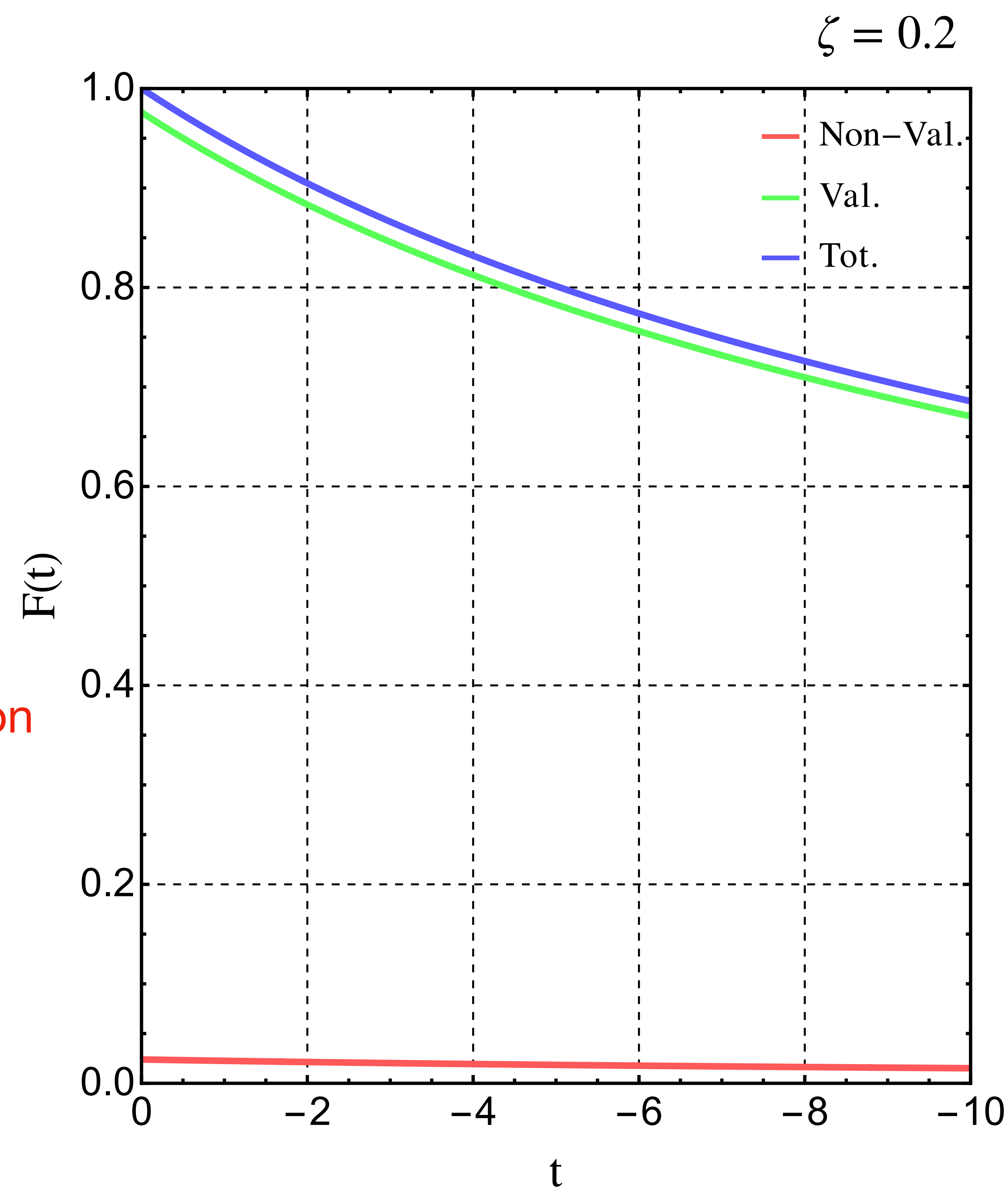
DGLAP ⟶ Valence contribution

ERBL ⟶ Non-valence contribution

Ratio depends on ζ,

If $\zeta \to 0$, only DGLAP survives.

But, total does not depends on ζ.



Energy-Momentum Tensor FFs

2nd Melin moment

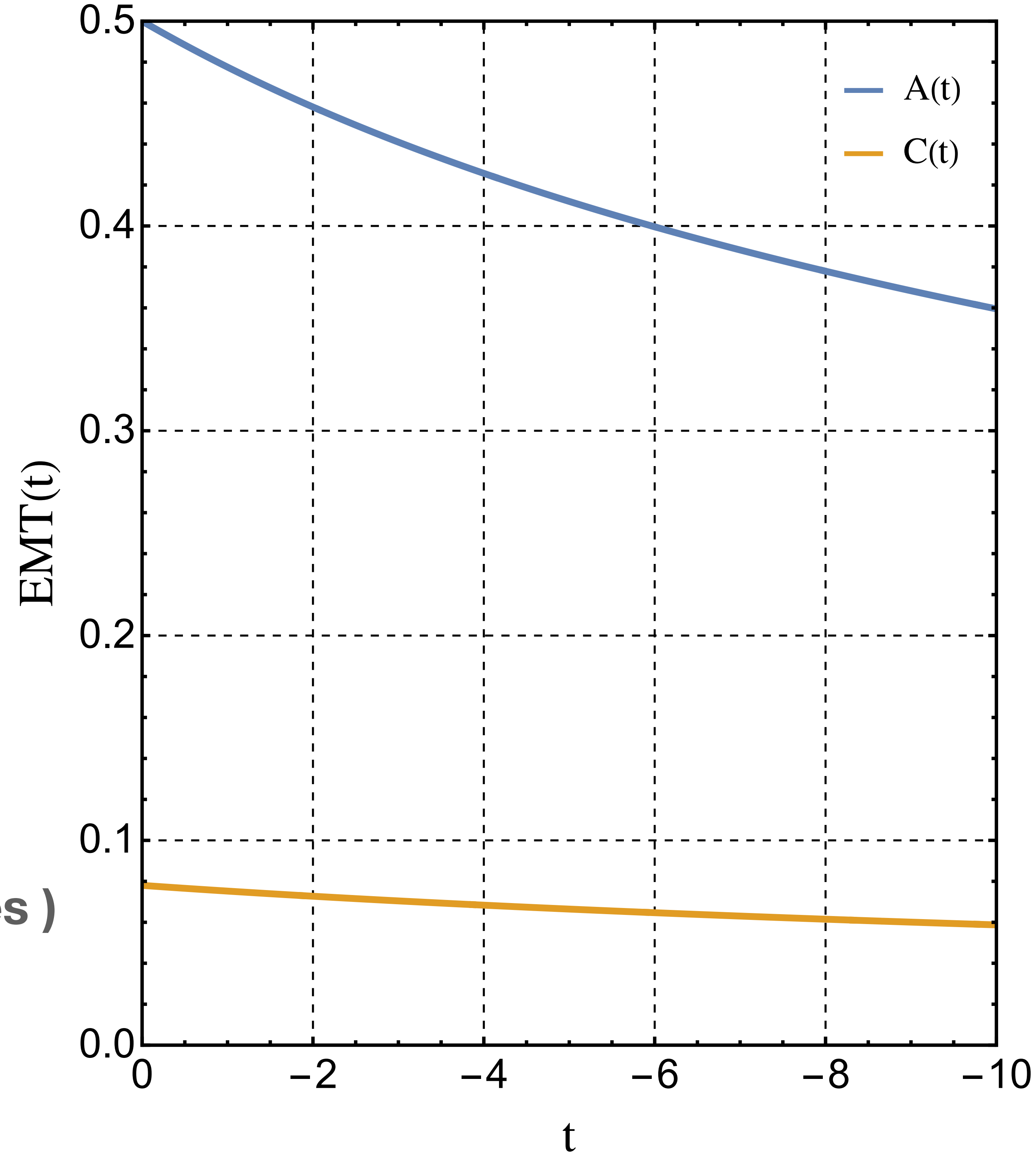
$$\int dx\ x\ H(x,\ \xi,\ t) = A(t) + 4\xi^2\ C(t)$$

$$= \boxed{A(t)} + 4\left(\frac{\zeta}{2-\zeta}\right)^2 \boxed{C(t)}$$

Mass → **(same masses of partons)**

D-term → **(pressure & shear forces)**

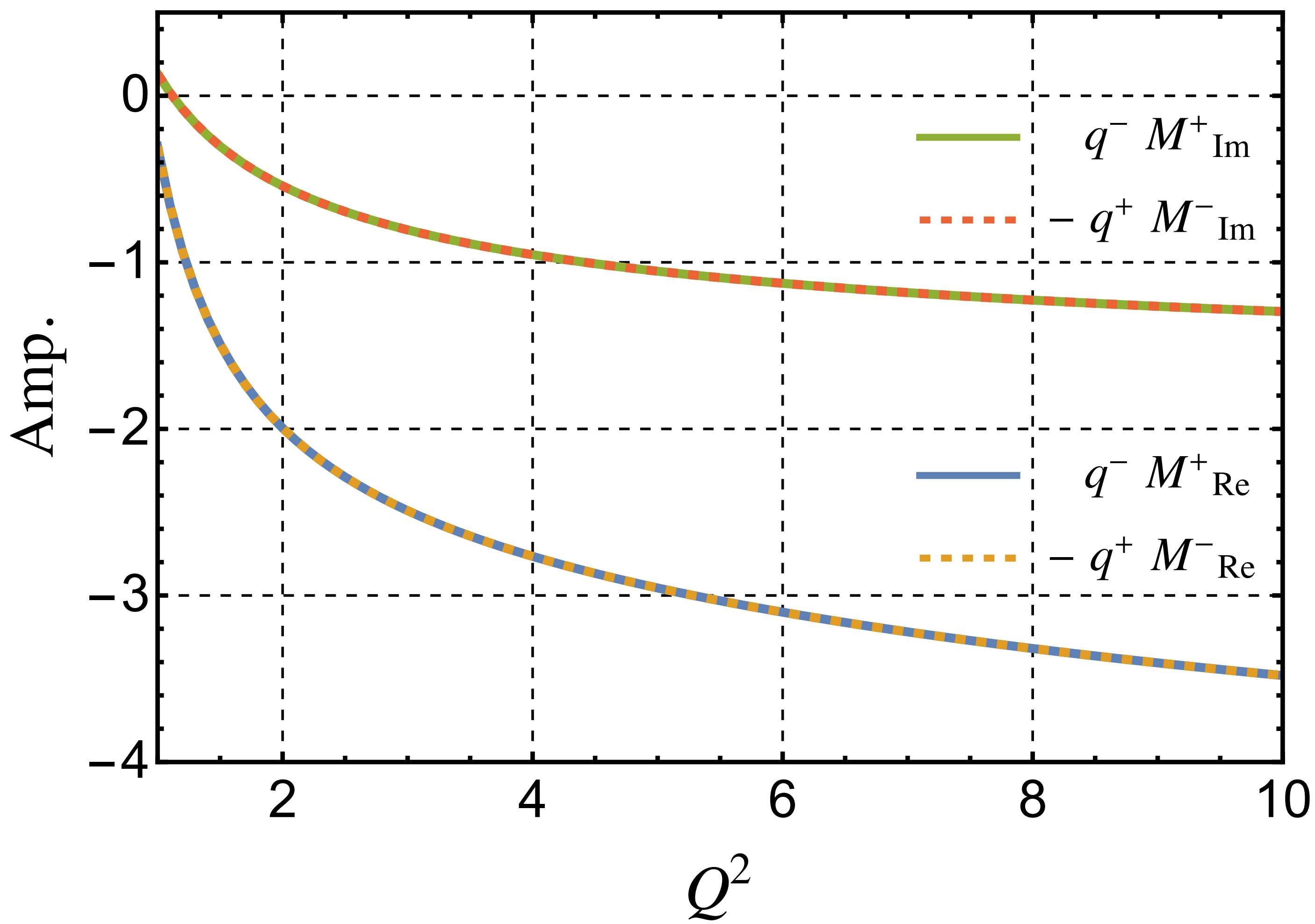
No QCD dynamics in soft parts?



IV. Results

Gauge Invariance (Ward Identity)

I. One-loop based model



Set $|\mathbf{q}_\perp| = 0$ for simplicity.

$$q_\mu \ \mathcal{M}^\mu = q^+\mathcal{M}^- + q^-\mathcal{M}^+ = 0$$

II. GPD based model

$$q_\mu \ \mathcal{M}^\mu_{DVMP} \sim \frac{1}{2} + \frac{\zeta}{2Q}$$

The gauge violation is decreased in large Q^2,

but, is not vanished even in very large Q^2.

Compton Form Factors

For scalar meson $f_0(980)$ production on spin-0 ^4He target ,

$$\mathscr{M}^\mu = A^\mu\ \mathscr{F}_1 + B^\mu\ \mathscr{F}_2$$

$$= \Big[\ (\Delta\cdot q)\ q^\mu - q^2\ \Delta^\mu\ \Big]\ \mathscr{F}_1$$

$$+\Big[\ (\Delta\cdot q)\ \mathscr{P}^\mu - (\mathscr{P}\cdot q)\ \Delta^\mu\ \Big]\ \mathscr{F}_2$$

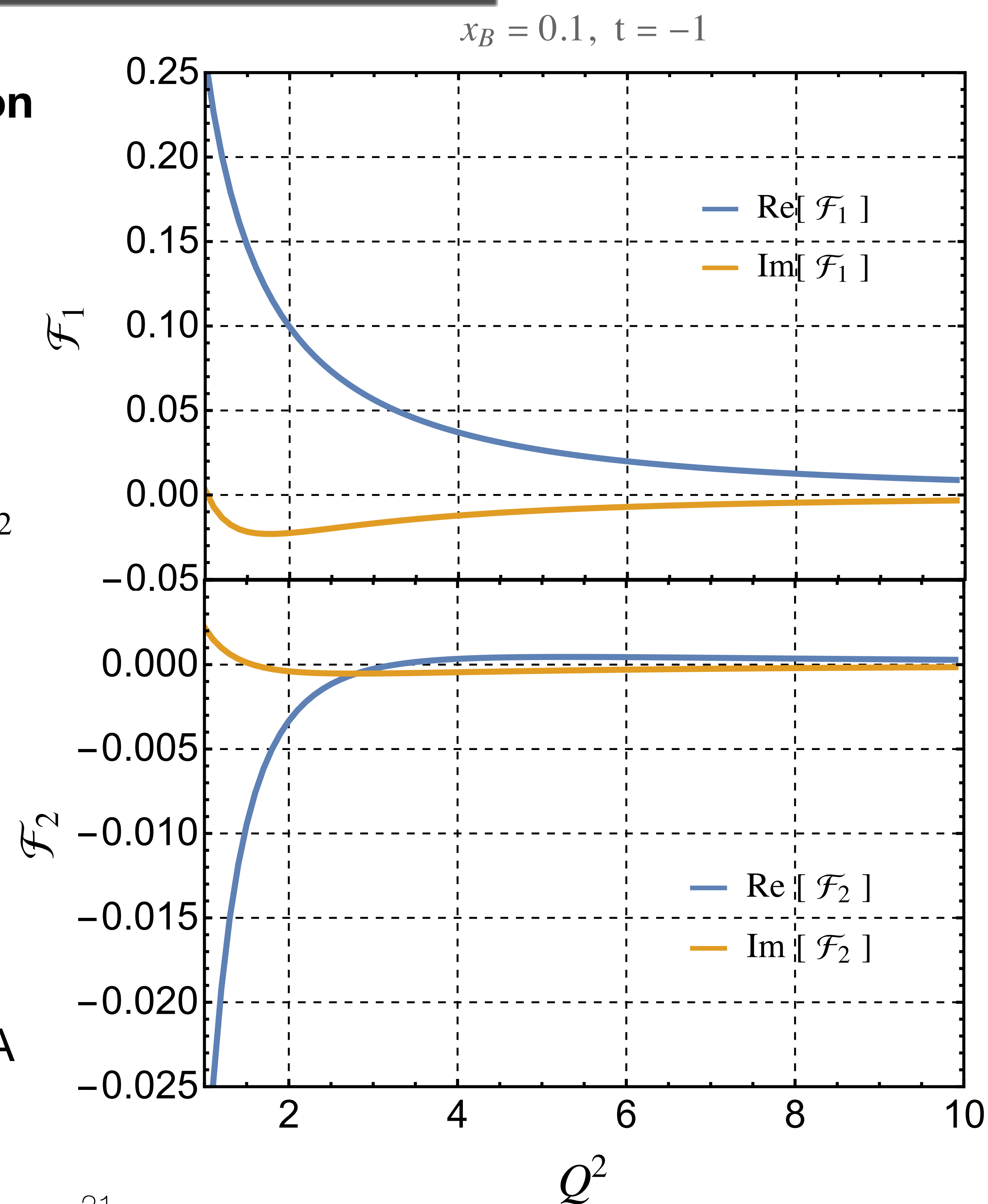
Remarks

Magnitude of $|\mathscr{F}_2|$ is quiet small

compared to $|\mathscr{F}_1|$,

but, **non-zero complex values**

provide interferences, $\mathscr{F}_1^*\mathscr{F}_2$, in BSA



Comparison two models for CFFs

$$\mathscr{M}^\mu = A^\mu\ \mathscr{F}_1 + B^\mu\ \mathscr{F}_2$$

$$= \boxed{\left(\mathcal{O}(Q^2)\right)\ \mathscr{F}_1} + \left(\mathcal{O}(Q^0)\right)\ \mathscr{F}_2$$

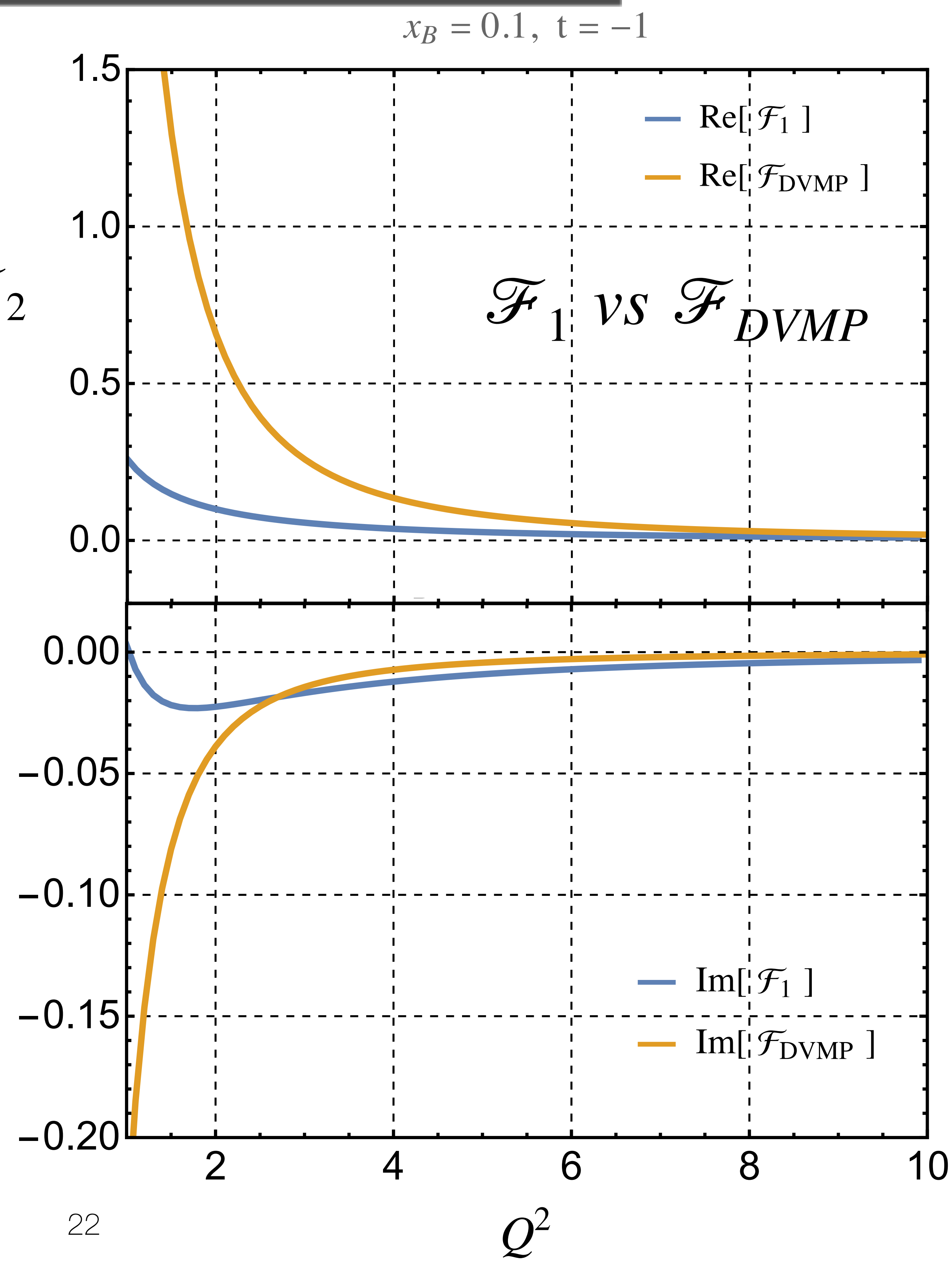
Moreover, **model-I** provides

$$\mathscr{F}_1 >> \mathscr{F}_2$$

$\mathscr{F}_1$ almost contributes to the scattering amplitude.

From a single GPD (model-II),

$$H(X, \zeta, t) \rightarrow \mathscr{F}_{DVMP}$$



Beam Spin Asymmetry

$$A^S_{LU}(\phi) = \frac{d\sigma^S_{BSA}}{d\sigma^S_T\,(1+\epsilon\,\cos(2\phi)) + d\sigma^S_L\,\epsilon_L + d\sigma^S_{LT}\,\cos(\phi)\sqrt{\epsilon_L(1+\epsilon)/2}}$$

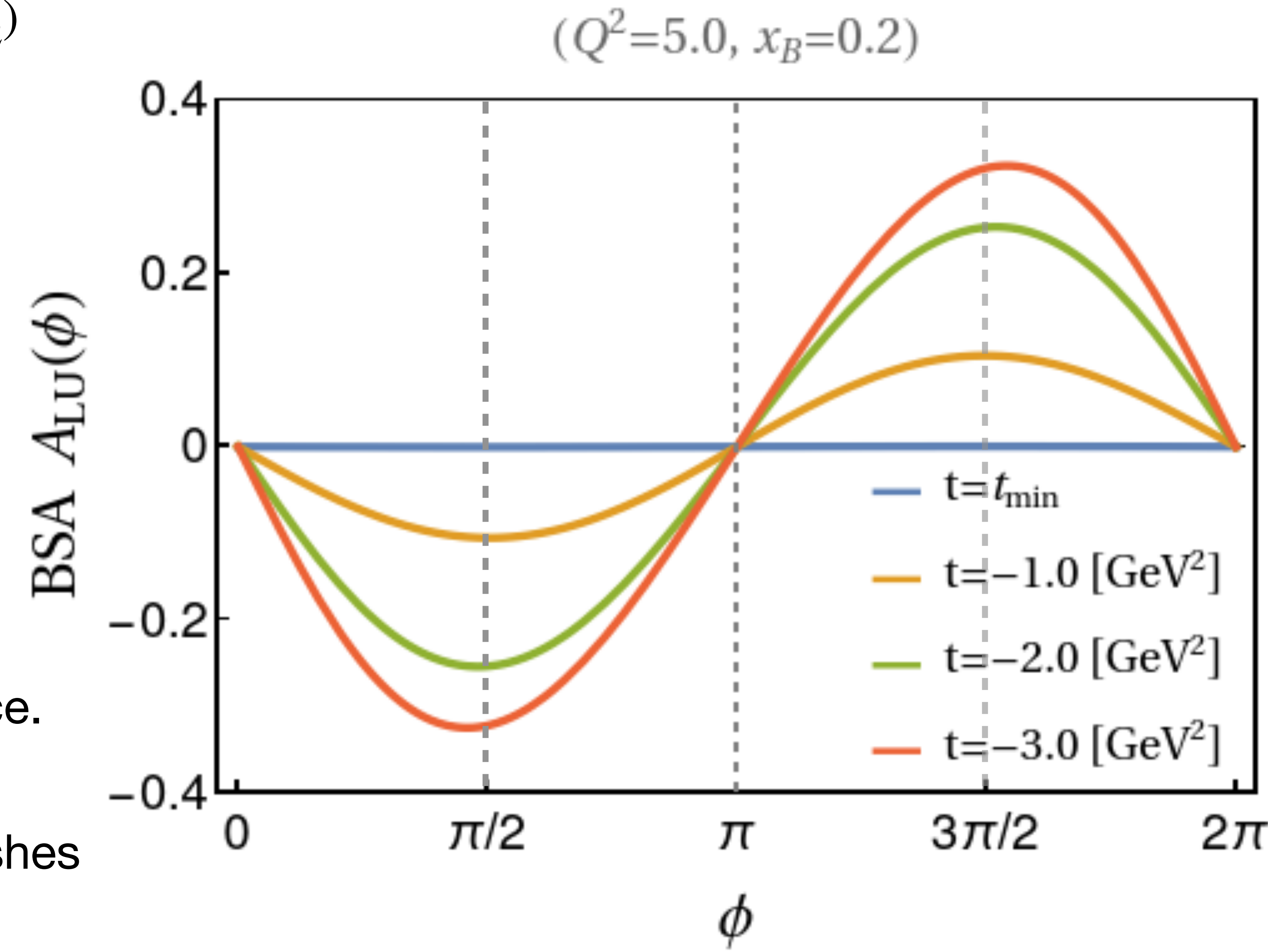
$$d\sigma^S_{BSA} = S_A\,(\mathscr{F}_1\mathscr{F}^*_2 - \mathscr{F}^*_1\mathscr{F}_2)$$

Remarks

1. As $|t|$ increases, magnitude of BSA increases.

2. At $t = t_{\min}$, BSA vanishes due to (1+1)-correspondence.

3. As Q^2 increases, BSA vanishes due to the small $\mathscr{F}_2$.



Conclusion

1. To invesgate the deeply virtual **scalar meson** (0^{++}) production on **helium target**, the simple theoretical test was performed.

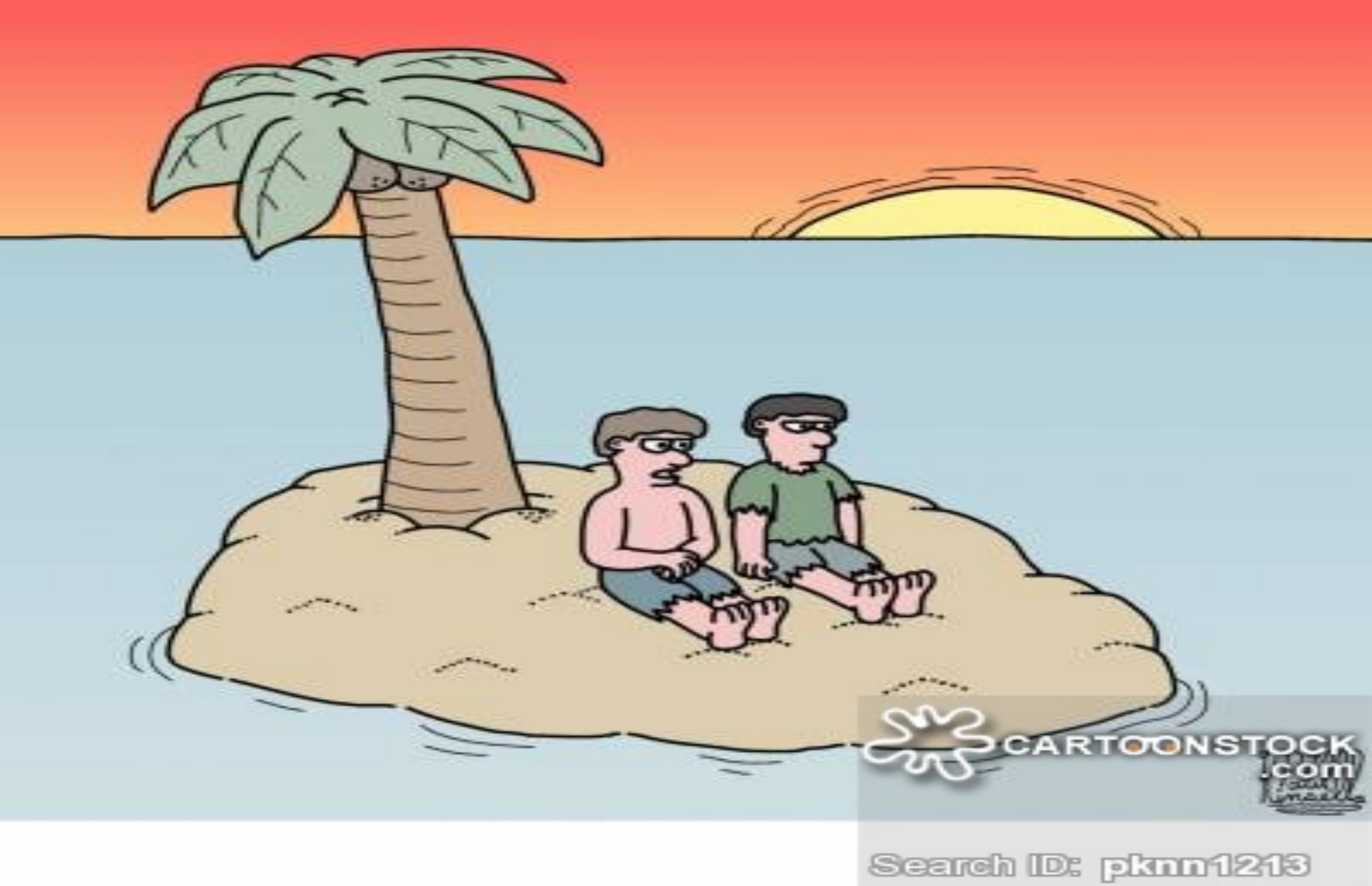
2. Two simple models are used, and the leading-twsit GPD can be obtained from the one-loop based model in the large Q^2 limit.

3. In the model calculation, we can obtain the GPD. and PDF, FF, and EMT are obtained by using the GPD sum rule.

4. At specific kinematical region, **non-vanishing BSA** implies that **the leading-twist approximation is not sufficient**.

Future work

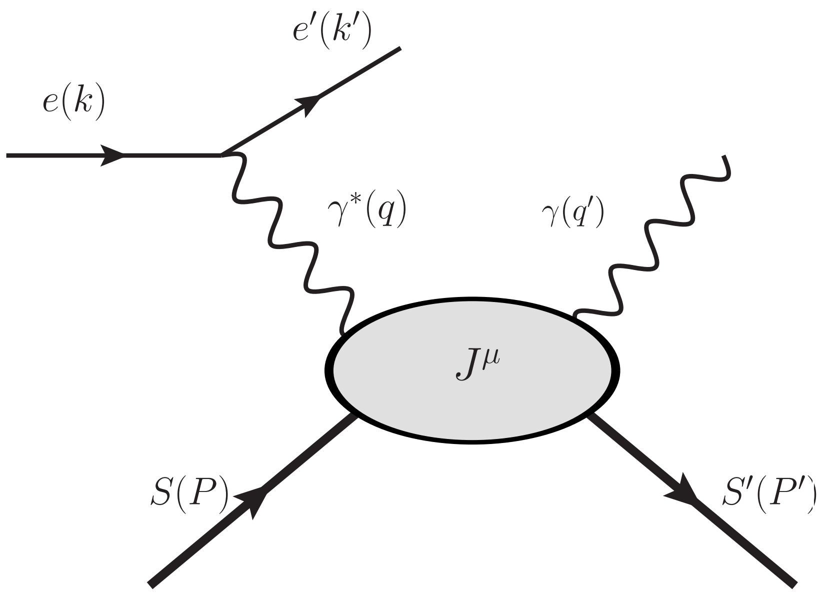
1. Realistic partons (quark propagator)
2. Realistic model for soft part (Liquid Instanton Model)
3. Coherent and incoherent processes
4. Twist-3 contribution (chiral-odd GPD)
5. Gluon contents



"Thank you for listening."

Backup

Exclusive Inelastic Scattering

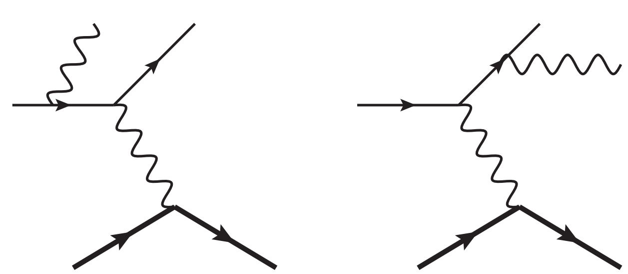


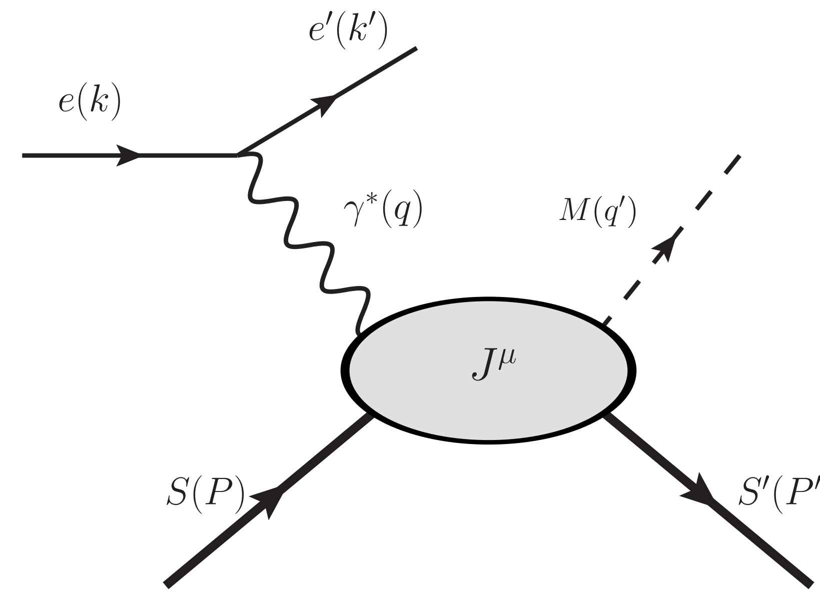
Deeply Virtual Compton Scattering (DVCS)

$\sigma\ (\gamma^* p \to \gamma p)\ \propto\ Q^{-4}$

Measures a combination of $H,\ E,\ \tilde{H},\ \tilde{E}$

Contribution of Bethe-Heitler process :





Deeply Virtual Meson Production (DVMP)

$\sigma\ (\gamma^* p \to M p)\ \propto\ Q^{-6}$

M = V : $H,\ E$

M = PS : $\tilde{H},\ \tilde{E}$

Managable complication :

the meson structure

Kinematics & GPDs limit

Kinematics in (1+1)-LFD

$$k = [\, xp^+,\ k^- \,],$$

$$p = \left[\, p^+,\ \frac{M_t}{p^+} \,\right],$$

$$q = \left[\, \left(\frac{\zeta' M_s^2}{Q^2} - \zeta\right) p^+,\ \left(\frac{1}{\zeta'} - \frac{t}{\zeta Q^2}\right) \frac{\boxed{Q^2}}{p^+} \,\right],$$

$$\Delta = q' - q = p - p' = \left[\, \zeta p^+,\ \frac{t}{\zeta p^+} \,\right],$$

$$p' = \left[\, (1-\zeta)\, p^+,\ \left(M_t^2 - \frac{t}{\zeta}\right) \frac{1}{p^+} \,\right],$$

$$q' = \left[\, \frac{\zeta' M_s^2 p^+}{Q^2},\ \frac{\boxed{Q^2}}{\zeta' p^+} \,\right],$$

where $\zeta = \dfrac{p^+ - p'^+}{p^+}$.

→

GPDs limit ($Q^2 >> |t|,\ M_s^2,\ \cdots$)

$$k = [\, xp^+,\ k^- \,],$$

$$p = \left[\, p^+,\ \frac{M_t}{p^+} \,\right],$$

$$q = \left[-\zeta p^+,\ \frac{Q^2}{\zeta' p^+} \,\right] \simeq \left[-\zeta p^+,\ \boxed{\frac{Q^2}{\zeta p^+}} \right],$$

$$\Delta = \left[\, \zeta p^+,\ \frac{t}{\zeta p^+} \,\right],$$

$$p' = \left[\, (1-\zeta)\, p^+,\ \left(M_t^2 - \frac{t}{\zeta}\right) \frac{1}{p^+} \,\right],$$

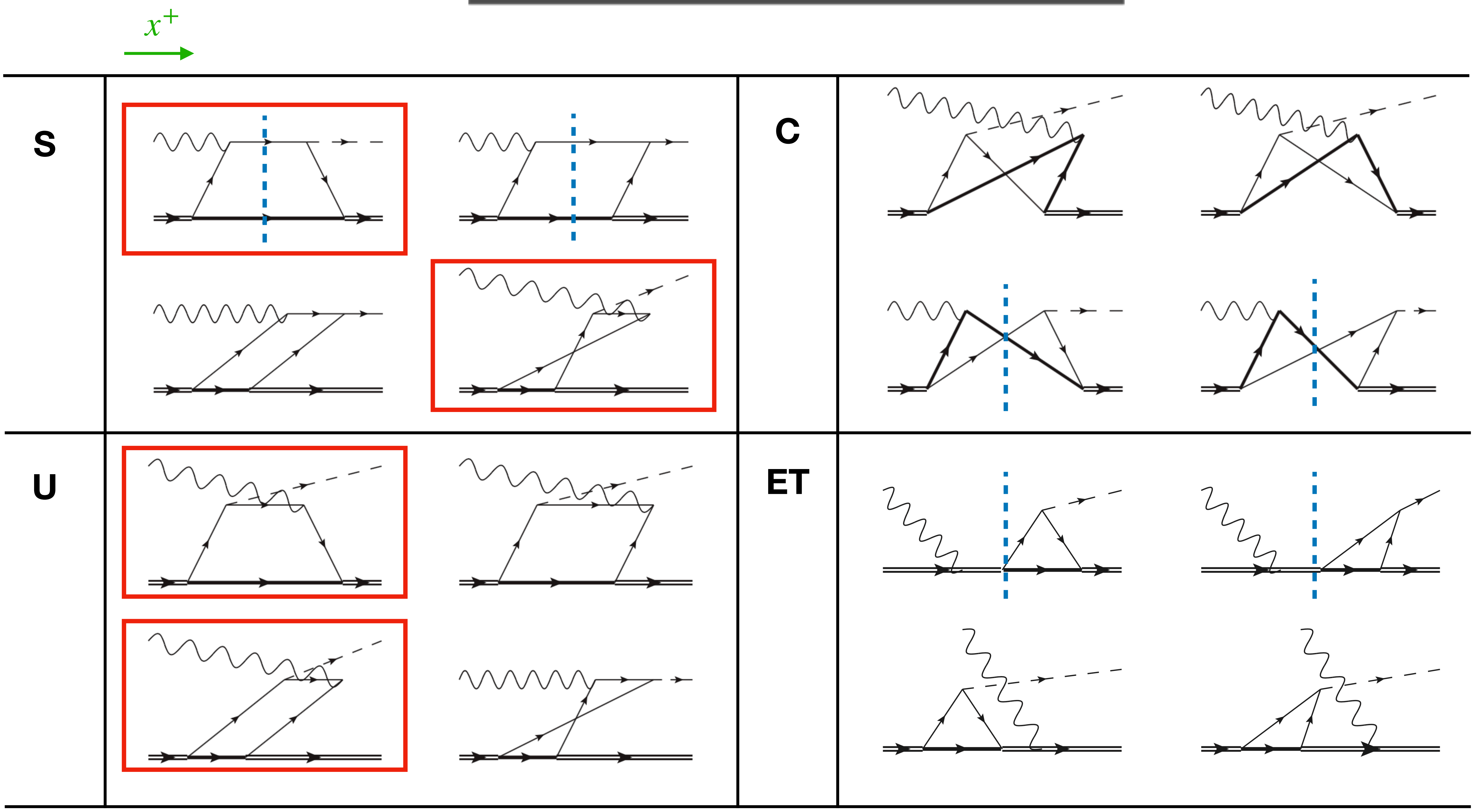
$$q' = \left[\, 0,\ \frac{Q^2}{\zeta' p^+} \,\right] \simeq \left[\, 0,\ \boxed{\frac{Q^2}{\zeta p^+}} \right],$$

where there is no M_s dependence, and $\zeta \simeq \zeta'$.

GPD does not distinguish between DVMP & DVCS.

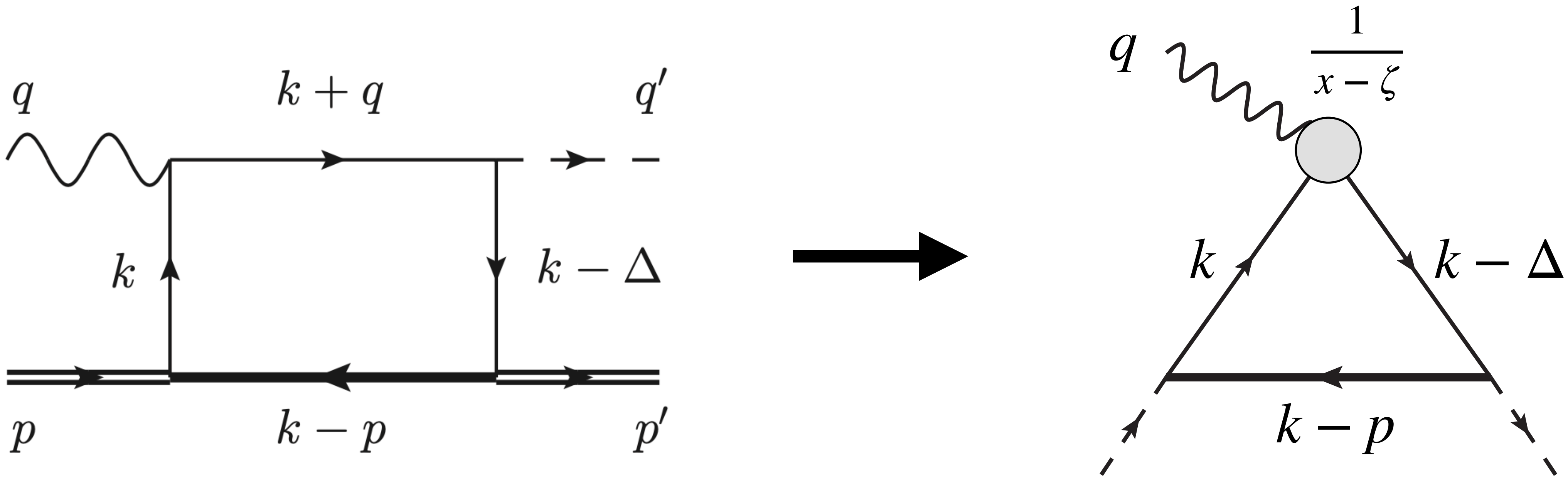
CR Ji, BLG Bakker,
Int. J. Mod. Phys. E 22, 1330002 (2013)

GPDs vs VMP in Diagrams



: GPDs formulation

: Occurring imaginary values



$$\mathscr{M}_s^\mu \sim \int \frac{d^2k}{(2\pi)^2}\,\frac{1}{k^2-m^2}\,\boxed{\frac{2k^\mu+q^\mu}{(k+q)^2-m^2}}\,\frac{1}{(k-\Delta)^2-m^2}\,\frac{1}{(k-p)^2-M^2}$$

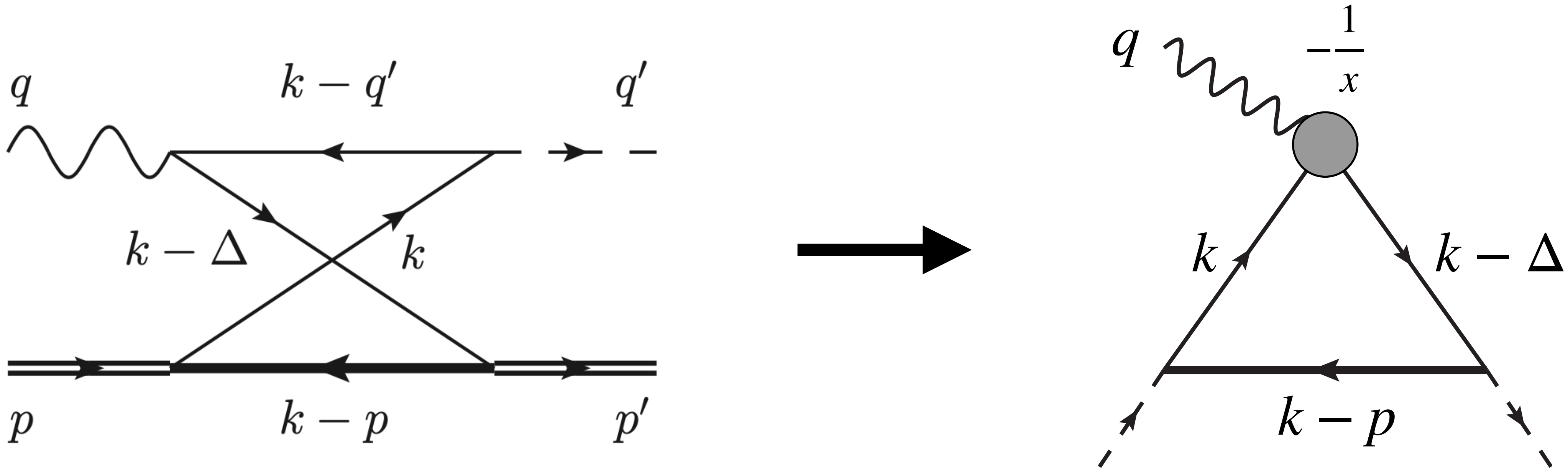
For a plus current of a virtual photon,

$$\boxed{\frac{2k^+ + q^+}{(k^+ + q^+)(k^- - k_t^-)}}\,\frac{1}{k^2-m^2}\,\frac{1}{(k-\Delta)^2-m^2}\,\frac{1}{(k-p)^2-M^2}$$

where $k_t^- = -q^- + \dfrac{m_{Q_1}^2}{k^+ + q^+} - i\dfrac{\epsilon}{k^+ + q^+}$

For large Q^2 : $\boxed{} \simeq \dfrac{1}{(x-\zeta)}\,\dfrac{\zeta'}{Q^2}\,(2x-\zeta) + \mathcal{O}\left(\dfrac{1}{Q^4}\right)$

$$= \boxed{\frac{1}{x-\zeta}}\,\frac{\zeta'}{Q^2}\,\boxed{\frac{1}{k^2-m^2}\,\frac{2k^+ - \Delta^+}{p^+}\,\frac{1}{(k-\Delta)^2-m^2}\,\frac{1}{(k-p)^2-M^2}}$$



$$\mathscr{M}_u^\mu \sim \int \frac{d^2k}{(2\pi)^2}\,\frac{1}{k^2-m^2}\,\frac{2k^\mu-\Delta^\mu-q'^\mu}{(k-q')^2-m^2}\,\frac{1}{(k-\Delta)^2-m^2}\,\frac{1}{(k-p)^2-M^2}$$

For a plus current of a virtual photon,

$$\frac{2k^\mu-\Delta^\mu-q'^\mu}{(k^+-q'^+)(k^- - k_u^-)}\,\frac{1}{k^2-m^2}\,\frac{1}{(k-\Delta)^2-m^2}\,\frac{1}{(k-p)^2-M^2}$$

where $k_u^- = q'^- + \frac{m_{Q_1}^2}{k^+-q'^+} - i\frac{\epsilon}{k^+-q'^+}$

For large Q^2 : $\qquad \simeq -\frac{1}{x}\,\frac{\zeta'}{Q^2}\,(2x-\zeta) + \mathcal{O}\left(\frac{1}{Q^4}\right)$

$$= -\frac{1}{x}\,\frac{\zeta'}{Q^2}\,\frac{1}{k^2-m^2}\,\frac{2k^+-\Delta^+}{p^+}\,\frac{1}{(k-\Delta)^2-m^2}\,\frac{1}{(k-p)^2-M^2}$$

Operator Product Expansion

Bilocal current can be expressible in the sum of local currents when the separation is going to zero :

$$\hat{T}\{J^\mu(y)\ J^\nu(0)\} = \sum_n C_n(y)\ \hat{\mathcal{O}}_n(0) \qquad \text{for } y^\mu \to 0$$

Phys. Rev. 179, 1499 (1969)

$$\hat{T}\{(\bar{\psi}_q(y)\gamma^\mu\psi_q(y))\ (\bar{\psi}_q(0)\gamma^\mu\psi_q(0))\}$$

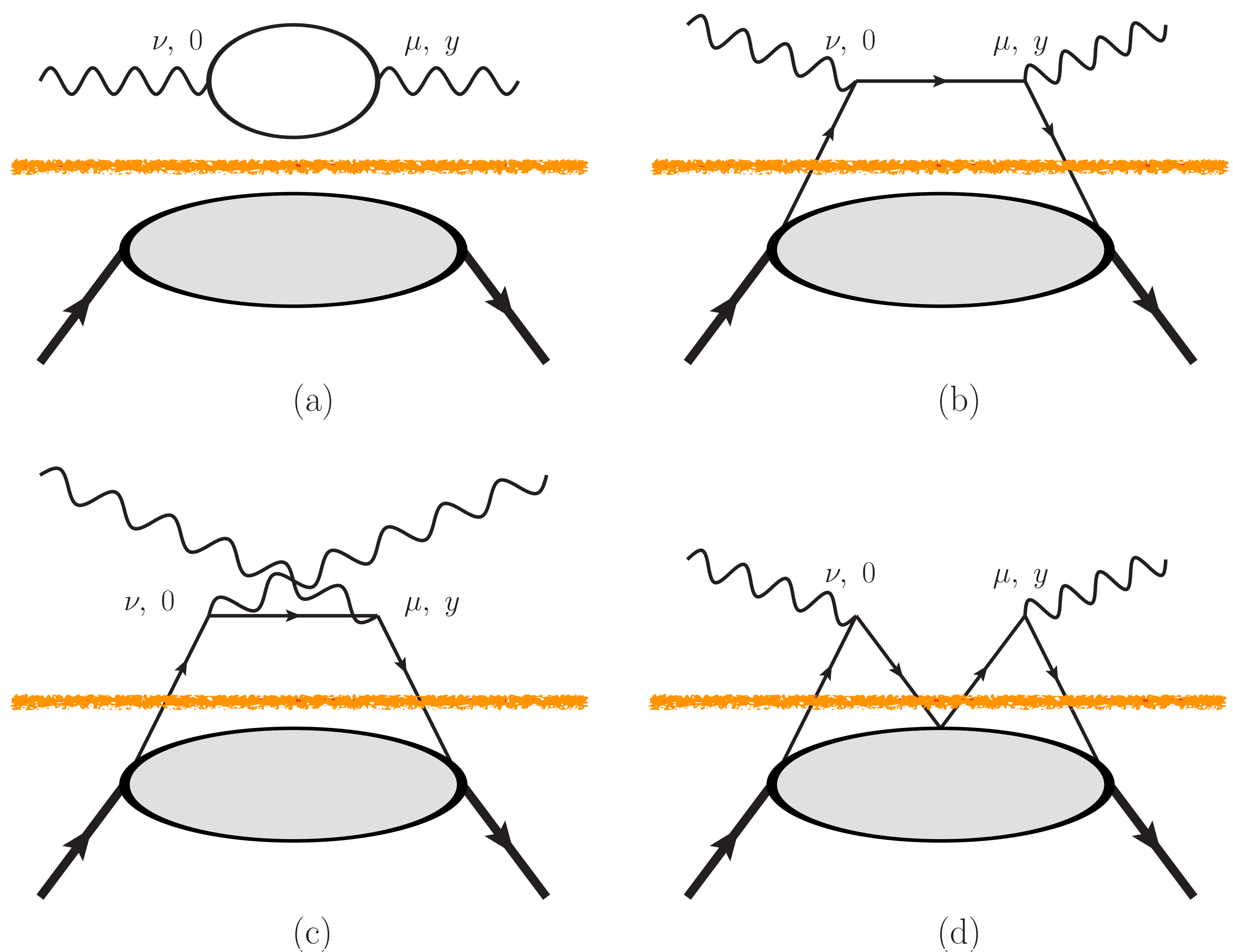
$$= -\,\mathrm{Tr}[\ \gamma^\mu S_F(y)\gamma^\nu S_F(-y)\ ]$$

$$+ :\bar{\psi}_q(y)\gamma^\mu S_F(y)\gamma^\nu\psi_q(0):$$

$$+ :\bar{\psi}_q(0)\gamma^\nu S_F(-y)\gamma^\mu\psi_q(y):$$

$$+ :\bar{\psi}_q(y)\gamma^\mu\psi_q(y)\bar{\psi}_q(0)\gamma^\nu\psi_q(0):$$

$S_F(y)$ is divergent as $y^\mu \to 0$



Twist Expansion - Factorization

What is **the twist**?

twist(τ) = dimension(d) - spin(s)

(Mass) dimension : naive canonical dimension of the operator from that action is dimensionless.

operator	$\mathcal{S}$	P_μ, D_μ, A_μ	ψ	$G_{\mu\nu}, \tilde{G}_{\mu\nu}$	$\overline{\psi}\gamma_\mu\psi$	$\mathcal{L}$
dimension	0	1	3/2	2	3	4

Spin : its transformation properties under the Lorentz group.
(the number of Lorentz indices)

In QCD, the leading twist is 2,
and hadronic tensor eventually can be expressed in terms of $(\Lambda/Q^2)^{\tau-2}$ powers.