Gravitational formfactors, equivalence principle and shear viscosity

3D Structure of the Nucleon via Generalized Parton Distributions

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Outline

Gravitational formfactors and

hadron spin structure

gravitational field

Spacelike vs timelike

Viscosity

Space ->Time: T-odd->exotics

Golographic bounds -> smallness of DA

Gravitational Formfactors (spin ½)

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with scalar and tensor particles, with both classical and "TeV" gravity

Gravity and hadron structure: (OT'99)

- Interaction – field vs metric deviation

$$M = \langle P'|J_q^\mu|P\rangle A_\mu(q)$$

$$M = \frac{1}{2}\sum_{q,G}\langle P'|T_{q,G}^{\mu\nu}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J_q^\mu|P\rangle = 2e_q P^\mu$$

$$\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J_q^\mu|P\rangle A_\mu = 2e_q M\,\phi(q)$$

$$M_0 = \frac{1}{2}\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle h_{\mu\nu} = 2M\cdot M\,\phi(q)$$

- Mass as charge – equivalence principle

EP and hadron structure

“Microscopic” EP (coupling of gravity to EMT)

+

Conservation law

(Momentum SR to get local from LC: $\int dx\, x\, (\Sigma\, q(x) + G(x)) = 1$)

=

“Macroscopic” EP (universal falling) :

Tested VERY precisely

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

  $$\vec{H}_J = \frac{1}{2} rot \vec{g};\ \vec{g}_i \equiv g_{0i}$$

  spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

  $$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot \vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle

Experimental test of PNEP

Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW
LETTERS

VOLUME 68 13 JANUARY 1992 NUMBER 2

Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson
Physics Department, FM-15, University of Washington, Seattle, Washington 98195
(Received 25 September 1991)

If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

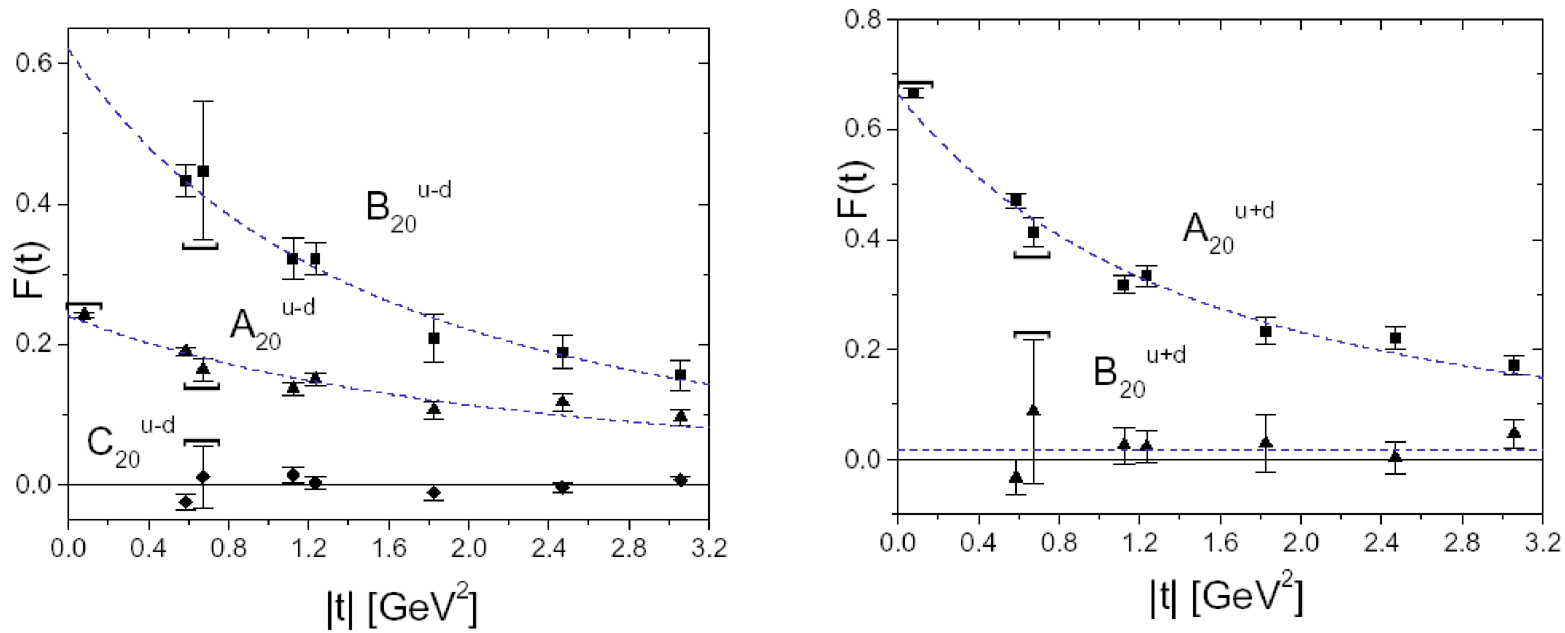
$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

Quantim measurement and EP

- If spin is just a (pseudo) vector : EP due to Earth rotation is trivial
- Crucial if measured by a device in rotating frame
- Quantum measurement problem becomes practical
- Cf Unruh effect in HIC (Prokhorov, OT, Zakharov'19 and in progress)

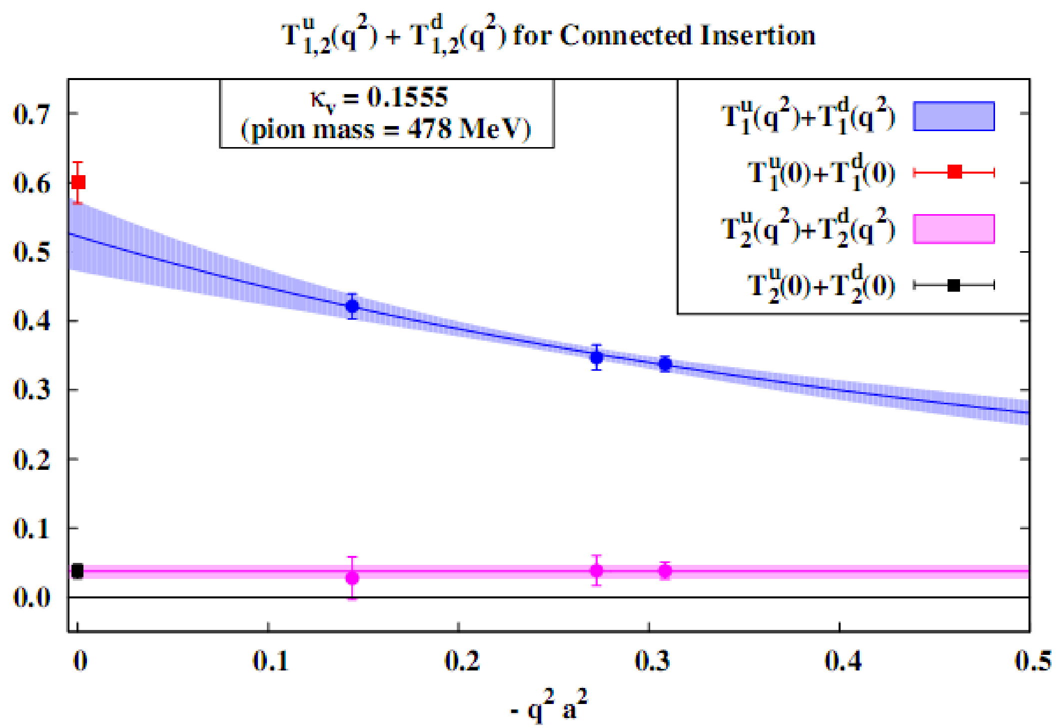
Generalization of Equivalence principle (smallness of B_G used om gluons FF extraxtion- talk of Z.-E. Meziani)

- Various arguments: AGM ≈0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



Further lattice study (M. Deka et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In QED, pQCD – violated (Brodsky et al)
- Reason – in the case of ExEP enforcing by subtracted DR - no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking (test in models? Cf talk of H.- Ch. Kim)
- Gravityproof confinement? Nucleons do not break even by black holes?
- Support by recent observations of smalness of (nucleon "cosmological constant") Cbar

One more gravitational formfactor (related to "D-term" of Maxim Polyakov and Christian Weiss)

- Quadrupole

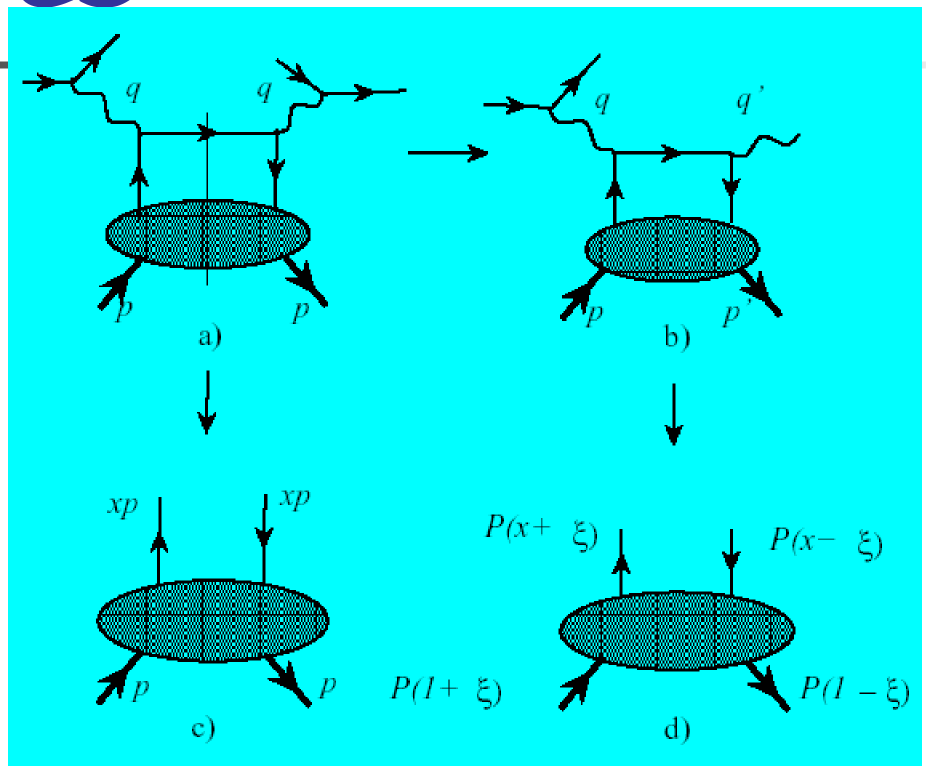
$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

- Cf vacuum matrix element – cosmological constant $\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$

$$\Lambda = C(q^2)q^2$$

- NO "vacuum-like" term – EP, Smallness -ExEP
- How to measure experimentally – DVCS (and DVMP?)

QCD Factorization for DIS and DVCS



- *Manifestly spectral*

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}.$$

- *Extra dependence on ξ*

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Unphysical regions

- DIS : Analytical function – polynomial in $1/x_B$ if $1 \leq |x_B|$

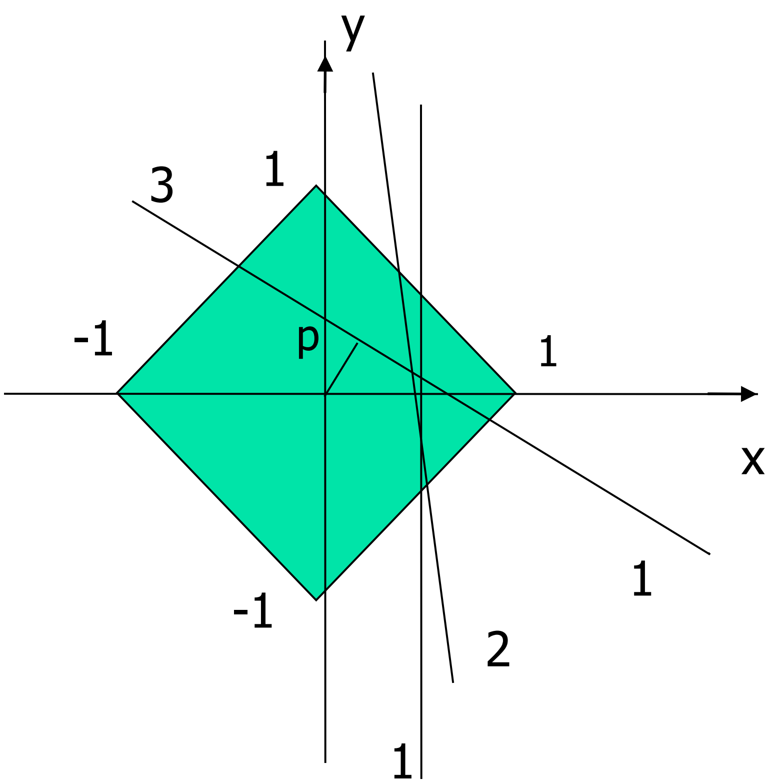
$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x, \xi)$
- Solved by using of Double Distributions Radon transform

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line-skewness
- Kinematics of DIS: $\xi = 0$
  ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$
  - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

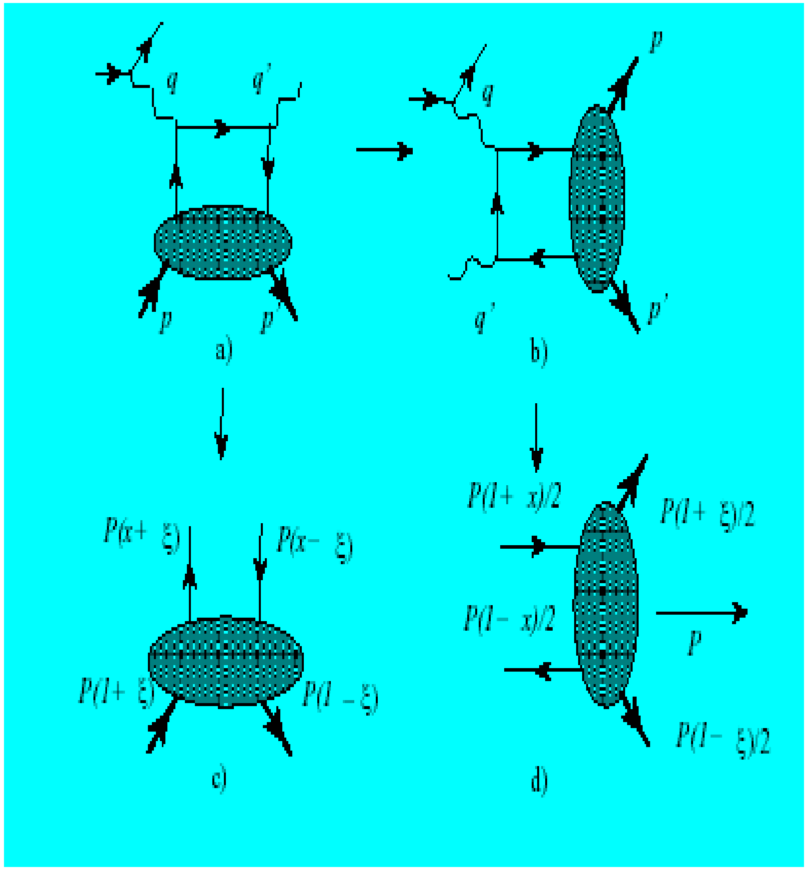


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi |cos\phi| (H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi (H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes
- Duality between s and t channels (Polyakov,Shuvaev, Guzey, Vanderhaeghen)



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in x weighted with x^n - are polynomials in 1/ ξ of power n+1
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Holographic property (OT'05)

- *Factorization Formula ->*

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial\xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

- *Analyticity -> Imaginary part -> Dispersion relation:*

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

- *"Holographic" equation*

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

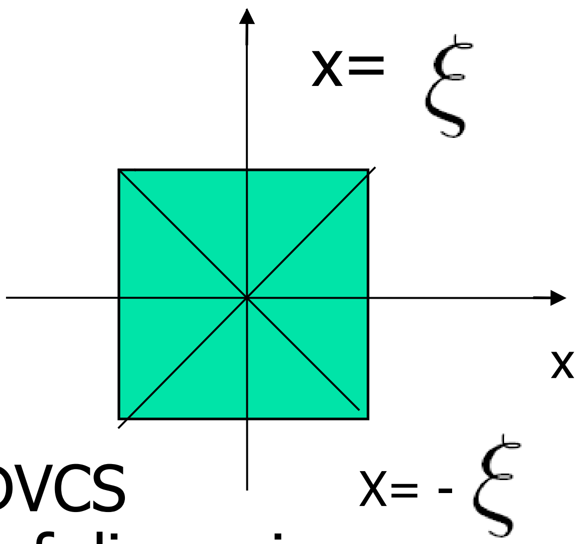
- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta \mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!



- Strategy (now adopted) of GPD's studies: start at diagonals (through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\mathcal{A}(\nu,Q^2)=\frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu',Q^2)}{(\nu'^2-\nu^2)}+\Delta \qquad \Delta=2\int_{-1}^{1}d\beta\frac{D(\beta)}{\beta-1}$$

$$\Delta^p_{\mathrm{CQM}}(2)=\Delta^n_{\mathrm{CQM}}(2)\approx 4.4, \qquad \Delta^p_{\mathrm{latt}}\approx\Delta^n_{\mathrm{latt}}\approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!

- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

From D-term to pressure

- *Inverse -> 1st moment (model)*
- *Kinematical factor – moment of pressure D~-<p r⁴> (<p r²> =0)   M.Polyakov (2003)*

$$T^{Q}_{\mu\nu}(\vec{r},\vec{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{\Delta}}\, \langle p', S'|\hat{T}^{Q}_{\mu\nu}(0)|p, S\rangle$$

$$T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\delta_{ij}$$

- *Possible justification: Born gravitational scattering*
- *Stable equilibrium D<0: Holds for quarks (or leptons) in photon*

Pressure in hadron pairs production

- Back to GDA region
- -> moments of H(x,x) - define the coefficients of powers of cosine!– $1/\xi$
- Higher powers of cosine in t-channel – threshold in s -channel
- Larger for pion than for nucleon pairs because of less fast decrease at x ->1
- Large ξ limit – access to D-term

$$\mathscr{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x) \frac{x^n}{\xi^{n+1}} + \Delta\mathscr{H}.$$

Gravitational FFs from Belle data on GDAs

S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

- Gravitational FFs are related to twist-2 GDAs

$$A_{\lambda_1\lambda_2} = T_{\mu\nu}\varepsilon^{\mu}(\lambda_1)\varepsilon^{\nu}(\lambda_2)/e^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz\,\frac{2z-1}{z(1-z)}\Phi^q\left(z,\xi,W^2\right)$$

$$\int dz(2z-1)\Phi_q^{+}\left(z,\xi,W^2\right) = \frac{2}{(P^+)^2}\left\langle \pi^+(p_1)\pi^-(p_2)\right|T_q^{++}(0)\left|0\right\rangle$$

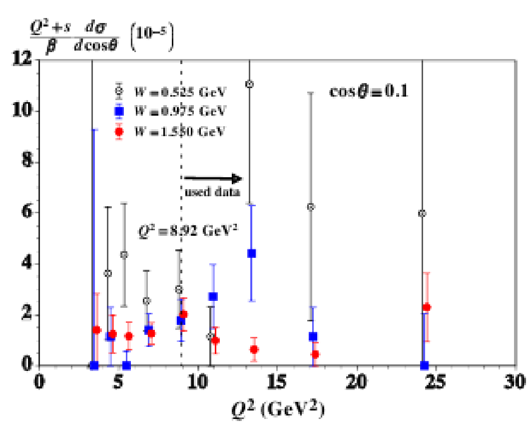
$$\left\langle \pi^0(p_1)\pi^0(p_2)\right|T^{\mu\nu}(0)\left|0\right\rangle = \frac{1}{2}\left[\left(sg^{\mu\nu}-P^{\mu}P^{\nu}\right)\Theta_1+\Delta^{\mu}\Delta^{\nu}\Theta_2\right]$$

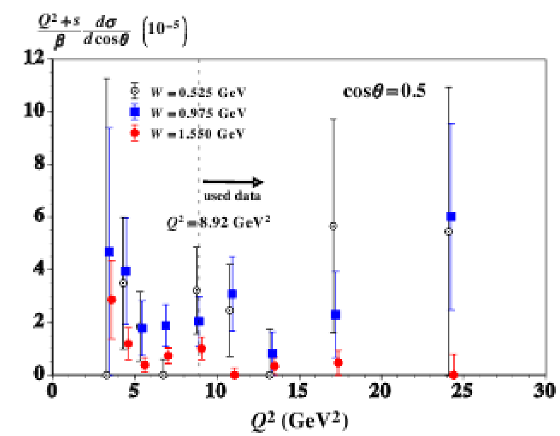
$$P=p_1+p_2\,,\ \Delta=p_1-p_2$$

M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003

- Belle data and scaling : W=0.525,0.975, 1.55 GeV

$$\frac{(Q^2+s)d\sigma}{\beta d|\cos\theta|} \propto \left|\Phi^{\pi^0\pi^0}(z,\,\cos\theta,\,W,\,Q)\right|^2$$





Phase shifts and resonances

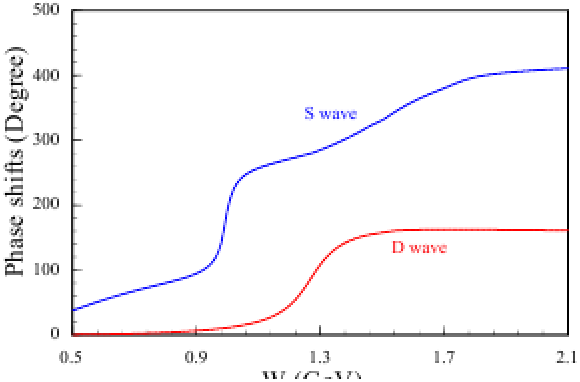
Leading harmonics

$$\sum_q \Phi_q^+\left(z,\xi,W^2\right)=18n_f z(1-z)(2z-1)[B_{10}(W)+B_{12}(W)P_2(2\xi-1)]$$

$$=18n_f z(1-z)(2z-1)[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)]$$

$$\tilde{B}_{10}(W)=\bar{B}_{10}(W)e^{i\delta_0},\tilde{B}_{12}(W)=\bar{B}_{12}(W)e^{i\delta_2}$$

S/D shifts



$f_0(500)$, $f_2(1270)$ contributions

$$\bar{B}_{12}(W)=\beta^2\frac{10g_{f_2\pi\pi}f_{f_2}M_{f_2}^2}{9\sqrt{2}\sqrt{(M_{f_2}^2-W^2)^2-\Gamma_{f_2}^2M_{f_2}^2}}$$

$$\bar{B}_{10}(W)=\frac{5g_{f_0\pi\pi}f_{f_0}}{3\sqrt{2}\sqrt{(M_{f_0}^2-W^2)^2-\Gamma_{f_0}^2M_{f_0}^2}}$$

Fits and results

Collection

$$\Phi_q^+\left(z,\xi,W^2\right)=N_h z^{\alpha}(1-z)^{\alpha}(2z-1)[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)]$$

$$\tilde{B}_{10}(W)=[\frac{-3+\beta^2}{2}\frac{5R_\pi}{9}F_h(W^2)+\frac{5g_{f_0\pi\pi}f_{f_0}}{3\sqrt{2}\sqrt{(M_{f_0}^2-W^2)^2-\Gamma_{f_0}^2M_{f_0}^2}}]e^{i\delta_0}$$

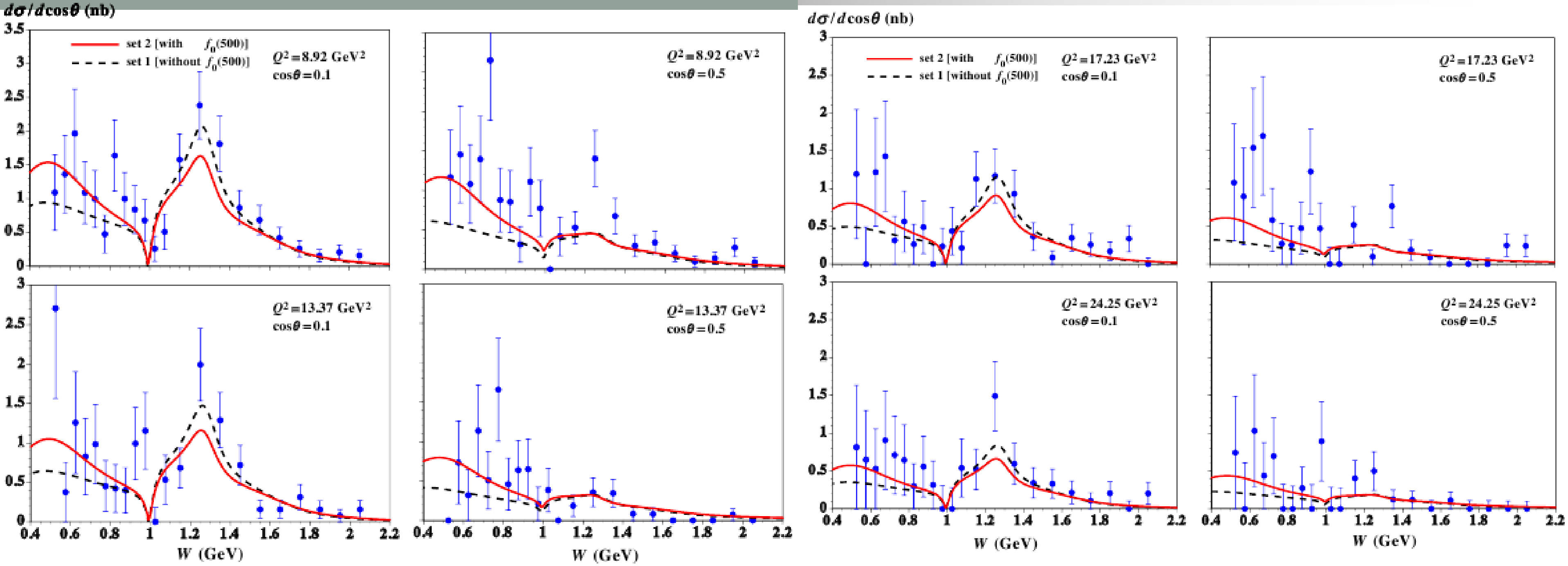
$$\tilde{B}_{12}(W)=[\beta^2\frac{5R_\pi}{9}F_h(W^2)+\beta^2\frac{10g_{f_2\pi\pi}f_{f_2}M_{f_2}^2}{9\sqrt{2}\sqrt{(M_{f_2}^2-W^2)^2-\Gamma_{f_2}^2M_{f_2}^2}}]e^{i\delta_2}$$

$$F_h(W^2)=\frac{1}{\left[1+\frac{W^2-4m_\pi^2}{\Lambda^2}\right]^{n-1}}$$

Best fit with (2) and without (1) f_0

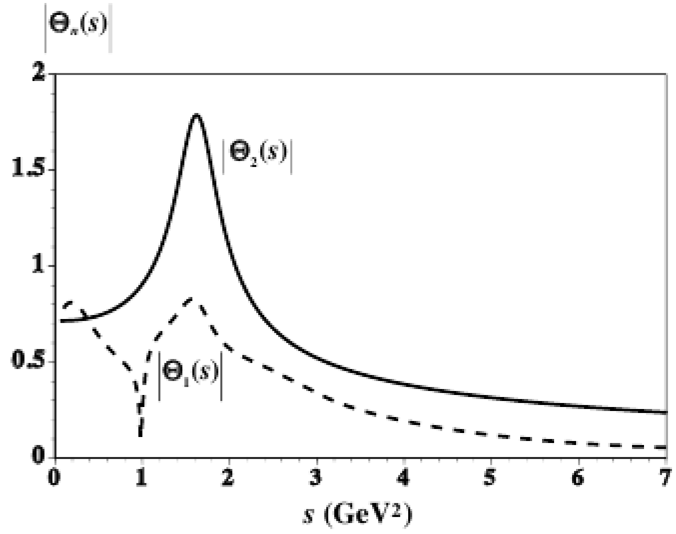
	Set 1	Set 2
α	0.801±0.042	1.157± 0.132
Λ	1.602±0.109	1.928±0.213
a	3.878± 0.165	3.800± 0.170
b	0.382± 0.040	0.407± 0.041
f_{f0}	-------	0.0184± 0.034
	$\frac{\chi^2}{NOF}=1.22$	$\frac{\chi^2}{NOF}=1.09$

Description of data



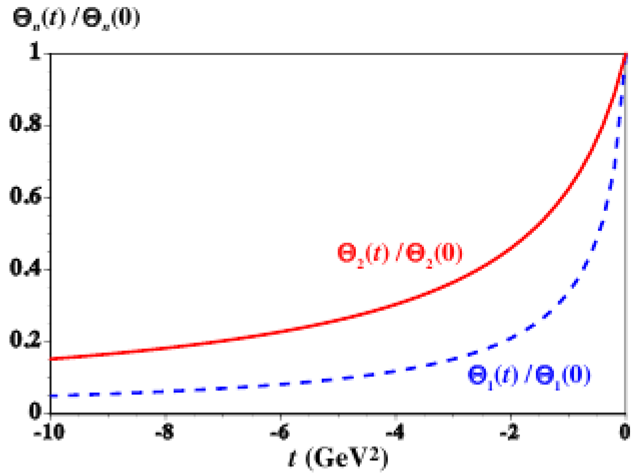
Formfactors

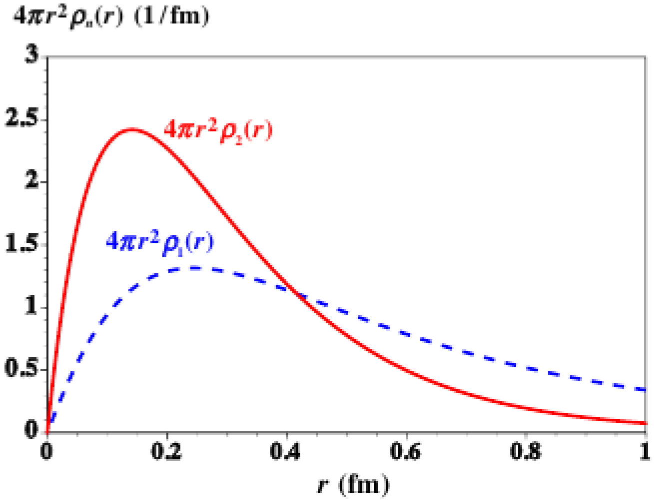
Resonance structure in pressure –related Θ_1



Time-like -> space-like

Dispersion relation and Fourier transform





Mass radius

$$< r^2 > = 6 \int_{4m^2}^{\infty} \frac{\mathrm{Im}(F(s))}{s^2}$$

$$\sqrt{\langle r^2 \rangle} = 0.69 \text{ fm for } \Theta_2$$

Shear – natural counterpart of pressure (talk of H.-Ch. Kim)

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Forces inside the nucleon on the light front from 3D Breit frame force distributions: Abel tomography case

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(Received 4 March 2021; accepted 15 June 2021; published 9 July 2021)

Other traceless structures in EMT?

Elastic medium -> liquid

(LL v. VII->v. VI)

Shear viscosity in liquid

- From spherically symmetric object to fluid (EoS!)
- $T^{\mu\lambda} = (e+p)\, v^{\mu} v^{\lambda} - p\, g^{\mu\lambda}$
- $V^{\mu} = P^{\mu}/M$: correct normalization but no coordinate dependence
- Another suggestion:
- $V^{\mu} = (P^{\mu} + a(t)\, k_T^{\ \mu}) / (M^2 + a^2(t)\, k_T^{\ 2})^{1/2}$
- Viscosity: $\sim E_{\eta}\, p^{[\mu} \Delta^{\lambda]}$
- Naïve T-oddness: phases
- NO such term in total EMT – violates ExEP (but can be for quarks separately)
- Phases <-> dissipation: polarization in pionic superfluidity model (V. I. Zakharov, OT' 17)

Viscosity in GDA channel

- Viscosity:will correspond to Exotic $J^{PC}=1^{-+}$ meson (already studied without reference to viscosity: Anikin, Pire, Szymanowski,OT, Wallon'06)
- Spin: related to structure of matrix element: One index of EMT (0^{th} in rest frame) is carried by momentum and other by polarization vector- just what we need for viscosity
- No zero-momentum (classical) limit -> quantum
- NO for conserved EMT (zero coupling!): violated ExEP
- $\pi\eta$ pairs observation instead of $\pi\pi$ required
- Smallness of viscosity: related to smallness of exotic GDAs and ExEP violation?!

Exotic hybrid meson production

On exotic hybrid meson production in $\gamma^*\gamma$ collisions

I.V. Anikin[1], B. Pire[2,a], L. Szymanowski[3,4,5], O.V. Teryaev[1], S. Wallon[5]

Eur. Phys. J. C 47, 71–79 (2006)
Digital Object Identifier (DOI) 10.1140/epjc/s2006-02533-7

Possible candidate
π_1 (1400)

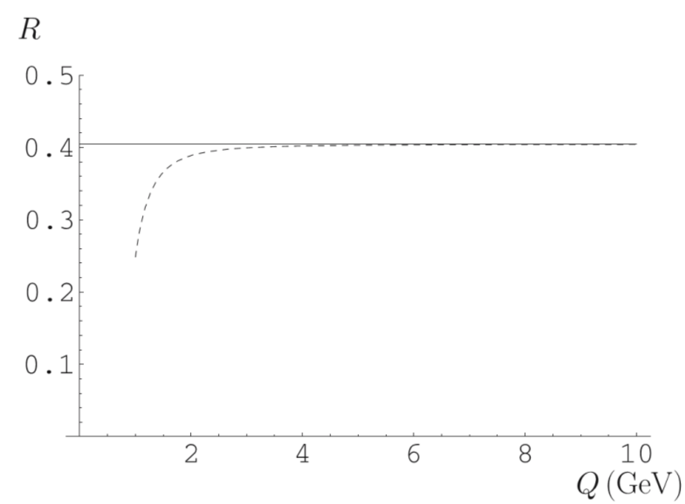

Fig. 2. The ratio $R(Q^2)$ of the squared amplitudes for H and π^0 production in $\gamma^*\gamma$ collisions at leading twist and zero-th order in α_s (*solid line*) and including twist three contributions in the numerator (*dashed line*)

Estimate of viscosity

Terms in EMT:

$(e+p)\, v^{\mu} v^{\lambda} \sim A\, P^{\mu} P^{\lambda}$

$\eta\, dv^{\mu}/d\, x_T^{\ \lambda} \sim E_{\eta}\, p^{[\mu} \Delta^{\lambda]}$

TD: $e+p \sim Ts$

$\eta/s\ (> 1/(4\pi)) \sim E_{\eta} T / AM$

Correct dependence on Planck constant recovered via Δ^{λ}

(cf K. Trachenko et al.)

DA vs holographic bound

$$\eta \frac{\partial v^\nu}{\partial x_\mu} \to \frac{P^\nu \Delta^\mu}{M} \sim E(t) P_\nu \Delta_\mu$$

$$v^\mu = \frac{P^\mu + a(t)\Delta^\mu}{\sqrt{M^2 - a^2(t)t}}, \qquad \frac{\partial}{\partial x_\mu} \to i\Delta^\mu$$

$$(e+p) v^\nu v^\mu \to T s \frac{P^\nu P^\mu}{M^2} \sim A(t) P^\nu P^\mu$$

$$\frac{\eta}{s} \sim \frac{E(t)}{A(t)} \cdot \frac{T}{M}$$

T~ < K_T>

Time-like

$$\langle \pi\eta(P,\Delta) | T_i^{\alpha\nu} | 0 \rangle_{\mu^2} = E_i(s,\mu^2) P^\alpha \Delta^\nu$$

Dimensionful $\frac{\eta}{s} \sim \hbar \frac{E(t)}{A(t)} \cdot \frac{T}{M} \sim \frac{\hbar}{k_B} \cdot \frac{E(t)}{A(t)} \cdot \frac{k_B T}{M}$

$$\hbar \frac{\partial}{\partial x_\mu} \to i\Delta^\mu$$

- Small bound => small exotic DA, small ExEP violation

Conclusions/Outlook

- ExEP (unique QCD/Gravity relation
- Time-like Gravitational FFs: may be studied in meson pairs production
- Exotic hybrid mesons: access to shear viscosity and interplay between hadronic and heavy-ion physics
- Holographic bound: related to smallness of exotic GDA and violation of ExEP?
- Shear from asymptotic transition gravitational FFs (Q.-T. Song, OT, work in progress)
- Medium viscosity from GrFFs?