New developments on proton Generalized Parton Distributions from lattice QCD

Martha Constantinou

Temple University

3D Structure of the Nucleon via Generalized Parton Distributions

June 26, 2024

Outline

- ★ Approaches to access information on GPDs from lattice QCD
- ★ Definition of light-cone GPDs vs Euclidean lattice definition (quasi GPDs)
- ★ New Lorentz covariant approach to access x-dependence of GPDs
- ★ Twist-3 GPDs
- ★ Future extensions - Other developments

Collaborators:

Twist-2

S. Bhattacharya, K. Cichy, J. Dodson, X. Gao, A. Metz, J. Miller, S. Mukherjee, P. Petreczky, A. Scapellato, F. Steffens, Y. Zhao

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

PHYSICAL REVIEW D **109,** 034508 (2024)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[2,†] Jack Dodson,[2] Xiang Gao,[3] Andreas Metz,[2] Joshua Miller,[2,‡] Swagato Mukherjee,[4] Peter Petreczky,[4] Fernanda Steffens,[5] and Yong Zhao[3]

Twist-3 S. Bhattacharya, K. Cichy, J. Dodson, A. Metz, J. Miller, A. Scapellato, F. Steffens

PHYSICAL REVIEW D **108,** 054501 (2023)

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

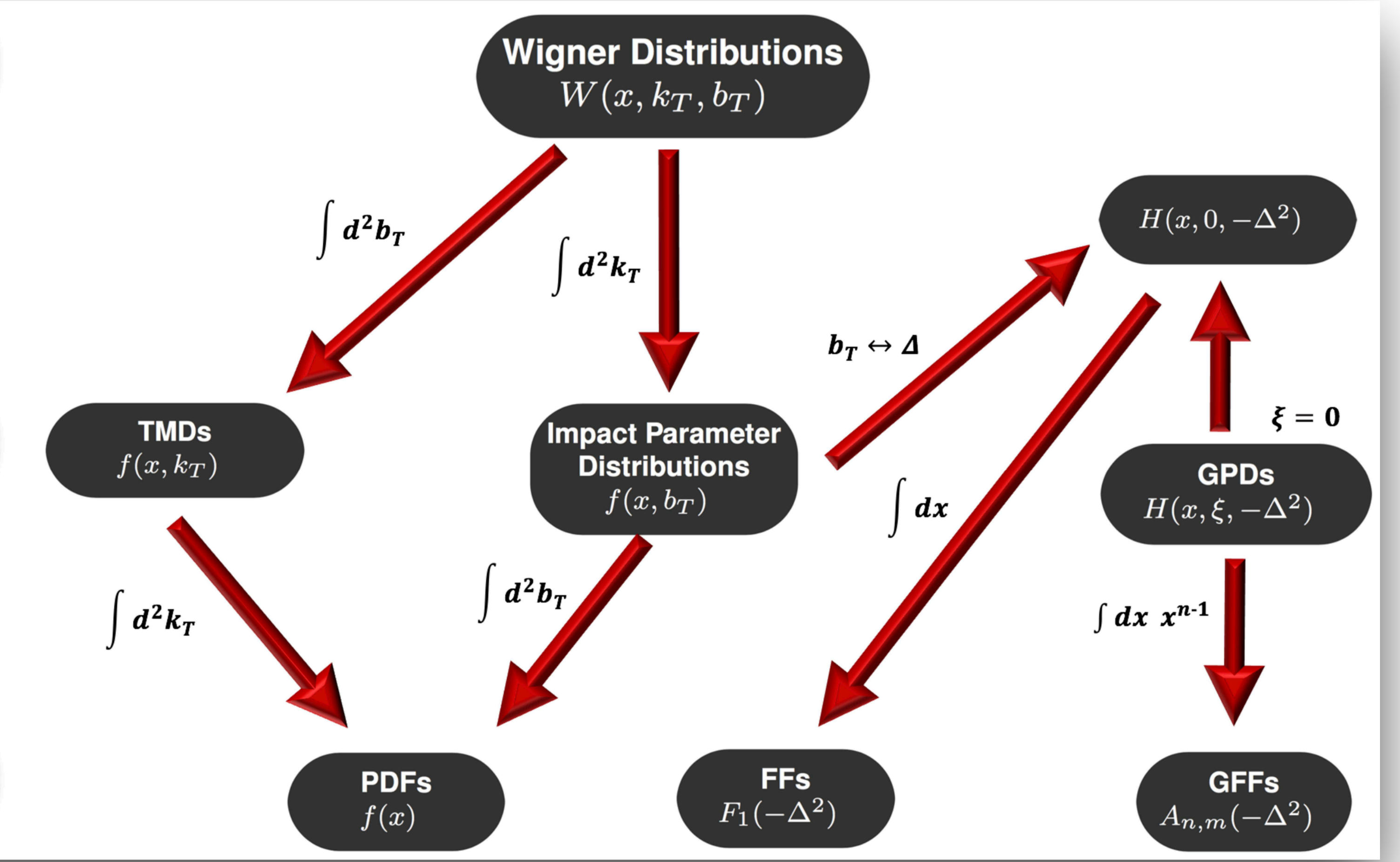
Shohini Bhattacharya,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[1] Jack Dodson,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[4]

Nucleon Characterization

Wigner distributions

★ provide multi-dim images of the parton distributions in phase space

★ encode both TMDs and GPDs in a unified picture

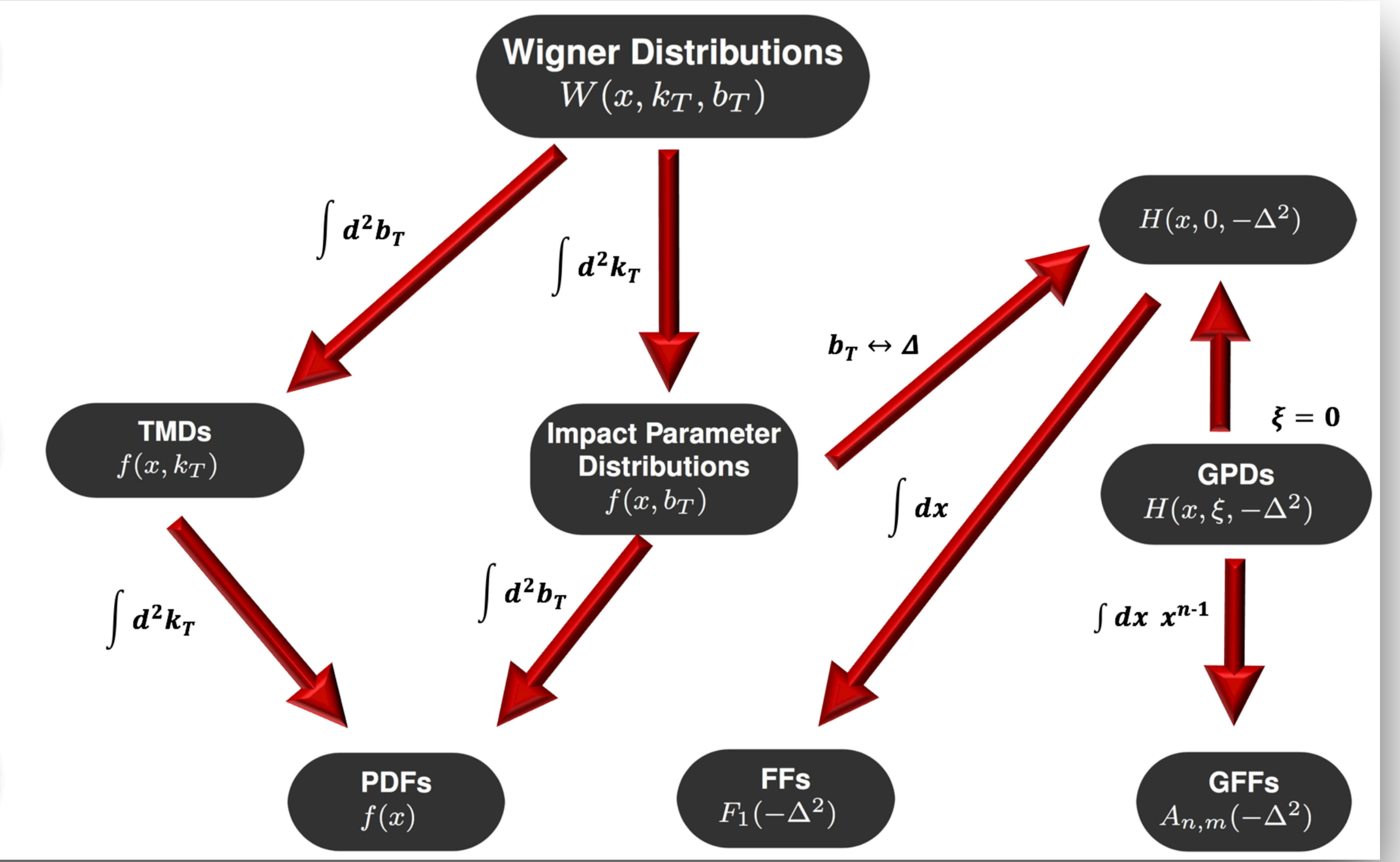


Nucleon Characterization

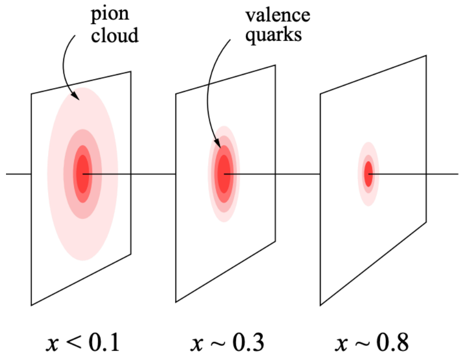
Wigner distributions

★ provide multi-dim images of the parton distributions in phase space

★ encode both TMDs and GPDs in a unified picture → crucial in mapping hadron tomography



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]

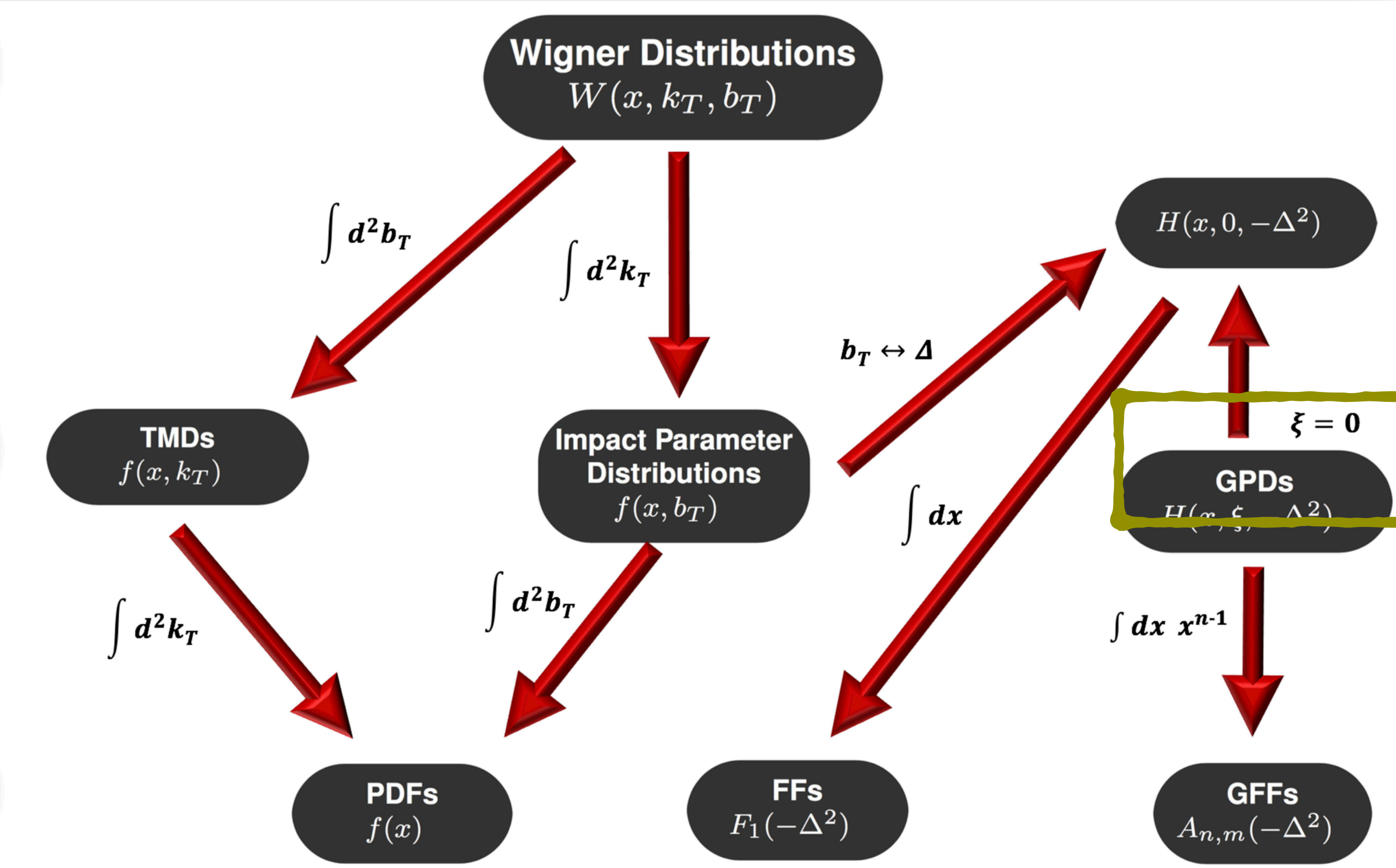


Nucleon Characterization

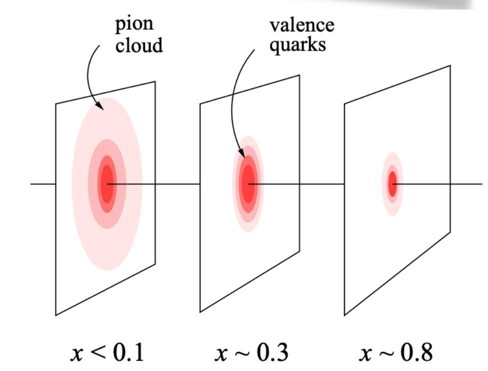
Wigner distributions

★ provide multi-dim images of the parton distributions in phase space

★ encode both TMDs and GPDs in a unified picture → crucial in mapping hadron tomography



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]



GPDs

★ "Parent" functions for PDFs, FFs, GFFs

★ Multi-dimensional objects

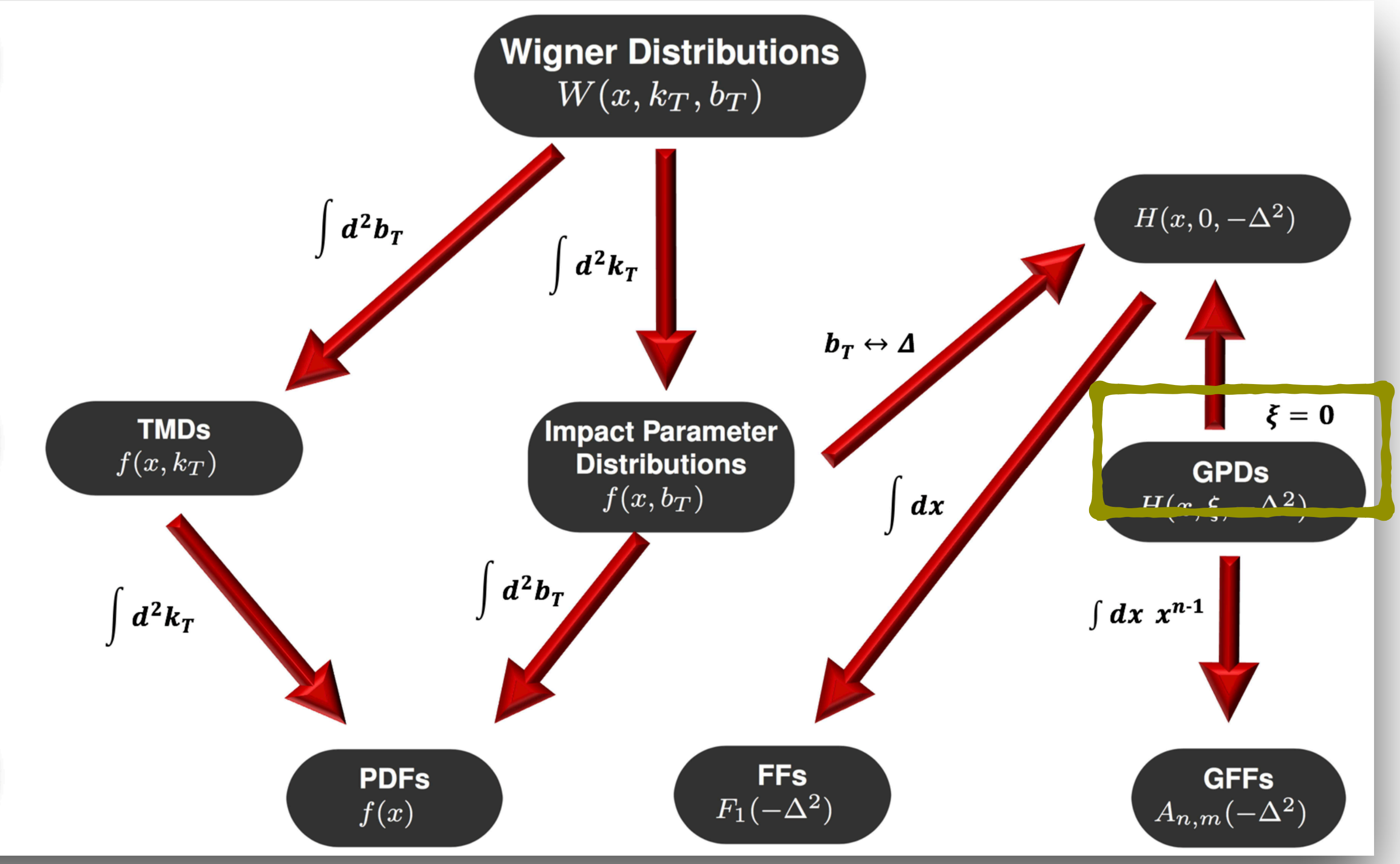
★ Provide correlation between transverse position & longitudinal momentum of the partons in the hadron

Nucleon Characterization

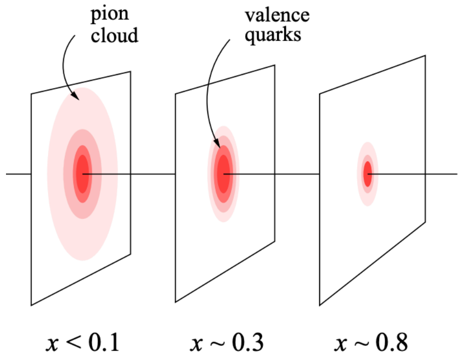
Wigner distributions

★ provide multi-dim images of the parton distributions in phase space

★ encode both TMDs and GPDs in a unified picture → crucial in mapping hadron tomography



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]



GPDs

★ "Parent" functions for PDFs, FFs, GFFs

★ Multi-dimensional objects

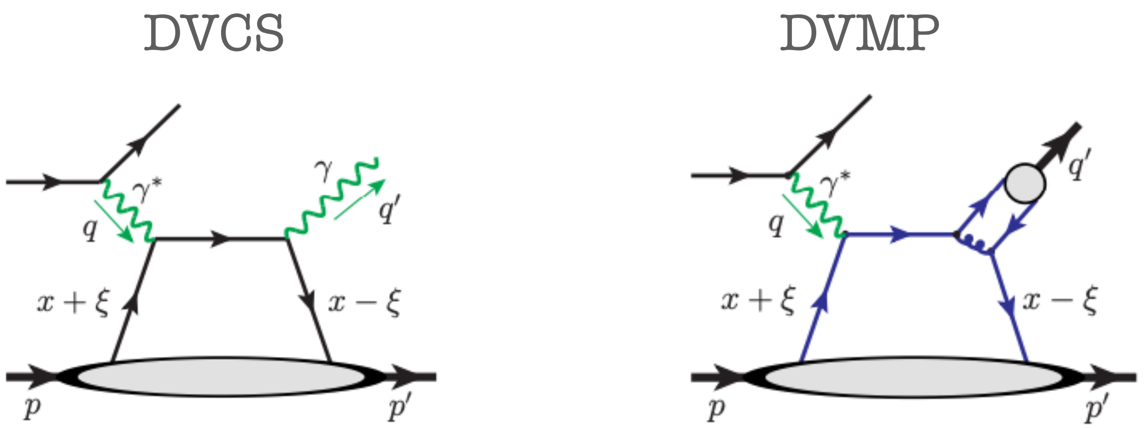
★ Provide correlation between transverse position & longitudinal momentum of the partons in the hadron

★ Information on the hadron's mechanical properties (OAM, pressure, etc.)

Experimental processes for GPDs

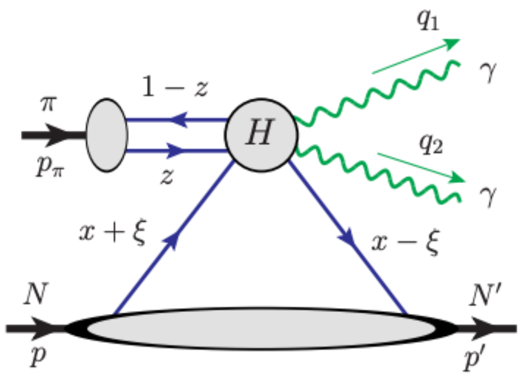
★ GPDs may be accessed via exclusive reactions (DVCS, DVMP)

[X.-D. Ji, PRD 55, 7114 (1997)]



★ exclusive pion-nucleon diffractive production of a γ pair of high $p_\perp$

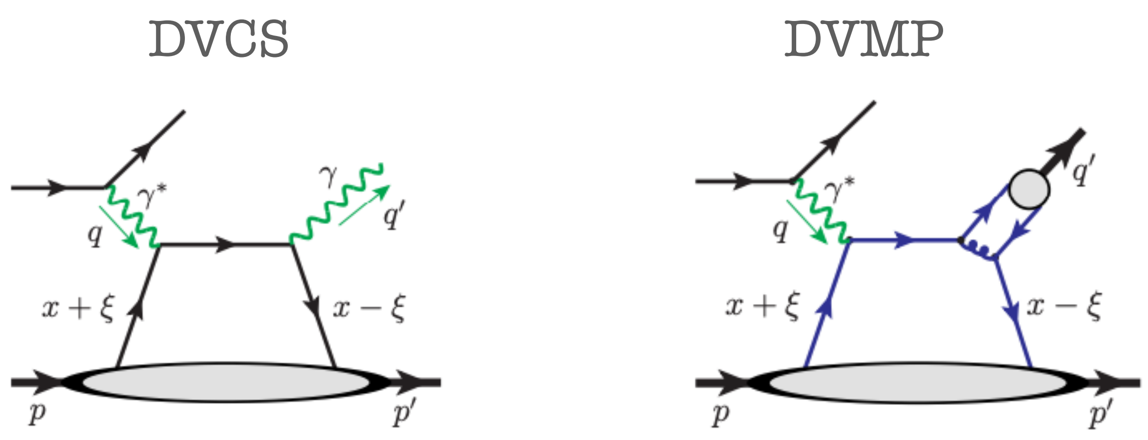
[J. Qiu et al, arXiv:2205.07846]



Experimental processes for GPDs

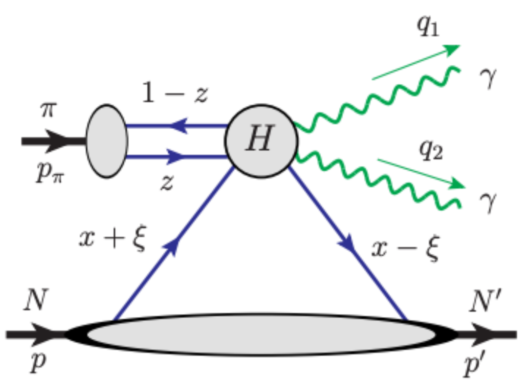
★ GPDs may be accessed via exclusive reactions (DVCS, DVMP)

[X.-D. Ji, PRD 55, 7114 (1997)]



★ exclusive pion-nucleon diffractive production of a γ pair of high $p_\perp$

[J. Qiu et al, arXiv:2205.07846]



★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

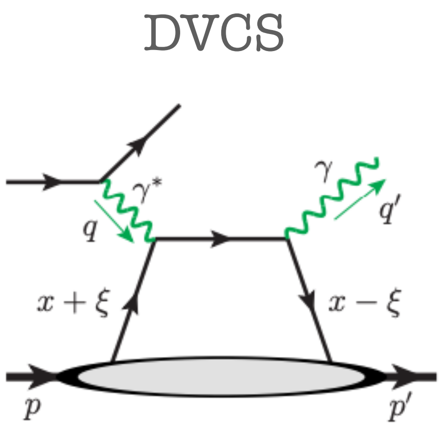
(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

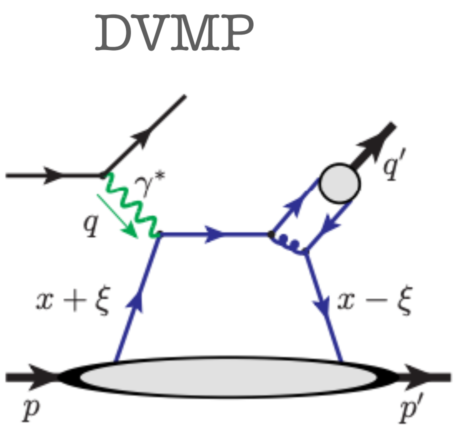
- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges …

Experimental processes for GPDs

★ GPDs may be accessed via exclusive reactions (DVCS, DVMP)

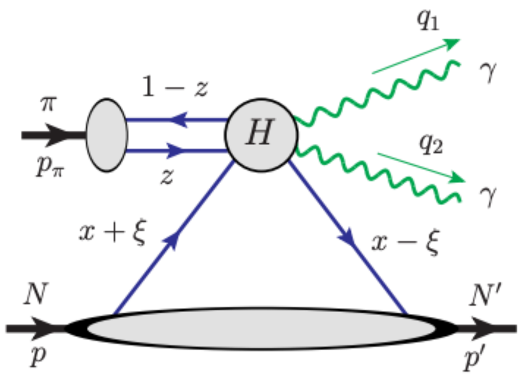
[X.-D. Ji, PRD 55, 7114 (1997)]





★ exclusive pion-nucleon diffractive production of a γ pair of high $p_\perp$

[J. Qiu et al, arXiv:2205.07846]



★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges …

Essential to complement the knowledge on GPD from lattice QCD

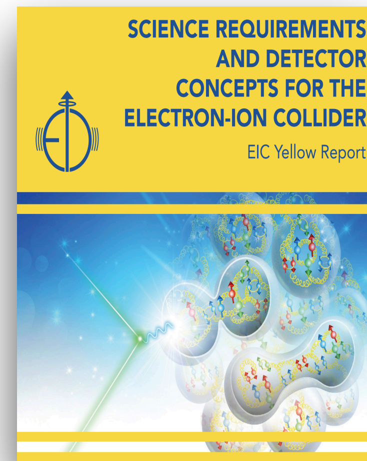
Hadron structure at core of nuclear physics

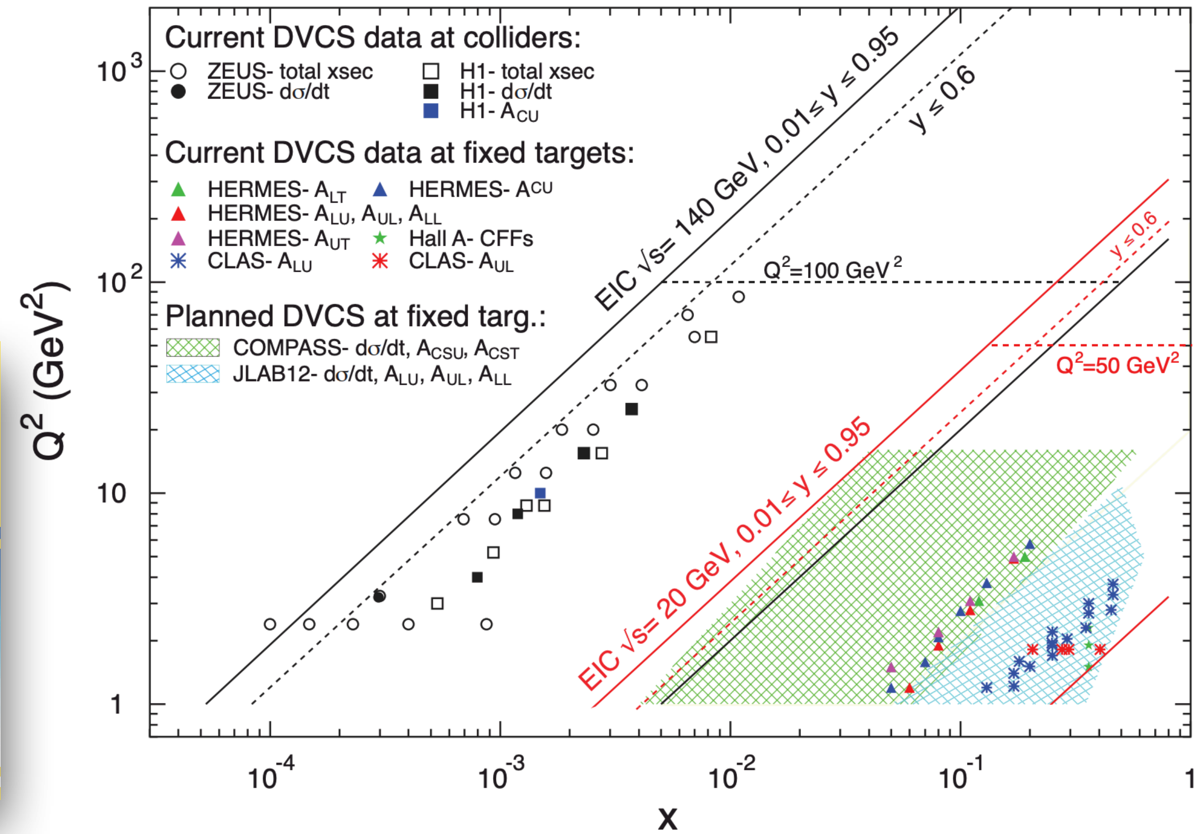
★ Tomographic imaging of proton has central role in the science program of EIC

GPDs, FFs, GFFs, TMDs, …

[R. Abdul Khalek et al.,

EIC Yellow Report 2021, arXiv:2103.05419]





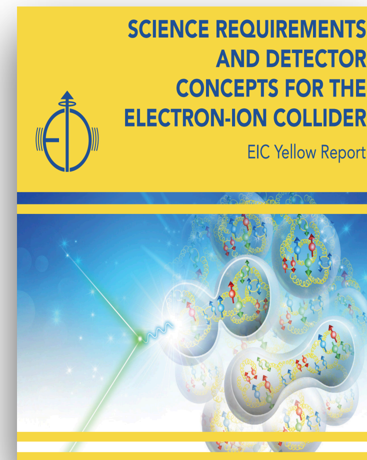
Hadron structure at core of nuclear physics

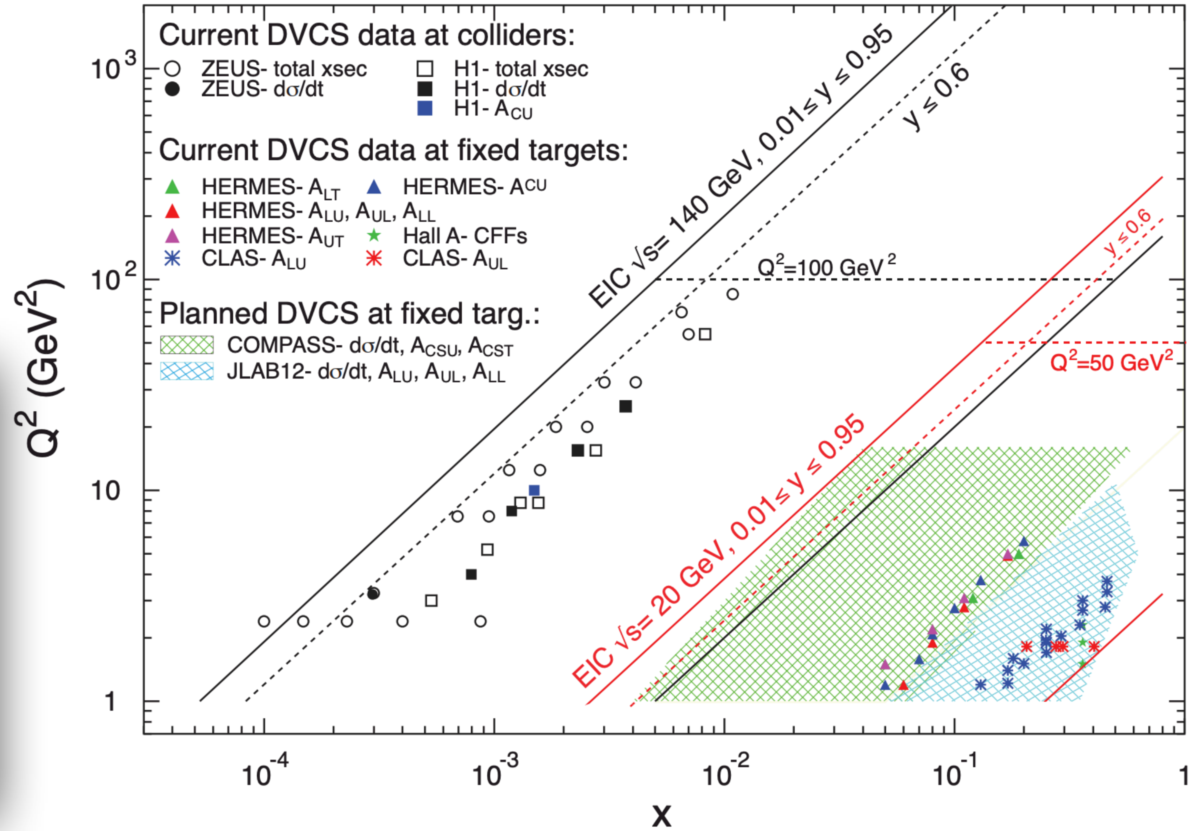
★ Tomographic imaging of proton has central role in the science program of EIC

GPDs, FFs, GFFs, TMDs, …

[R. Abdul Khalek et al.,

EIC Yellow Report 2021, arXiv:2103.05419]





U.S. DEPARTMENT OF ENERGY | Office of Science

QUARK-GLUON TOMOGRAPHY COLLABORATION

Award Number:
DE-SC0023646

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

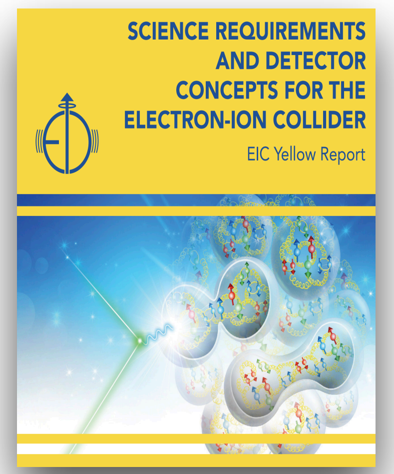
Hadron structure at core of nuclear physics

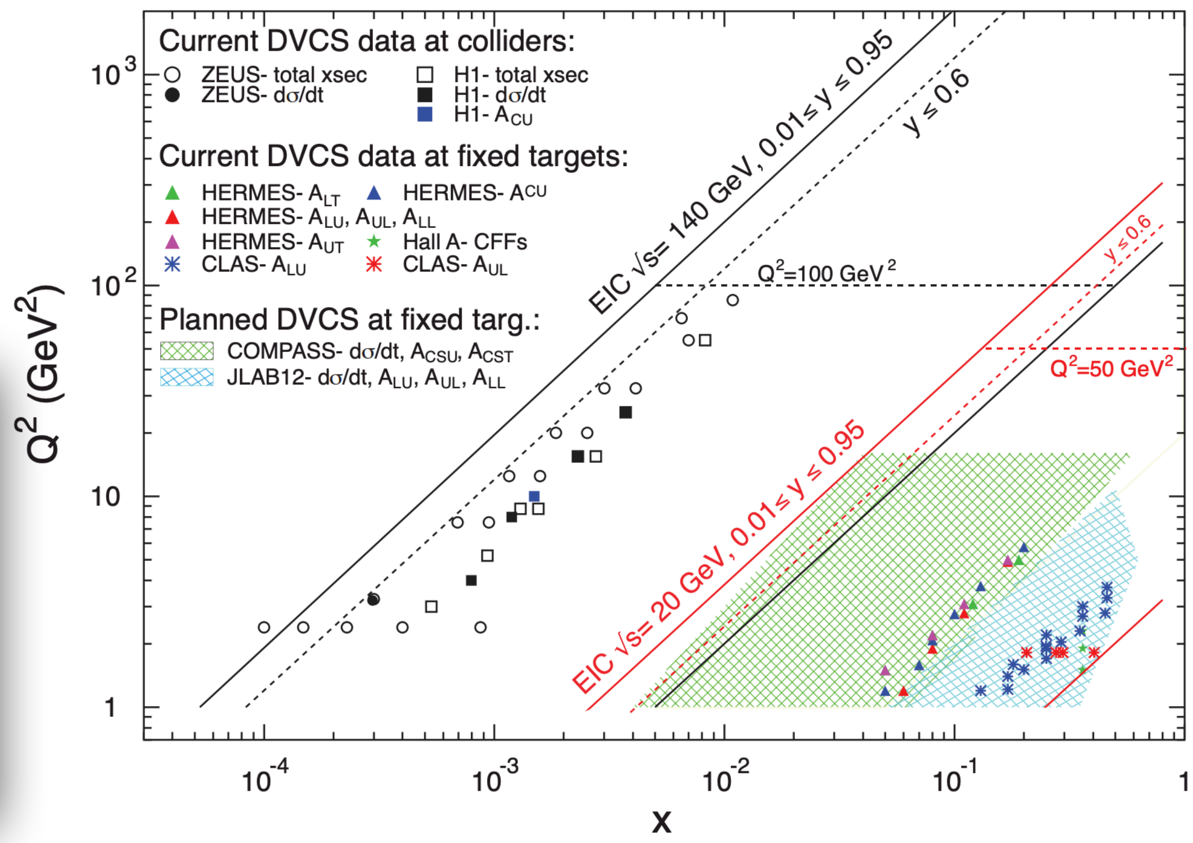
★ Tomographic imaging of proton has central role in the science program of EIC

GPDs, FFs, GFFs, TMDs, …

[R. Abdul Khalek et al.,

EIC Yellow Report 2021, arXiv:2103.05419]





U.S. DEPARTMENT OF ENERGY | Office of Science

QUARK-GLUON TOMOGRAPHY COLLABORATION

Award Number: DE-SC0023646

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

Advances of lattice QCD are timely

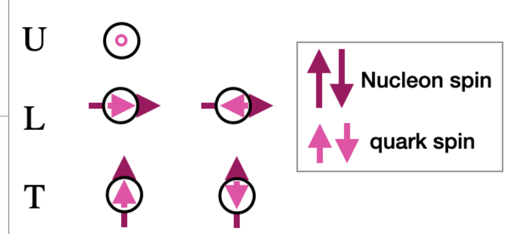
Twist-classification of PDFs, GPDs, TMDs

★ Twist: specifies the order in $1/Q$ at which the function enters factorization formula for a given observable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Nucleon \ Quark	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) Twist-3 $(f_i^{(1)})$

Nucleon \ $\mathcal{O}$	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

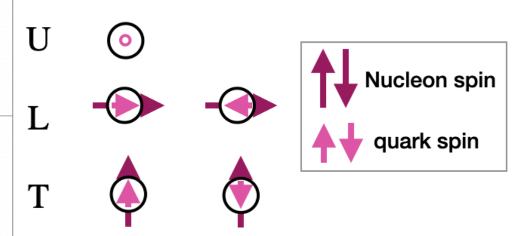
Twist-classification of PDFs, GPDs, TMDs

★ Twist: specifies the order in $1/Q$ at which the function enters factorization formula for a given observable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Nucleon \ Quark	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) **Twist-3** $(f_i^{(1)})$

Nucleon \ $\mathcal{O}$	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

★ Twist-2: probabilistic densities - a wealth of information exists (mostly on PDFs)

★ Twist-3: poorly known, but very important:

- as sizable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g. g_2)
- certain twist-3 PDFs are related to the TMDs
- physical interpretation (e.g. average force on partons inside hadron)

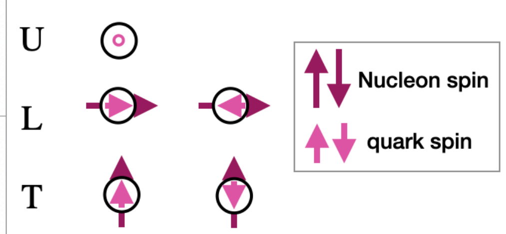
Twist-classification of PDFs, GPDs, TMDs

★ Twist: specifies the order in $1/Q$ at which the function enters factorization formula for a given observable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Nucleon \ Quark	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) **Twist-3** $(f_i^{(1)})$

Nucleon \ $\mathcal{O}$	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

★ Twist-2: probabilistic densities - a wealth of information exists (mostly on PDFs)

★ Twist-3: poorly known, but very important:
- as sizable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g. g_2)
- certain twist-3 PDFs are related to the TMDs
- physical interpretation (e.g. average force on partons inside hadron)

While twist-3 $f_i^{(1)}$ share some similarities with twist-2 $f_i^{(0)}$ in their extraction, there are several challenges both experimentally and theoretically

Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\underbrace{\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]}_{\text{local operators}}$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t)-i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\}+\frac{\Delta^{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Big\}$$

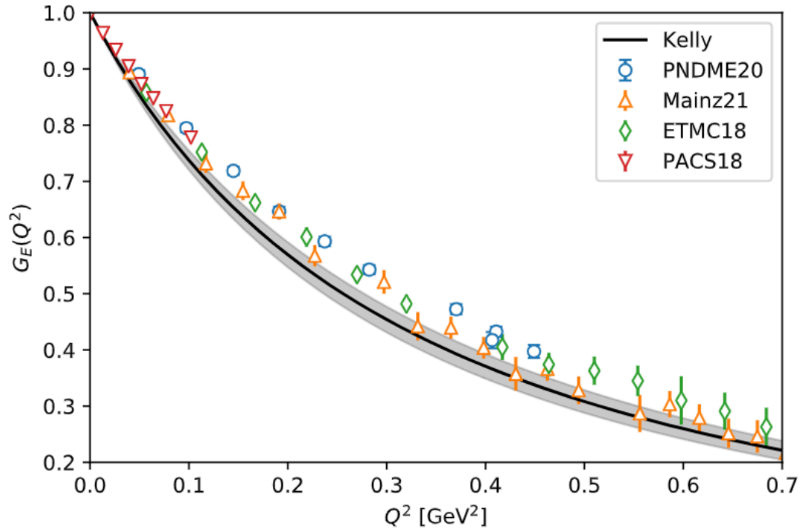
Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\dots z_{\alpha_n}\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\dots\overleftrightarrow{D}^{\alpha_n}q\right]$$

local operators

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\dots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\dots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\dots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\dots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^\mu\Delta^{\mu_1}\dots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Big\}$$



Wide -t range that comes at the cost of 1 (in the majority of cases)

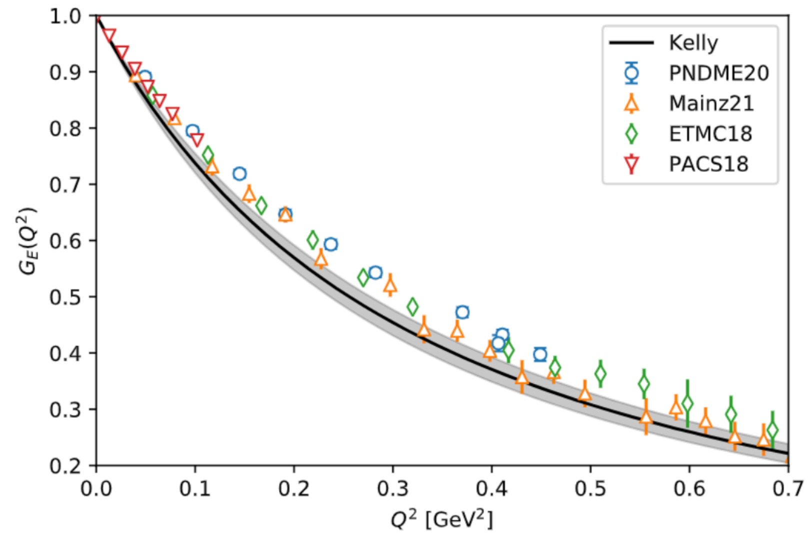
Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\underbrace{\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]}_{\text{local operators}}$$

local operators

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t)\;-i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^\mu\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Bigg\}$$



Wide -t range that comes at the cost of 1 (in the majority of cases)

★ **Matrix elements of non-local operators**
(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\underbrace{\mathcal{W}(z,0)}_{\text{Wilson line}}\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

GPDs

Through non-local matrix elements

of fast-moving hadrons

Access of PDFs/GPDs on a Euclidean Lattice

- ★ Matrix elements of momentum-boosted hadrons coupled to nonlocal (equal-time) operators
- ★ Connection to light-cone GPDs through LaMET [X. Ji, PRL 110 (2013) 262002], SDF [A. Radyushkin, PRD 96, 034025 (2017)]

$$\tilde{q}_{\Gamma}^{\mathrm{GPD}}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_{\mu}$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

Access of PDFs/GPDs on a Euclidean Lattice

★ Matrix elements of momentum-boosted hadrons coupled to nonlocal (equal-time) operators

★ Connection to light-cone GPDs through LaMET [X. Ji, PRL 110 (2013) 262002], SDF [A. Radyushkin, PRD 96, 034025 (2017)]

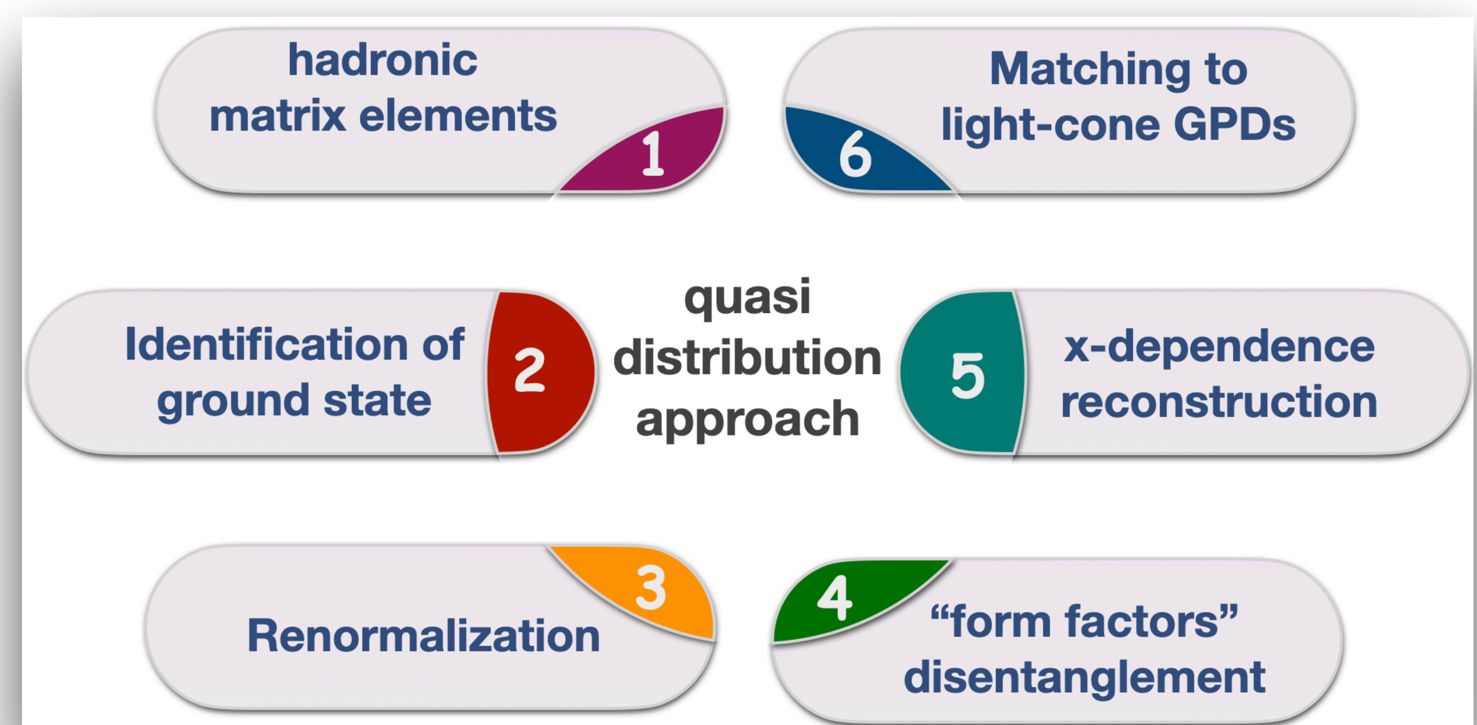
$$\tilde{q}^{\mathrm{GPD}}_{\Gamma}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i x P_3 z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Accessing -t dependence: Computationally intensive

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$



Access of PDFs/GPDs on a Euclidean Lattice

- ★ Matrix elements of momentum-boosted hadrons coupled to nonlocal (equal-time) operators
- ★ Connection to light-cone GPDs through LaMET [X. Ji, PRL 110 (2013) 262002], SDF [A. Radyushkin, PRD 96, 034025 (2017)]

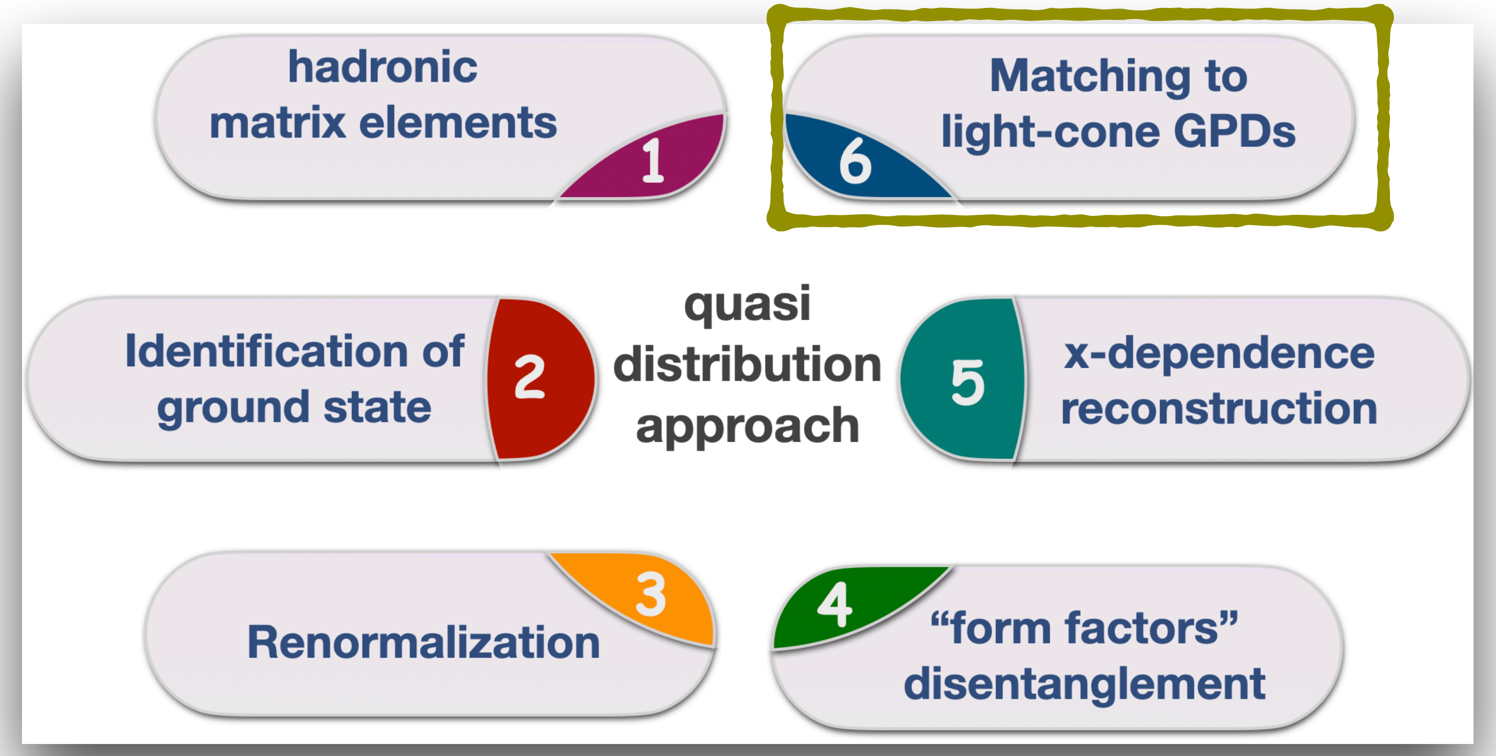
$$\tilde{q}^{\rm GPD}_{\Gamma}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Accessing -t dependence: Computationally intensive

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$



New parametrization of GPDs

PHYSICAL REVIEW D **106**, 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

PHYSICAL REVIEW D **109**, 034508 (2024)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[2,†] Jack Dodson,[2] Xiang Gao,[3] Andreas Metz,[2] Joshua Miller,[2,‡] Swagato Mukherjee,[4] Peter Petreczky,[4] Fernanda Steffens,[5] and Yong Zhao[3]

Theoretical setup

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q(0)}}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M} E_{\mathrm{Q(0)}}(x,\xi,t;P^3)\right]u(p,\lambda)$$

Theoretical setup

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q}(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

Theoretical setup

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x, \Delta; \lambda, \lambda'; P^3) = \frac{1}{2P^0}\bar{u}(p', \lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(x, \xi, t; P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M} E_{\mathrm{Q}(0)}(x, \xi, t; P^3)\right]u(p, \lambda)$$

★ Lorentz-invariant parametrization

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p', \lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu MA_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z} MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p, \lambda)$$

Goals

★ Extraction of standard GPDs using A_i obtained from any frame

★ quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone

Theoretical setup

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{Q(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{Q(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

★ Lorentz-invariant parametrization

$$F^\mu_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu MA_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z}MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Goals

★ Extraction of standard GPDs using A_i obtained from any frame

★ quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone

➡ **Proof-of-concept calculation ($\xi = 0$):**

- **symmetric frame:** $\vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}$, $\vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2}$ $\quad -t^s = \vec{Q}^2 = 0.69\,GeV^2$
- **asymmetric frame:** $\vec{p}^{\,a}_f = \vec{P}$, $\vec{p}^{\,a}_i = \vec{P} - \vec{Q}$ $\quad t^a = -\vec{Q}^2 + (E_f - E_i)^2 = 0.65\,GeV^2$

Theoretical setup

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q}(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

★ Lorentz-invariant parametrization

$$F^\mu_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu MA_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z}MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Goals

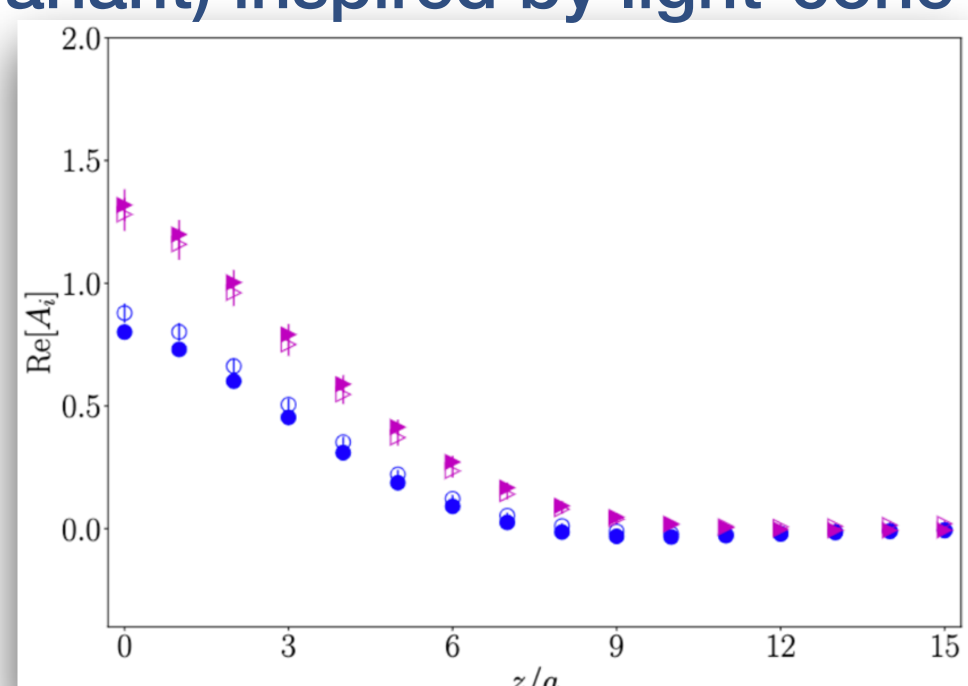
★ Extraction of standard GPDs using A_i obtained from any frame

★ quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone

➡ **Proof-of-concept calculation ($\xi = 0$):**

- **symmetric frame:** $\vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}, \quad \vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2}$

- **asymmetric frame:** $\vec{p}^{\,a}_f = \vec{P}, \quad \vec{p}^{\,a}_i = \vec{P} - \vec{Q}$



eV^2

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

Zero-skewness calculation

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

Zero-skewness calculation

Symmetric frame: each momentum requires separate computational resources

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

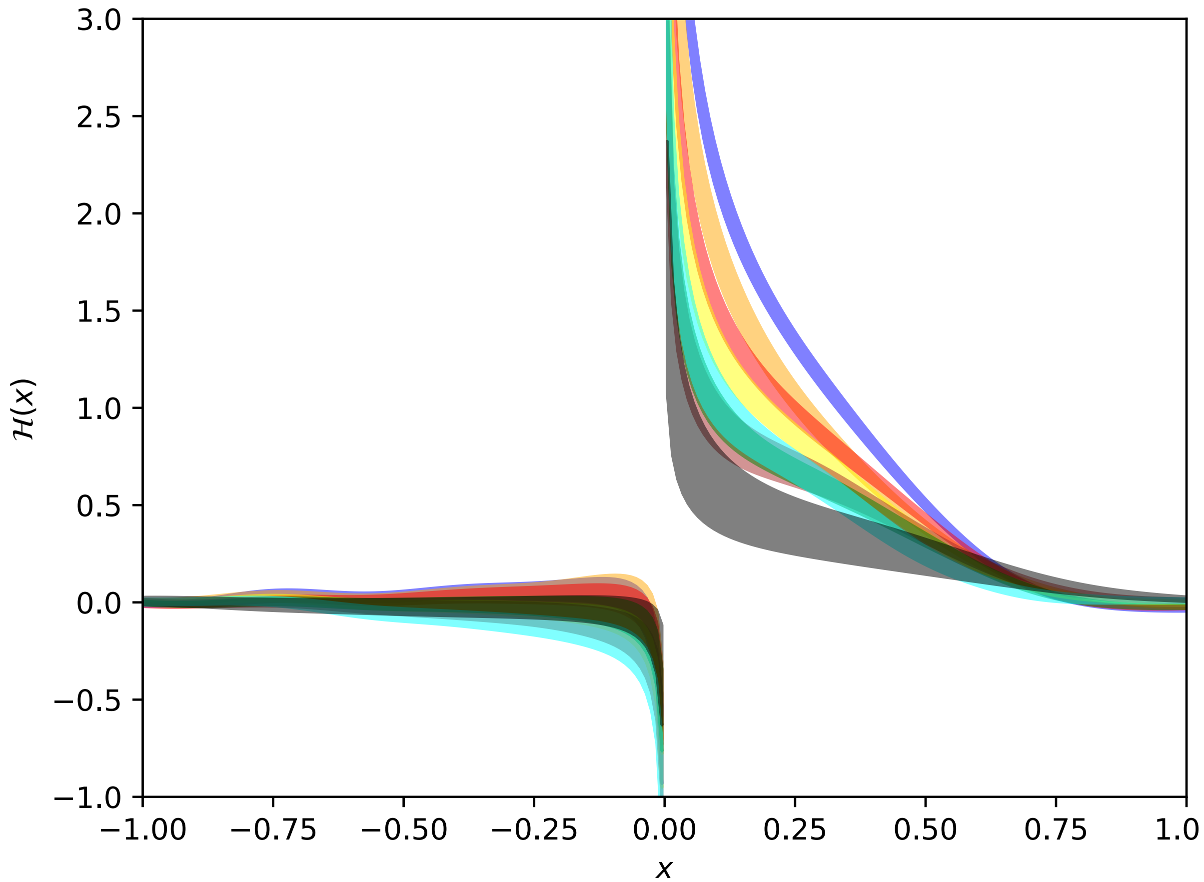
frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

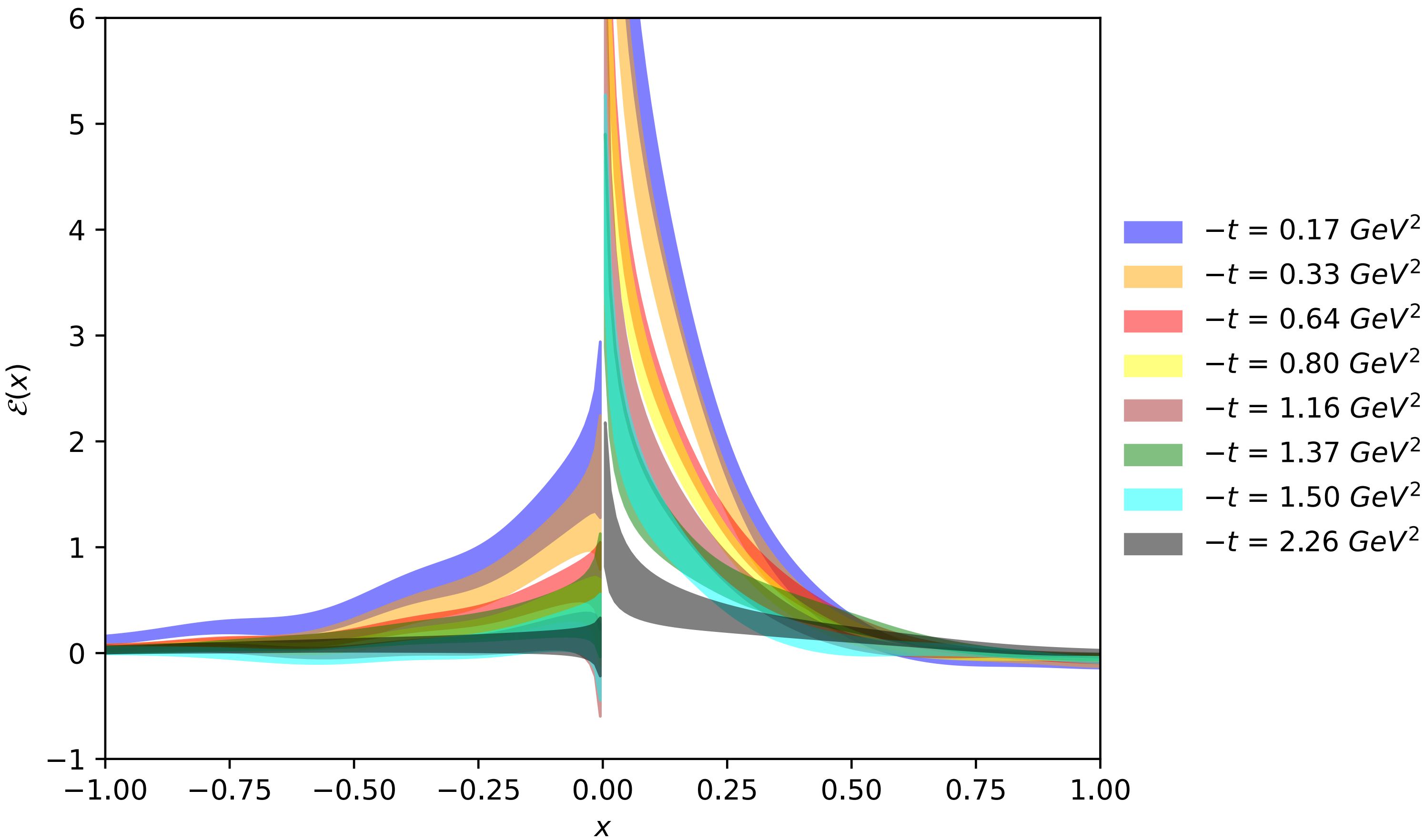
Zero-skewness calculation

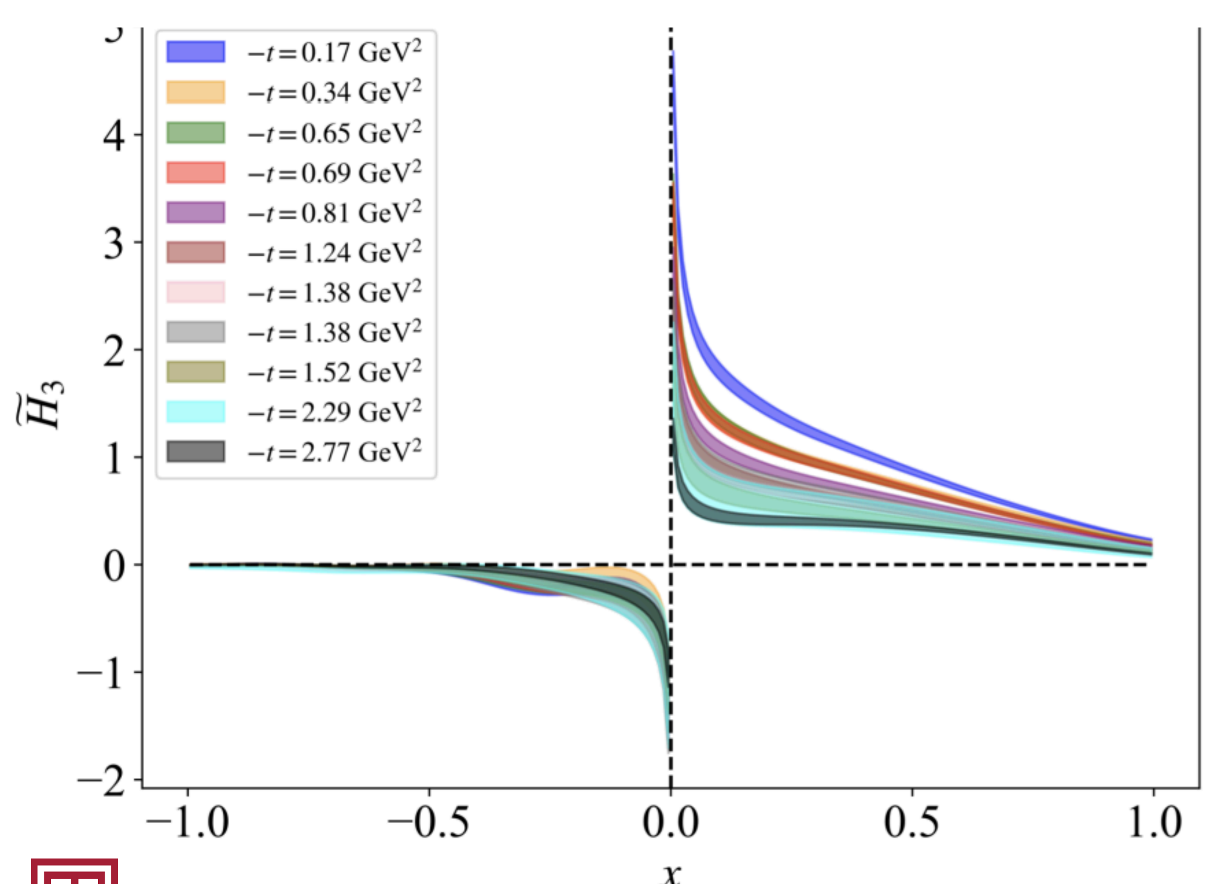
! Symmetric frame: each momentum requires separate computational resources

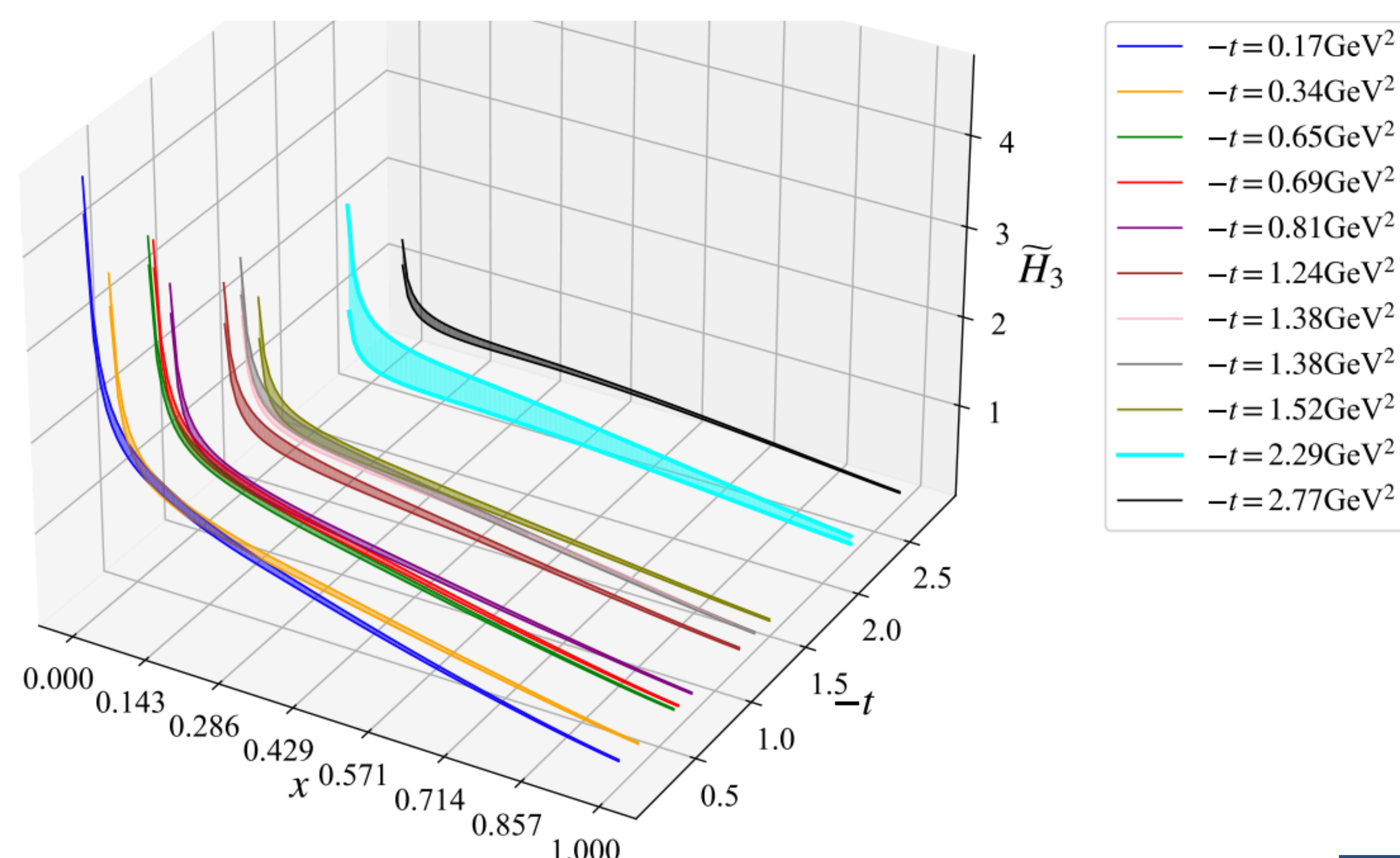
! Asymmetric frame: momenta grouped in 2 sets of runs [(Q,0,0), (Qx,Qy,0)]

Light-cone GPDs







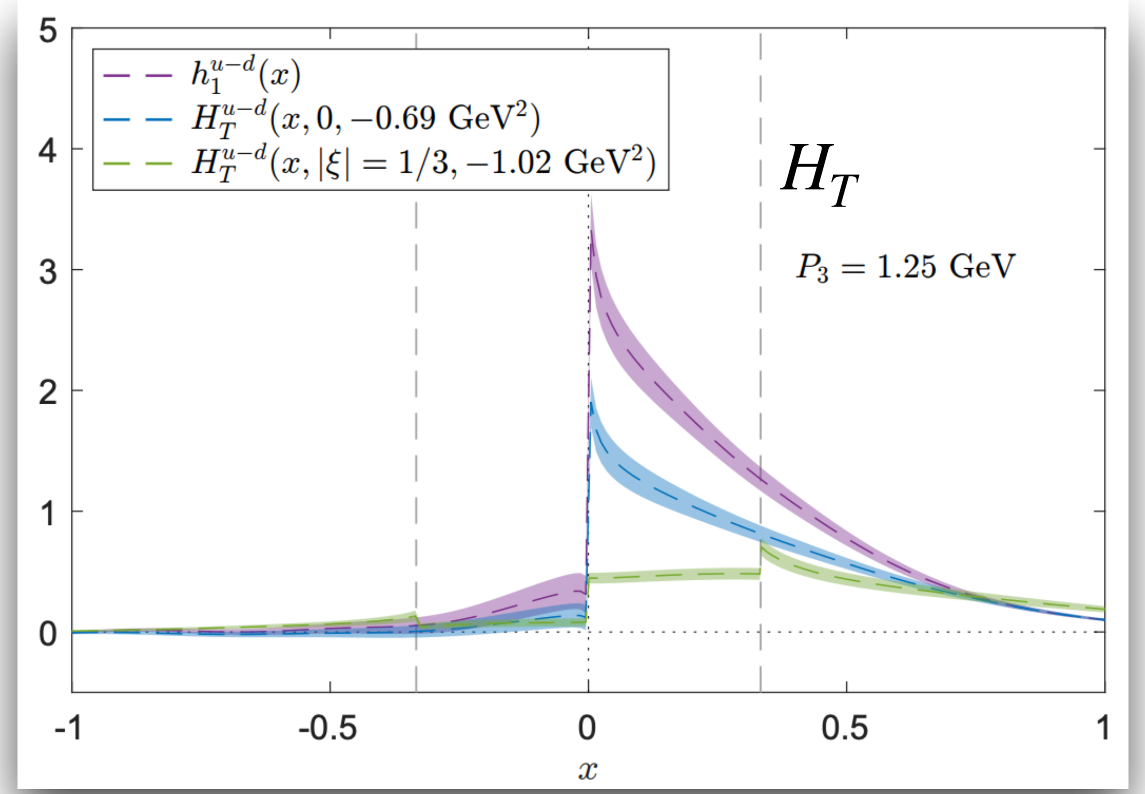


Transversity GPDs

Standard parametrization

$$h_T^j(\Gamma_\nu, z, P_f, P_i) = \langle\langle \sigma^{3j} \rangle\rangle F_{H_T}(z,\xi,t,P_3) + \frac{i}{2m}\langle\langle \gamma^3\Delta_j - \gamma^j\Delta_3 \rangle\rangle F_{E_T}(z,\xi,t,P_3)$$
$$+ \frac{P_3\Delta_j - P_j\Delta_3}{m^2}\langle\langle \hat{1} \rangle\rangle F_{\widetilde{H}_T}(z,\xi,t,P_3) + \frac{1}{m}\langle\langle \gamma^3 P_j - \gamma^j P_3 \rangle\rangle F_{\widetilde{E}_T}(z,\xi,t,P_3)$$

[C. Alexandrou et al., PRD 105, 034501 (2022)]



Symmetric frame

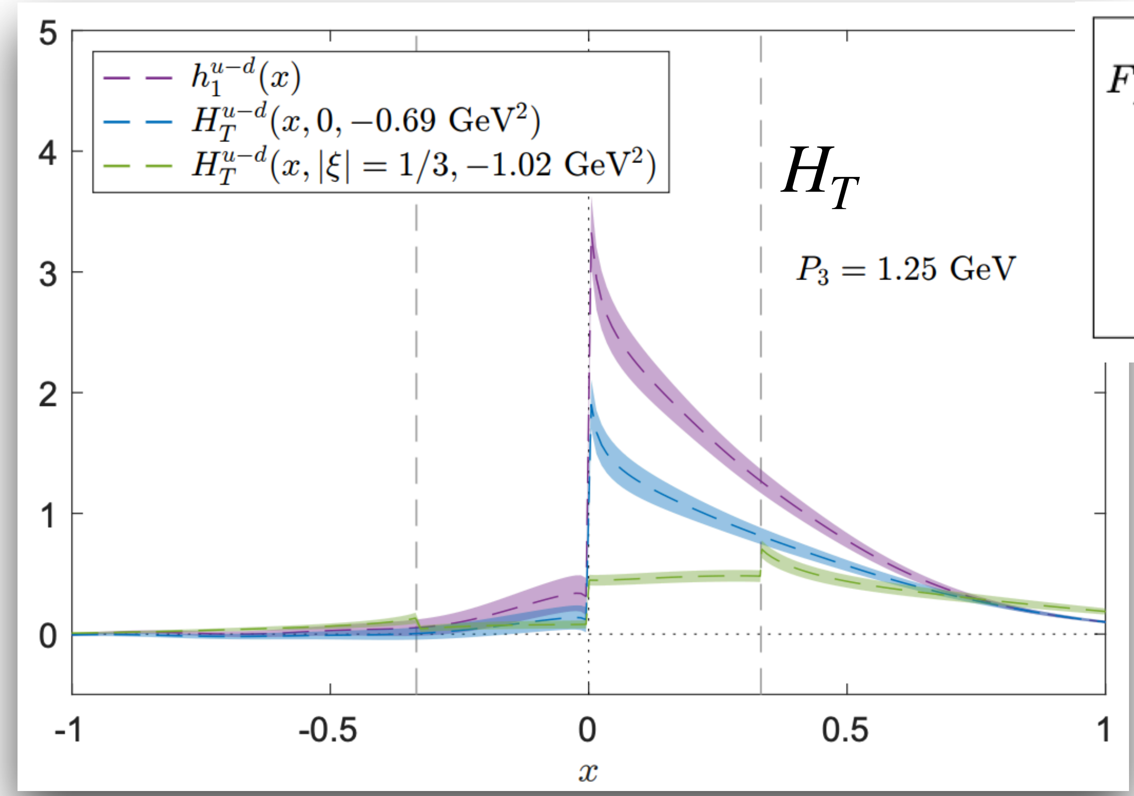
Transversity GPDs

On-going work

Standard parametrization

$$h^j_T(\Gamma_\nu, z, P_f, P_i) = \langle\langle \sigma^{3j} \rangle\rangle F_{H_T}(z,\xi,t,P_3) + \frac{i}{2m}\langle\langle \gamma^3\Delta_j - \gamma^j\Delta_3\rangle\rangle F_{E_T}(z,\xi,t,P_3) + \frac{P_3\Delta_j - P_j\Delta_3}{m^2}\langle\langle \hat{1}\rangle\rangle F_{\widetilde{H}_T}(z,\xi,t,P_3) + \frac{1}{m}\langle\langle \gamma^3 P_j - \gamma^j P_3\rangle\rangle F_{\widetilde{E}_T}(z,\xi,t,P_3)$$

[C. Alexandrou et al., PRD 105, 034501 (2022)]

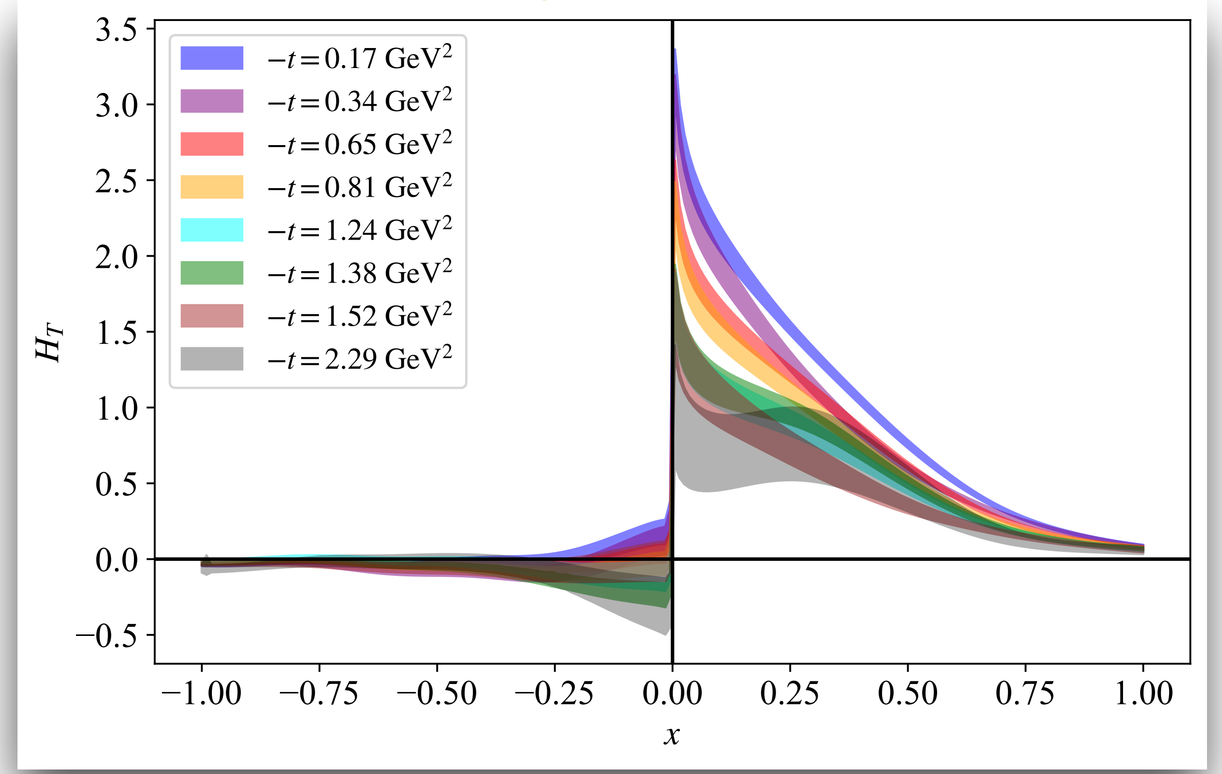


Symmetric frame

Lorentz covariant parametrization

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}\gamma_5 A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + M z^{\nu]} A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 + M \not{z}\gamma_5\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10} + i\epsilon^{\mu\nu P z}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

Asymmetric frame



Twist-3 GPDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

PHYSICAL REVIEW D **108**, 054501 (2023)

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[1] Jack Dodson,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[4]

+ Josh Miller (Temple graduate student)

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$\begin{aligned} F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[& P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3) \\ & + \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3) \\ & + \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda) \end{aligned}$$

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$\begin{aligned}F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[&P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)\\ &+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)\\ &+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)\end{aligned}$$

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar\psi(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$\begin{aligned}F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar u(p_f,\lambda')\Big[&P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde H}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde E}(x,\xi,t;P^3)\\ &+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde E+\widetilde G_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde H+\widetilde G_2}(x,\xi,t;P^3)\\ &+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde G_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde G_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)\end{aligned}$$

★ Nf=2+1+1 twisted mass fermions with a clover term

[ETMC, Phys. Rev. D 104, 074515 (2021)]

Name	β	N_f	$L^3\times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u,d,s,c	$32^3\times 64$	0.093	260 MeV	4

P_3 [GeV]	$\vec{q}\,[\frac{2\pi}{L}]$	$-t$ [GeV^2]	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm total}$
±0.83	(0,0,0)	0	2	194	8	3104
±1.25	(0,0,0)	0	2	731	16	23392
±1.67	(0,0,0)	0	2	1644	64	210432
±0.83	(±2,0,0)	0.69	8	67	8	4288
±1.25	(±2,0,0)	0.69	8	249	8	15936
±1.67	(±2,0,0)	0.69	8	294	32	75264
±1.25	(±2,±2,0)	1.38	16	224	8	28672
±1.25	(±4,0,0)	2.76	8	329	32	84224

Symmetric frame

Consistency Checks

★ Sum Rules (generalization of Burkhardt-Cottingham)

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

$$\int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t) = G_A(t)\,, \quad \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

★ Sum Rules (generalization of Efremov-Leader-Teryaev)

[A. Efremov, O. Teryaev , E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_3(x,0,t) = \frac{\xi}{4} G_E \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,0,t) = \frac{1}{4} G_E(t)$$

G_E : electric FF

Consistency Checks

★ Sum Rules (generalization of Burkhardt-Cottingham)

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

$$\int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t) = G_A(t)\,, \quad \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

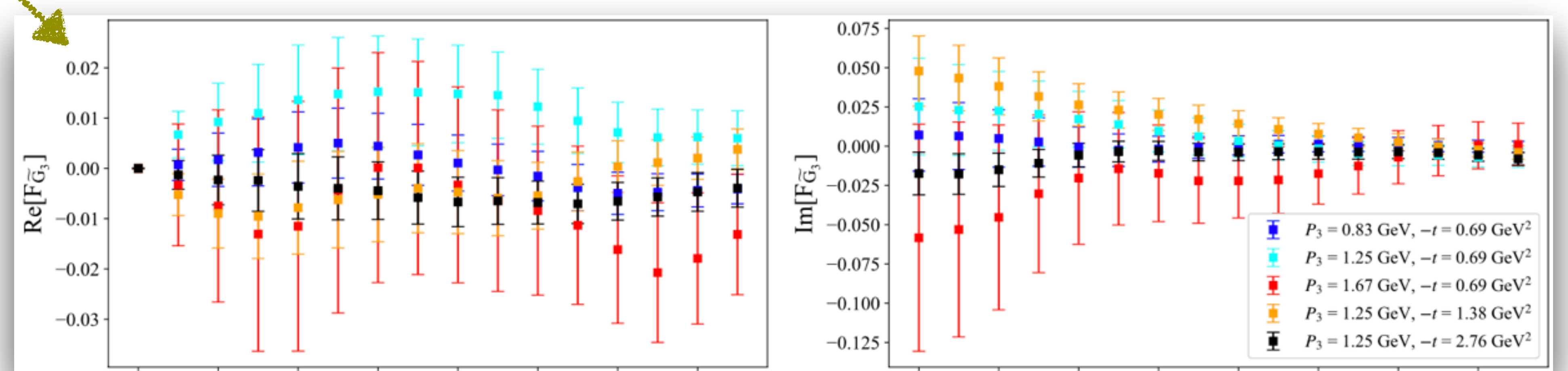
★ Sum Rules (generalization of Efremov-Leader-Teryaev)

[A. Efremov, O. Teryaev , E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_3(x,0,t) = \frac{\xi}{4} G_E \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,0,t) = \frac{1}{4} G_E(t)$$

G_E : electric FF

Indeed, numerically found to be zero within uncertainties at ξ=0



Reconstruction of x-dependence & matching

★ quasi-GPDs transformed to momentum space using Backus Gilbert
[G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)]

★ Matching formalism to 1 loop accuracy level

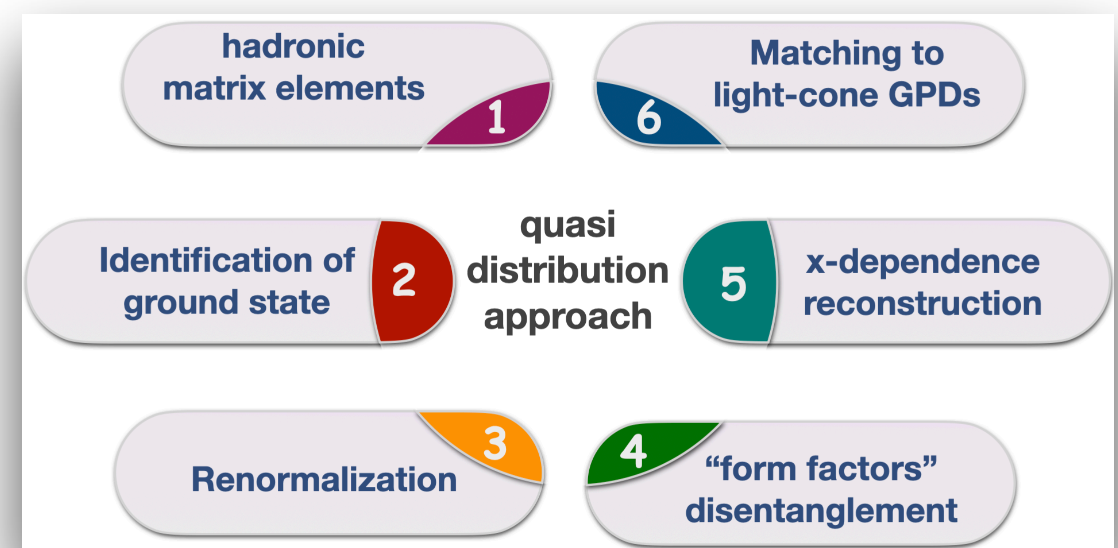
$$F_X^{\mathrm{M\overline{MS}}}(x,t,P_3,\mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C_{\gamma_j\gamma_5}^{\mathrm{M\overline{MS}},\overline{\mathrm{MS}}}\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) G_X^{\overline{\mathrm{MS}}}(y,t,\mu) \; + \mathcal{O}\left(\frac{m^2}{P_3^2}, \frac{t}{P_3^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{x^2P_3^2}\right)$$

★ Operator dependent kernel

PHYSICAL REVIEW D **102,** 034005 (2020)

One-loop matching for the twist-3 parton distribution $g_T(x)$

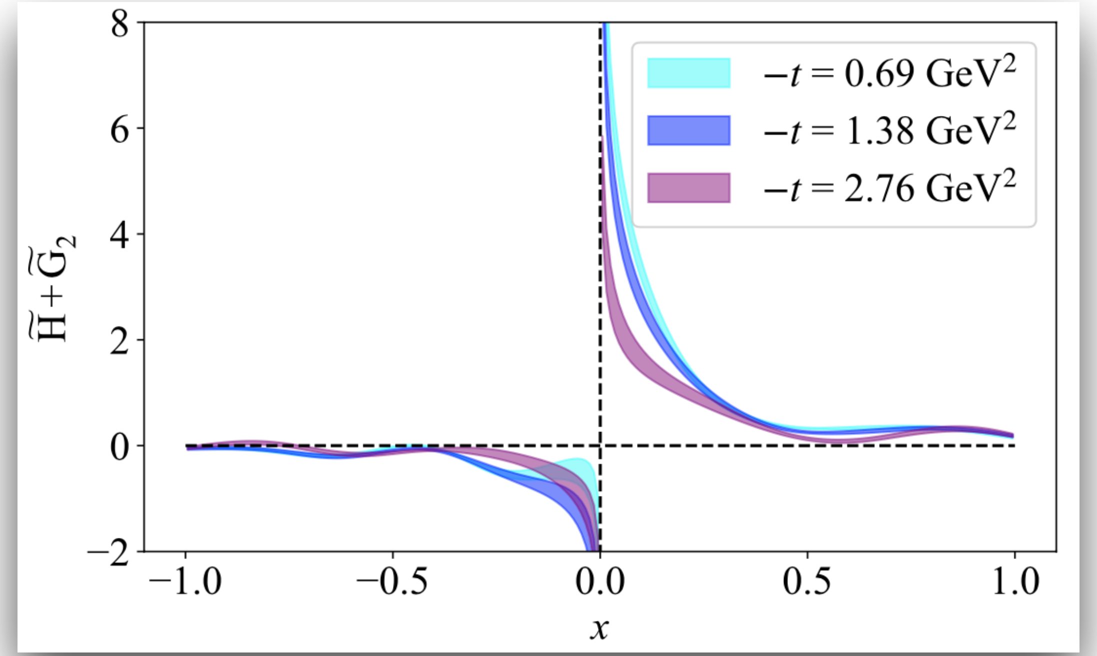
Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1]
Aurora Scapellato,[2] and Fernanda Steffens[3]



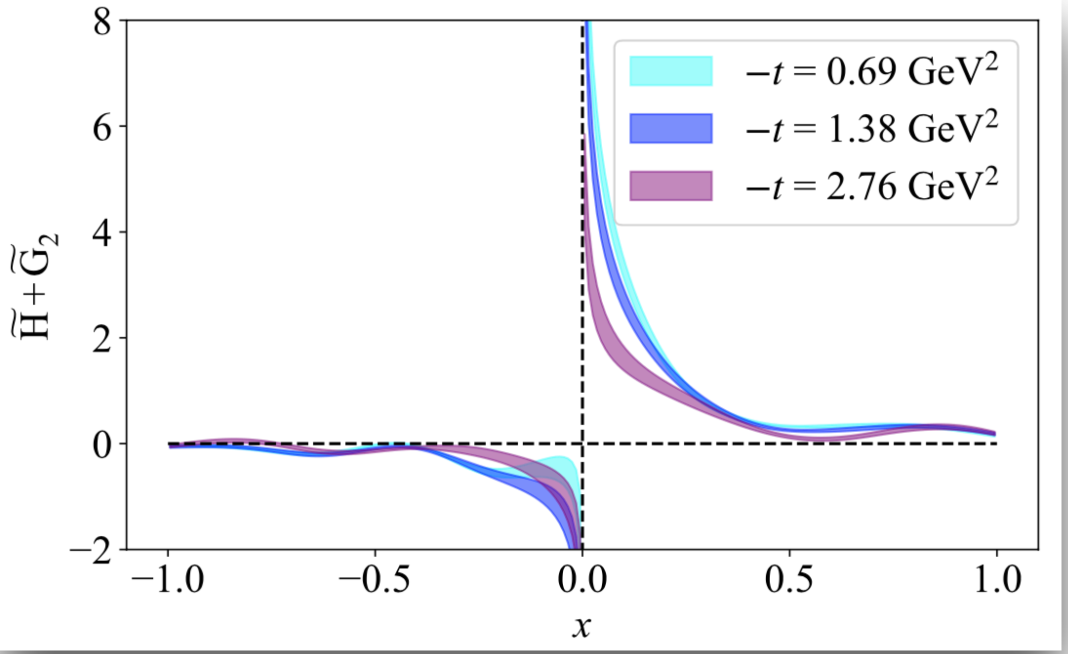
$$C_{\mathrm{M\overline{MS}}}^{(1)}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} 0 \\ \delta(\xi) \\ 0 \end{cases} + \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ & \xi > 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)(xP_3)^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0 < \xi < 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0\,, \end{cases}$$

★ Matching does not consider mixing with q-g-q correlators
[V. Braun et al., JHEP 05 (2021) 086]

Lattice Results - light-cone GPDs

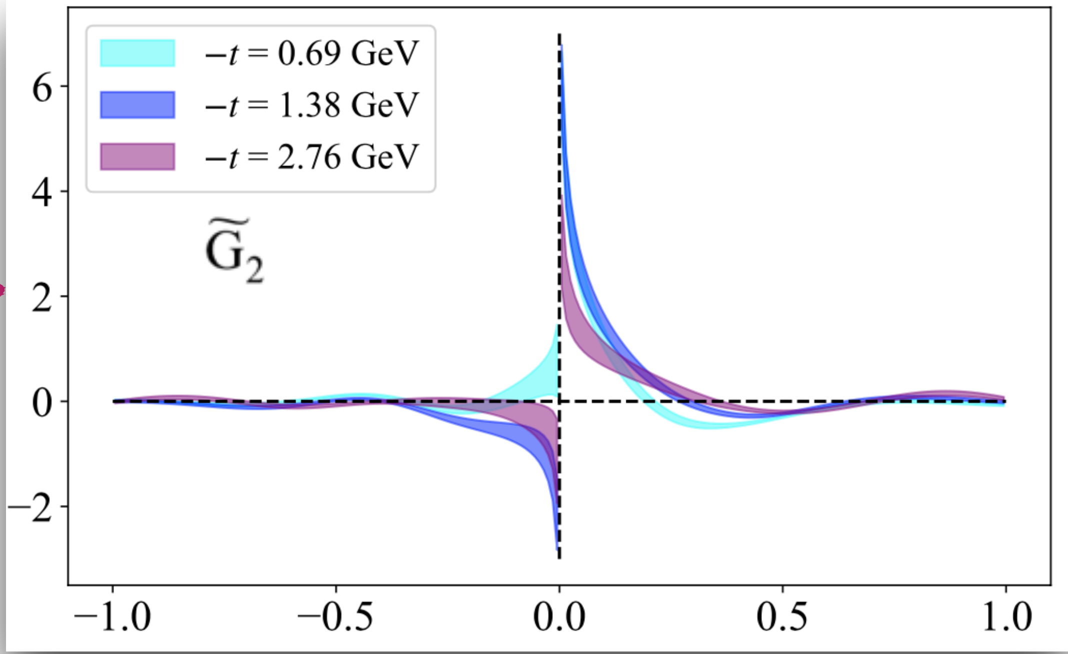


Lattice Results - light-cone GPDs

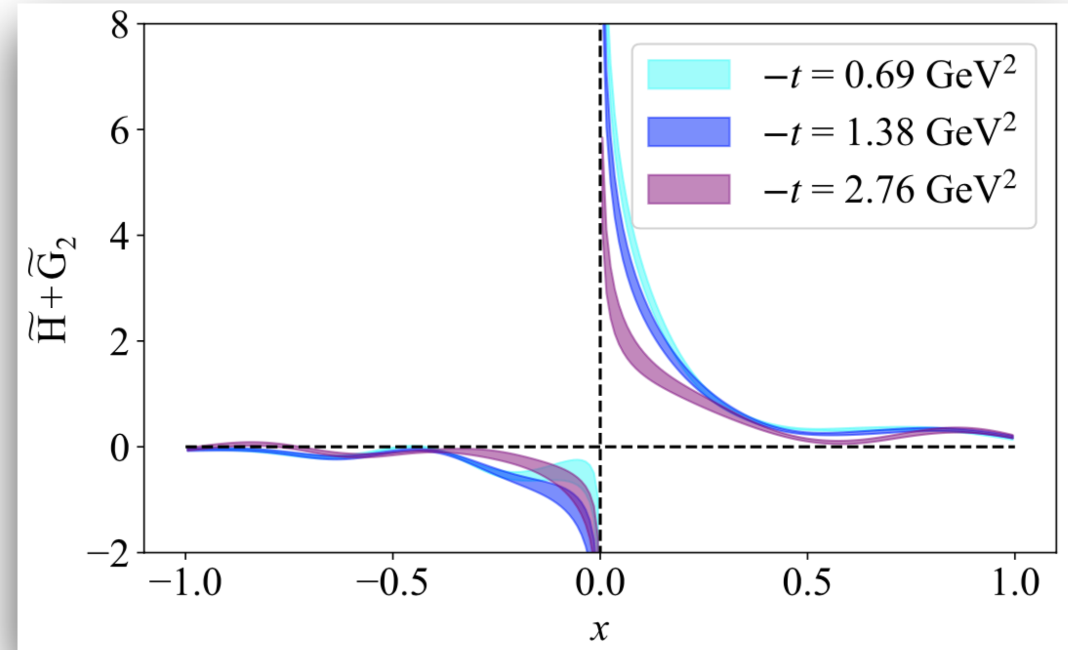


Isolating $\widetilde{G}_2$

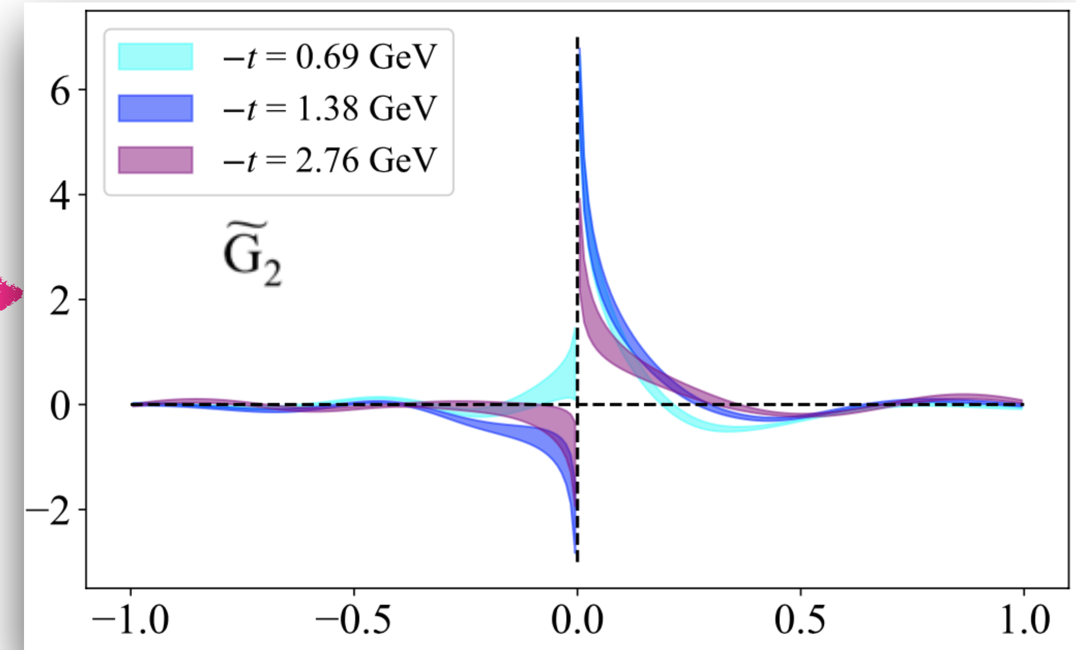
using $\widetilde{H}$

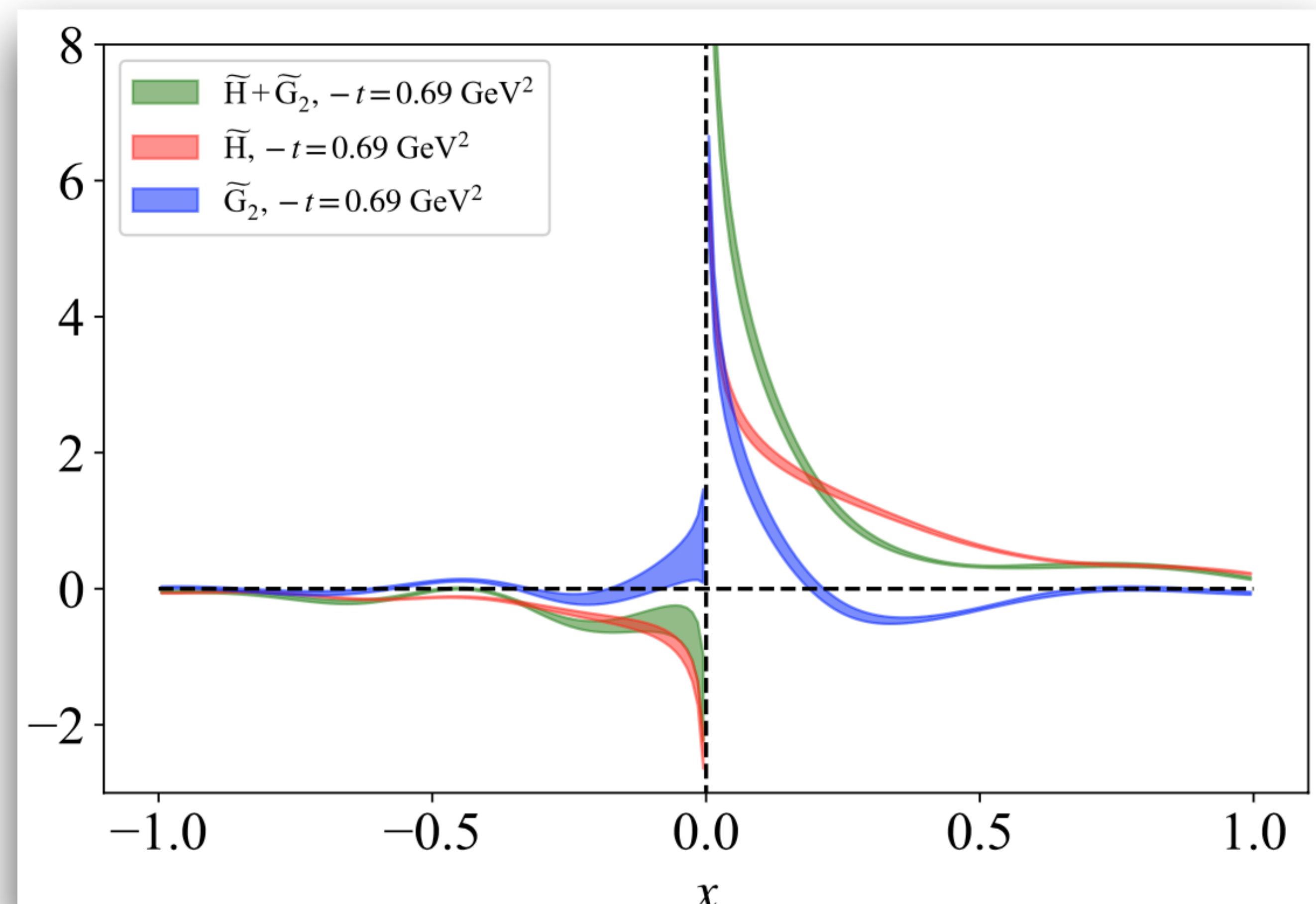


Lattice Results - light-cone GPDs



Isolating $\widetilde{G}_2$ using $\widetilde{H}$





Negative areas in $\widetilde{G}_2$ theoretically anticipated:

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

Lattice Results - light-cone GPDs

★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

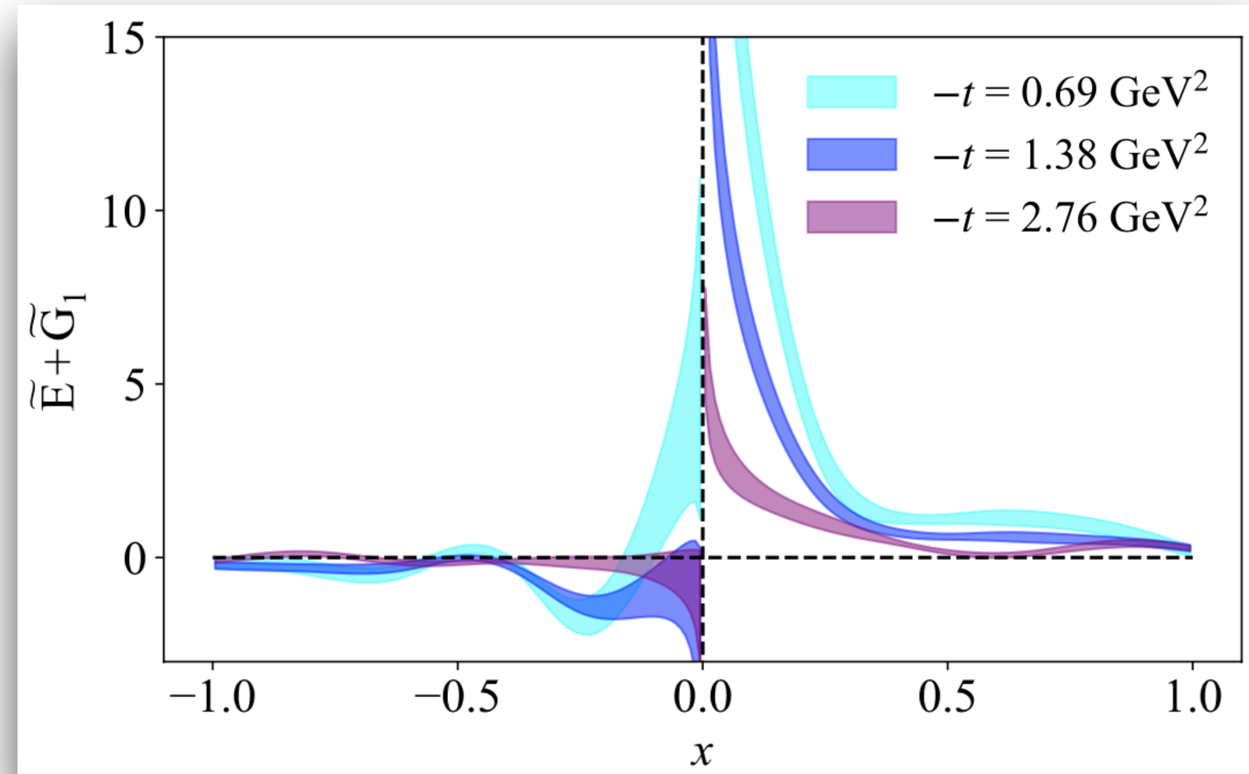
$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

Lattice Results - light-cone GPDs

★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

$$P^\mu \frac{\Delta^3\gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

★ Glimpse into $\widetilde{E}$-GPD through twist-3 :



★ Sizable contributions as expected

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

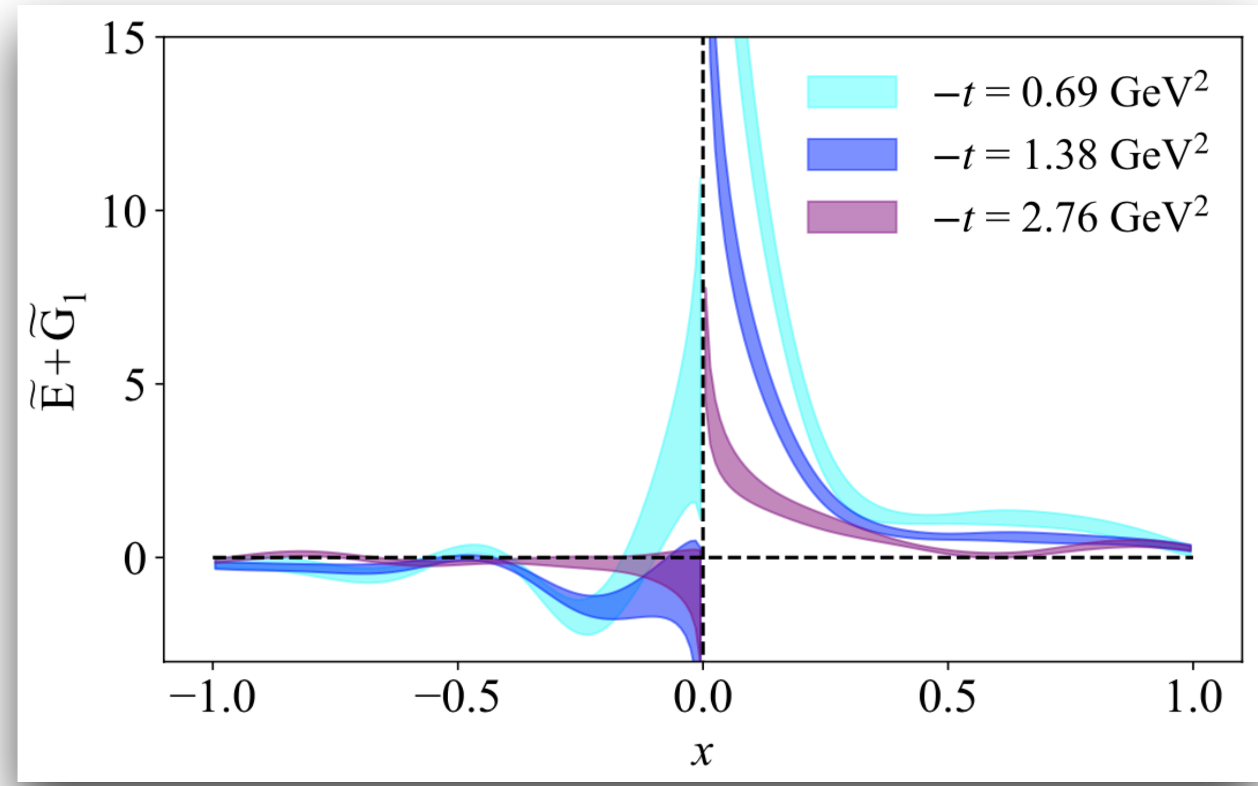
$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

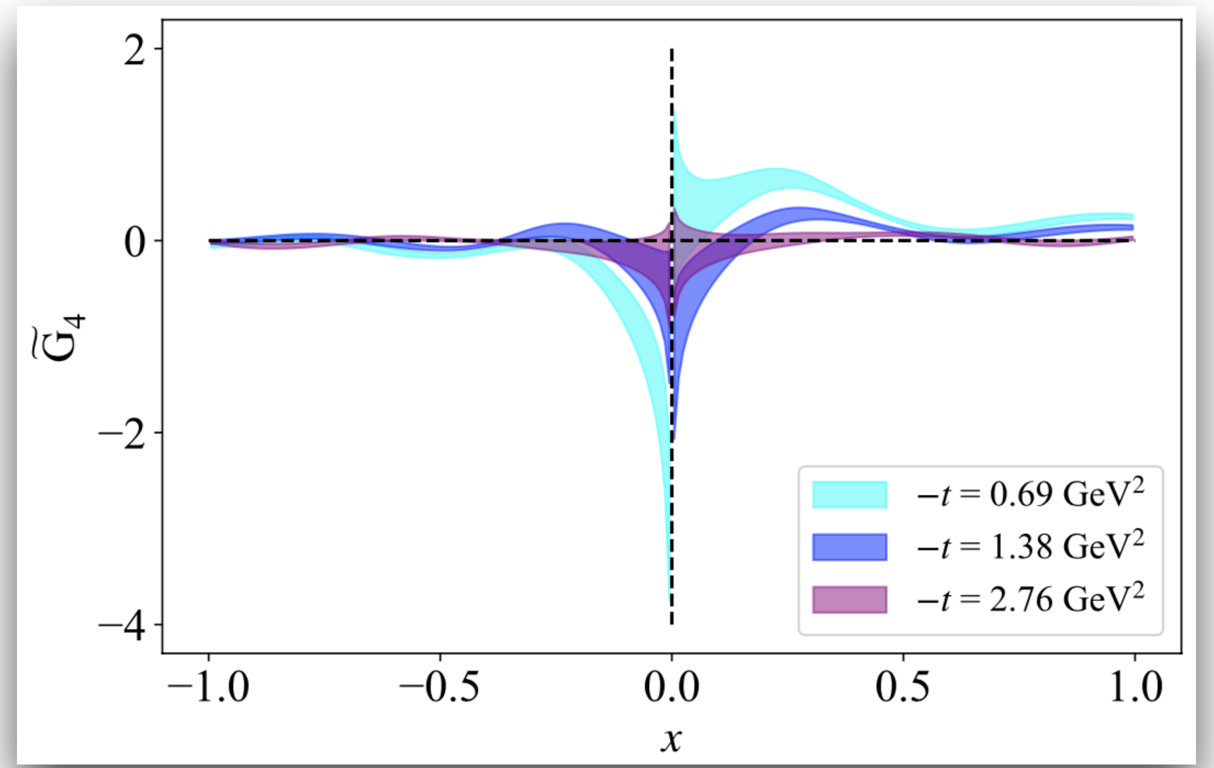
Lattice Results - light-cone GPDs

★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

★ Glimpse into $\widetilde{E}$-GPD through twist-3 :





★ Sizable contributions as expected

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

★ $\widetilde{G}_4$ very small; no theoretical argument to be zero

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E$$

Extension of calculation

★ Alternative kinematic setup can be utilized

$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2 (\Delta_\perp)^2}{P_3} + A_2$$

$$F_{\widetilde{E}+\widetilde{G}_1} = \frac{2 z_3 P_0^2}{P_3} + 2A_5$$

$$F_{\widetilde{G}_3} = \frac{1}{2m^2}\left(z_3 P_0^2 \Delta_3 - z_3 P_3 P_0 \Delta_0\right) A_1 - z_3 P_3 A_8$$

$$F_{\widetilde{G}_3} = \frac{1}{m^2}\left(z_3 P_0 P_3^2 - z_3 P_0^3\right) A_1$$

Extension of calculation

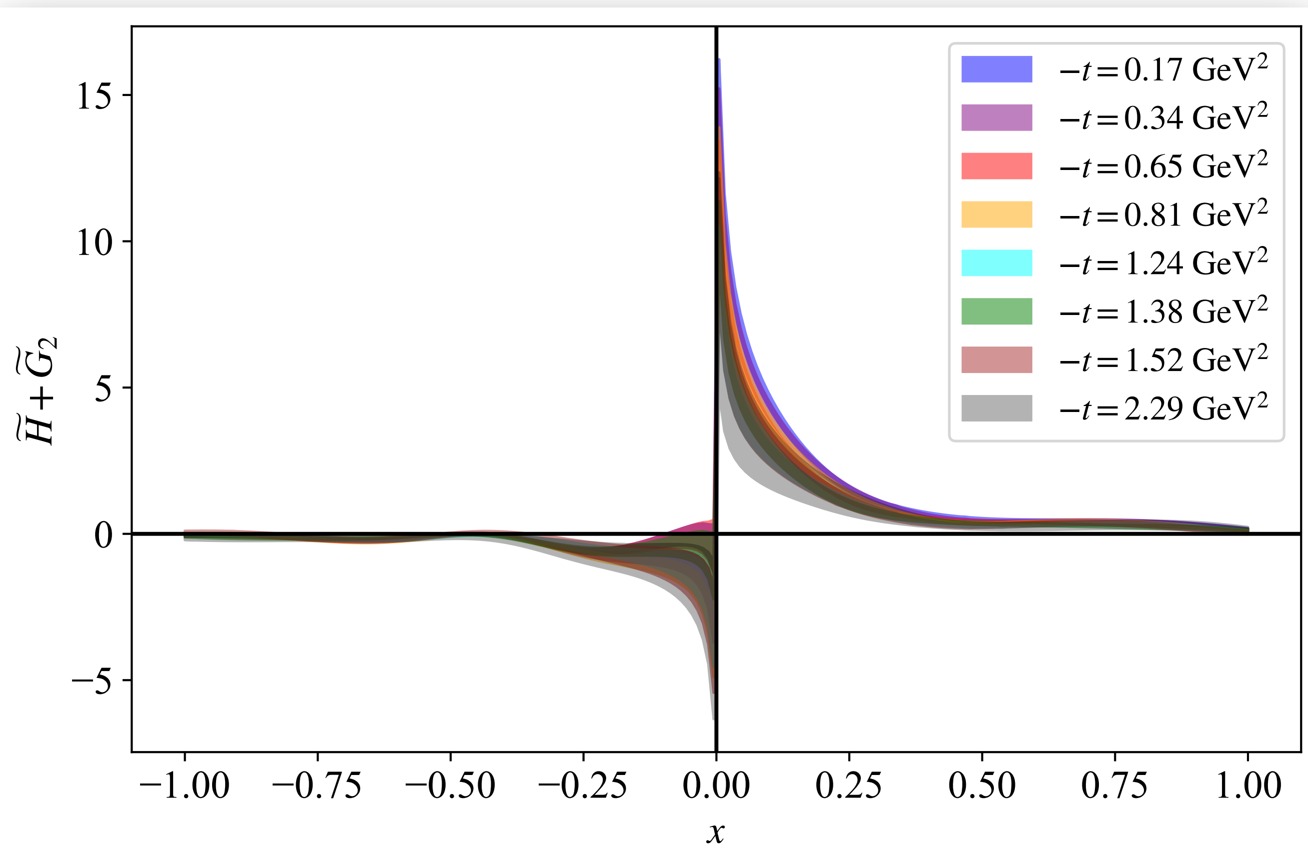
★ Alternative kinematic setup can be utilized

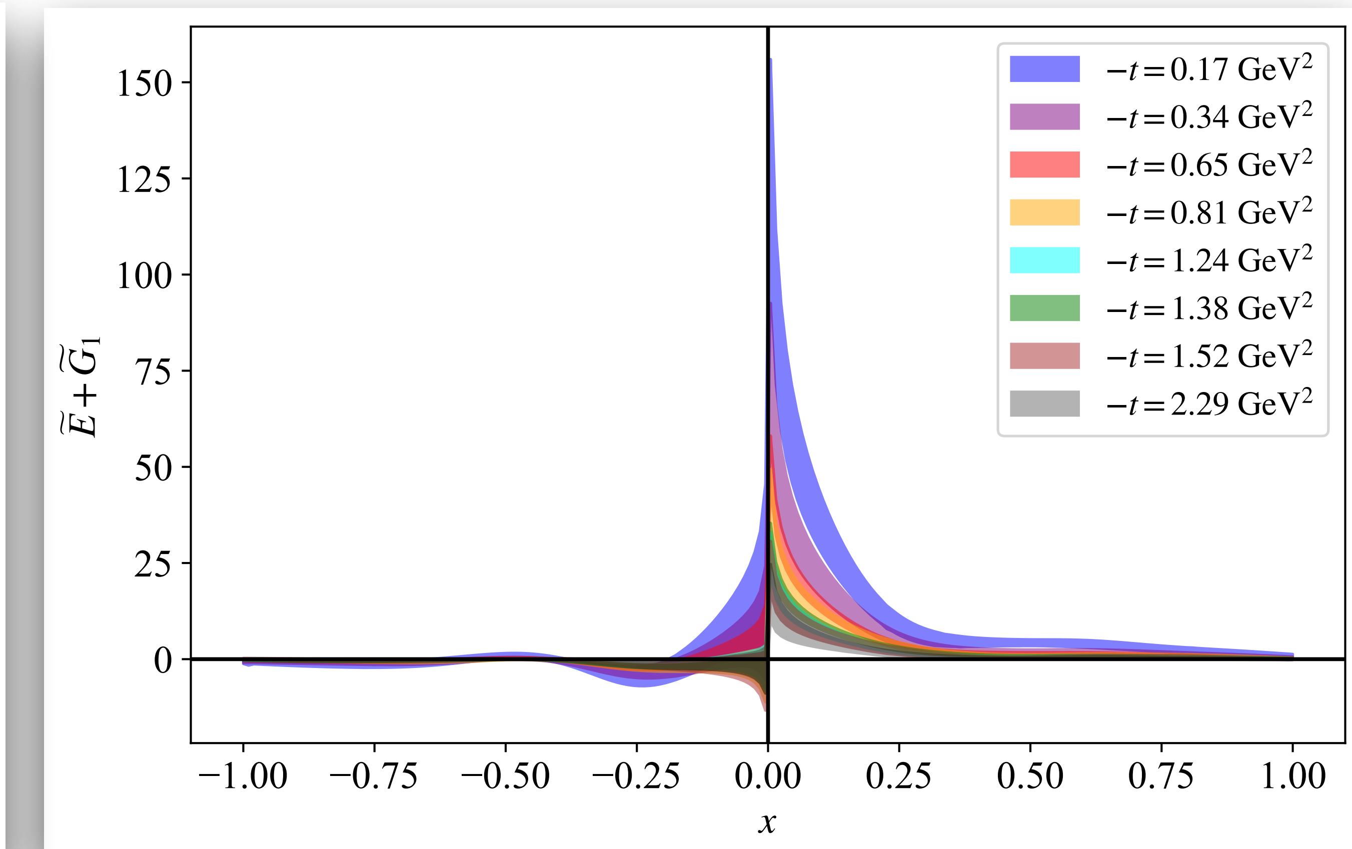
$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2 (\Delta_\perp)^2}{P_3} + A_2$$

$$F_{\widetilde{G}_3} = \frac{1}{2m^2}\Big(z_3 P_0^2 \Delta_3 - z_3 P_3 P_0 \Delta_0\Big) A_1 - z_3 P_3 A_8$$

$$F_{\widetilde{E}+\widetilde{G}_1} = \frac{2 z_3 P_0^2}{P_3} + 2A_5$$

$$F_{\widetilde{G}_3} = \frac{1}{m^2}\Big(z_3 P_0 P_3^2 - z_3 P_0^3\Big) A_1$$





On-going work

How to lattice QCD data fit into the overall effort for hadron tomography

How to lattice QCD data fit into the overall effort for hadron tomography

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

How to lattice QCD data fit into the overall effort for hadron tomography

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

PHENOMENOLOGY
LATTICE QCD
THEORY

QUARK-GLUON TOMOGRAPHY
COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number:
DE-SC0023646

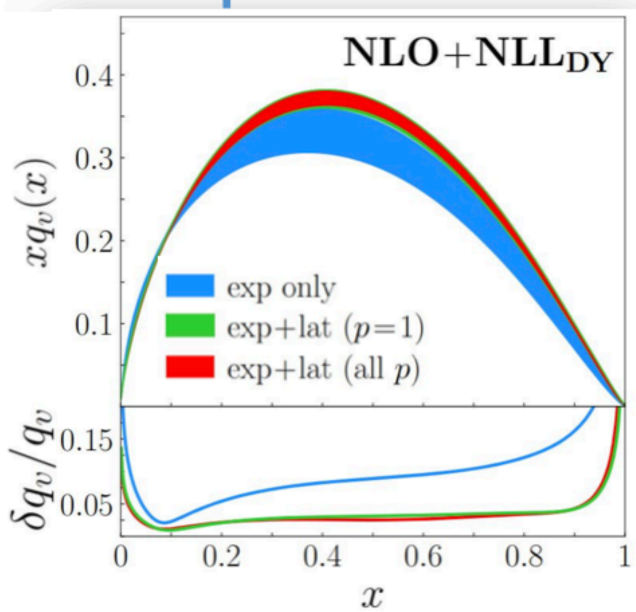
1. **Theoretical studies** of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties

2. **Lattice QCD** calculations of GPDs and related structures

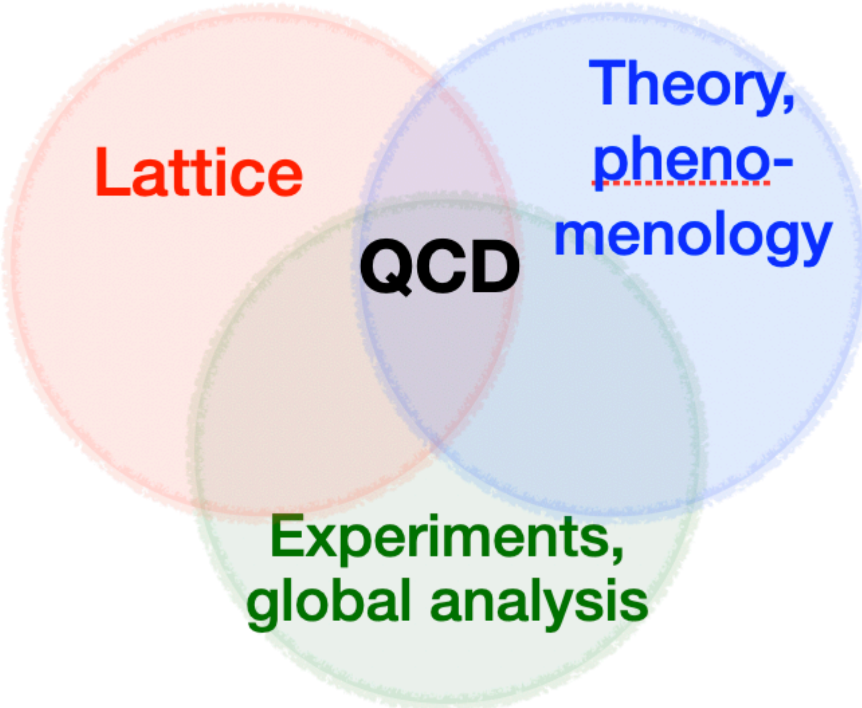
3. **Global analysis** of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

Synergies: constraints & predictive power of lattice QCD

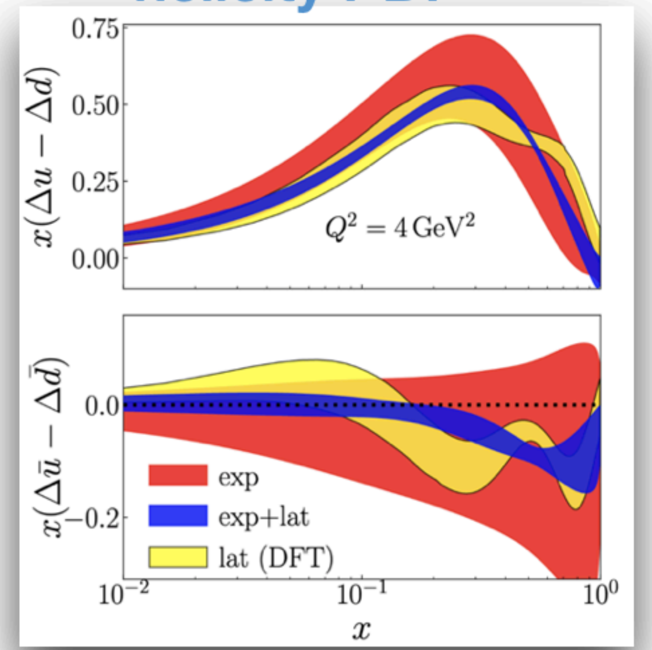
pion PDF



[JAM/HadStruc, PRD105 (2022) 114051]

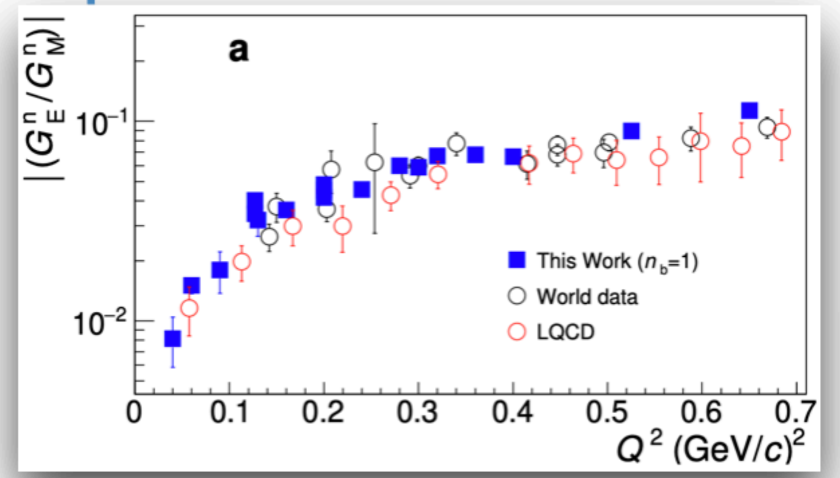


helicity PDF



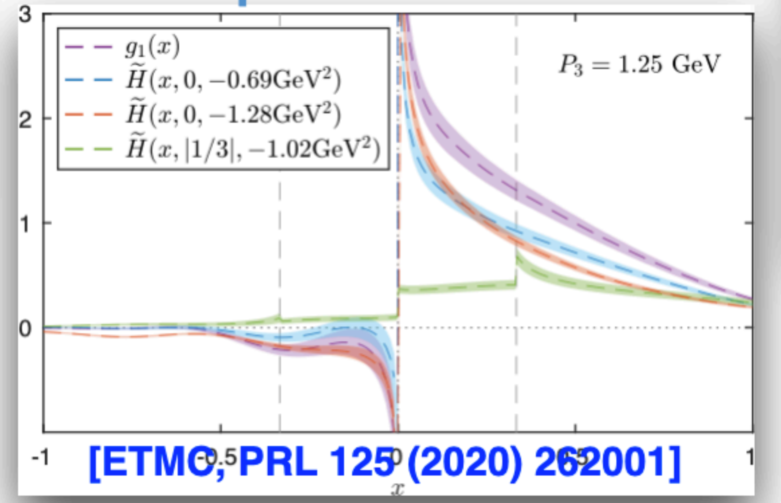
[JAM & ETMC, PRD 103 (2021) 016003]

proton & neutron radius



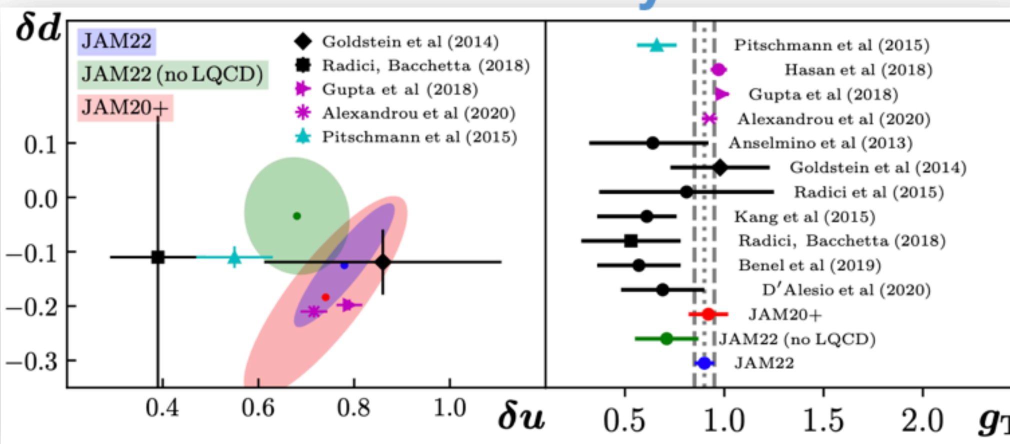
[Atac et al., Nature Comm. 12, 1759 (2021)]

proton GPDs



[ETMC, PRL 125 (2020) 262001]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

And many more!

Summary

★ Definitions of quasi-GPDs on Euclidean lattice: intrinsically frame dependent. Historically used symmetric frame is computationally very expensive

★ Novel Lorentz covariant decomposition has great advantages:
- access to symmetric-frame GPDs from matrix elements in any frame
- significant reduction of computational cost
- access to a broad range of t and ξ

★ Numerical results demonstrate the validity of the approach

★ Future calculations have the potential to transform the field of GPDs

★ Lattice QCD data on GPDs will play an important role in the pre-EIC era and can complement experimental efforts of JLab@12GeV.

★ Synergy with phenomenology is an exciting prospect!

Summary

- ★ Definitions of quasi-GPDs on Euclidean lattice: intrinsically frame dependent. Historically used symmetric frame is computationally very expensive

- ★ Novel Lorentz covariant decomposition has great advantages:
  - access to symmetric-frame GPDs from matrix elements in any frame
  - significant reduction of computational cost
  - access to a broad range of t and ξ

- ★ Numerical results demonstrate the validity of the approach

- ★ Future calculations have the potential to transform the field of GPDs

- ★ Lattice QCD data on GPDs will play an important role in the pre-EIC era and can complement experimental efforts of JLab@12GeV.

- ★ Synergy with phenomenology is an exciting prospect!

Thank you

PHENOMENOLOGY LATTICE QCD THEORY
U.S. DEPARTMENT OF ENERGY Office of Science
QUARK-GLUON TOMOGRAPHY COLLABORATION
Award Number: DE-SC0023646

U.S. DEPARTMENT OF ENERGY Office of Science
DOE Early Career Award (NP)
Grant No. DE-SC0020405

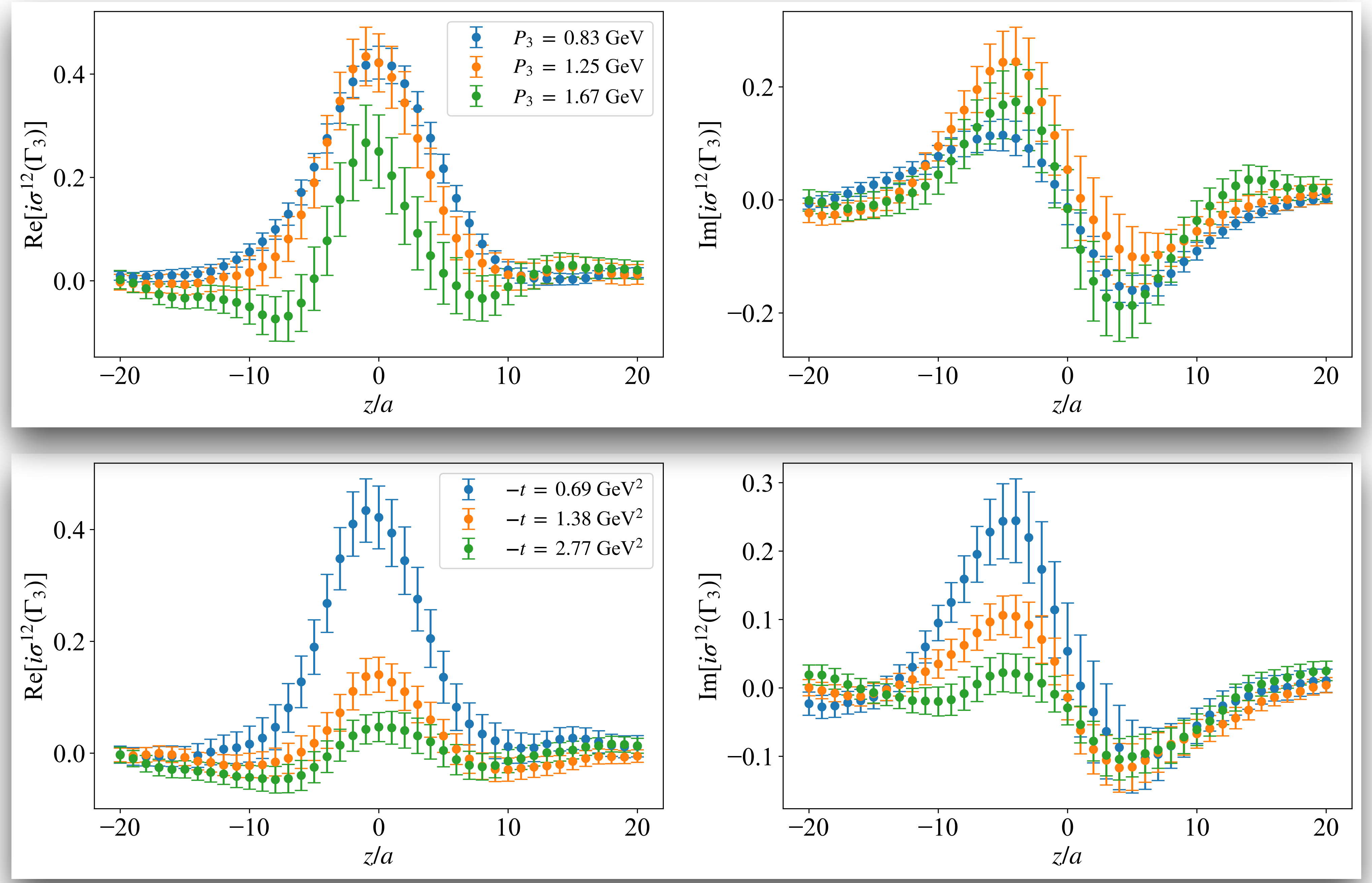
U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING
NERSC
ASCR Leadership Computing Challenge ALCC

Miscellaneous

Extension to twist-3 tensor GPDs

★ Parametrization [Meissner et al., *JHEP* 08 (2009) 056]

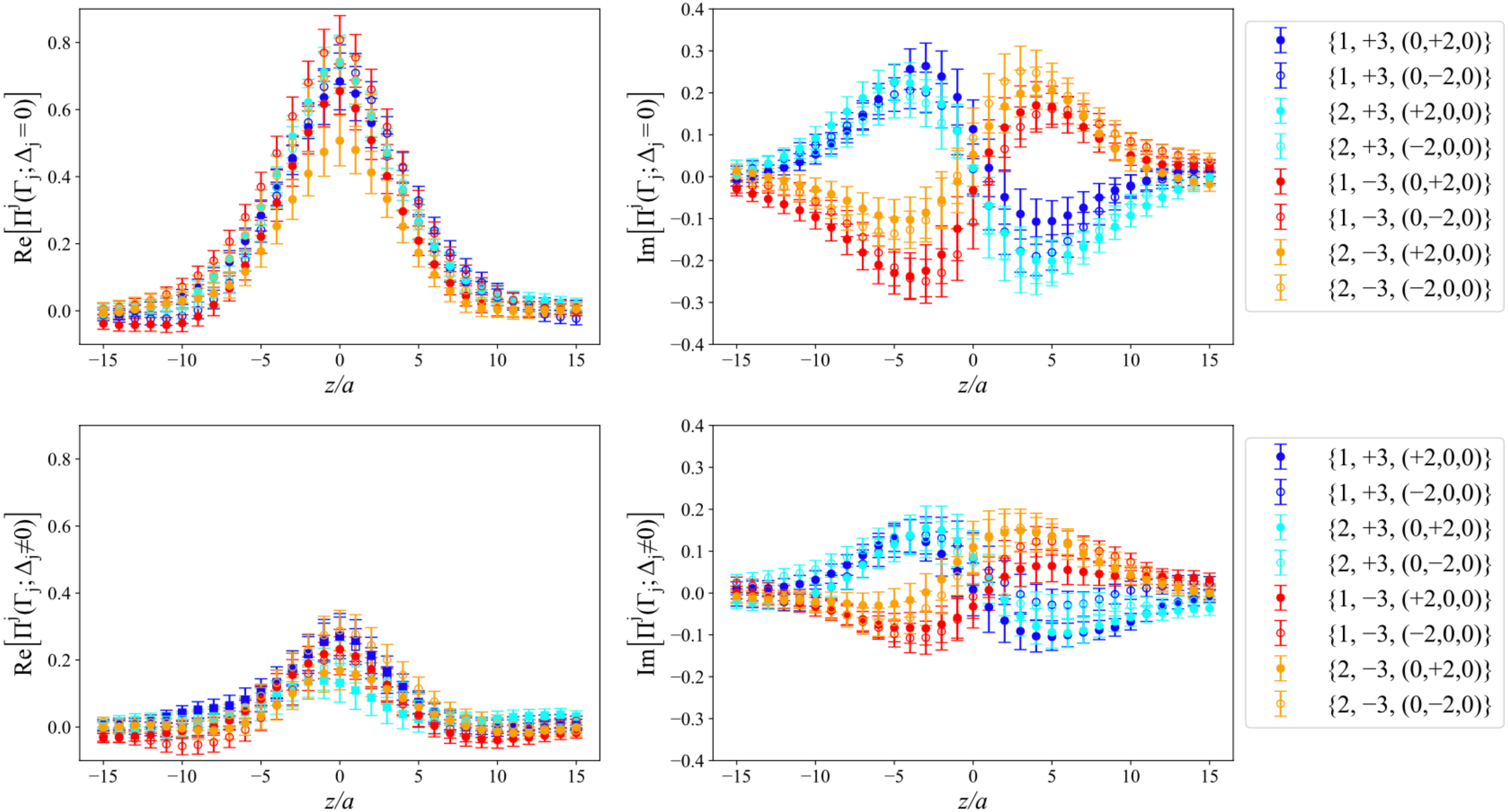
$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p')\left(\gamma^+\gamma_5\,\widetilde{H}_2' + \frac{P^+\gamma_5}{M}\,\widetilde{E}_2'\right)u(p)$$



Lattice Results - Matrix Elements

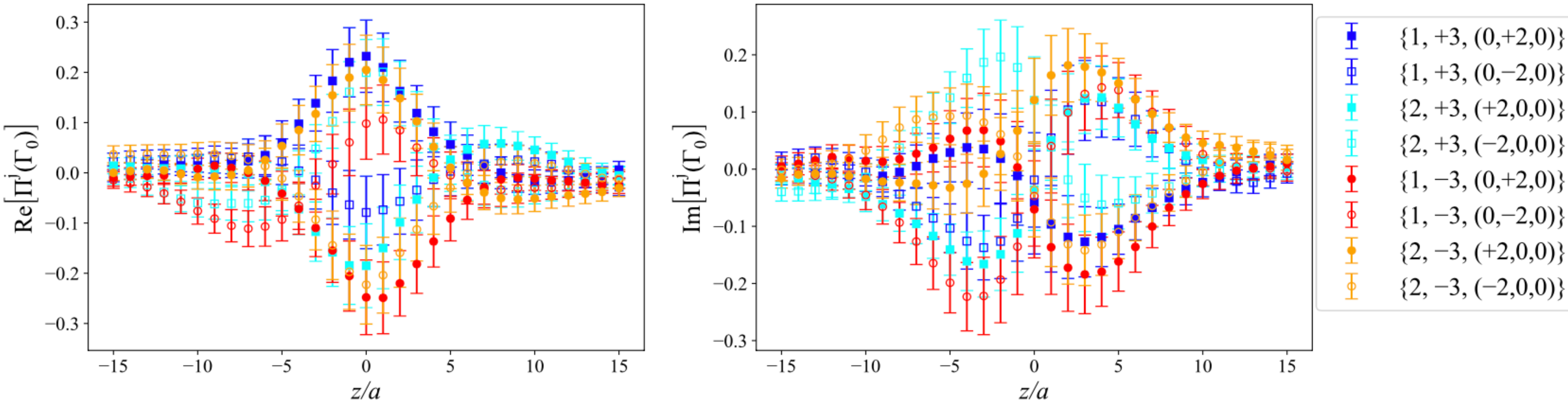
★ Bare matrix elements

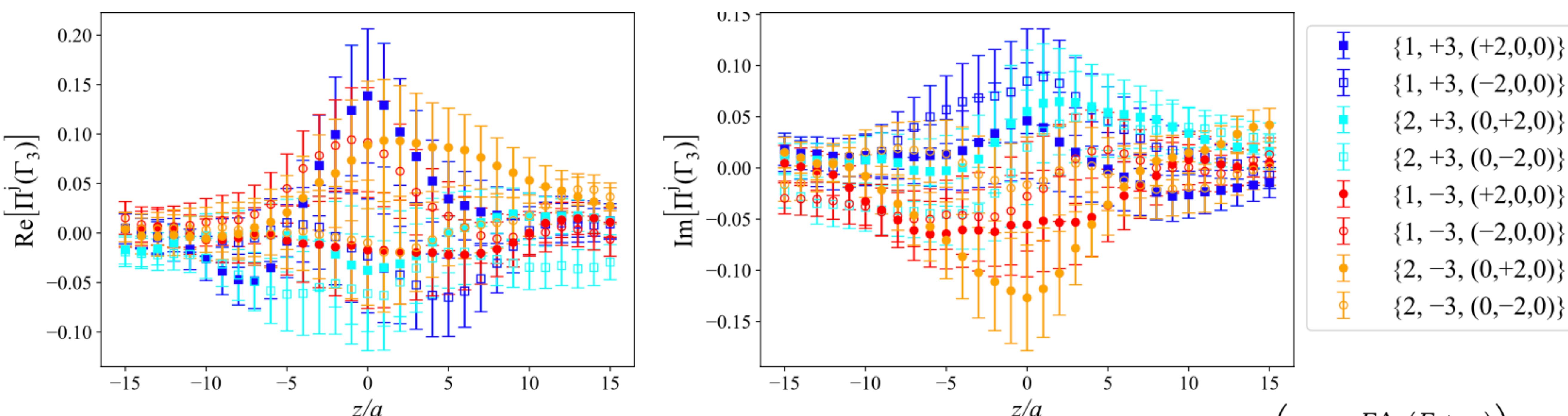
$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{(4m(E+m)+\Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\frac{\text{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2P_3}\right)$$



Lattice Results - Matrix Elements

★ Bare matrix elements





$$\Pi^1(\Gamma_3) = C\left(-F_{\widetilde{G}_3}\frac{E\Delta_x(E+m)}{2m^2P_3}\right)$$

★ Suppressed signal compared to $\gamma_+\gamma_5$ operators

Consistency checks

★ Norms satisfied encouraging results

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

Consistency checks

★ Norms satisfied encouraging results

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

★ Alternative kinematic setup can be utilized

[Fernanda Steffens]

$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2 (\Delta_\perp)^2}{P_3} + A_2$$

$$F_{\widetilde{G}_3} = \frac{1}{2m^2}\Big(z_3 P_0^2 \Delta_3 - z_3 P_3 P_0 \Delta_0\Big) A_1 - z_3 P_3 A_8$$

$$F_{\widetilde{E}+\widetilde{G}_1} = \frac{2 z_3 P_0^2}{P_3} + 2A_5$$

$$F_{\widetilde{G}_3} = \frac{1}{m^2}\Big(z_3 P_0 P_3^2 - z_3 P_0^3\Big) A_1$$

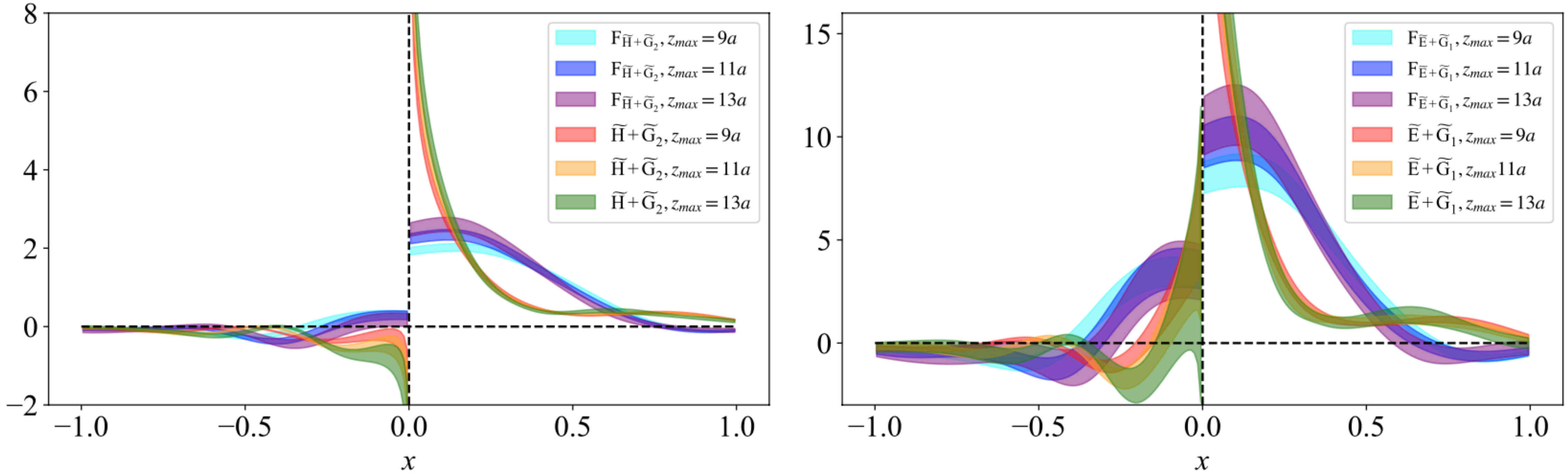


FIG. 10. $z_{\rm max}$ dependence of $F_{\widetilde{H}+\widetilde{G}_2}$ and $\widetilde{H}+\widetilde{G}_2$ (left), as well as $F_{\widetilde{E}+\widetilde{G}_1}$ and $\widetilde{E}+\widetilde{G}_1$ (right) at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in the $\overline{\rm MS}$ scheme at a scale of 2 GeV.

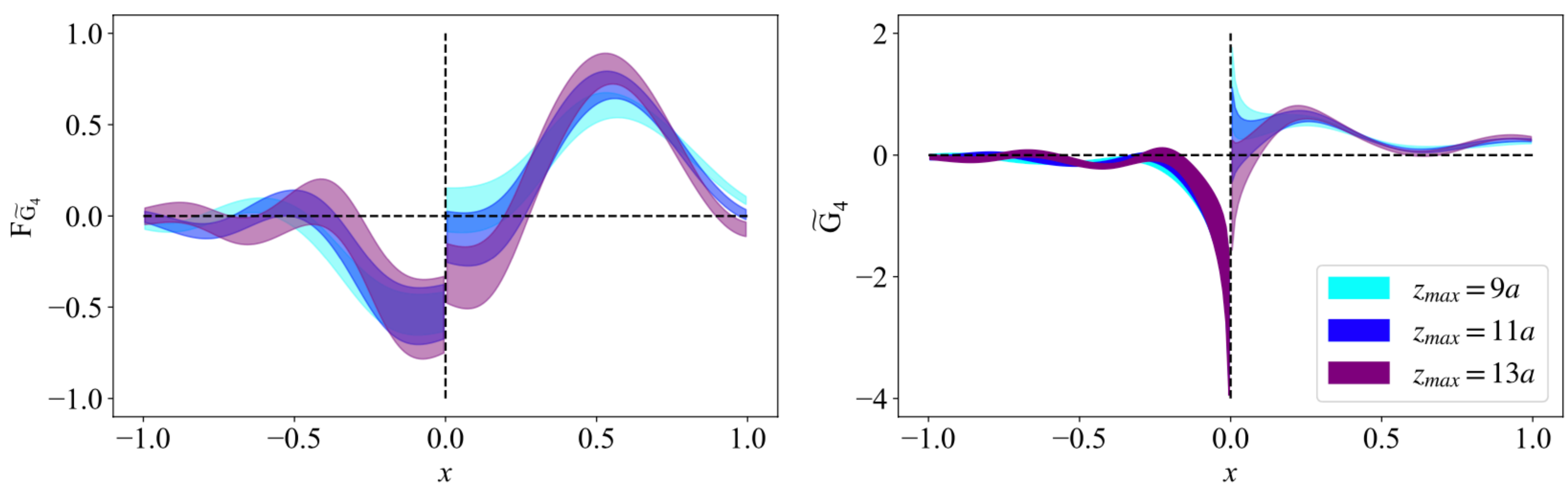


FIG. 11. $z_{\rm max}$ dependence of $F_{\widetilde{G}_4}$ and $\widetilde{G}_4$ at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in $\overline{\rm MS}$ scheme at a scale of 2 GeV.