Phenomenology of virtual Compton scattering processes in the era of new experiments

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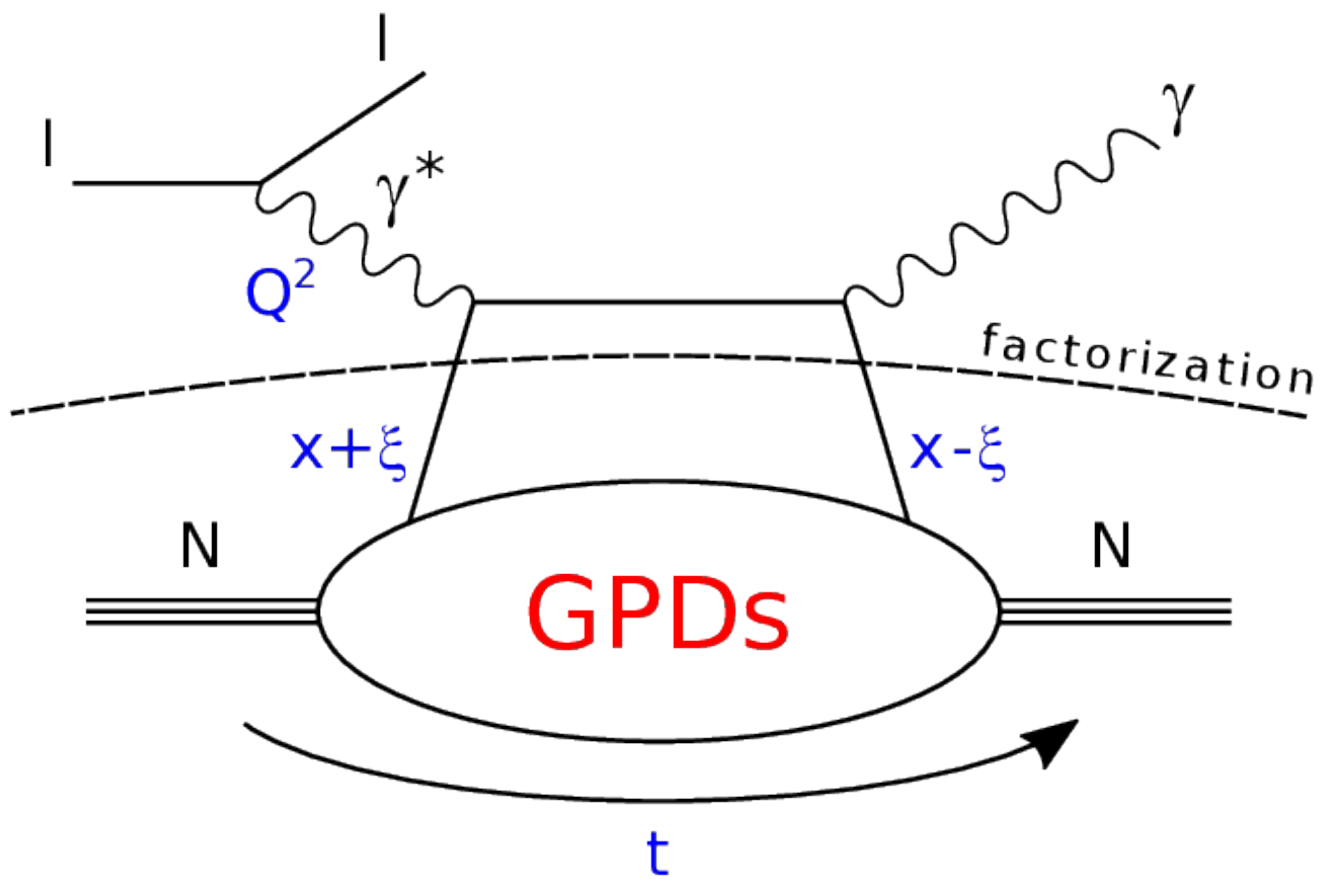
3D Structure of the Nucleon via Generalized Parton Distributions,
Incheon, Republic of Korea, June 25th, 2024

Outline

- Introduction and motivation
- Virtual Compton scattering processes
- Deconvolution problem
- Double Deeply Virtual Scattering
- Inclusion lattice-QCD results
- Summary

Deeply Virtual Compton Scattering (DVCS)



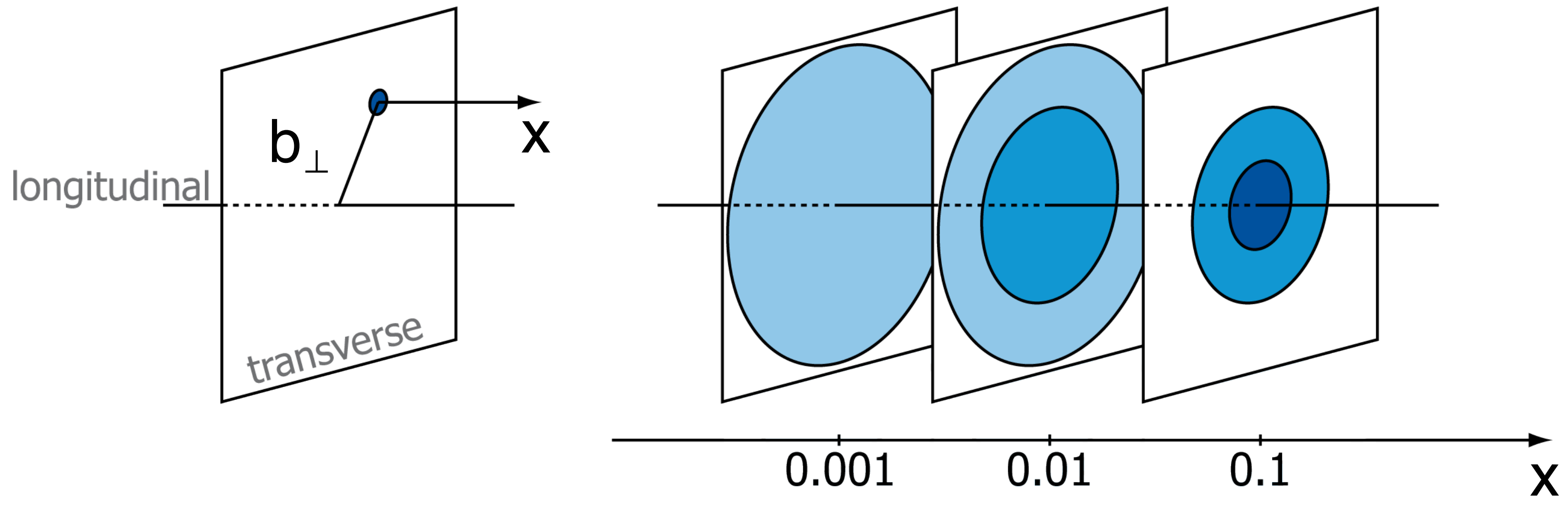
factorisation for |t|/Q² ≪ 1

Chiral-even GPDs:
(helicity of parton conserved)

$H^{q,g}(x,\xi,t)$	$E^{q,g}(x,\xi,t)$	*for sum over parton helicities*
$\widetilde{H}^{q,g}(x,\xi,t)$	$\widetilde{E}^{q,g}(x,\xi,t)$	*for difference over parton helicities*
nucleon helicity conserved	*nucleon helicity changed*	

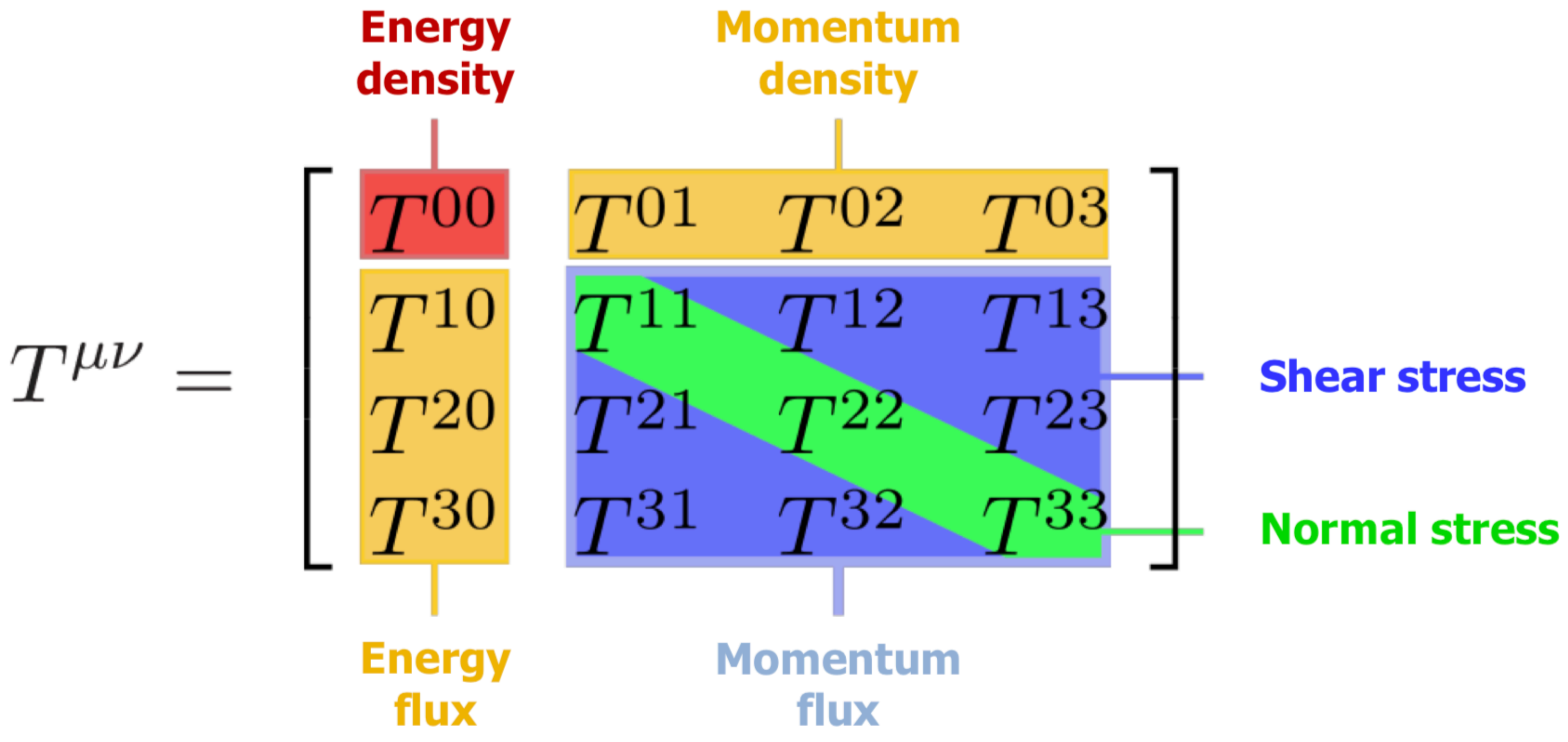
Introduction

Nucleon tomography:



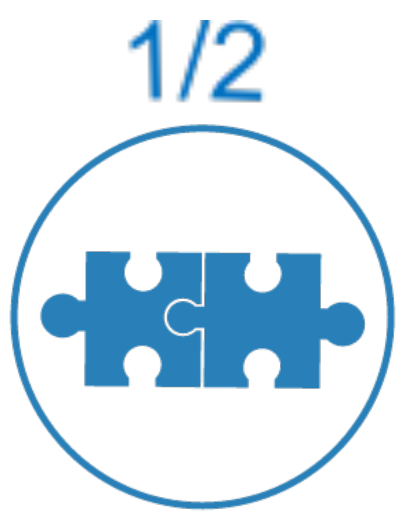
$$q(x,\mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{4\pi^2} e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}} H^q(x,0,t=-\boldsymbol{\Delta}^2)$$

Energy momentum tensor in terms of form factors (OAM and mechanical forces):



$$\langle p',s'|\widehat{T}^{\mu\nu}|p,s\rangle = \bar{u}(p',s')\left[\frac{P^\mu P^\nu}{M}A(t) + \frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M}C(t) + M\eta^{\mu\nu}\bar{C}(t) + \frac{P^\mu i\sigma^{\nu\lambda}\Delta_\lambda}{4M}[A(t)+B(t)+D(t)] + \frac{P^\nu i\sigma^{\mu\lambda}\Delta_\lambda}{4M}[A(t)+B(t)-D(t)]\right]u(p,s)$$

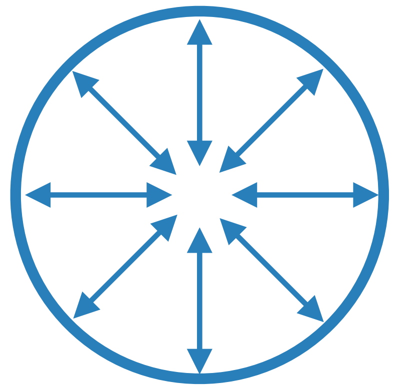
Total angular momentum:



$$A^q(0) + B^q(0) = \int_{-1}^{1} x \left[H^q(x, \xi, 0) + E^q(x, \xi, 0)\right] = 2J^q$$

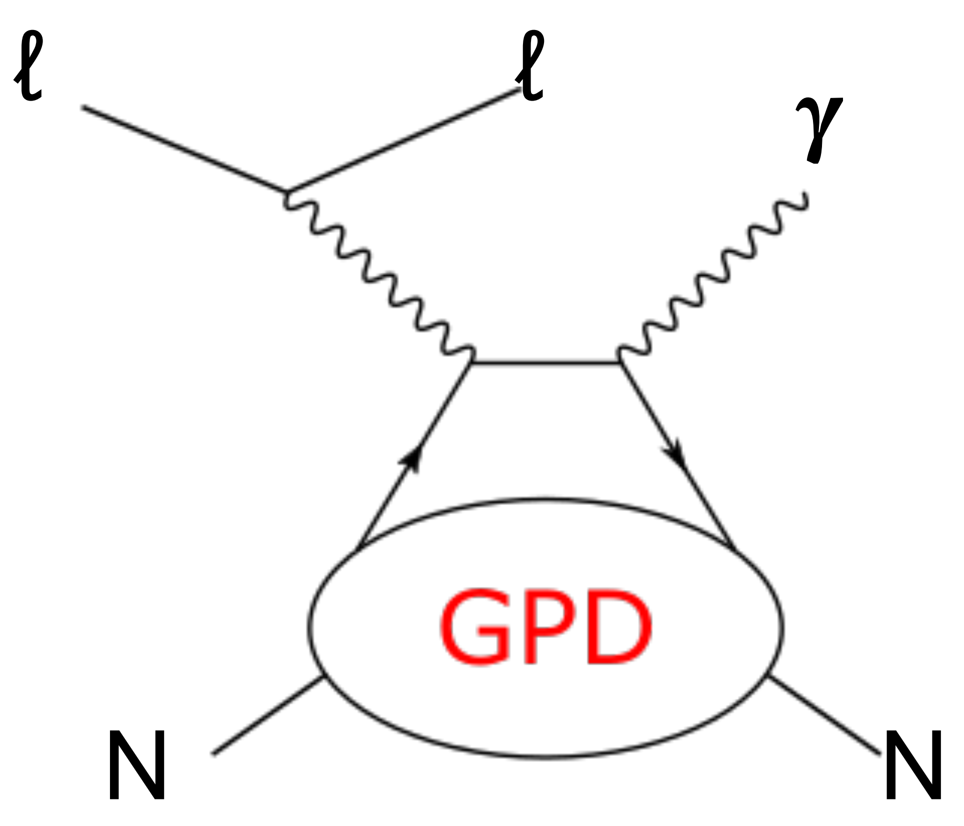
Ji's sum rule

"Mechanical" forces acting on quarks, e.g. pressure in nucleon center:



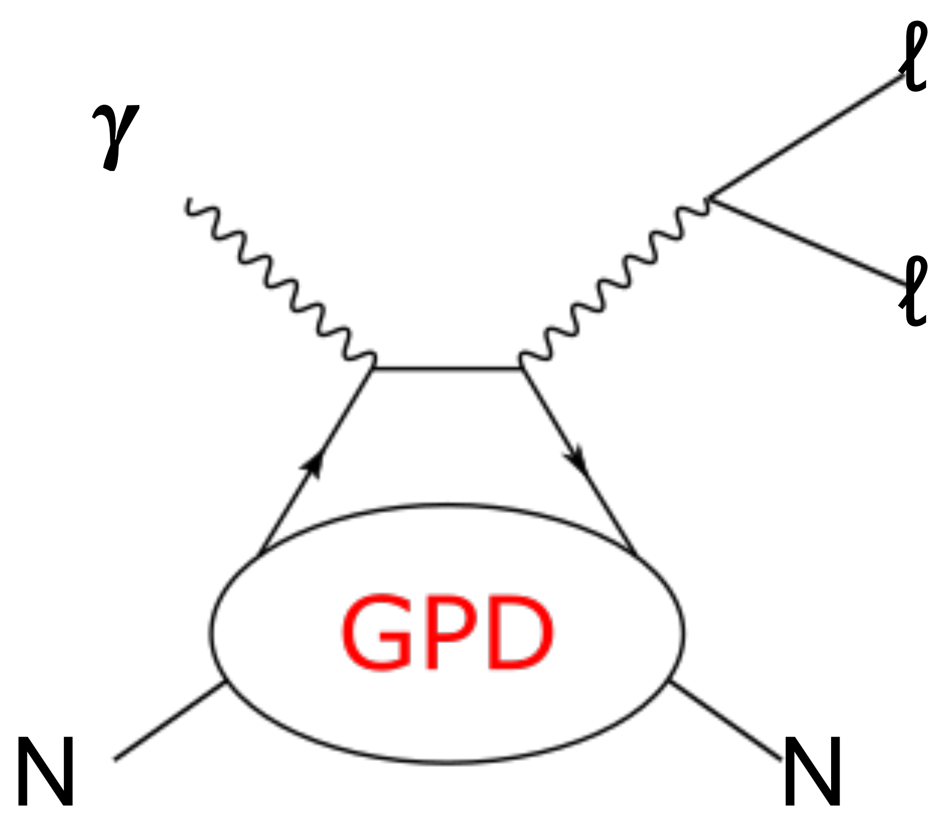
$$p(0) = \frac{1}{6\pi^2 M} \int_{-\infty}^{0} dt \sqrt{-t}\, t C(t)$$

VCS processes provide the most straightforward way to access GPDs (at least from theory side...)



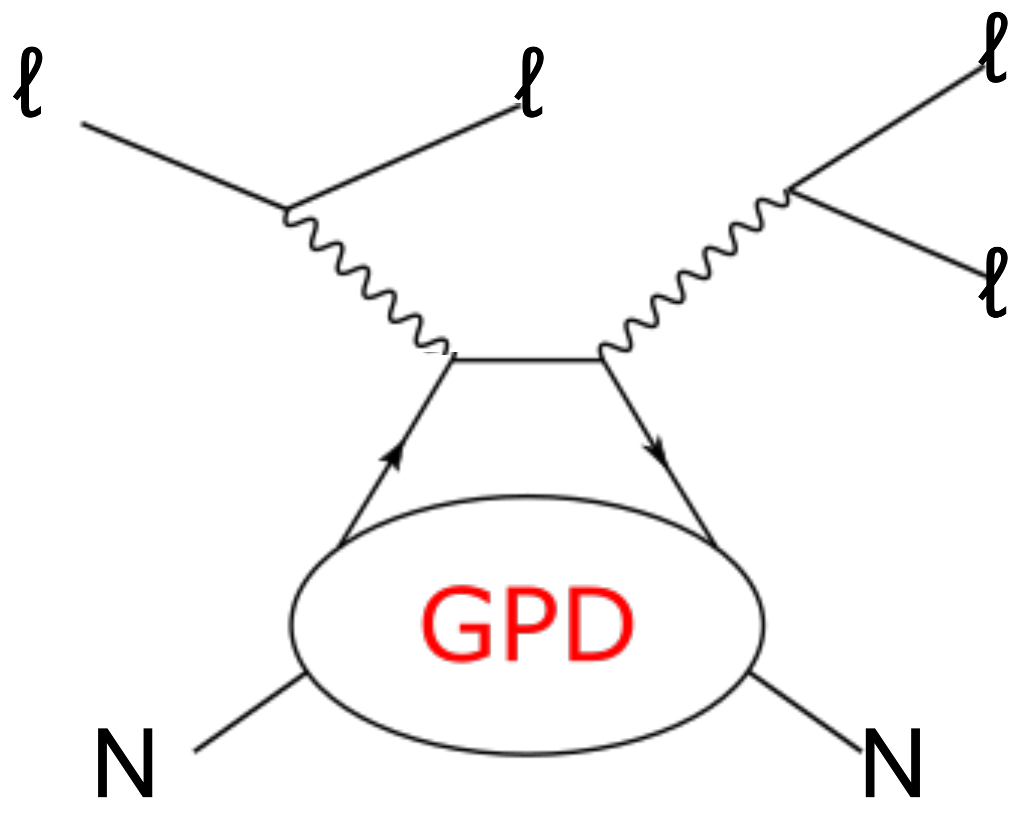
DVCS
Deeply Virtual Compton Scattering

- **many measurements, see e.g.:**
  EPJA 52 (2016) 6, 157
- **description up to NNLO and twist-4 available**
  PRL 129 (2022) 17, 172001
  JHEP 01 (2023) 078



TCS
Timelike Compton Scattering

- **first measurement by CLAS**
  PRL 127 (2021) 26, 262501
- **description up to NLO and twist-2 available**
  (preliminary tw-4)



DDVCS
Double Deeply Virtual Compton Scattering

- **never measured**
- **description up to NLO and twist-2 available**
  (preliminary tw-4)

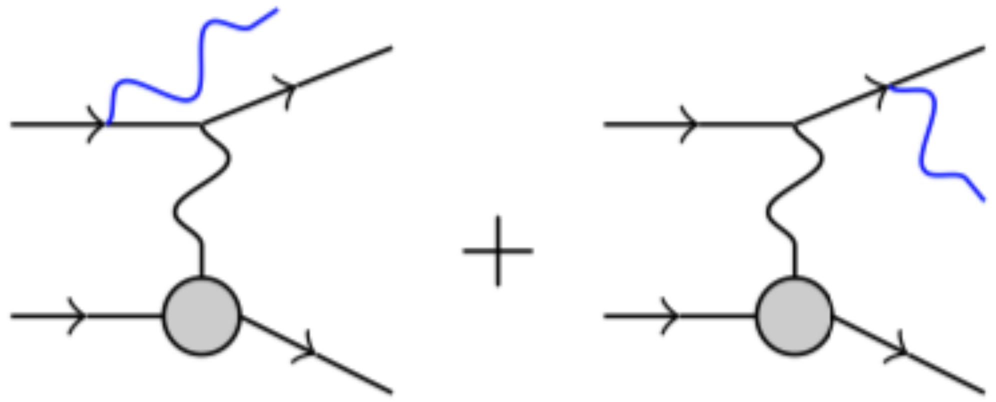
more production channels sensitive to GPDs exist!

DVCS Compton Form Factors

Cross-section for single photon production ($l + N \to l + N + \gamma$):

$$\sigma \propto |\mathscr{A}|^2 = |\mathscr{A}_{BH} + \mathscr{A}_{DVCS}|^2 = |\mathscr{A}_{BH}|^2 + |\mathscr{A}_{DVCS}|^2 + \mathscr{I}$$

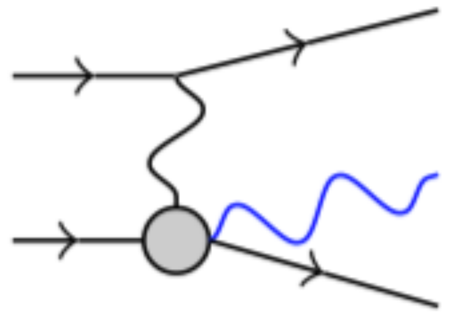
Bethe-Heitler process



calculable within QED

parametrised by elastic FFs

DVCS



calculable within QCD

parametrised by CFFs

see e.g. NPB 878 (2014) 214 for more details

$$\mathrm{Im}\mathscr{H}(\xi, t) \stackrel{\text{LO}}{=} \pi \sum_q e_q^2 H^{q(+)}(\xi, \xi, t)$$

$$\mathrm{Re}\mathscr{H}(\xi, t) = \mathrm{PV} \int_0^1 \frac{d\xi'}{\pi} \mathrm{Im}\mathscr{H}(\xi', t) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) + C_H(t)$$

Shadow GPDs

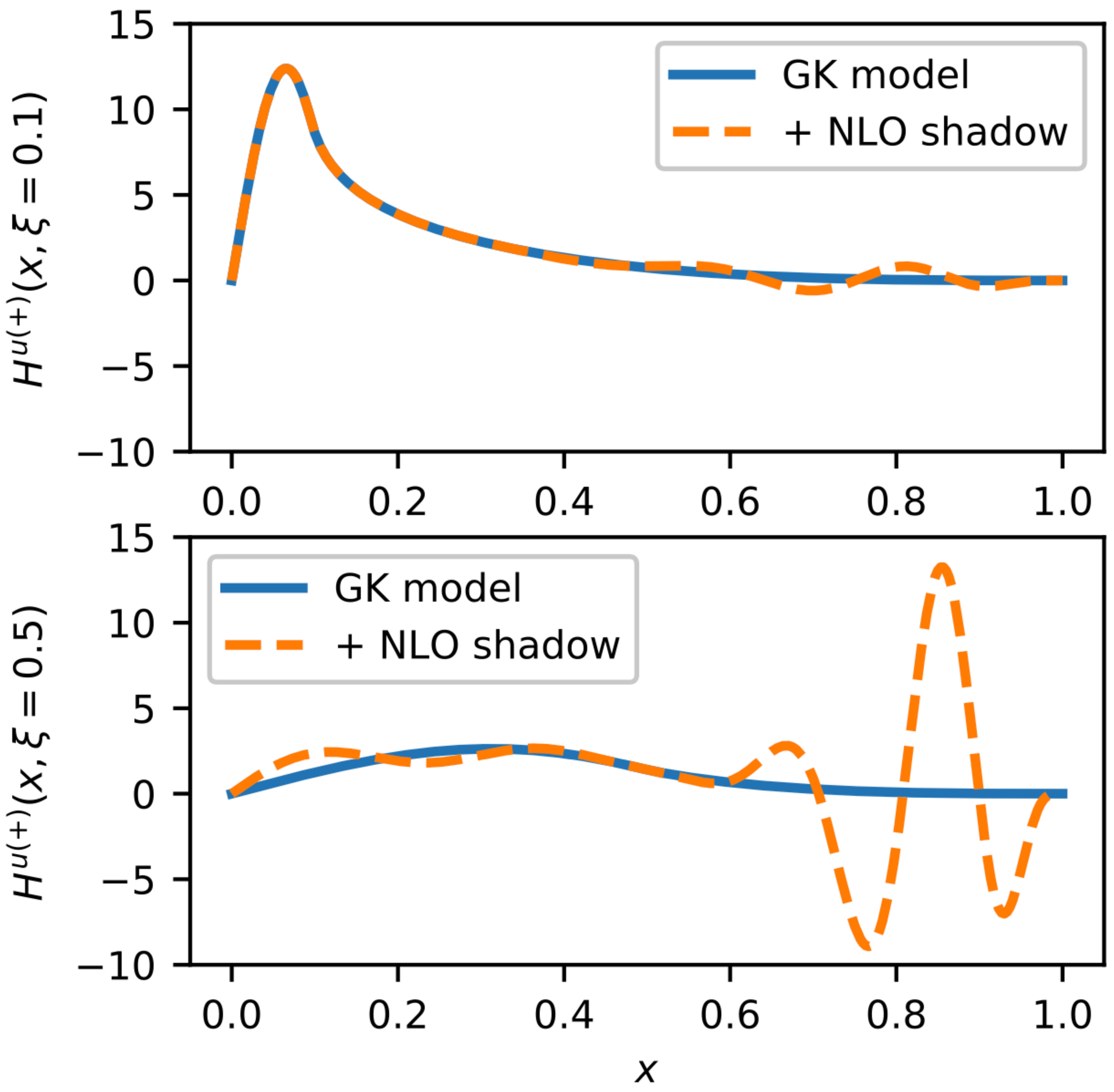
V. Bertone et al.,
Phys. Rev. D 103 (2021) 11, 114019

Shadow GPDs have considerable size, but:

- at arbitrary initial scale do not contribute to PDFs and CFFs
- at other scales contribute negligibly

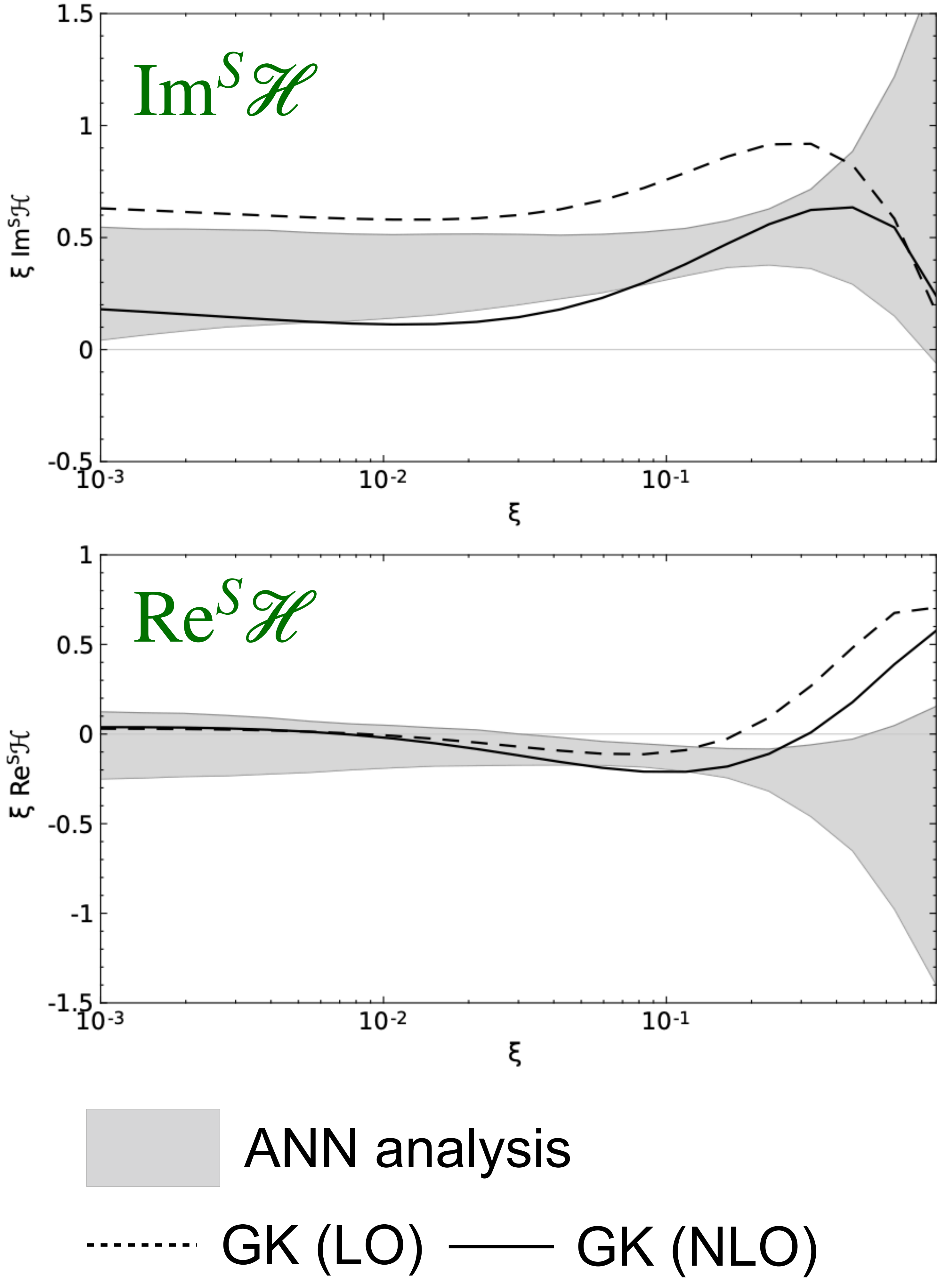
making the deconvolution of CFFs ill-posed problem

We found such GPDs for DVCS for both LO and NLO
(for discussion see also PRD 108 (2023) 3, 036027)



TCS Compton form factors

O. Grocholski et al.,
Eur. Phys. J. C 80 (2020) 2, 171



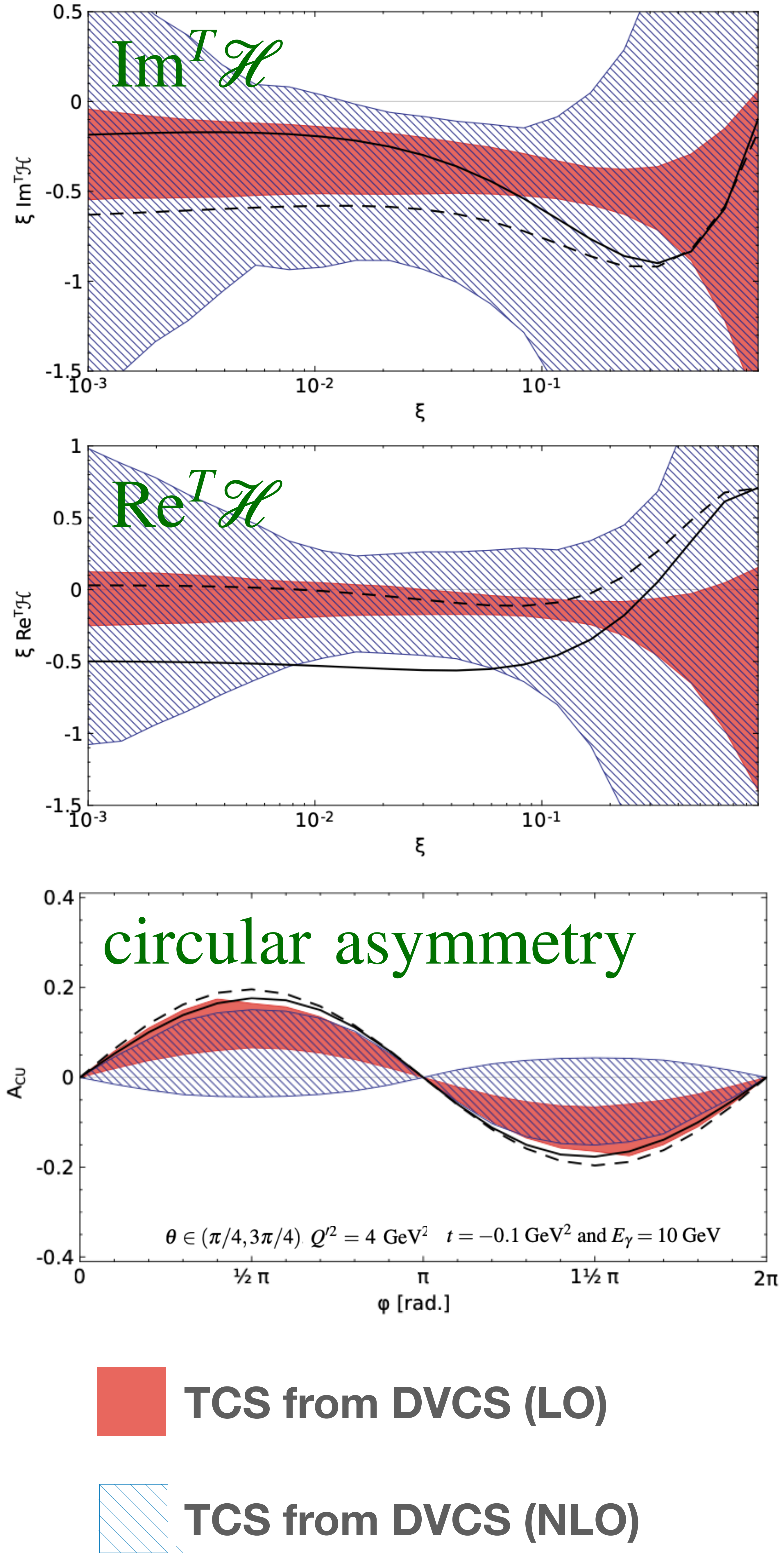
Relation between space-like and time-like amplitudes

$$^{T}\mathcal{H} \overset{\mathrm{LO}}{=} {}^{S}\mathcal{H}^{*}$$

$$^{T}\widetilde{\mathcal{H}} \overset{\mathrm{LO}}{=} -{}^{S}\widetilde{\mathcal{H}}^{*}$$

$$^{T}\mathcal{H} \overset{\mathrm{NLO}}{=} {}^{S}\mathcal{H}^{*} - i\pi\,\mathcal{Q}^{2}\frac{\partial}{\partial\mathcal{Q}^{2}}\,{}^{S}\mathcal{H}^{*}$$

$$^{T}\widetilde{\mathcal{H}} \overset{\mathrm{NLO}}{=} -{}^{S}\widetilde{\mathcal{H}}^{*} + i\pi\,\mathcal{Q}^{2}\frac{\partial}{\partial\mathcal{Q}^{2}}\,{}^{S}\widetilde{\mathcal{H}}^{*}$$

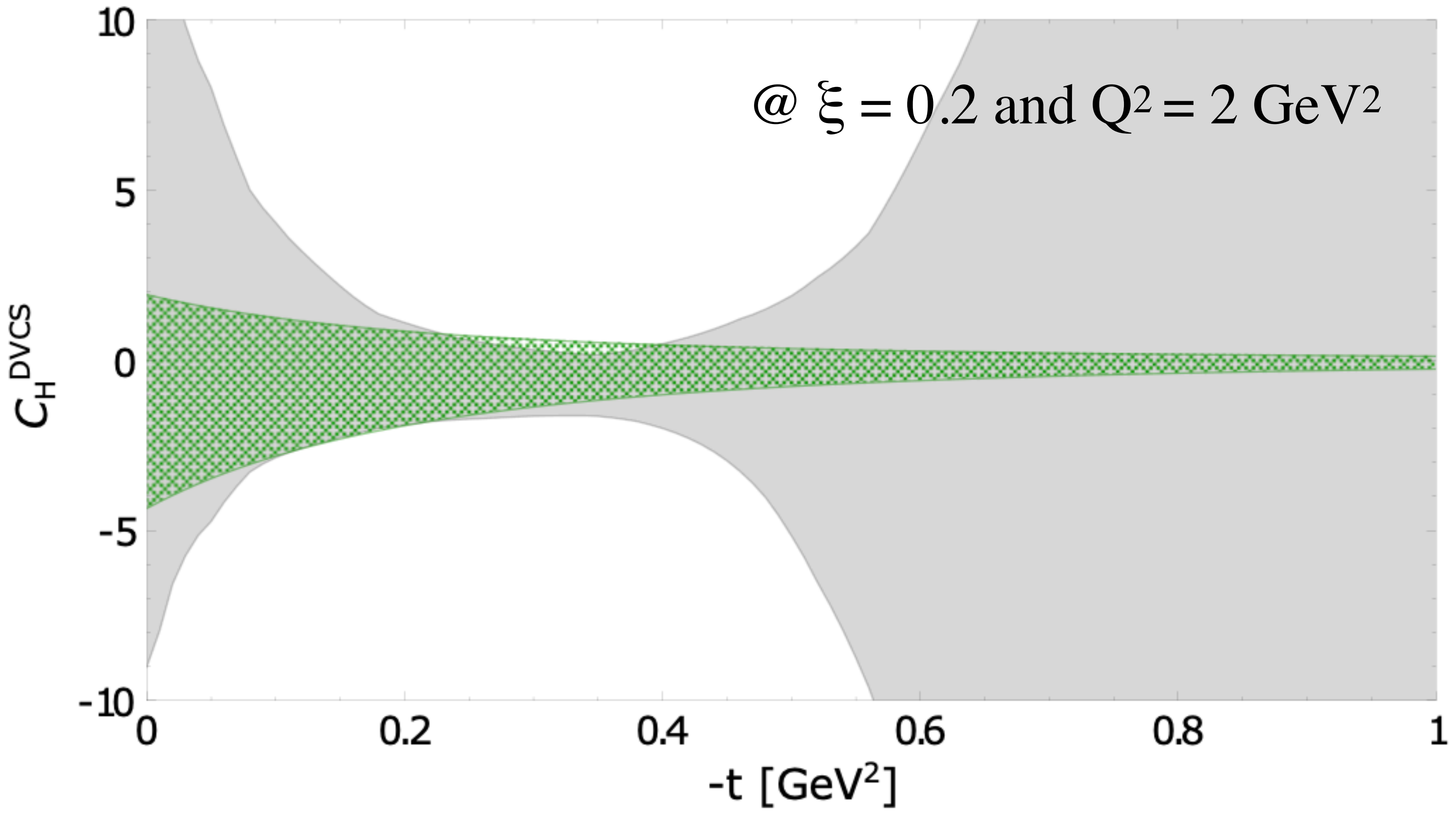


Analyses not requiring de-convolution

- probing nucleon tomography at low-xB (see: *JHEP 09 (2013) 093*)

$$\mathrm{d}^3\sigma/(\mathrm{d}x_{\mathrm{Bj}}\,\mathrm{d}Q^2\,\mathrm{d}t) \propto (\mathrm{Im}\mathcal{H}(\xi,t))^2 \propto \left(\sum_q e_q^2 H^{q(+)}(\xi,\xi,t)\right)^2 \propto \left(\sum_q e_q^2 H^{q(+)}(\xi,0,t)\right)^2$$

- extraction of D-term (see: *Nature 570 (2019) 7759, E1*, *EPJC 81 (2021) 4, 300*)

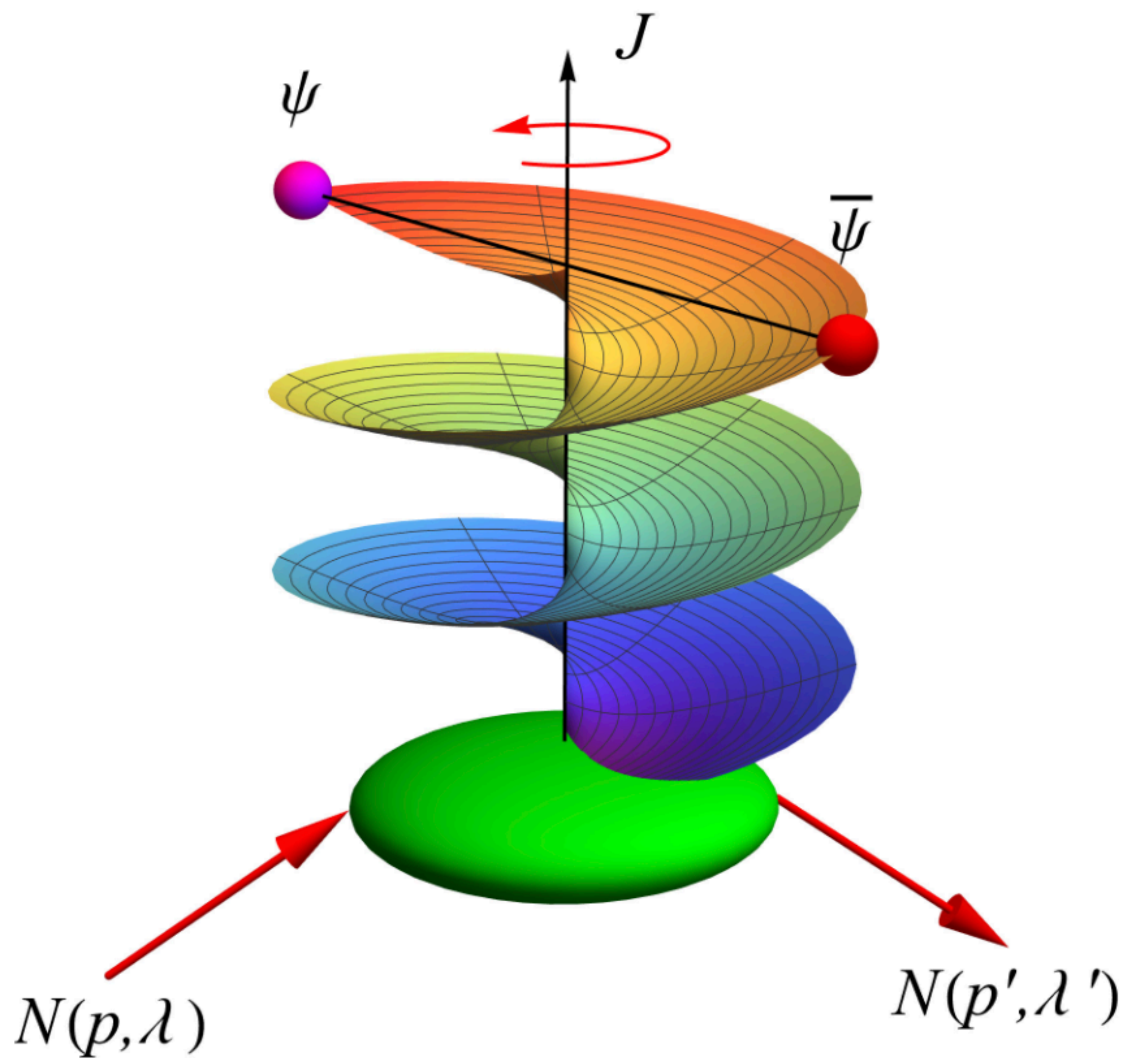


ANN analysis

Model dependent extraction

$$d_1^{uds}(t,\mu_F^2) = d_1^{uds}(\mu_F^2)\left(1-\frac{t}{\Lambda^2}\right)^{-\alpha}$$

$\alpha = 3 \quad \Lambda = 0.8$ GeV



- Froissart-Gribov projections (see: *PRD 109 (2024) 5, 054010*)

Froissart-Gribov projections - spin 0

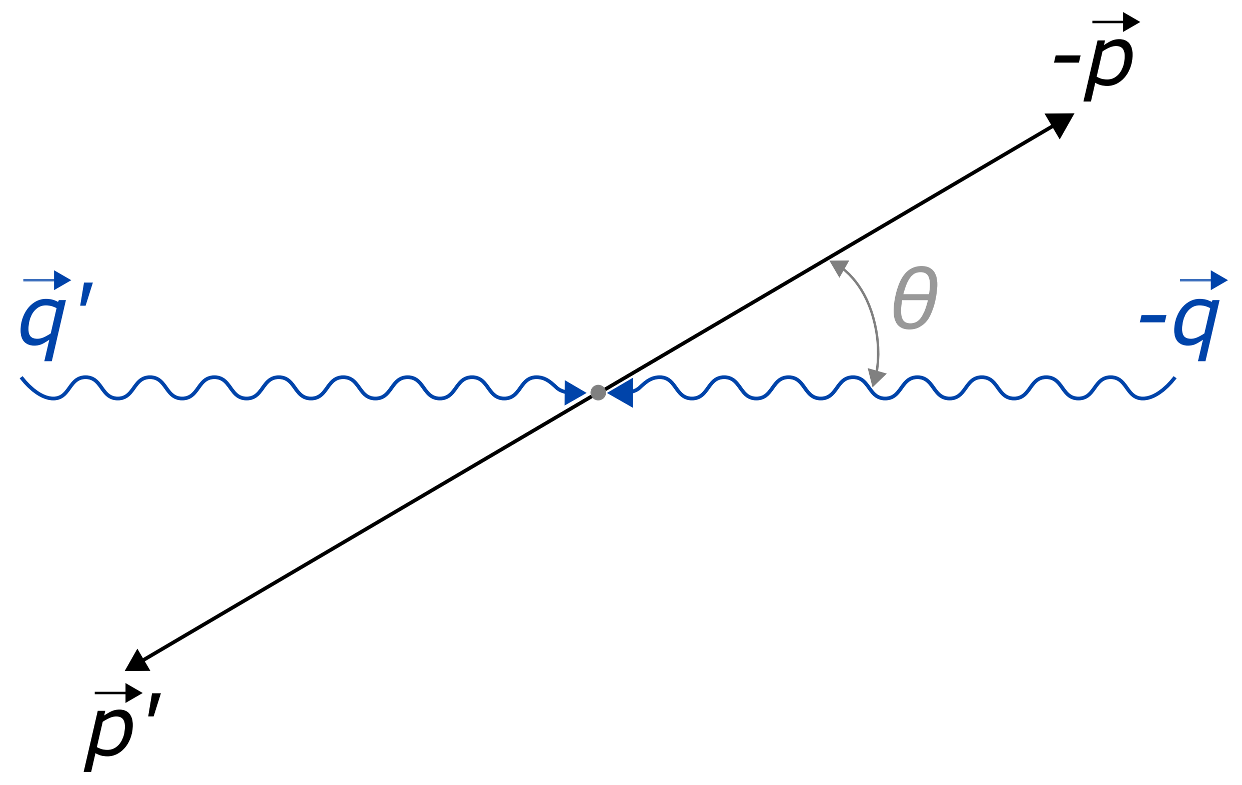
K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

FG projections are obtained by reconstructing cross-channel partial wave expansion amplitudes from the dispersive representation of the amplitude in the direct channel.

In cross-channel: $\gamma^*(q) + \gamma\,(-q') \to h\,(p') + \bar{h}(-p)$

Expansion in the cross channel SO(3) partial waves: $$\mathcal{H}_+\,(\cos\theta_t, t) = \sum_{\substack{J=0\\ \text{even}}}^{\infty} F_J(t) P_J(\cos\theta_t)$$

which gives: $$F_J(t) = \frac{2J+1}{2}\int_{-1}^{1} d\,(\cos\theta_t)\, P_J\,(\cos\theta_t)\,\mathcal{H}_+\,(\cos\theta_t, t)$$



In direct-channel: $\gamma^*(q) + h(p) \to \gamma\,(q') + h\,(p')$

Dispersion relation: $$\mathrm{Re}\,\mathcal{H}_+(\xi, t) = \mathcal{P}\int_0^1 dx \frac{2x H_+(x, x, t)}{\xi^2 - x^2} + 4D(t)$$

where: $$\cos\theta_t \to -\frac{1}{\xi\beta} + \mathcal{O}\left(1/Q^2\right) \qquad \beta = \sqrt{1 - \frac{4m^2}{t}}$$

$\beta = 1$
in the current analysis (see the publication for discussion of consequences)

Froissart-Gribov projections - spin 0

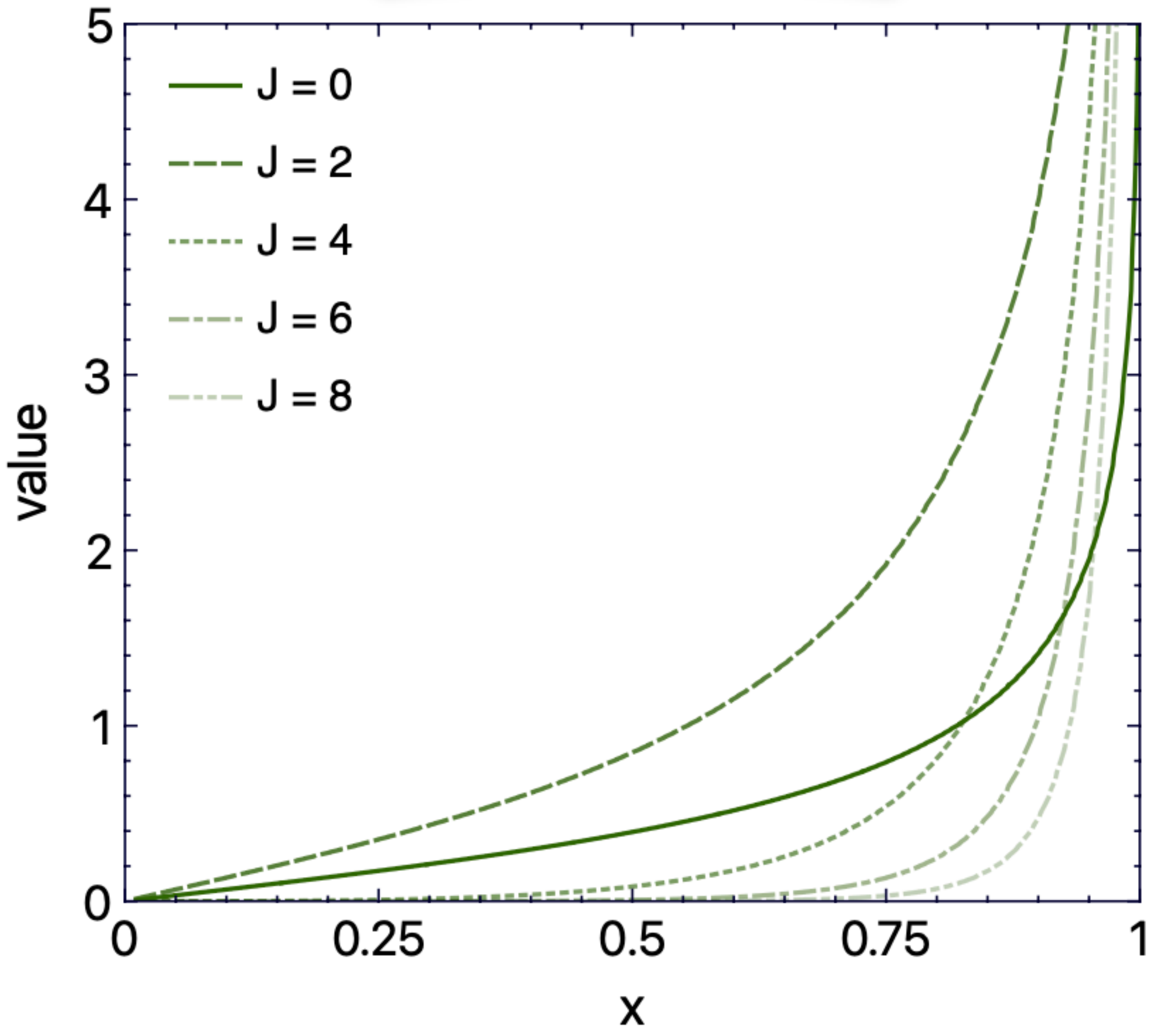
K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

Final result:

$$F_{J=0}(t) = 2\int_0^1 dx \left(\frac{\mathcal{Q}_0(1/x)}{x^2} - \frac{1}{x}\right) H_+(x,x,t) + 4D(t)$$

$$F_{J>0}(t) = 2(2J+1)\int_0^1 dx \frac{\mathcal{Q}_J(1/x)}{x^2} H_+(x,x,t)$$

weight functions



Froissart-Gribov projections - spin ½

K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

Electric combination:

$$H_{\pm}^{(E)}(x, \cos\theta_t, t) = H_{\pm}(x, \cos\theta_t, t) + \tau E_{\pm}(x, \cos\theta_t, t)$$

$$\tau \equiv t/(4m^2)$$

helicities of $p\bar{p}$ couple to |λ−λ'| = 0

has to be expanded in $P_J(\cos\theta_t)$ rotation function

Magnetic combination:

$$H_{\pm}^{(M)}(x, \cos\theta_t, t) = H_{\pm}(x, \cos\theta_t, t) + E_{\pm}(x, \cos\theta_t, t)$$

helicities of $p\bar{p}$ couple to |λ−λ'| = 1

has to be expanded in $\sin\theta_t P_J'(\cos\theta_t)/\sqrt{J(J+1)}$ rotation function

Froissart-Gribov projections - spin ½

K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

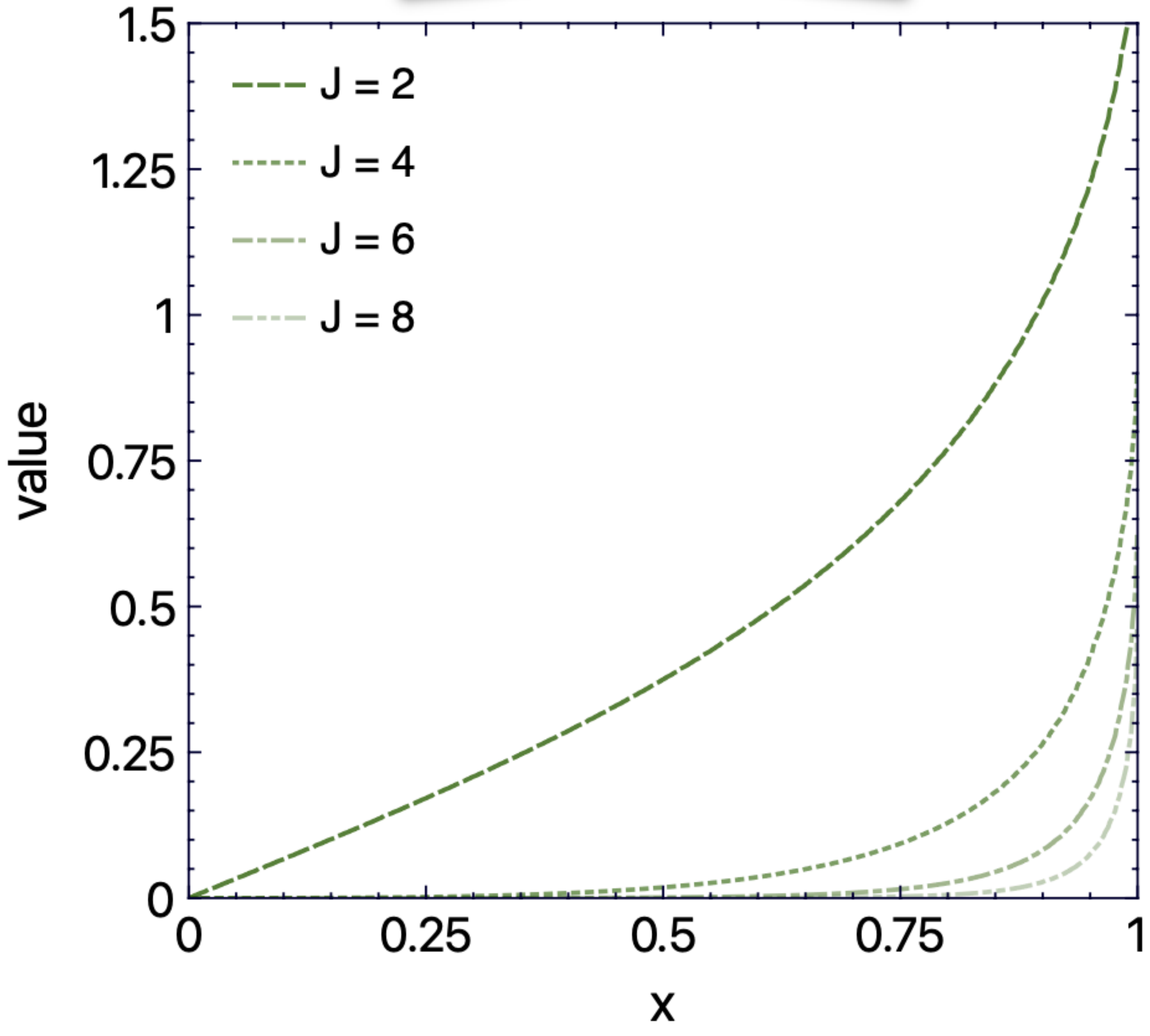
Final result:

$$F^{(E)}_{J=0}(t) = 2\int_0^1 dx \left[\frac{\mathcal{Q}_0(1/x)}{x^2} - \frac{1}{x}\right] H^{(E)}_{+}(x,x,t) + 4(1-\tau)D(t)$$

$$F^{(E)}_{J>0}(t) = 2(2J+1)\int_0^1 dx \frac{\mathcal{Q}_0(1/x)}{x^2} H^{(E)}_{+}(x,x,t)$$

$$F^{(M)}_{J}(t) = 2\int_0^1 dx H^{(M)}_{+}(x,x,t)\frac{2J+1}{J(J+1)}\frac{(-1)}{x}\sqrt{\frac{1}{x^2}-1}\,\mathcal{Q}^1_J(1/x)$$

weight functions
(for $F^{(M)}_J$)

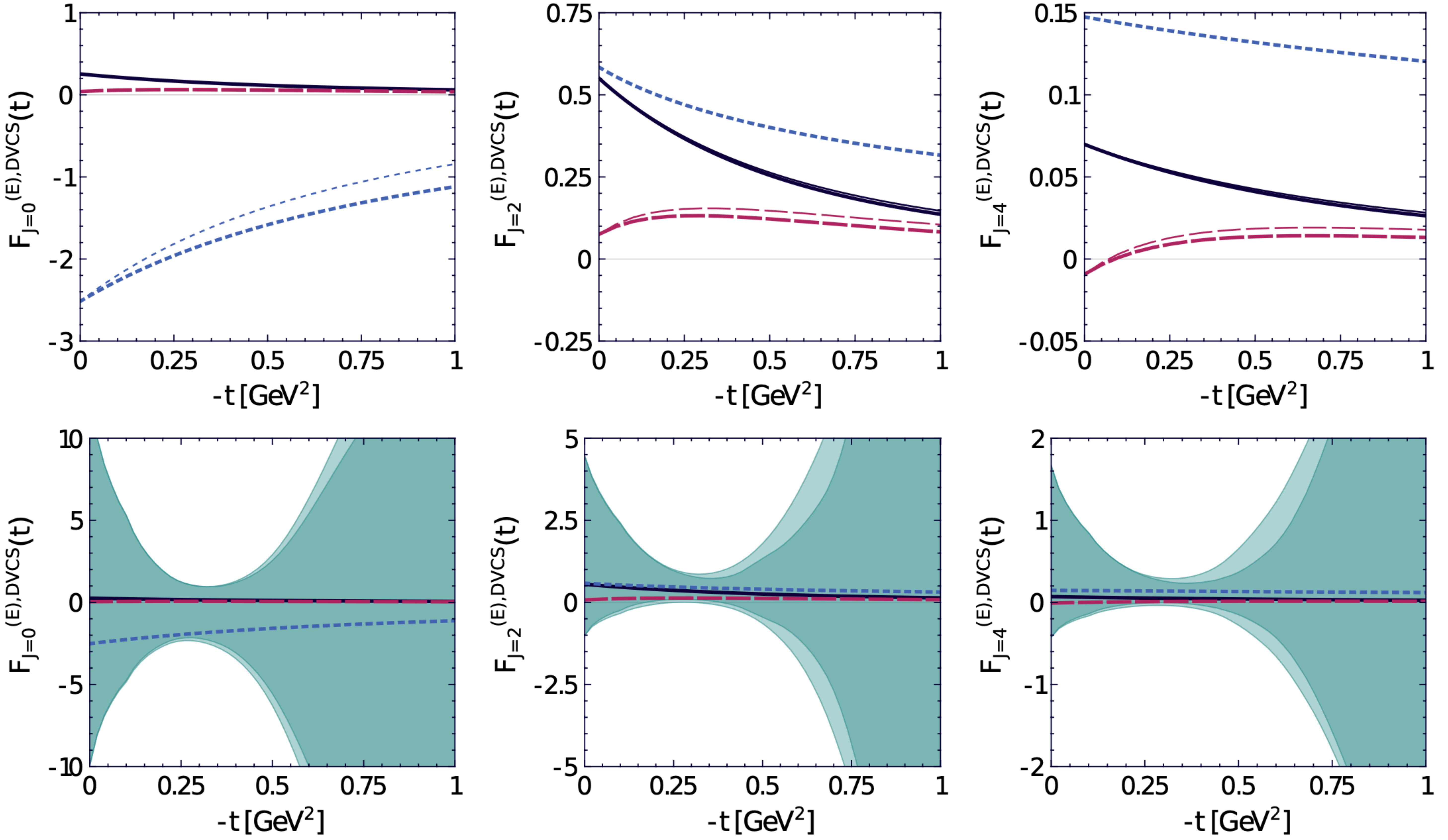


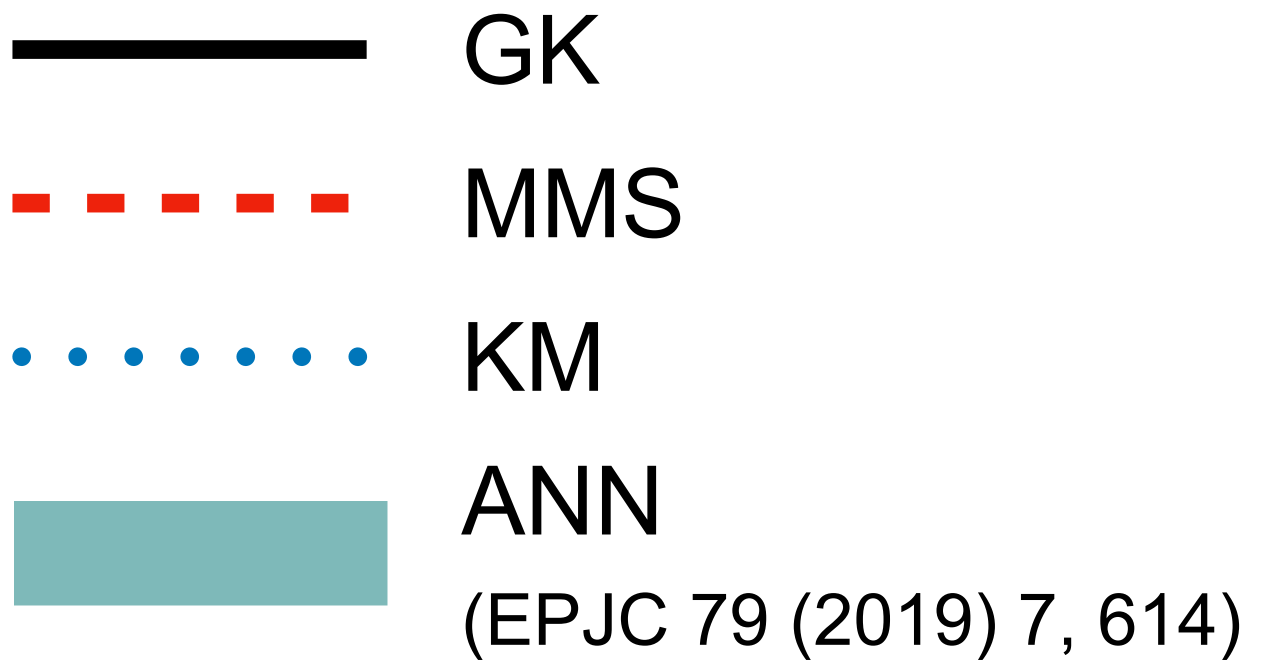
Froissart-Gribov projections - spin ½

K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

Numerical estimates - electric case:

J=0 | J=2 | J=4





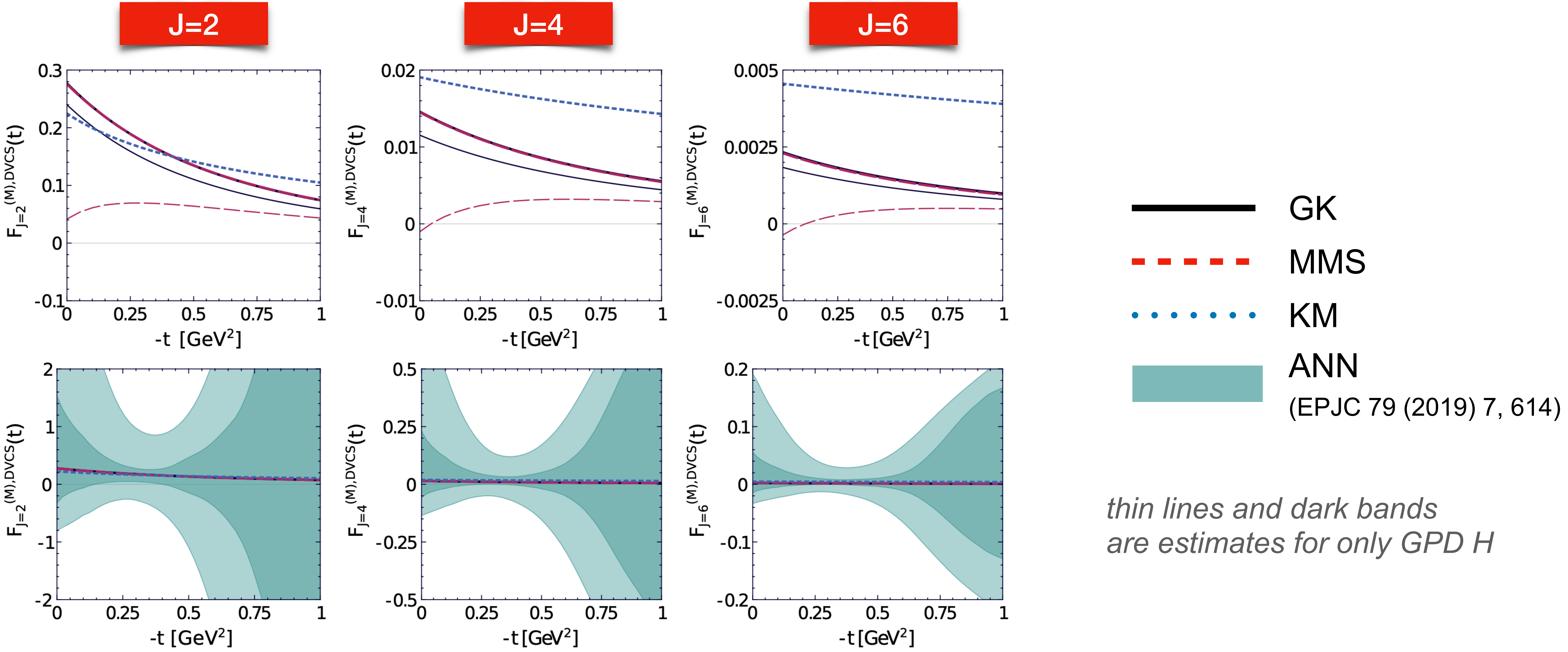
thin lines and dark bands are estimates for only GPD H

plots for Q^2 = 2 GeV^2

Froissart-Gribov projections - spin ½

K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

Numerical estimates - magnetic case:



thin lines and dark bands are estimates for only GPD H

plots for $Q^2 = 2$ GeV^2

See the publication for more,
in particular for sum rules connecting FG projections with Mellin moments

Double DVCS

K. Deja, V. Martínez-Fernández,
B. Pire, PS, J. Wagner
Phys. Rev. D 107 (2023) 9, 094035

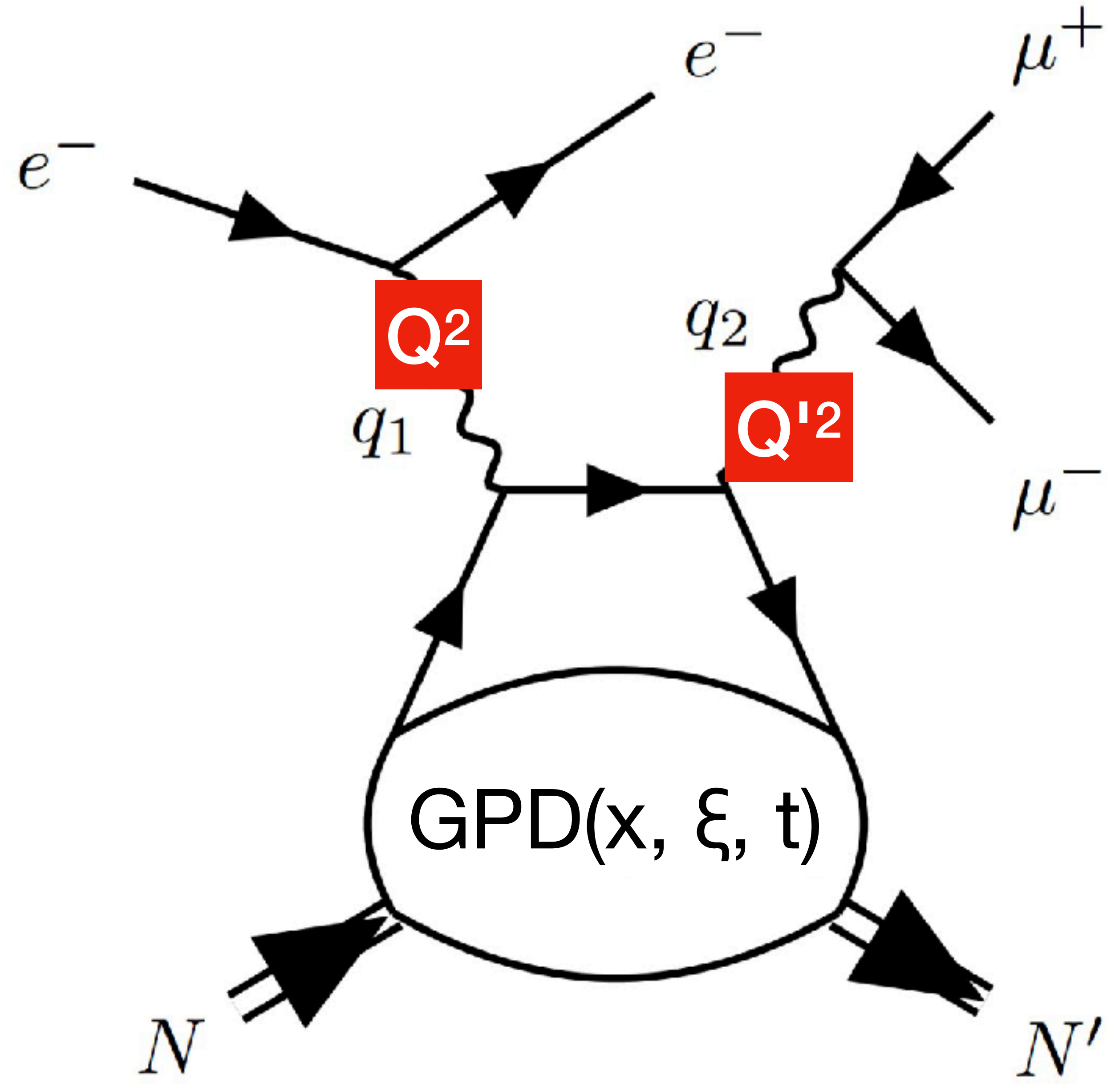
- The process allows to directly probe GPDs outside x=ξ line, but is much more challenging experimentally

$$\xi = \frac{Q^2 + Q'^2}{2Q^2/x_B - Q^2 - Q'^2}$$

$$(\mathcal{H}, \mathcal{E})(\rho, \xi, t) = \sum_{f=\{u,d,s\}} \int_{-1}^{1} dx\, C_f^{(-)}(x, \rho)(H_f, E_f)(x, \xi, t)$$

$$\rho = \xi \frac{Q^2 - Q'^2}{Q^2 + Q'^2}$$

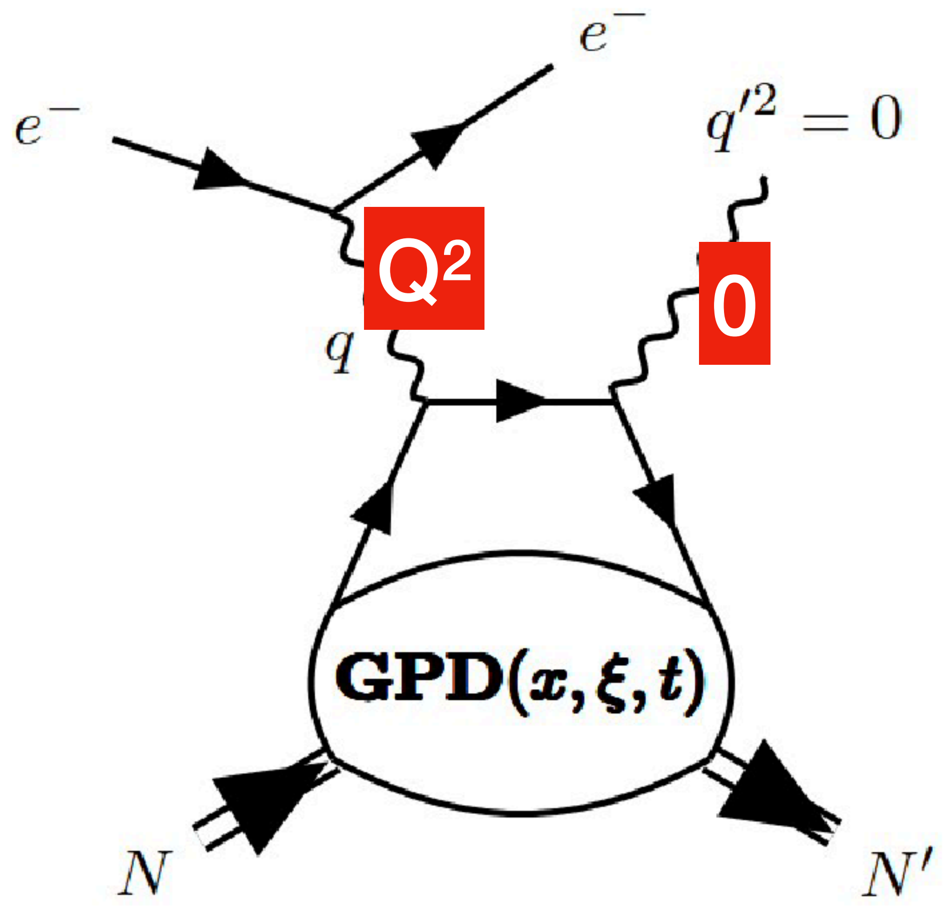
$$C_f^{(\pm)}(x, \rho) \overset{LO}{=} \left(\frac{e_f}{e}\right)^2 \left(\frac{1}{\rho - x - i0} \pm \frac{1}{\rho + x - i0}\right)$$

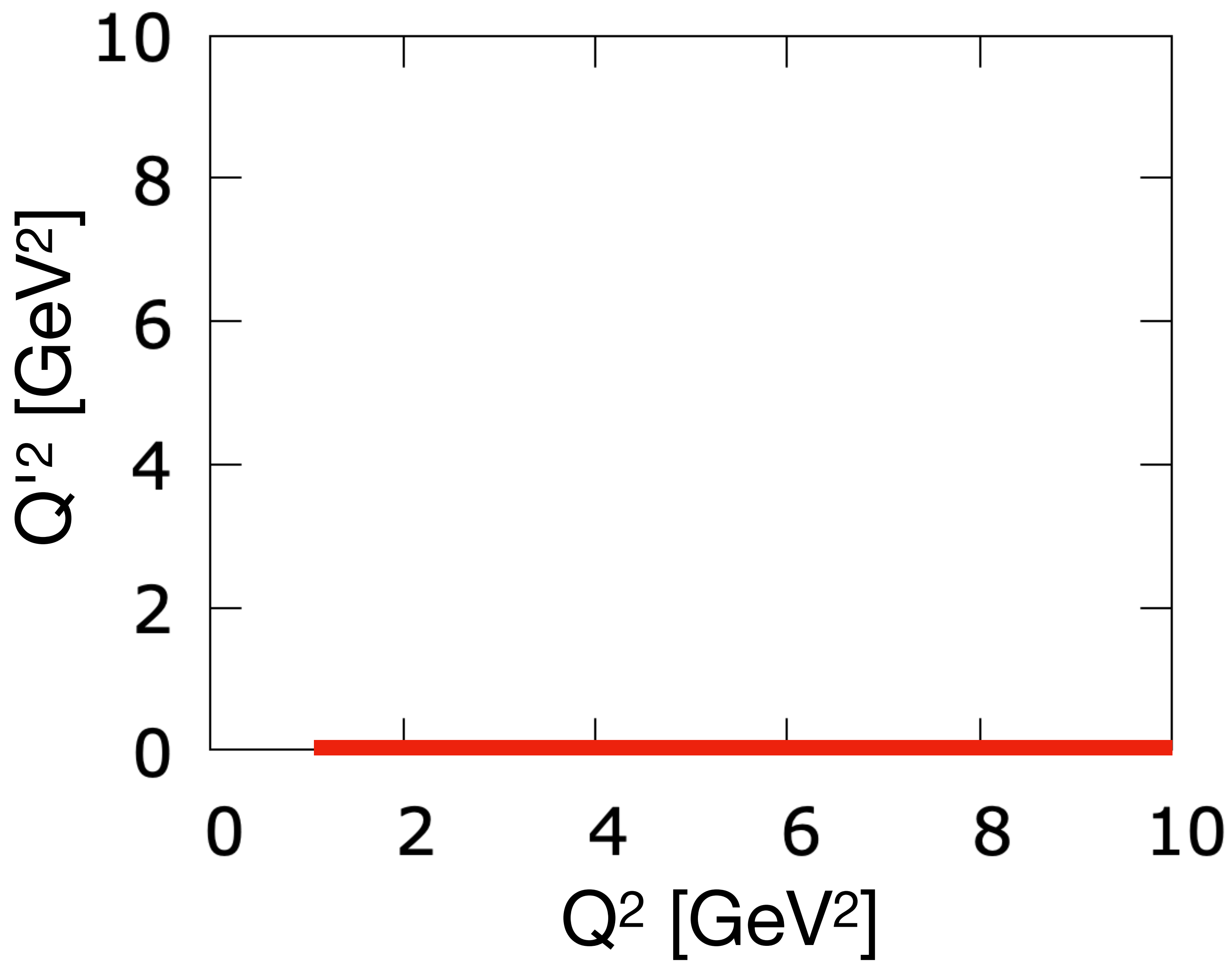


- We revisit DDVCS phenomenology in view of new experiments, including reevaluation of DDVCS and BH cross-sections with Kleiss-Stirling spinor techniques

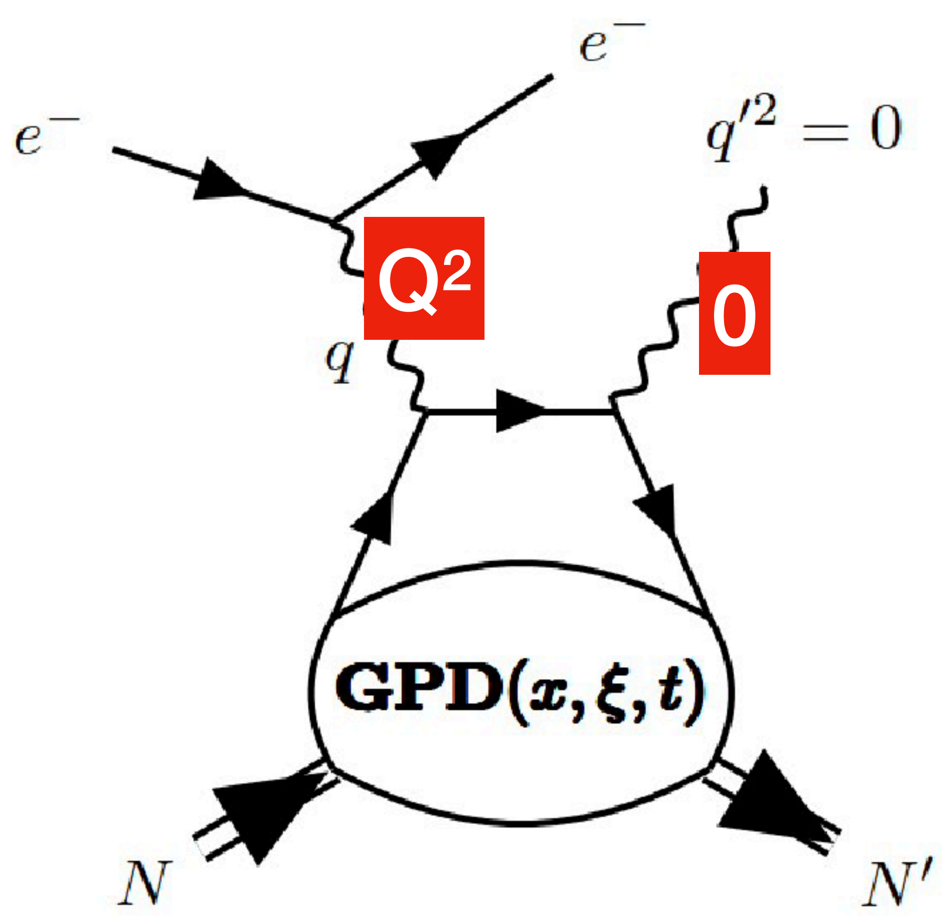
- Obtained results are available in PARTONS and EpIC MC generator

DVCS



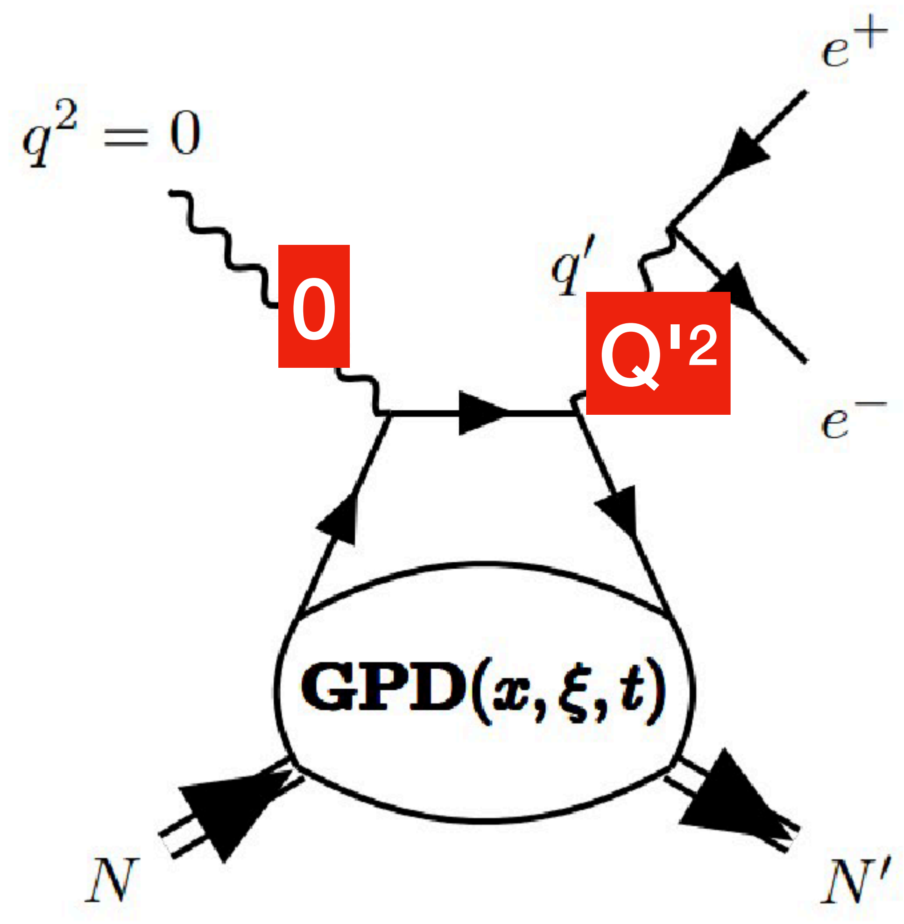


DVCS

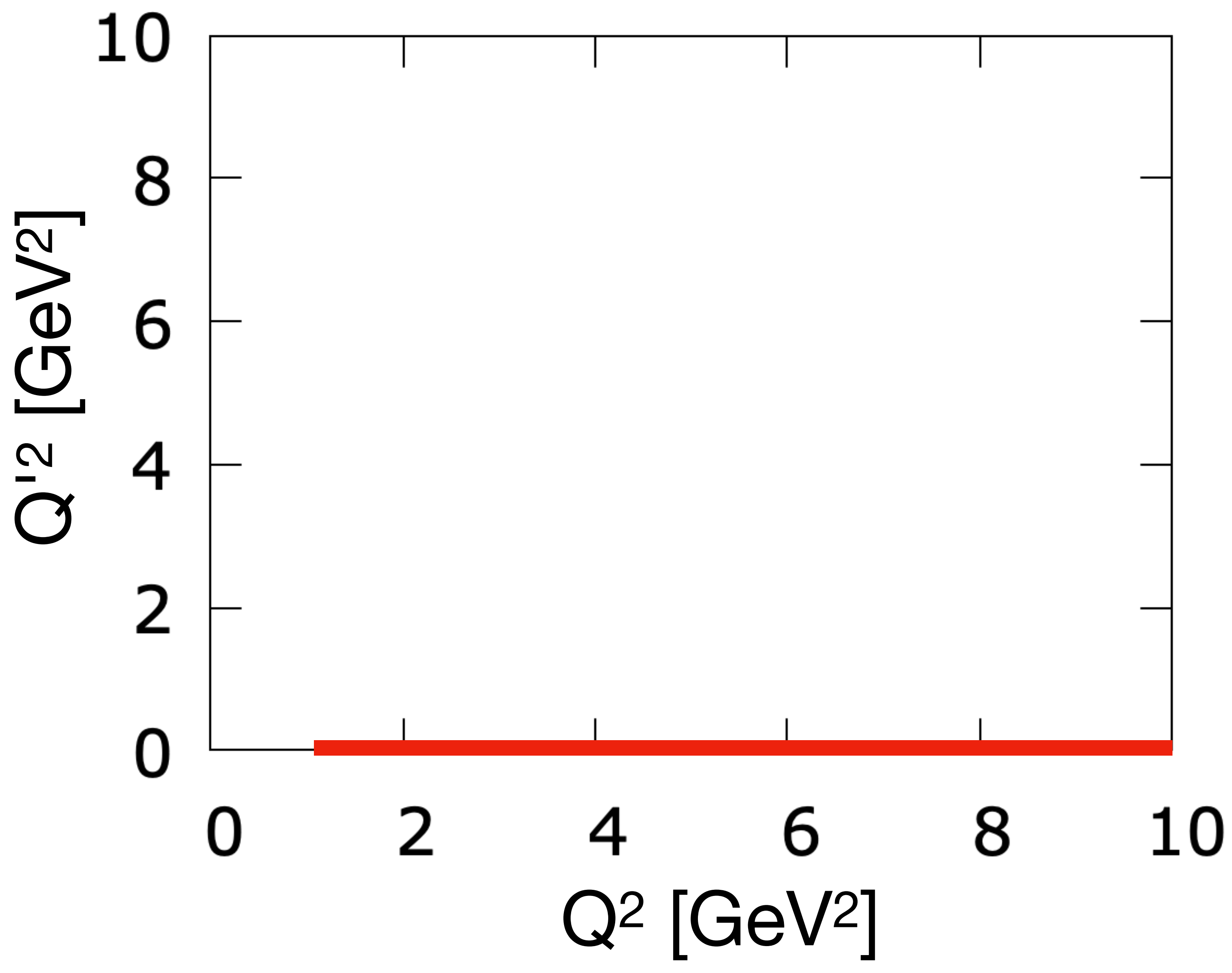


e^-
e^-
$q'^2 = 0$
Q²
0
q
GPD(x, ξ, t)
N
N'

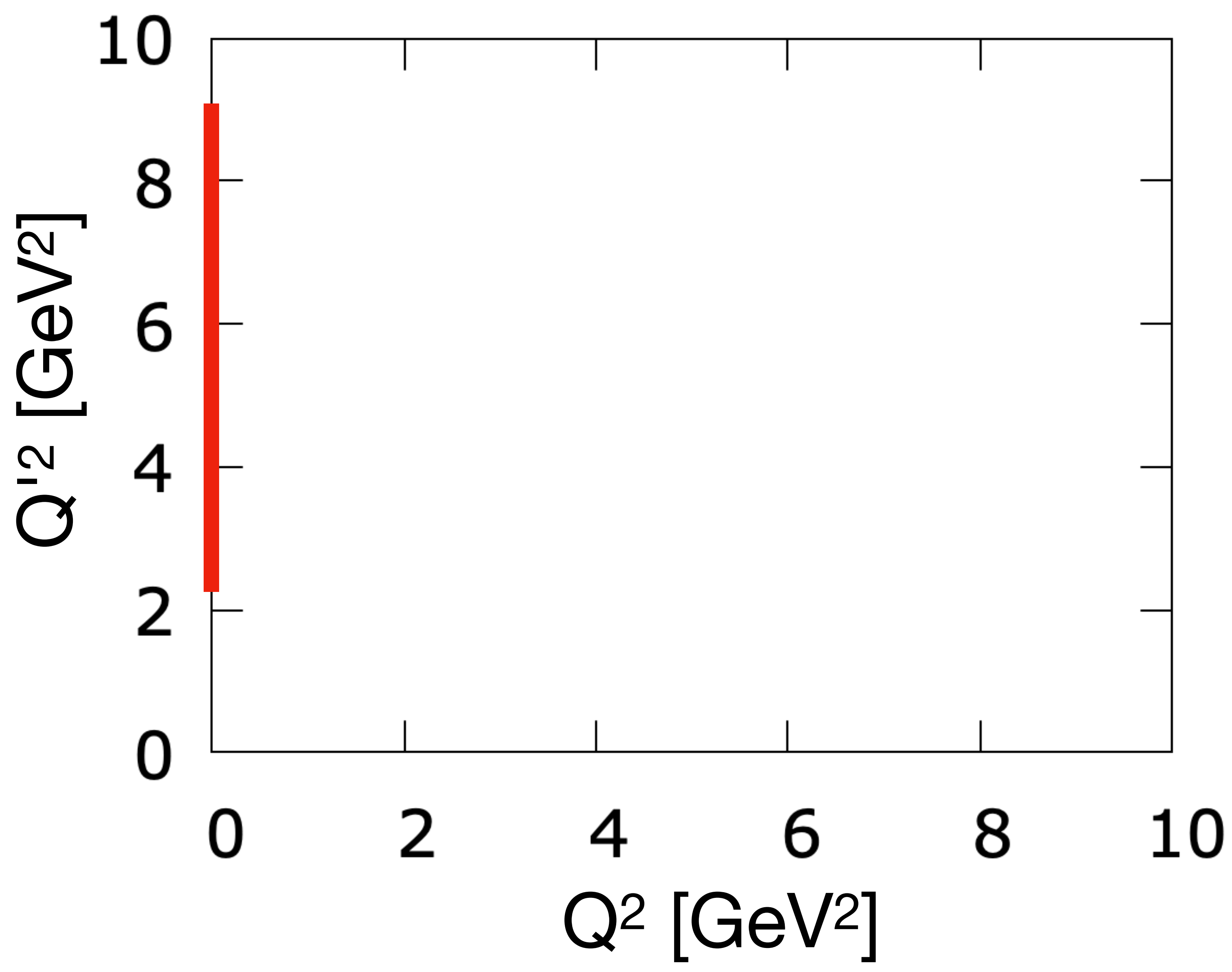
TCS



e^+
$q^2 = 0$
0
q'
Q'²
e^-
GPD(x, ξ, t)
N
N'

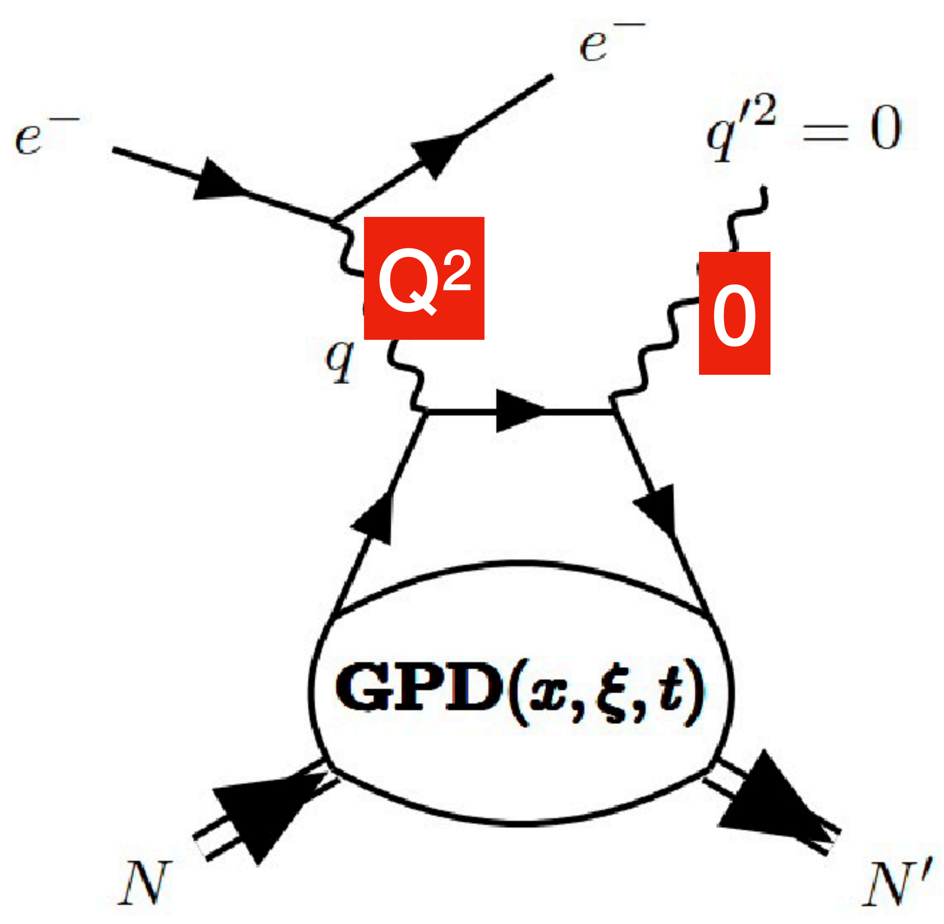


Q'² [GeV²]
Q² [GeV²]
0
2
4
6
8
10

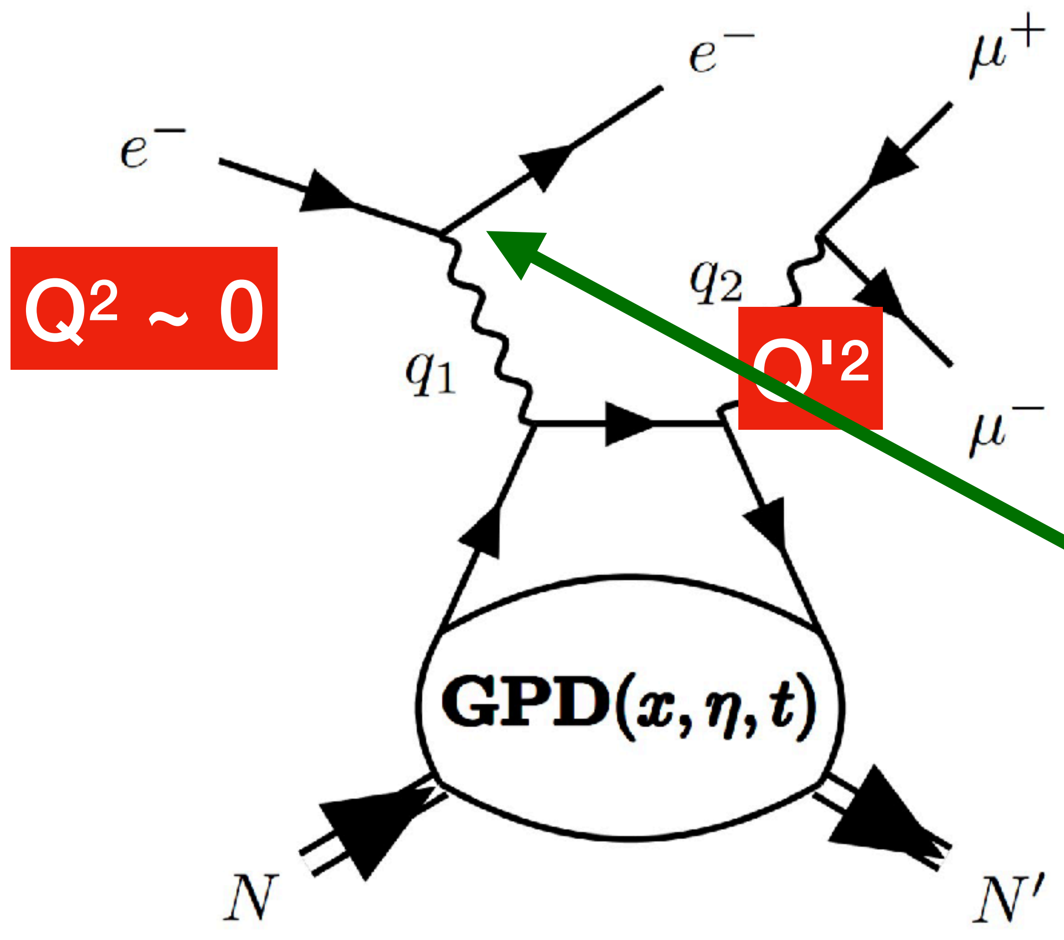


Q'² [GeV²]
Q² [GeV²]
0
2
4
6
8
10

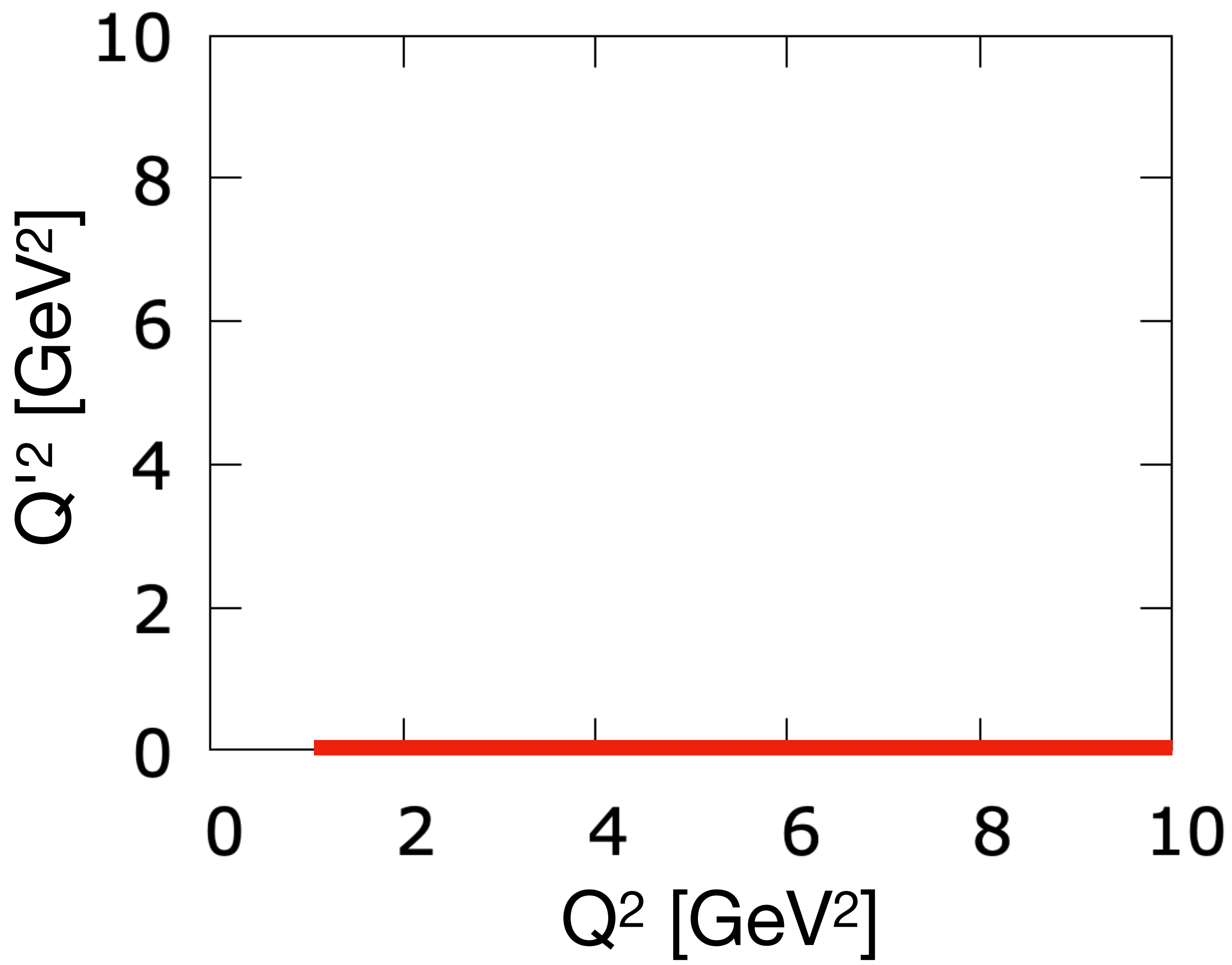
DVCS

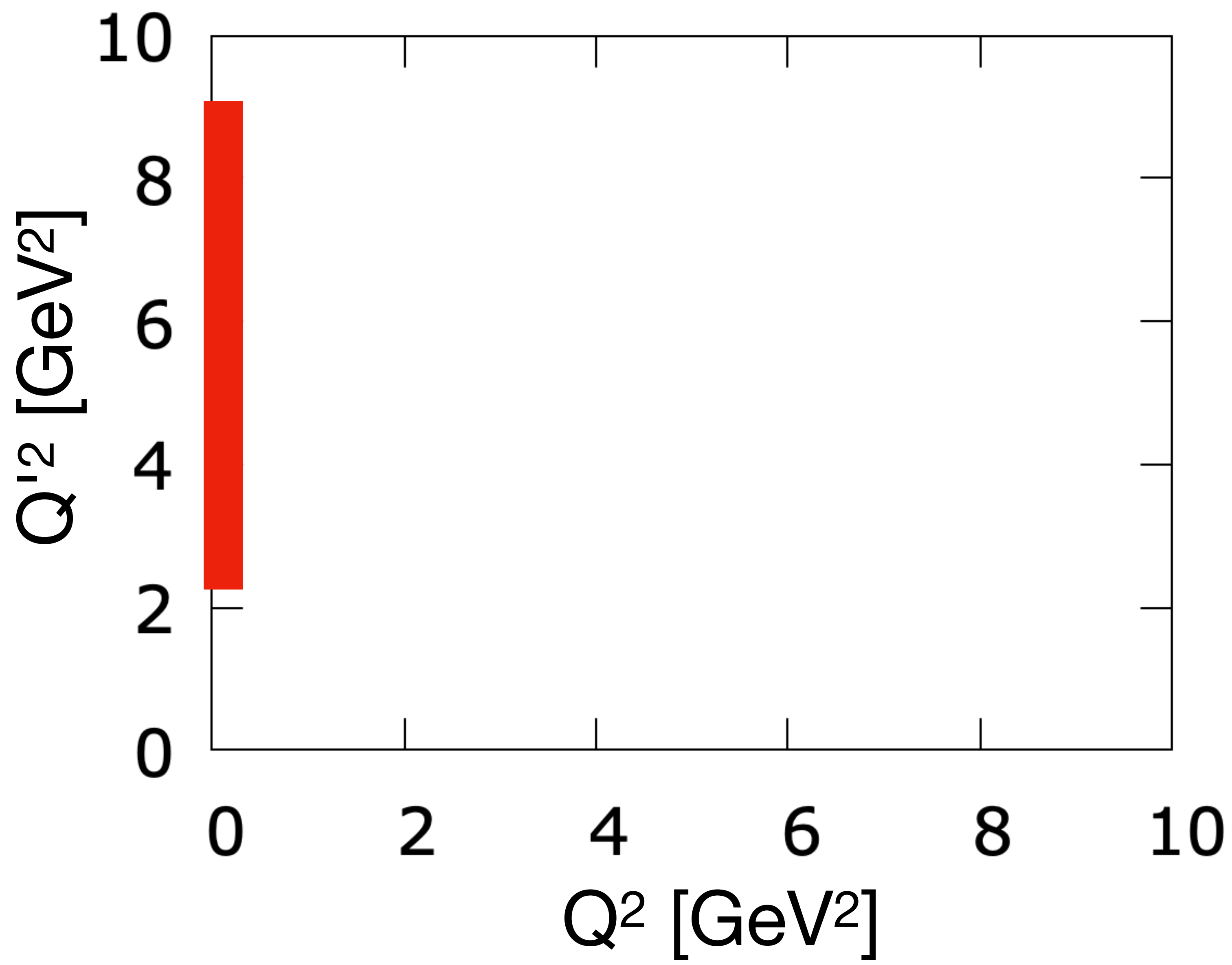


TCS in ep experiments

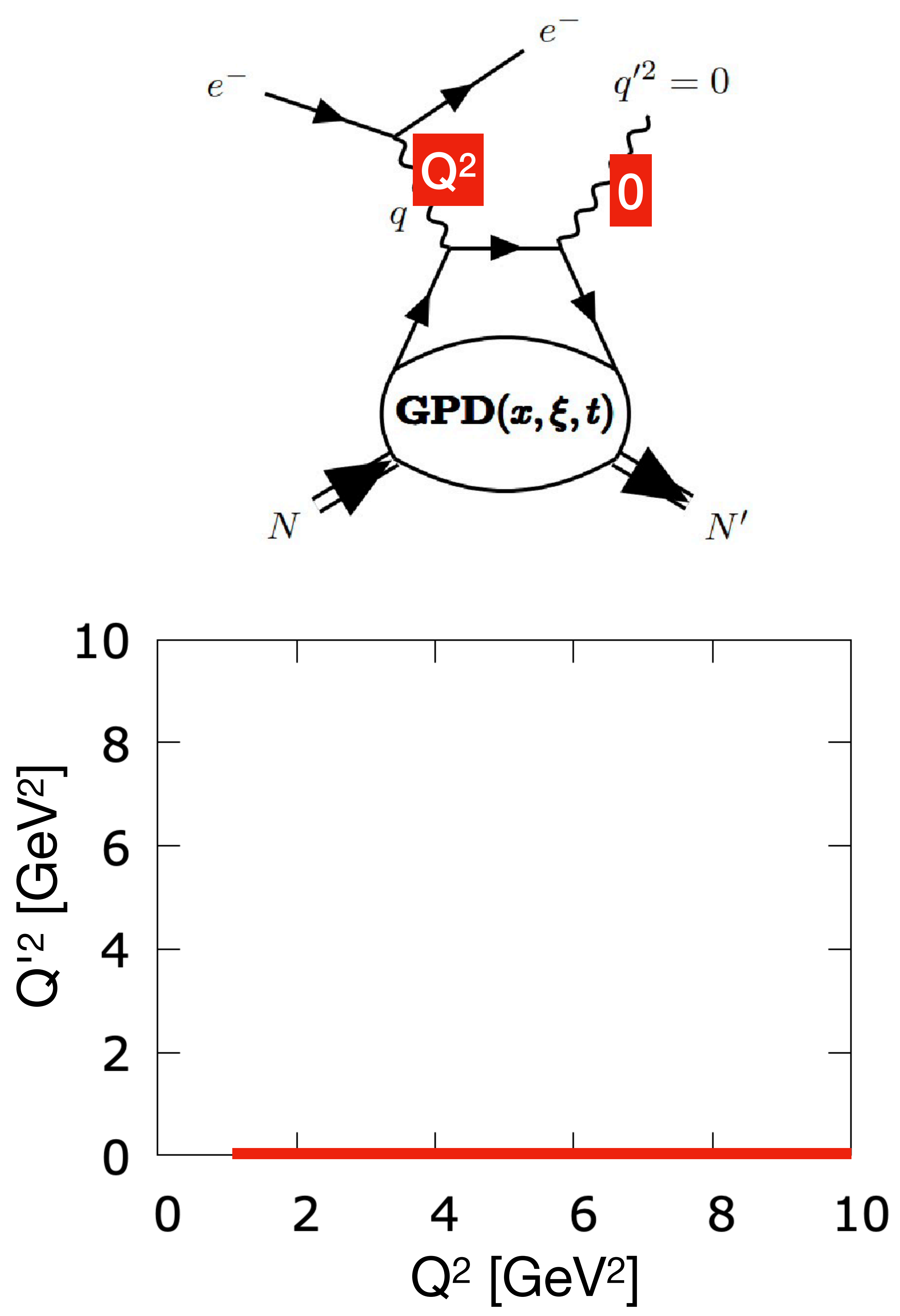


α_{EM} already here (in Γ)!

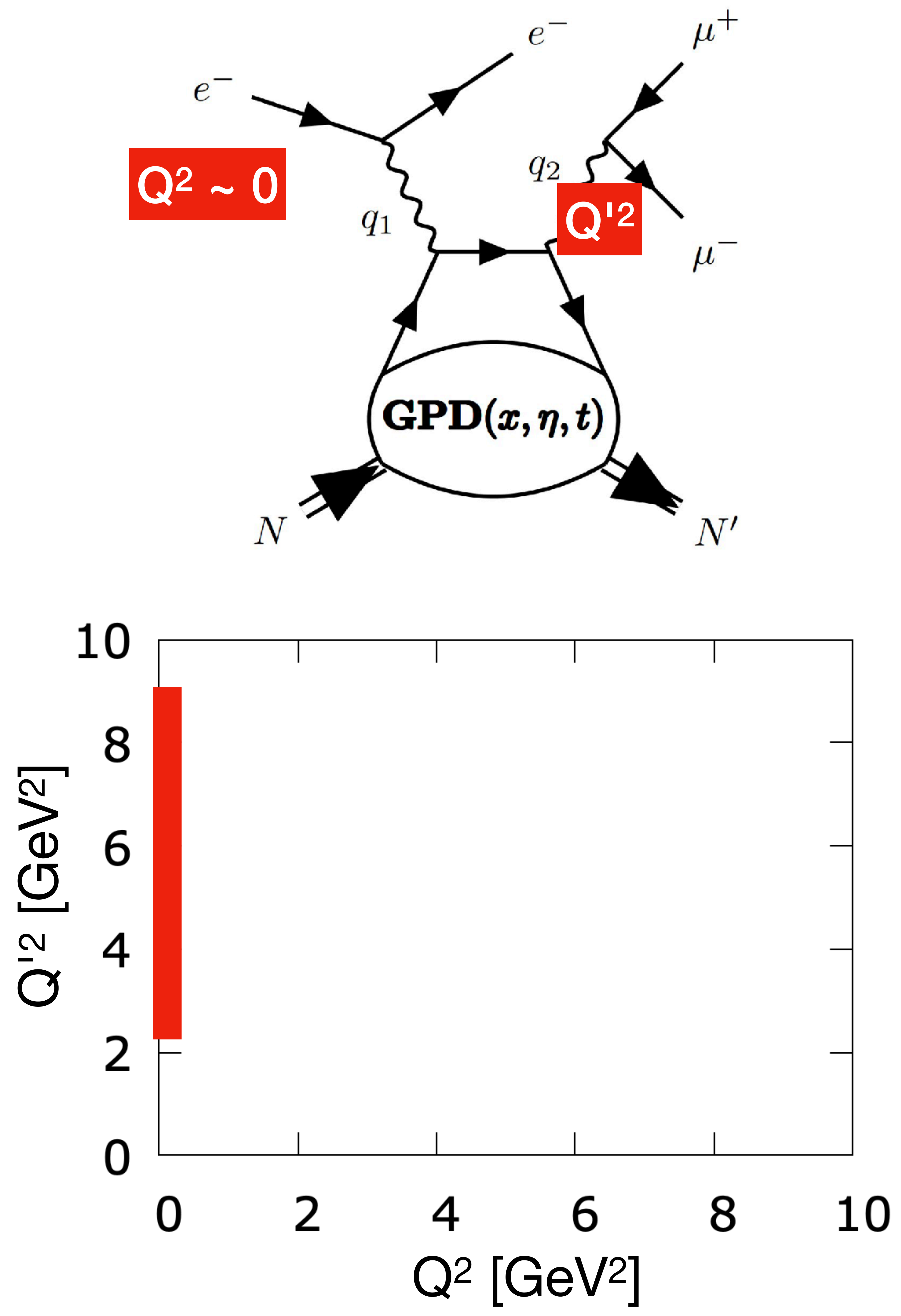




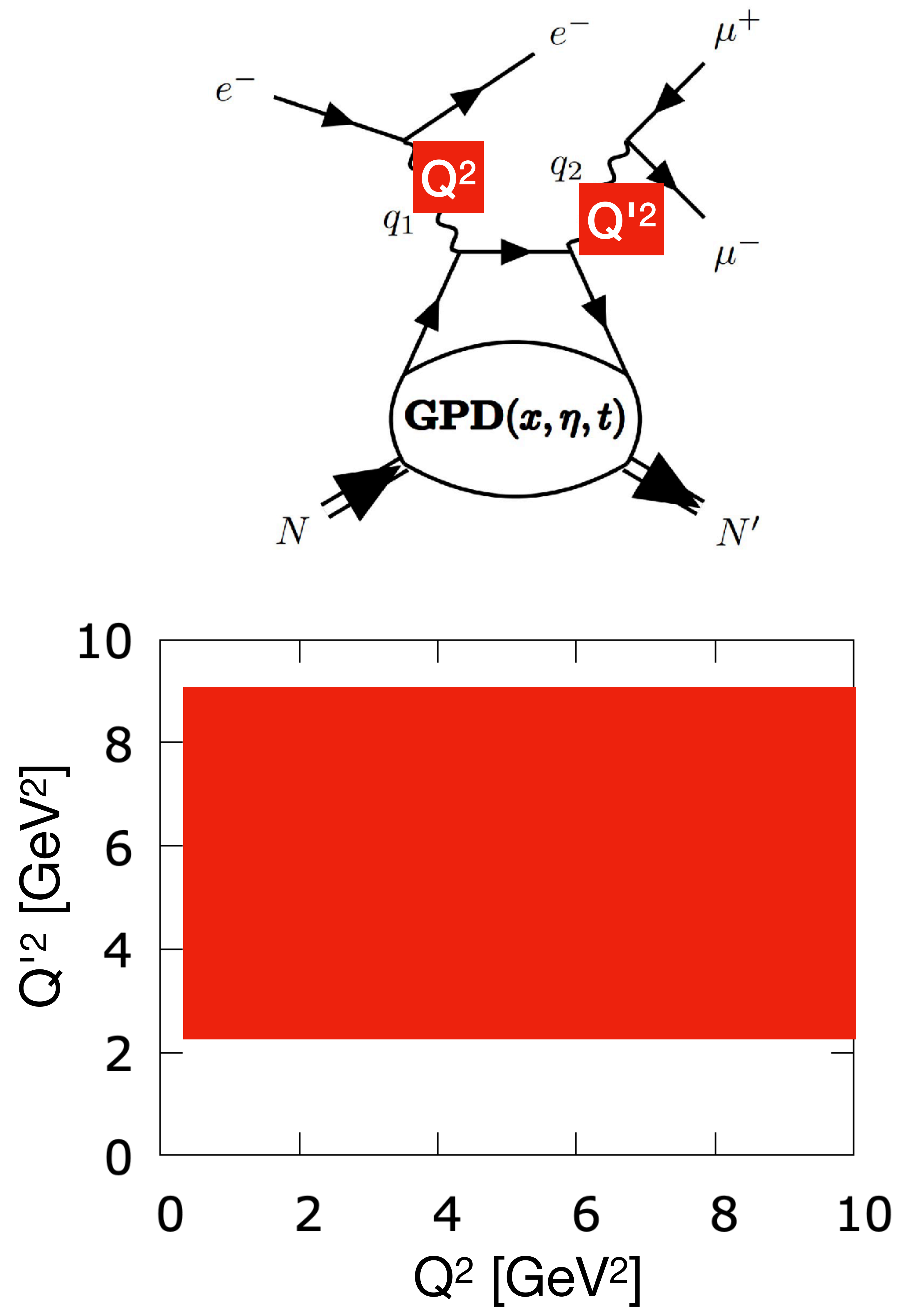
DVCS



TCS in ep experiments



DDVCS



Double DVCS

K. Deja, V. Martínez-Fernández,
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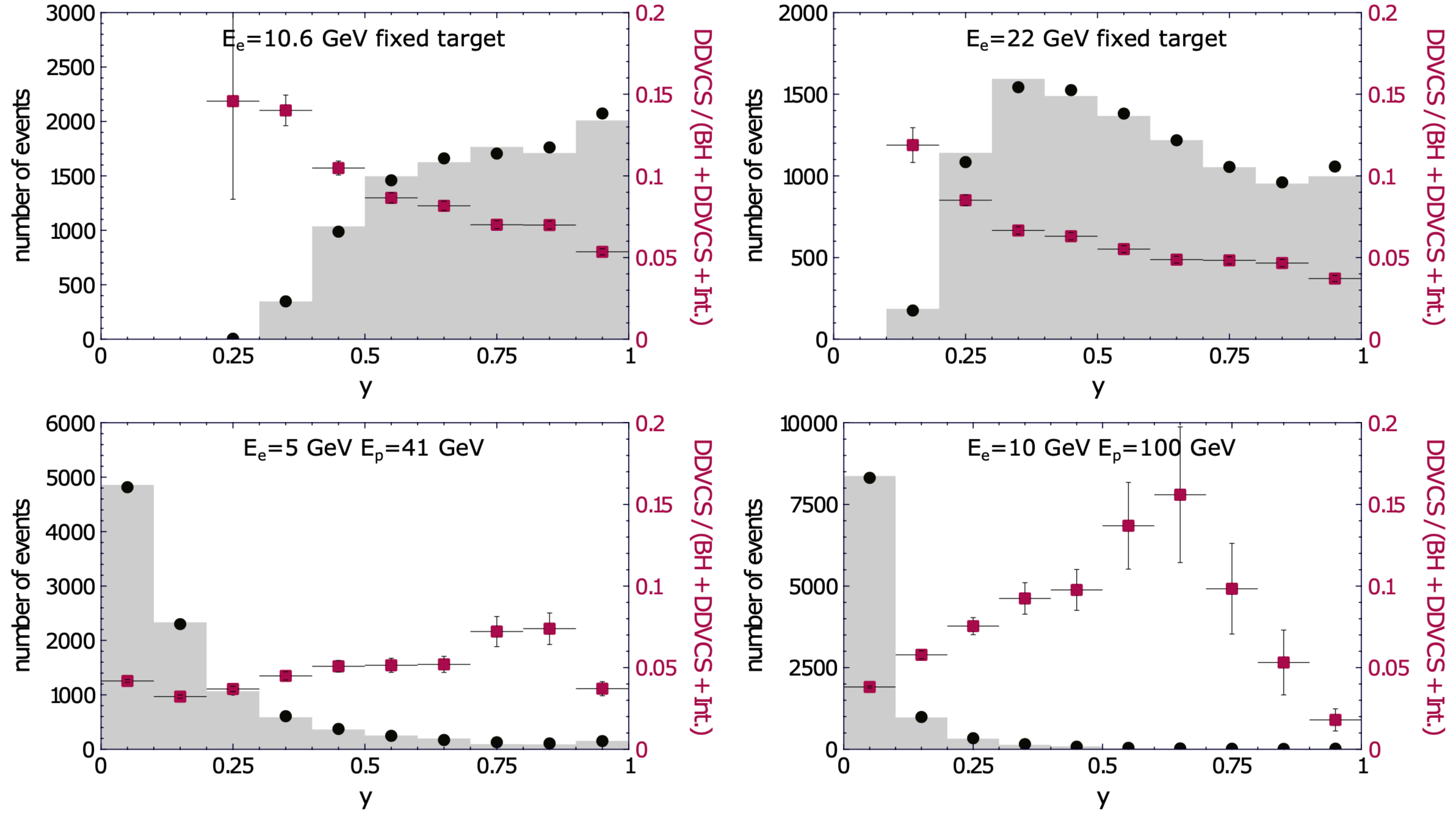
Phys. Rev. D 107 (2023) 9, 094035



EpIC MC
● integrated cross-section
■ pure DDVCS contribution

Kinematic cuts:

- 0.15 GeV^2 < Q^2 < 5 GeV^2
- 2.25 GeV^2 < Q'^2 < 9 GeV^2
- 0.1 GeV^2 < t < 0.8 GeV^2 (JLab)
- 0.05 GeV^2 < t < 1 GeV^2 (EIC)
- 0.1 < φ, φ_l < 2π − 0.1
- π/4 < θ_l < 3π/4
- 0.1 < y < 1 (JLab)
- 0.05 < y < 1 (EIC)

Experiment	Beam energies [GeV]	Range of $\|t\|$ [GeV^2]	$\sigma\|_{0<y<1}$ [pb]	$\mathcal{L}^{10k}\|_{0<y<1}$ [fb^{-1}]	$y_{\min}$	$\sigma\|_{y_{\min}<y<1}/\sigma\|_{0<y<1}$
JLab12	$E_e = 10.6,\ E_p = M$	$(0.1, 0.8)$	0.14	70	0.1	1
JLab2+	$E_e = 22,\ E_p = M$	$(0.1, 0.8)$	0.46	22	0.1	1
EIC	$E_e = 5,\ E_p = 41$	$(0.05, 1)$	3.9	2.6	0.05	0.73
EIC	$E_e = 10,\ E_p = 100$	$(0.05, 1)$	4.7	2.1	0.05	0.32

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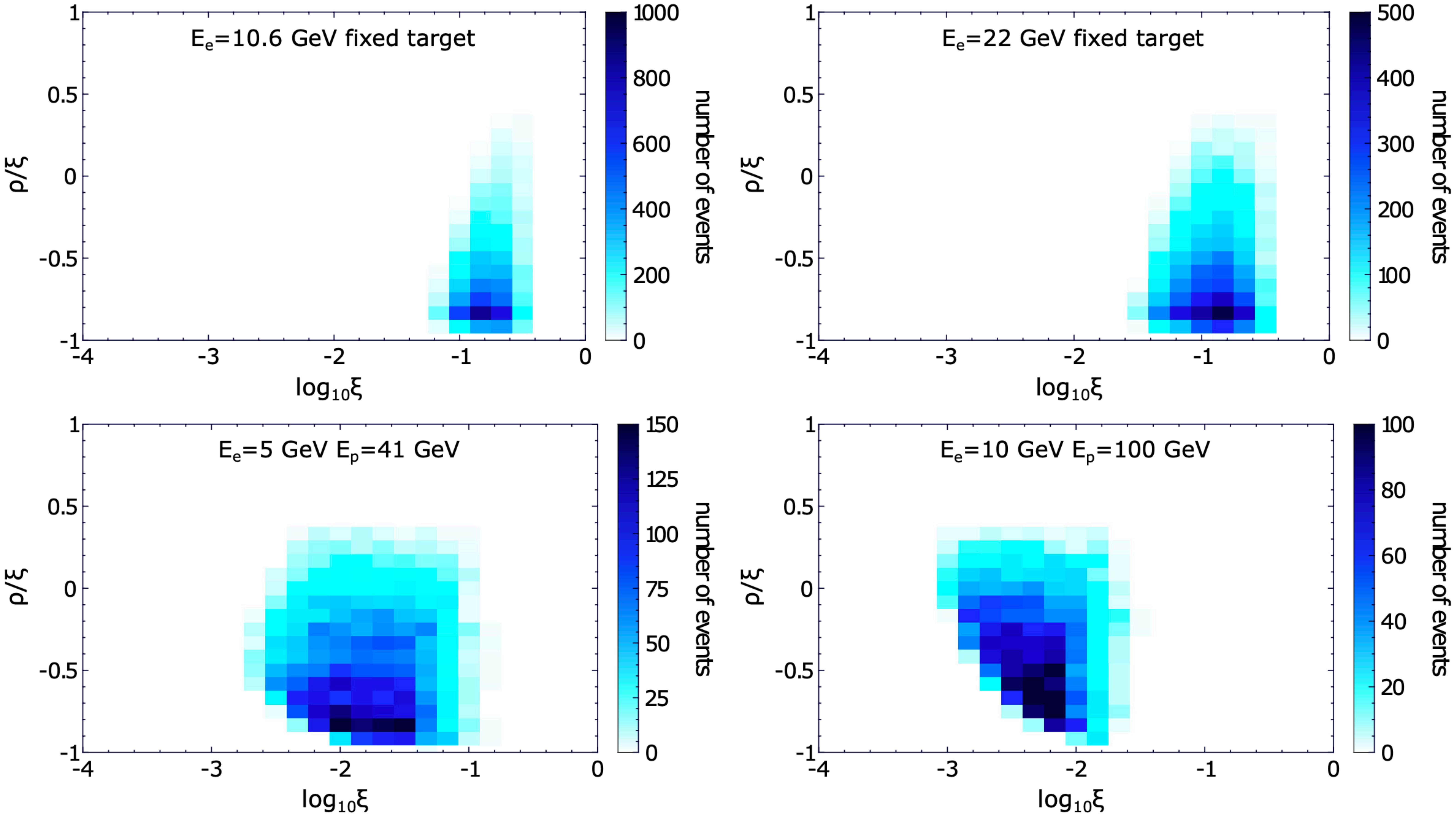


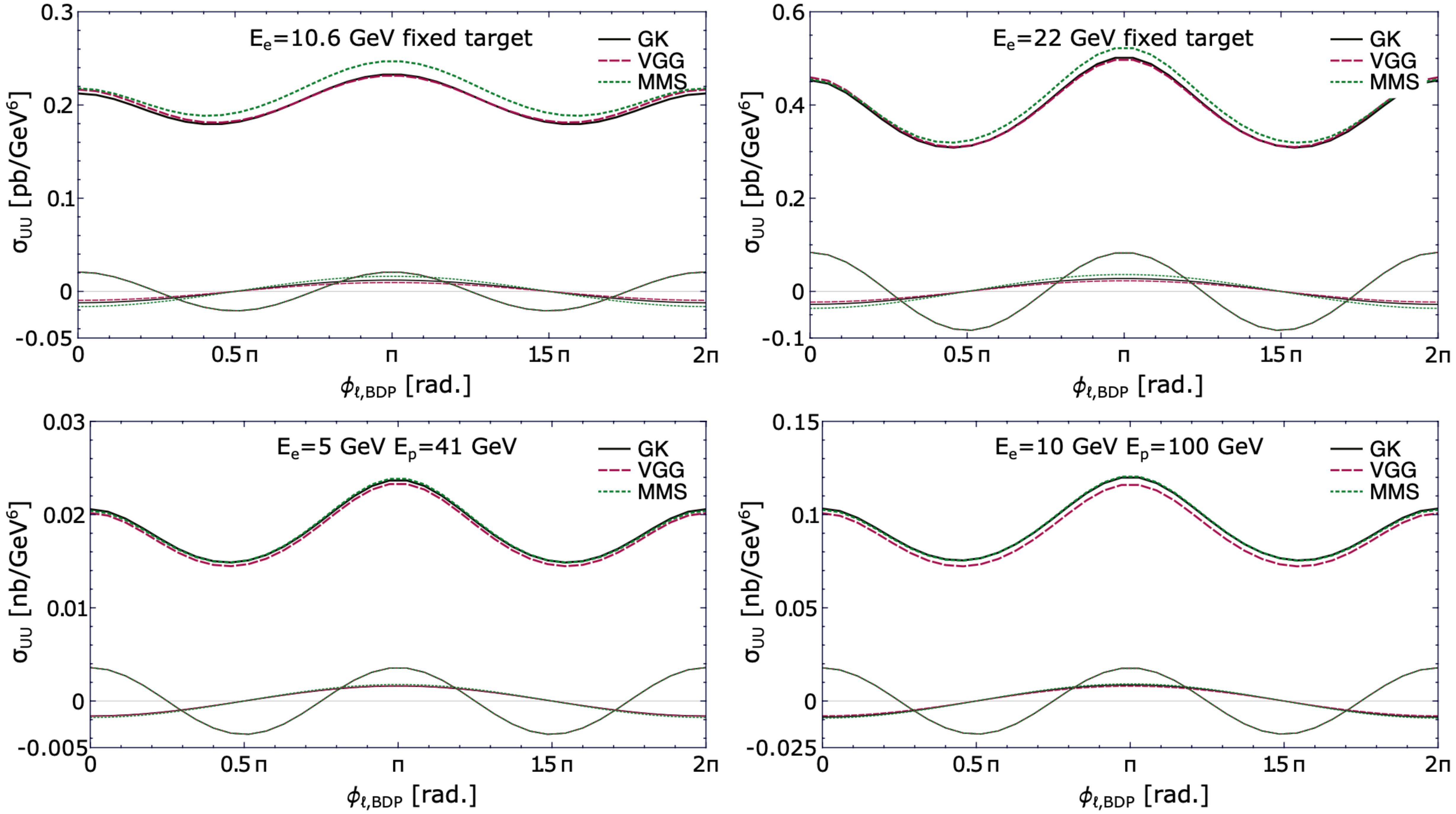
Kinematic cuts:

- $0.15\ \mathrm{GeV}^2 < Q^2 < 5\ \mathrm{GeV}^2$
- $2.25\ \mathrm{GeV}^2 < Q'^2 < 9\ \mathrm{GeV}^2$
- $0.1\ \mathrm{GeV}^2 < t < 0.8\ \mathrm{GeV}^2$ (JLab)
- $0.05\ \mathrm{GeV}^2 < t < 1\ \mathrm{GeV}^2$ (EIC)
- $0.1 < \varphi, \varphi_l < 2\pi - 0.1$
- $\pi/4 < \theta_l < 3\pi/4$
- $0.1 < y < 1$ (JLab)
- $0.05 < y < 1$ (EIC)

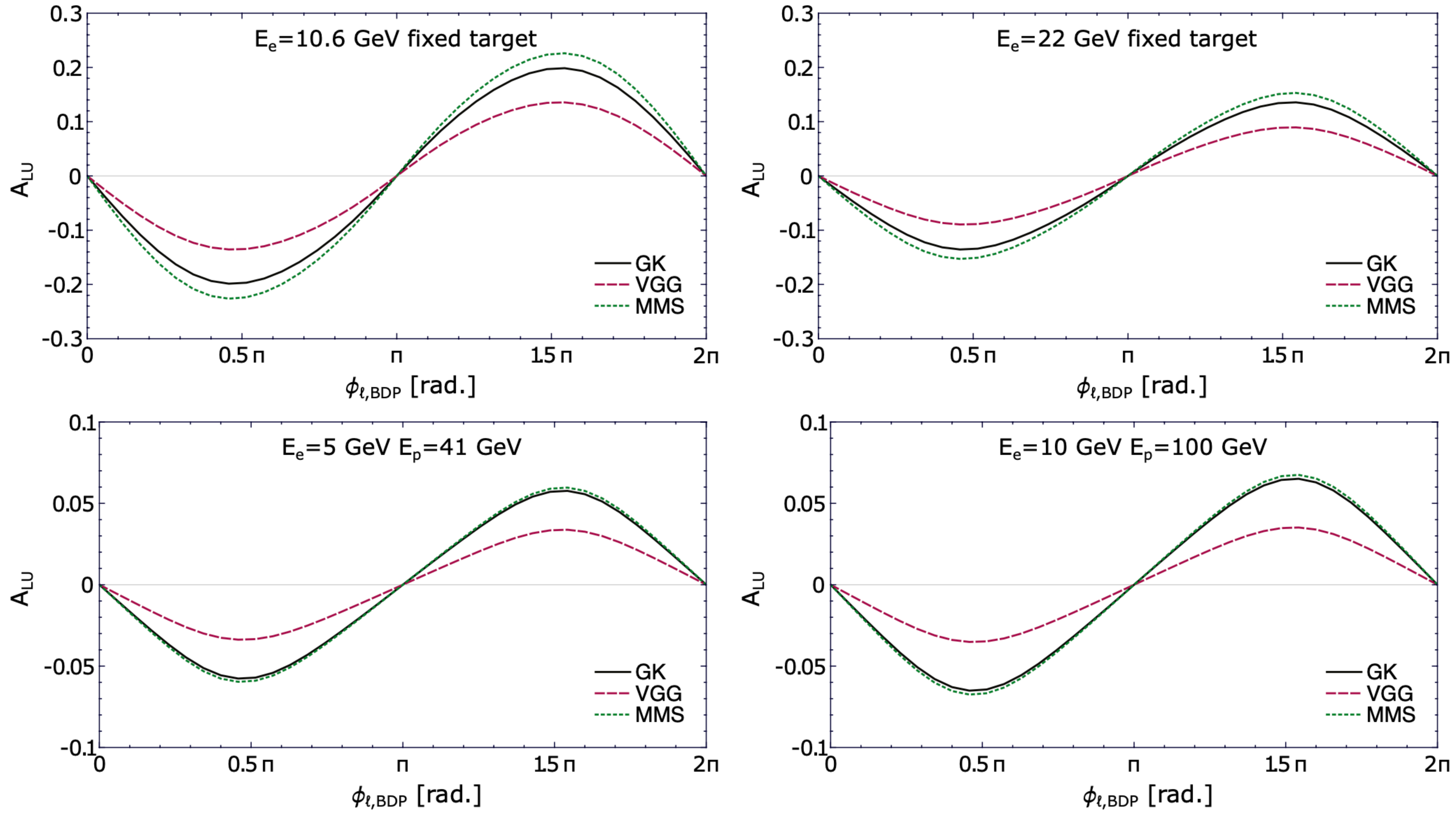
Double DVCS

K. Deja, V. Martínez-Fernández,
B. Pire, PS, J. Wagner
Phys. Rev. D 107 (2023) 9, 094035

Unpolarised cross-section integrated over
$0 < \phi < 2\pi$ and $\pi/4 < \theta_l < 3\pi/4$



corresponding ALU asymmetry



Experiment	Beam energies [GeV]	y	$\|t\|$ [GeV^2]	Q^2 [GeV^2]	Q'^2 [GeV^2]
JLab12	$E_e = 10.6,\ E_p = M$	0.5	0.2	0.6	2.5
JLab2+	$E_e = 22,\ E_p = M$	0.3	0.2	0.6	2.5
EIC	$E_e = 5,\ E_p = 41$	0.15	0.1	0.6	2.5
EIC	$E_e = 10,\ E_p = 100$	0.15	0.1	0.6	2.5

GK
VGG
MMS

Double DVCS

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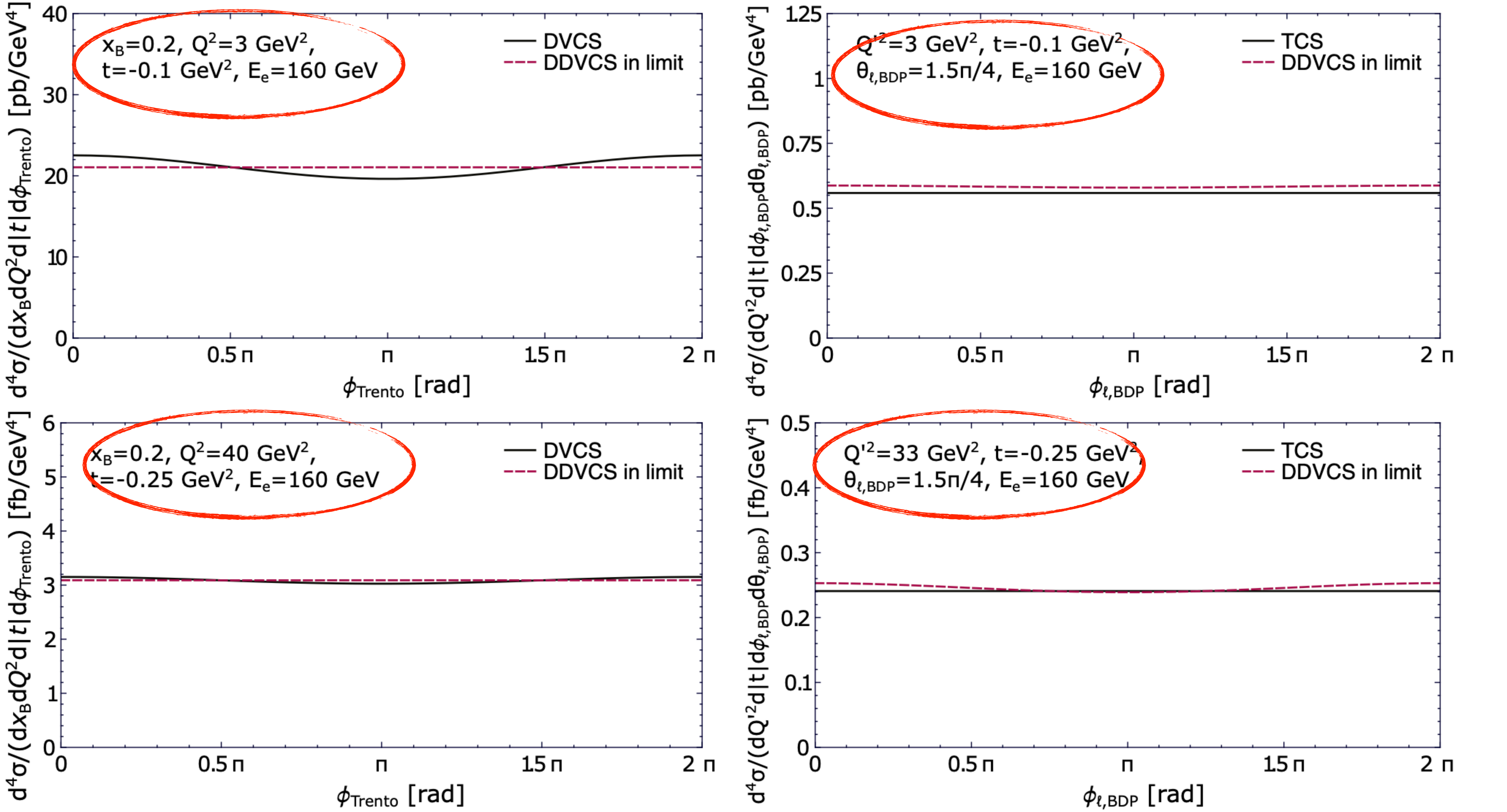
DVCS limit

TCS limit



Double DVCS at twist-4

V. Martínez-Fernández,
B. Pire, PS, J. Wagner
preliminary

- Starting point: OPE + CFT (Braun-Ji-Manashov result)
  (see: JHEP 03 (2021) 051 and JHEP 01 (2023) 078)

$$T^{\mu\nu} = i \int d^4z \, e^{iq'z} \langle p' | \mathcal{T}\{ j^\mu(z) j^\nu(0) \} | p \rangle$$

$$= \frac{1}{i\pi^2} i \int d^4z \, e^{iq'z} \left\{ \frac{1}{(-z^2 + i0)^2} \left[g^{\mu\nu} \mathcal{O}(1,0) - z^\mu \partial^\nu \int_0^1 du \, \mathcal{O}(\bar{u},0) - z^\nu (\partial^\mu - i\Delta^\mu) \int_0^1 dv \, \mathcal{O}(1,v) \right] \right. + \ldots$$

where $\mathcal{O}, \mathcal{O}_1, \mathcal{O}_2$ are matrix elements $\langle p'|\mathcal{O}|p\rangle, \langle p'|\mathcal{O}_1|p\rangle, \langle p'|\mathcal{O}_2|p\rangle$ containing information about GPDs

- For spin-0 target:

$$T^{\mu\nu} = \mathcal{A}^{00} \frac{-i}{QQ'R^2} \left[(qq')(Q'^2 q^\mu q^\nu - Q^2 q'^\mu q'^\nu) + Q^2 Q'^2 q^\mu q'^\nu - (qq')^2 q'^\mu q^\nu \right]$$
$$+ \mathcal{A}^{+0} \frac{i\sqrt{2}}{R|\bar{p}_\perp|} \left[Q' q^\mu - \frac{qq'}{Q'} q'^\mu \right] \bar{p}_\perp^\nu + \mathcal{A}^{0+} \frac{\sqrt{2}}{R|\bar{p}_\perp|} \bar{p}_\perp^\mu \left[\frac{qq'}{Q} q^\nu + Q q'^\nu \right]$$
$$+ \mathcal{A}^{+-} \frac{1}{|\bar{p}_\perp|^2} \left[\bar{p}_\perp^\mu \bar{p}_\perp^\nu - \tilde{\bar{p}}_\perp^\mu \tilde{\bar{p}}_\perp^\nu \right] + \mathcal{A}^{++} g_\perp^{\mu\nu} ,$$

$$R = \sqrt{(qq')^2 + Q^2 Q'^2}$$

Double DVCS at twist-4

V. Martínez-Fernández,
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preliminary

$$
\begin{aligned}
\mathcal{A}^{++} &= \mathcal{A}^{++}\big|_{\mathrm{LT}} + I_{(0)} + I_{(\mathrm{i})} + I_{(\mathrm{ii})} + I_{(\mathrm{iii})} + I_{(\mathrm{iv})} + I_{(\mathrm{v})} + I_{(\mathrm{vi})} + I_{(\mathrm{vii})} \\
&= \frac{1}{2}\int_{-1}^{1} dx \left\{ -\left(1 - \frac{t}{2\mathbb{Q}^2} + \frac{t(\xi-\rho)}{\mathbb{Q}^2}\partial_\xi\right) \frac{H^{(+)}}{x-\rho+i0} \right. \\
&+ \frac{t}{\xi\mathbb{Q}^2}\left[\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\widetilde{\mathbb{P}}_{(\mathrm{i})} - \widetilde{\mathbb{P}}_{(\mathrm{iii})}}{2} - \frac{\xi(J+L)}{2} \right. \\
&- \left. \frac{\xi}{x+\xi}\left(\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right) - \frac{\xi+\rho}{2\xi}\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \widetilde{\mathbb{P}}_{(\mathrm{i})}\right)\right] H^{(+)} \\
&- \frac{t}{\mathbb{Q}^2}\partial_\xi\left[\left(\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\xi L}{2} - \frac{\xi}{x+\xi}\left(\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right) - \frac{\xi+\rho}{2\xi}\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \widetilde{\mathbb{P}}_{(\mathrm{i})}\right)\right) H^{(+)}\right] \\
&+ \left. \frac{\vec{p}_\perp^{\,2}}{\mathbb{Q}^2} 2\xi^3 \partial_\xi^2 \left[\left(\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\widetilde{\mathbb{P}}_{(\mathrm{i})} - \widetilde{\mathbb{P}}_{(\mathrm{iii})}}{2} - \frac{\xi L}{2} + \ln\left(\frac{x-\rho+i0}{x-\xi+i0}\right)\right) H^{(+)}\right]\right\} + O(\text{tw-6})
\end{aligned}
$$

Double DVCS at twist-4

V. Martínez-Fernández, B. Pire, PS, J. Wagner preliminary

$$L = \int_0^1 dw\, \frac{-4}{x-\xi-w(x+\xi)} \int_0^1 du\, \ln\left(\xi-\rho+i0+\bar{u}(x-\xi-w(x+\xi))\right)\left[\ln\left(\frac{\bar{u}(1-w)}{1-\bar{u}w}\right)+\frac{1}{1-\bar{u}w}\right]$$

$$J = \widetilde{J} - \frac{2(\xi-\rho)}{-\xi-\rho+i0}\frac{1}{x-\xi}\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right)$$

$$\begin{aligned}
\widetilde{J} = {} & \frac{2(\xi-\rho)}{-\xi-\rho+i0}\frac{1}{x-\xi}\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right) \\
& + \frac{2(x+\rho)}{(x-\xi)(x+\xi)}\Bigg[-\mathrm{Li}_2\left(-\frac{x+\rho}{-\xi-\rho+i0}\right) - \mathrm{Li}_2\left(\frac{2\xi}{x+\xi}\right) + \mathrm{Li}_2\left(-\frac{x+\rho}{-\xi-\rho+i0}\frac{2\xi}{x+\xi}\right) \\
& \quad + \mathrm{Li}_2\left(\frac{x-\rho}{\xi-\rho+i0}\right) + \mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right) + \mathrm{Li}_2\left(\frac{x-\rho}{x+\xi}\right) \\
& \quad - \mathrm{Li}_2\left(\frac{\xi-\rho+i0}{x+\xi}\right) - \mathrm{Li}_2\left(\frac{x+\rho}{x+\xi}\right) + \mathrm{Li}_2\left(-\frac{-\xi-\rho+i0}{x+\xi}\right) \\
& \quad + \mathrm{Li}_2\left(-\frac{x-\xi}{-x-\rho+i0}\right) + \frac{1}{2}\ln^2\left(\frac{-\xi-\rho+i0}{-x-\rho+i0}\right)\Bigg] \\
& + \frac{\xi-\rho}{\xi(x+\xi)}\Bigg[-\mathrm{Li}_2\left(\frac{2\xi}{\xi-\rho+i0}\right) - \mathrm{Li}_2\left(-\frac{2\xi}{-\xi-\rho+i0}\right) + 2\,\mathrm{Li}_2(1) \\
& \quad - \mathrm{Li}_2\left(\frac{x-\xi}{x+\xi}\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \mathrm{Li}_2\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) + \mathrm{Li}_2\left(\frac{x-\xi}{x+\xi}\frac{\xi-\rho+i0}{-\xi-\rho+i0}\right) \\
& \quad - \mathrm{Li}_2\left(\frac{\xi-\rho+i0}{-\xi-\rho+i0}\right) - \ln\left(\frac{2\xi}{x+\xi}\right)\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) \\
& \quad - \ln\left(-\frac{2\xi}{x+\xi}\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right)\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \mathrm{Li}_2\left(\frac{2\xi}{x+\xi}\frac{x-\rho+i0}{\xi-\rho+i0}\right) + \mathrm{Li}_2\left(\frac{2\xi}{x+\xi}\right)\Bigg] \\
& - \frac{\xi+\rho}{\xi(x-\xi)}\Bigg[\mathrm{Li}_2\left(\frac{x-\xi}{x+\xi}\right) - \mathrm{Li}_2\left(\frac{x-\xi}{x+\xi}\frac{\xi-\rho+i0}{-\xi-\rho+i0}\right) + \ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right)\ln\left(\frac{2\xi}{x+\xi}\right) \\
& \quad - \ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right)\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\frac{x-\xi}{x+\xi}\right)\Bigg].
\end{aligned}$$

$$\mathbb{P}_{(\mathrm{i})}(x/\xi,\rho/\xi) = \frac{\xi-\rho}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right),$$

$$\widetilde{\mathbb{P}}_{(\mathrm{i})}(x/\xi,\rho/\xi) = -\frac{\xi-\rho}{x-\xi}\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right),$$

$$\mathbb{P}_{(\mathrm{ii})}(x/\xi,\rho/\xi) = \frac{\xi-\rho}{x+\xi}\left[\mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right) - (x\to-\xi)\right],$$

$$\widetilde{\mathbb{P}}_{(\mathrm{iii})}(x/\xi,\rho/\xi) = -\frac{\xi+\rho}{x+\xi}\ln\left(\frac{x-\rho+i0}{-\xi-\rho+i0}\right).$$

Double DVCS at twist-4

V. Martínez-Fernández,
B. Pire, PS, J. Wagner
preliminary

- In $Q'^2 \to 0$ limit (DVCS) expression agrees with Braun-Ji-Manashov result

$$
\begin{aligned}
\mathcal{A}^{++}_{\mathrm{DVCS}} =& \lim_{\rho\to\xi} \mathcal{A}^{++} \\
=& \frac{1}{2}\int_{-1}^{1} dx \Bigg\{ -\left(1-\frac{t}{2\mathbb{Q}^2_{\mathrm{DVCS}}}\right)\frac{H^{(+)}}{x-\xi+i0} \\
&- \frac{2t}{\mathbb{Q}^2_{\mathrm{DVCS}}}\left[\frac{1}{x+\xi}\ln\left(\frac{x-\xi+i0}{-2\xi}\right)+\frac{1}{x-\xi}\left(\mathrm{Li}_2\left(\frac{x+\xi}{2\xi}\right)-\mathrm{Li}_2(1)\right)\right]H^{(+)} \\
&+ \frac{t}{\mathbb{Q}^2_{\mathrm{DVCS}}}\partial_\xi\left[\left(\frac{2\xi}{x-\xi}\left(\mathrm{Li}_2\left(\frac{x+\xi}{2\xi}\right)-\mathrm{Li}_2(1)\right)+\frac{\xi}{x+\xi}\ln\left(\frac{x-\xi+i0}{-2\xi}\right)\right)H^{(+)}\right] \\
&- \frac{\bar{p}_\perp^2}{\mathbb{Q}^2_{\mathrm{DVCS}}}2\xi^3\partial_\xi^2\left[\left(\frac{2\xi}{x-\xi}\left(\mathrm{Li}_2\left(\frac{x+\xi}{2\xi}\right)-\mathrm{Li}_2(1)\right)+\frac{\xi}{x+\xi}\ln\left(\frac{x-\xi+i0}{-2\xi}\right)\right)H^{(+)}\right]\Bigg\} \\
&+ O(\text{tw-6})\,,
\end{aligned}
$$

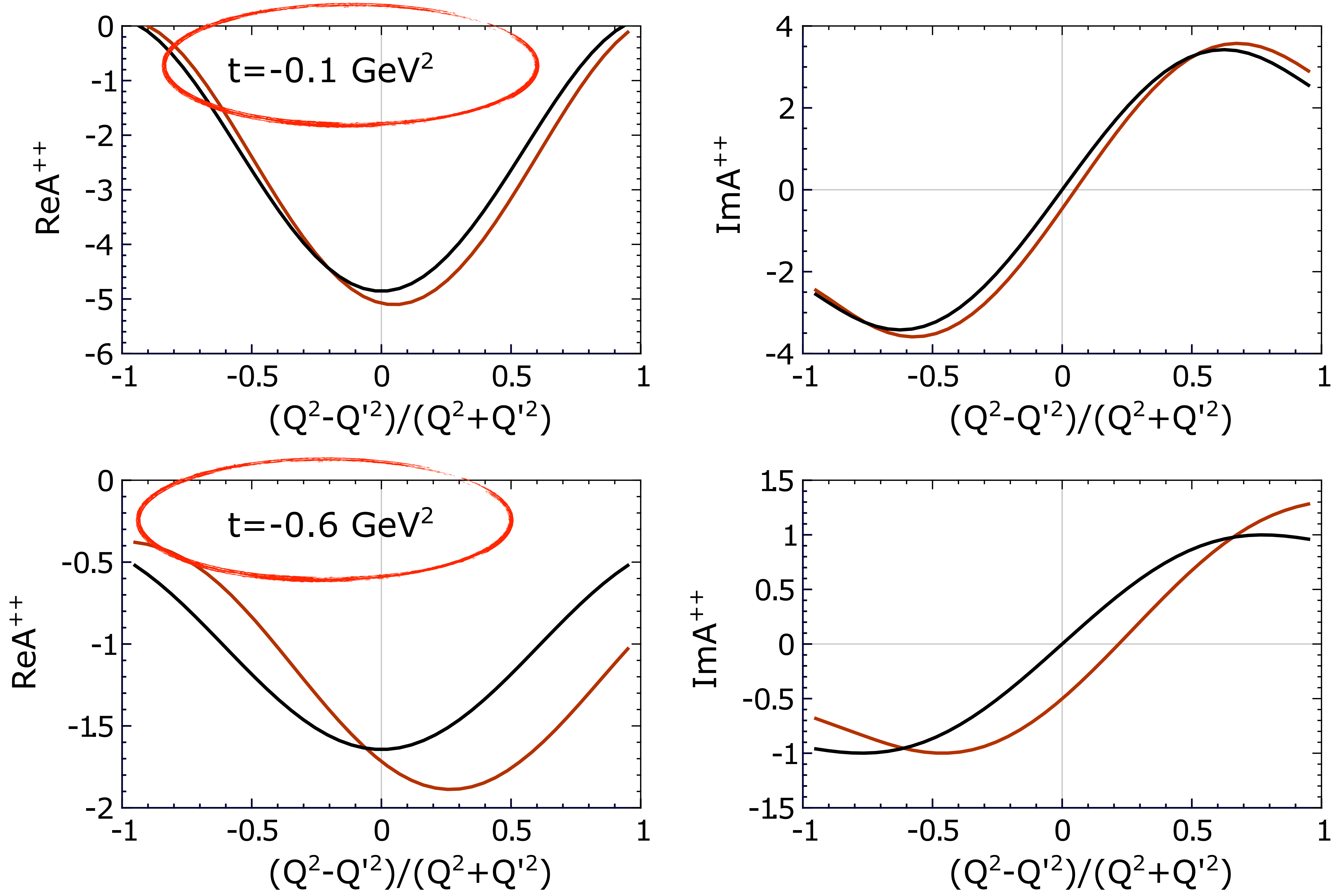
- New result for $Q^2 \to 0$ limit (TCS)

$$
\begin{aligned}
\mathcal{A}^{++}_{\mathrm{TCS}} =& \lim_{Q^2\to 0} \mathcal{A}^{++} \\
=& \frac{1}{2}\int_{-1}^{1} dx \Bigg\{ -\left(1-\frac{5}{2}\frac{t}{\mathbb{Q}^2_{\mathrm{TCS}}}\right)\frac{H^{(+)}}{x+\xi(1-2t/Q'^2)+i0} \\
&+ \frac{2t}{\mathbb{Q}^2_{\mathrm{TCS}}}\left[\frac{1}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right)+\frac{1}{x+\xi}\left(\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right)-\mathrm{Li}_2(1)\right)-\frac{\xi}{4}(L_{\mathrm{TCS}}+J_{\mathrm{TCS}})\right]H^{(+)} \\
&- \frac{t}{\mathbb{Q}^2_{\mathrm{TCS}}}\partial_\xi\Bigg[\left(\frac{2\xi}{x+\xi+i0}+\frac{2\xi}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right)+\frac{2\xi}{x+\xi}\left(\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right)-\mathrm{Li}_2(1)\right)-\frac{\xi L_{\mathrm{TCS}}}{2} \right.\\
&\left. -\frac{\xi}{x-\xi}\ln\left(\frac{x+\xi+i0}{2\xi}\right)\right)H^{(+)}\Bigg] \\
&+ \frac{\bar{p}_\perp^2}{\mathbb{Q}^2_{\mathrm{TCS}}}2\xi^3\partial_\xi^2\Bigg[\left(\frac{2\xi}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right)+\frac{2\xi}{x+\xi}\left(\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right)-\mathrm{Li}_2(1)\right)+\frac{\xi}{x-\xi}\ln\left(\frac{x+\xi+i0}{2\xi}\right)\right. \\
&\left.-\frac{\xi L_{\mathrm{TCS}}}{2}+\ln\left(\frac{x+\xi+i0}{x-\xi+i0}\right)\right)H^{(+)}\Bigg]\Bigg\} + O(\text{tw-6})\,,
\end{aligned}
$$

Double DVCS at twist-4

V. Martínez-Fernández,
B. Pire, PS, J. Wagner
preliminary

- Numerical estimate:



LT

LT+HT corrections
(up to tw-4)

$\xi = 0.2$

$\mu^2 = 1.9\ \mathrm{GeV}^2$

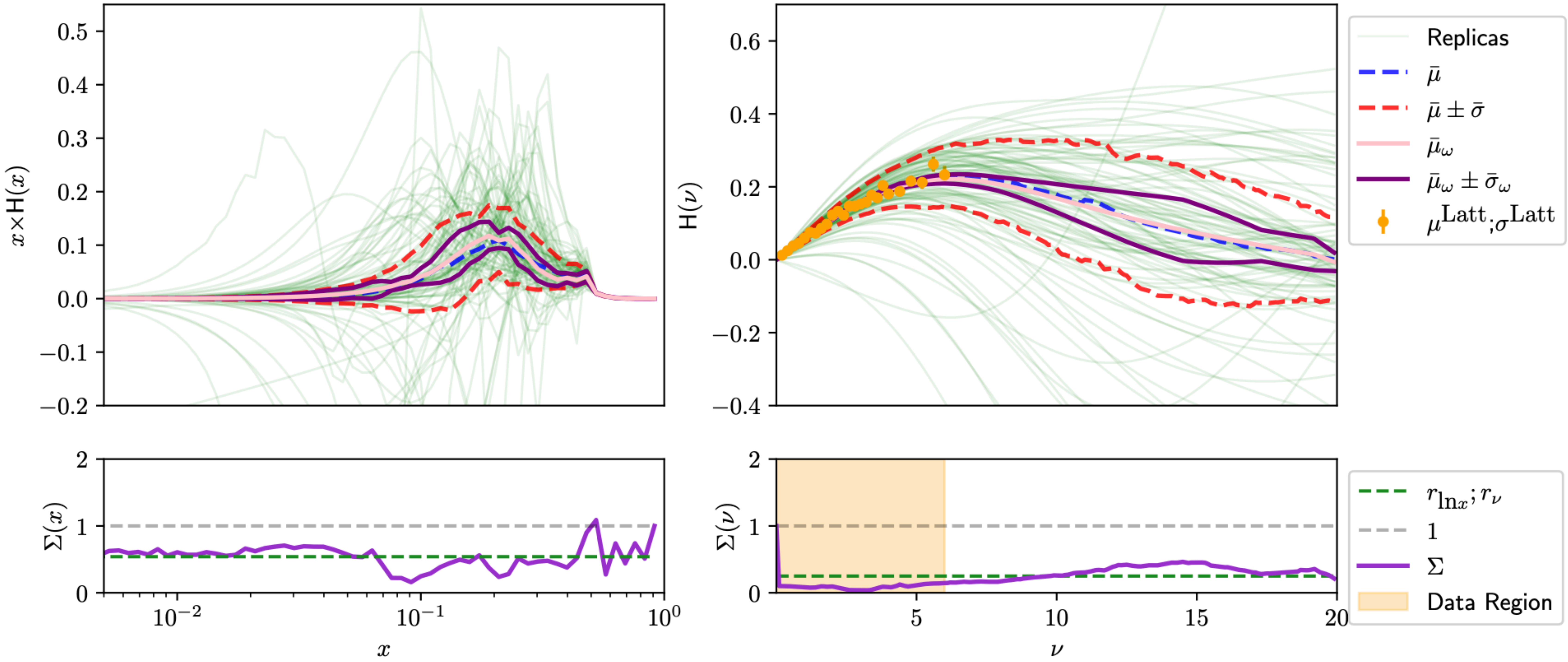
GPD model from:
PRD 105, 094012 (2022)

Lattice-QCD

M. J. Riberdy, H. Dutrieux, C. Mezrag, PS,
EPJC 84 (2024) 2, 201

- Exploratory study to include lattice-QCD results!

Reduction of GPD model uncertainties due to inclusion of pseudo-latticeQCD results



Summary

- Take-away messages:

  - virtual Compton scattering processes are important for measuring GPDs.
  - DVCS and TCS only give limited access to GPDs,
    but still offer a wealth of important information:
    - nucleon tomography at low-xB
    - "mechanical" properties
    - Froissart-Gribov projections (now also for spin 1/2 target)
  - DDVCS allows to avoid limitations of DVCS and TCS, but is much more difficult to measure
  - new DDVCS description (including pheno. studies) available
  - new evaluation of DDVCS (and also TCS) amplitudes in terms of twist expansion will become available soon
  - new impact studies of lattice QCD data inclusion

Froissart-Gribov projections - spin 0

K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

Dual parameterisation:

$$H_{+}(x,\xi,t) = 2\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{n,l}(t)\,\theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right)C_n^{3/2}\left(\frac{x}{\xi}\right)P_l\left(\frac{1}{\xi}\right)$$

Coefficients of Mellin moments:

$$\int_0^1 dx x^N H_{+}(x,\xi,t) = \sum_{\substack{k=0\\ \text{even}}}^{N+1} h_{N,k}(t)\xi^k$$

where:

$$h_{N,k}(t) = \sum_{\substack{n=1\\ \text{odd}}}^{N}\sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{n,l}(t)(-1)^{\frac{k+l-N-1}{2}}\frac{\Gamma\left(1-\frac{k-l-N}{2}\right)}{2^k\Gamma\left(\frac{1}{2}+\frac{k+l-N}{2}\right)\Gamma(2-k+N)}\frac{(n+1)(n+2)\Gamma(N+1)}{\Gamma\left(1+\frac{N-n}{2}\right)\Gamma\left(\frac{5}{2}+\frac{N+n}{2}\right)}$$

E.g.:

$$\int_0^1 dx\,x\,H_{+}(x,\xi,t) = \frac{6B_{1,2}(t)}{5} + \xi^2\left(\frac{4B_{1,0}(t)}{5} - \frac{2B_{1,2}(t)}{5}\right)$$

Froissart-Gribov projections - spin 0

K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

This gives:

$$F_{J=0}(t) = 4 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} B_{n,0}(t) = 4 \sum_{\nu=1}^{\infty} B_{2\nu-1,0}(t)$$

$$F_{J>0}(t) = 4 \sum_{\substack{n=J-1 \\ \text{odd}}}^{\infty} B_{n,J}(t) = 4 \sum_{\nu=0}^{\infty} B_{J+2\nu-1,J}(t)$$

The relations allow us to define "sum rules", e.g. for nu = 1:

$$F_{J=0}(t) = 4\left(B_{1,0}(t) + \ldots\right) = \frac{5}{3} h_{1,0}(t) + 5 h_{1,2}(t) + \left\{ \begin{matrix} \text{contribution of conformal PWs} \\ \text{with } \nu \geq 2 \end{matrix} \right\}$$

$$F_{J=2}(t) = 4\left(B_{1,2}(t) + B_{3,2}(t) + \ldots\right) = -\frac{7}{6} h_{1,0}(t) + 9 h_{3,0}(t) + \frac{21}{2} h_{3,2}(t) + \left\{ \begin{matrix} \text{contribution of conformal PWs} \\ \text{with } \nu \geq 2 \end{matrix} \right\}$$

Froissart-Gribov projections - spin 0

K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

Modified dual parameterisation:

$$H_+(x,\xi,t) = 2\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1}\beta^l \bar{B}_{n,l}(t)\,\theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right)C_n^{3/2}\left(\frac{x}{\xi}\right)P_l\left(\frac{1}{\xi\beta}\right)$$

$$\beta = \sqrt{1-\frac{4m^2}{t}}$$

Coefficients of Mellin moments:

$$\int_0^1 dx x^N H_+(x,\xi,t) = \sum_{\substack{k=0\\ \text{even}}}^{N+1} h_{N,k}(t)\xi^k$$

$$h_{N,k}(t) = \sum_{\substack{n=1\\ \text{odd}}}^{N}\sum_{\substack{l=0\\ \text{even}}}^{n+1}\beta^{l+k-N-1}\bar{B}_{n,l}(t)(-1)^{\frac{k+l-N-1}{2}}\frac{\Gamma\left(1-\frac{k-l-N}{2}\right)}{2^k\Gamma\left(\frac{1}{2}+\frac{k+l-N}{2}\right)\Gamma(2-k+N)}\frac{(n+1)(n+2)\Gamma(N+1)}{\Gamma\left(1+\frac{N-n}{2}\right)\Gamma\left(\frac{5}{2}+\frac{N+n}{2}\right)}$$

To keep these coefficients regular at t=0 one has to assume:

$$B_{n,n+1}(t) = \bar{B}_{n,n+1}(t)$$

$$B_{n,n-1}(t) = \bar{B}_{n,n-1}(t) - (1-\beta^2)\left(\frac{1}{2}-n\right)\bar{B}_{n,n+1}(t)$$

spin J=n-1 spin J=n+1

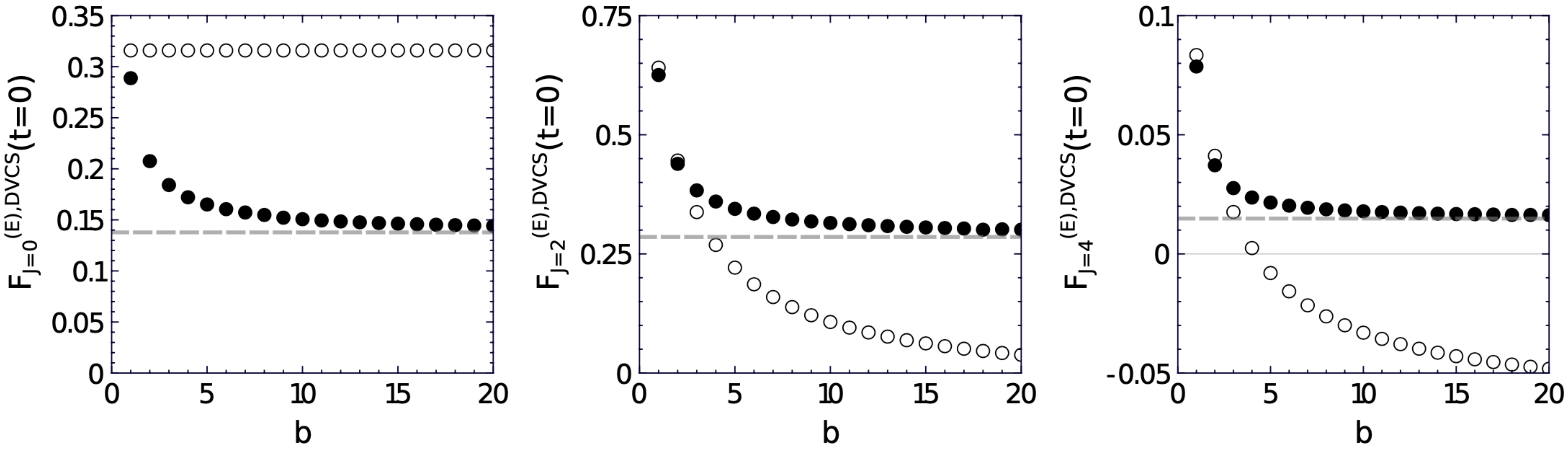
For $\beta \neq 1$ FG projections get admixture from higher spins

Froissart-Gribov projections - spin ½

K. Semenov-Tian-Shansky and P.S.
hep-ph/2312.09624

Sensitivity of FG projections on the shape of DD profile function

$$H(x,\xi,t=0) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-x)\, q(\beta) \frac{\Gamma(2b+2)}{2^{2b+1}\Gamma^2(b+1)} \frac{\left((1-|\beta|)^2-\alpha^2\right)^b}{(1-|\beta|)^{2b+1}}$$



● projections from GPD

b → ∞ limit (no skewness effect)

○ projections from sum rules (i.e. from certain Mellin moments) for $\nu = 1$