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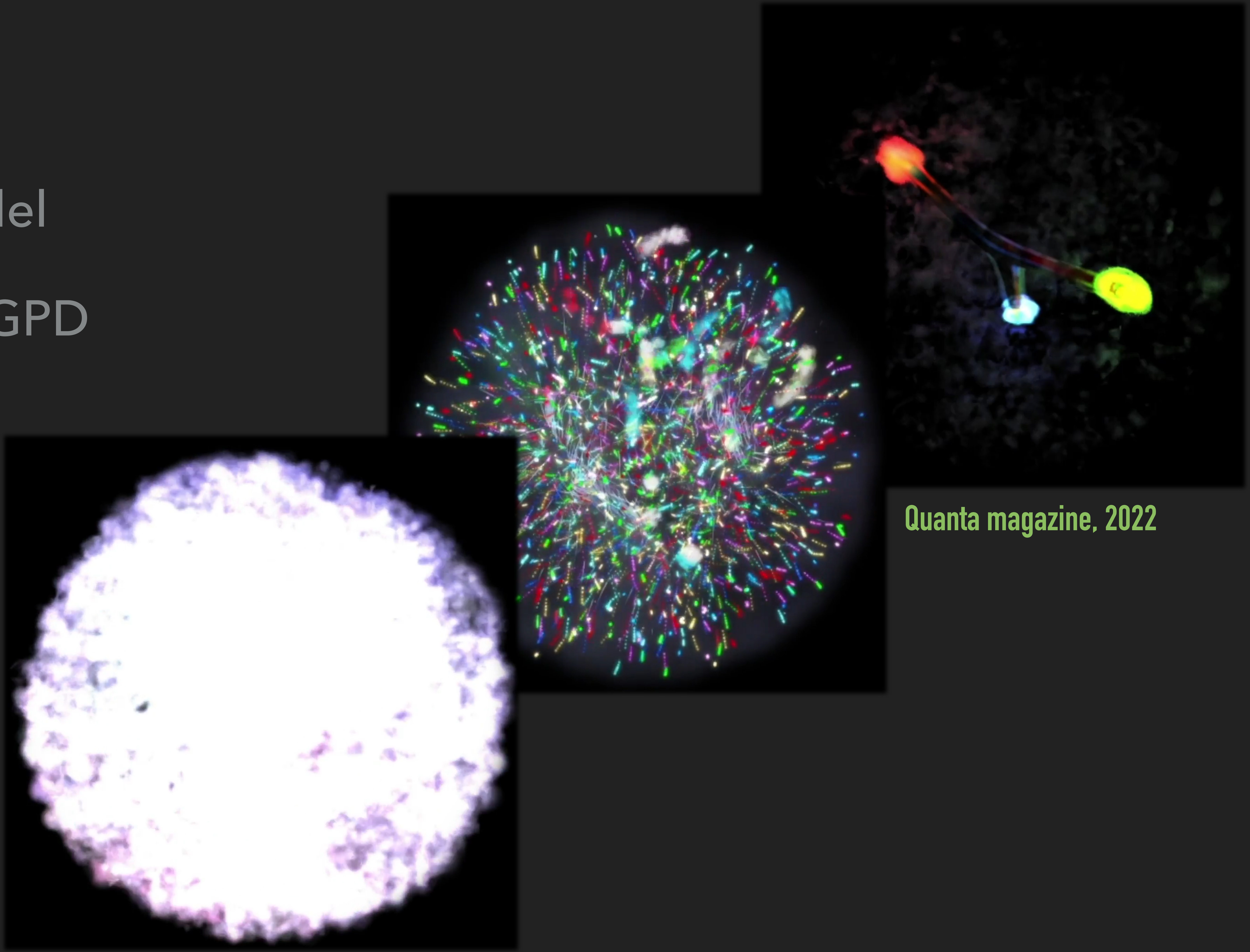
GPD AND DEVELOPMENT OF ITS EXTRACTION TECHNIQUE

OUTLINE

- GPD: Inspired QCD model
- Extraction technique of GPD from DVCS & DVMP
- Summary and outlook

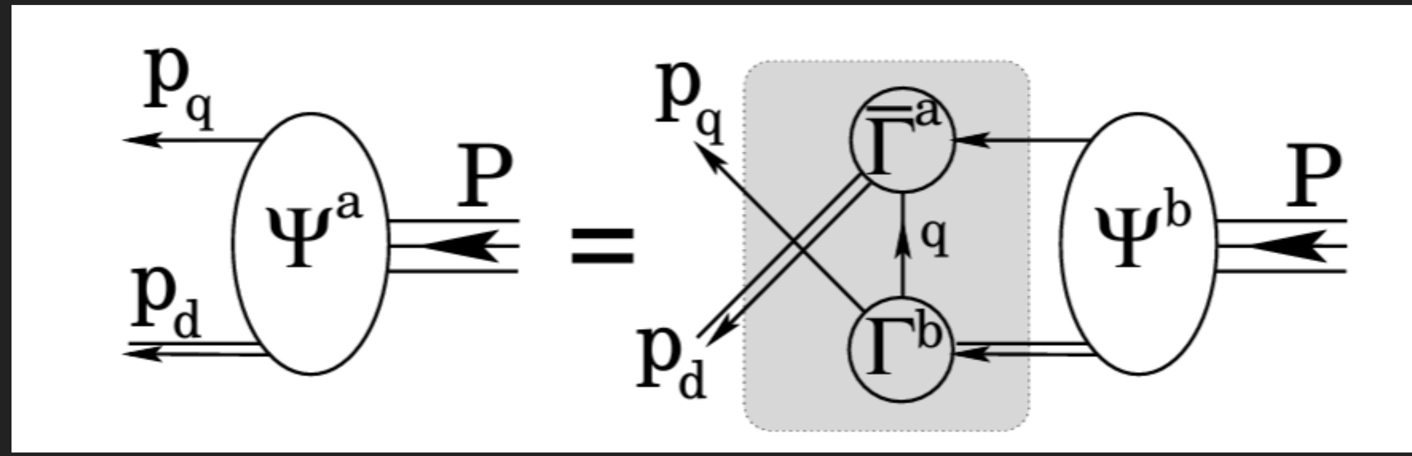


Quanta magazine, 2022

GPD

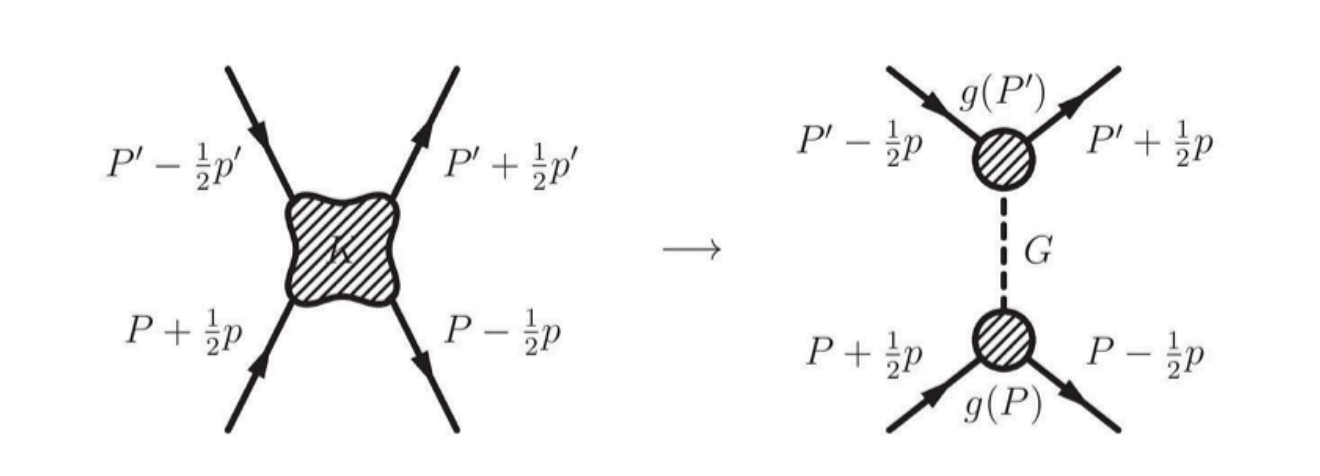
- GPD is a tool to study the 3D structure of hadrons – connecting with the chiral symmetry-breaking – one of the features of QCD PRD 56 (1997), PR 388 (2003), PR 418 (2005)
- Hard exclusive processes: Deep virtual Compton scattering (DVCS) and deeply virtual meson production (DVMP) – GPDs
- Experimentally, the study of GPDs could be challenging and useful for modern experiments such as COMPASS, JLAB22 Upgrade, JPARC, EIiC, and EIC

NONLOCAL CHIRAL QUARK AND NJL MODEL



- The NJL model is a phenomenological field theory inspired by QCD——preserves the basic symmetry of QCD in the quark sector
- In the NJL model, the quarks are not confined like in the case of low-energy QCD
- The basic ingredient of the NJL model is a zero-range interaction containing four fermion interactions
- The NJL model is not renormalizable—a regularization method is needed
- A relativistic Faddeev equation for a nucleon-bound state is solved in the covariant diquark-quark picture: After truncation of two-body channels to the scalar and axial vector diquarks PLB 286, 29 (1992), PRC 49, 1702 (1994), NPA 587, 617 (1995), PRC 51, 3388 (1995), PLB 344, 55 (1995), PRC 58, 2459 (1998), NPA 627, 679 (1997)

NONLOCAL CHIRAL QUARK AND NJL MODEL



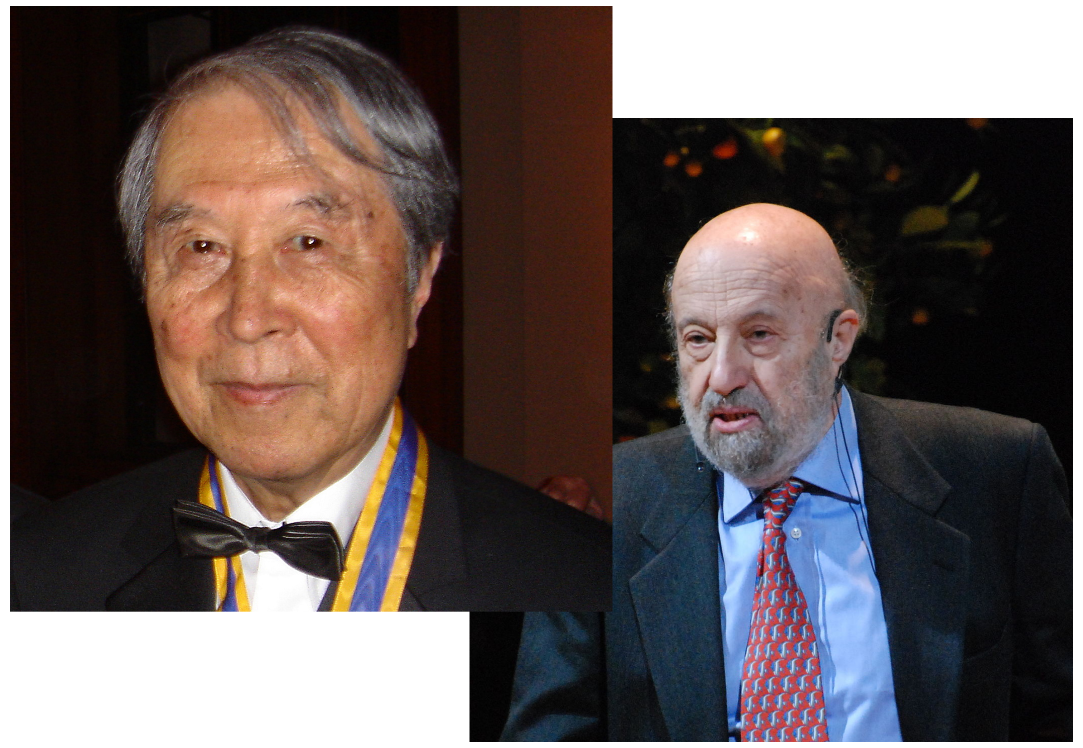
- A nonlocal version of the model has several advantages: the dynamical model quark mass is momentum-dependent, which is consistent with the lattice QCD simulation of QCD
- Introducing additional non-local terms in the currents, one can preserve the gauge invariance and anomalies
- The regulator makes the theory finite to all orders in the loop expansion and leads to NLO corrections
- This non-locality also emerges in DS resummation and gluon field configuration in the vacuum NPA 582, 655 (1995), NPA 628, 607 (1998) & NPA 703, 717 (2002)

NJL model

The Lagrangian NJL model—contain local four-fermion interactions—**PTPH**, Ian Cloet, Anthony Thomas, PRC94(2016), Ian Cloet PRC90(2014)

$$\mathscr{L}_{\mathrm{NJL}} = \bar{\psi}[i\partial\!\!\!/ - \hat{m}]\psi + G_\pi \sum_{a=0}^{8} [(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\lambda_a\gamma_5\psi)^2] + G_\rho \sum_{a=0}^{8} [(\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2] - G_\omega(\bar{\psi}\gamma^\mu\psi)^2$$

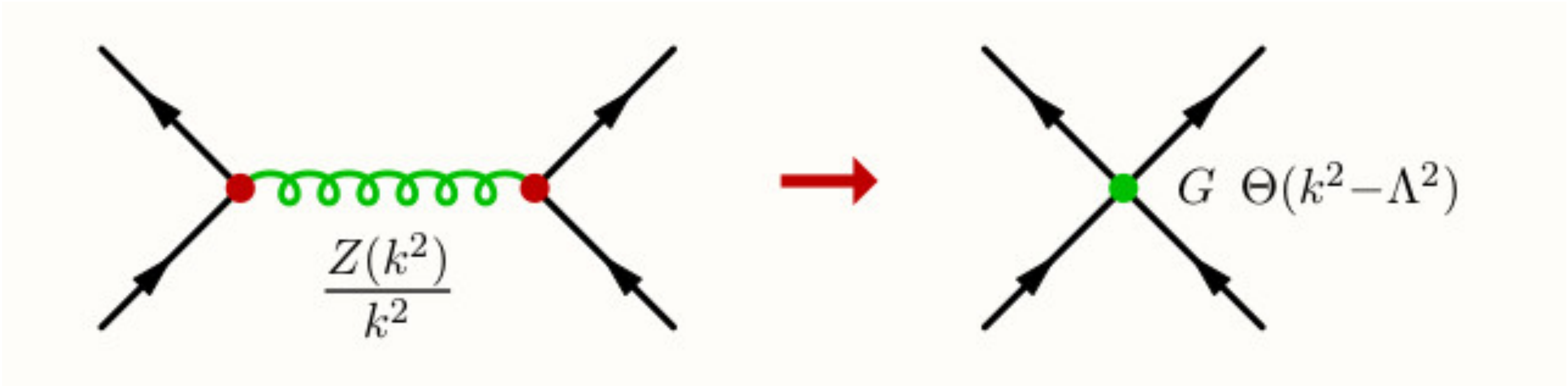
where



- $\psi = (u, d, s)^T$ is the quark field with the flavor components
- $G_\pi, G_\rho,$ and G_ω are local four-fermion coupling constants
- $\hat{m}_q = \mathrm{diag}[m_u,\ m_d,\ m_s]$ is the current quark mass matrix

NJL model

- In the NJL model, the gluon fields are integrated out and absorbing in the G_π coupling constants—Local four-fermion contact interactions—**PTPH**, Ian Cloet, Anthony Thomas, PRC94(2016), Ian Cloet PRC90(2014), S.Klevansky, RMP64(1992), Vogl &Weise, PPNP27(1991), Hatsuda&Kunihiro, PR247(1994)



- NJL model — lack of the confinement and divergence (pole in quark propagator) — We perform the Proper-time regularization (PTR) scheme — Simulating the confinement of QCD—**PTPH**, Ian Cloet, Anthony Thomas, PRC94(2016), Ian Cloet PRC90(2014)

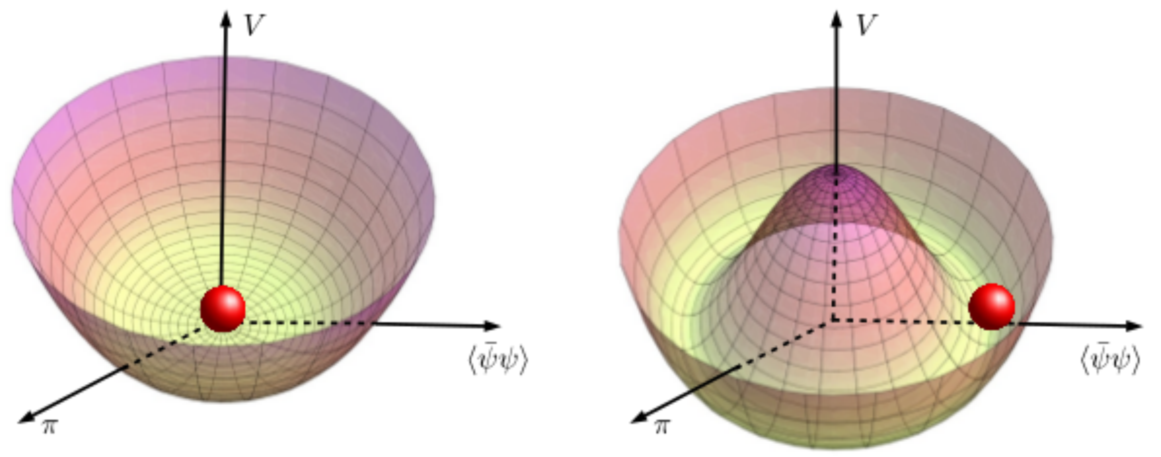
$$\frac{1}{(G)^n} = \frac{1}{[n-1]!}\int_0^\infty d\tau\tau^{[n-1]}\exp[-\tau G] \rightarrow \frac{1}{[n-1]}\int_{\tau_{UV}}^{\tau_{IR}} d\tau\tau^{[n-1]}\exp[-\tau G]$$

NJL model

- Where $\tau_{\mathrm{UV}} = \dfrac{1}{\Lambda_{\mathrm{UV}}^2}$ and $\tau_{\mathrm{IR}} = \dfrac{1}{\Lambda_{\mathrm{IR}}^2}$ with $\Lambda_{\mathrm{IR}} \simeq \Lambda_{\mathrm{QCD}} \simeq 240$ MeV and Λ_{UV} is determined to fit the pion mass and pion weak decay constant ($m_\pi = 140$ MeV and $f_\pi = 93$ MeV)

- NJL gap equation —dynamical quark mass— is determined through the quark propagator in momentum space

$$M_q = m_q + M_q \frac{3G_\pi}{\pi^2} \int_{\tau_{\mathrm{UV}}}^{\tau_{\mathrm{IR}}} \frac{d\tau}{\tau^2} \exp[-\tau M_q^2] = m_q - 2G_\pi \langle \bar{\psi}\psi \rangle$$

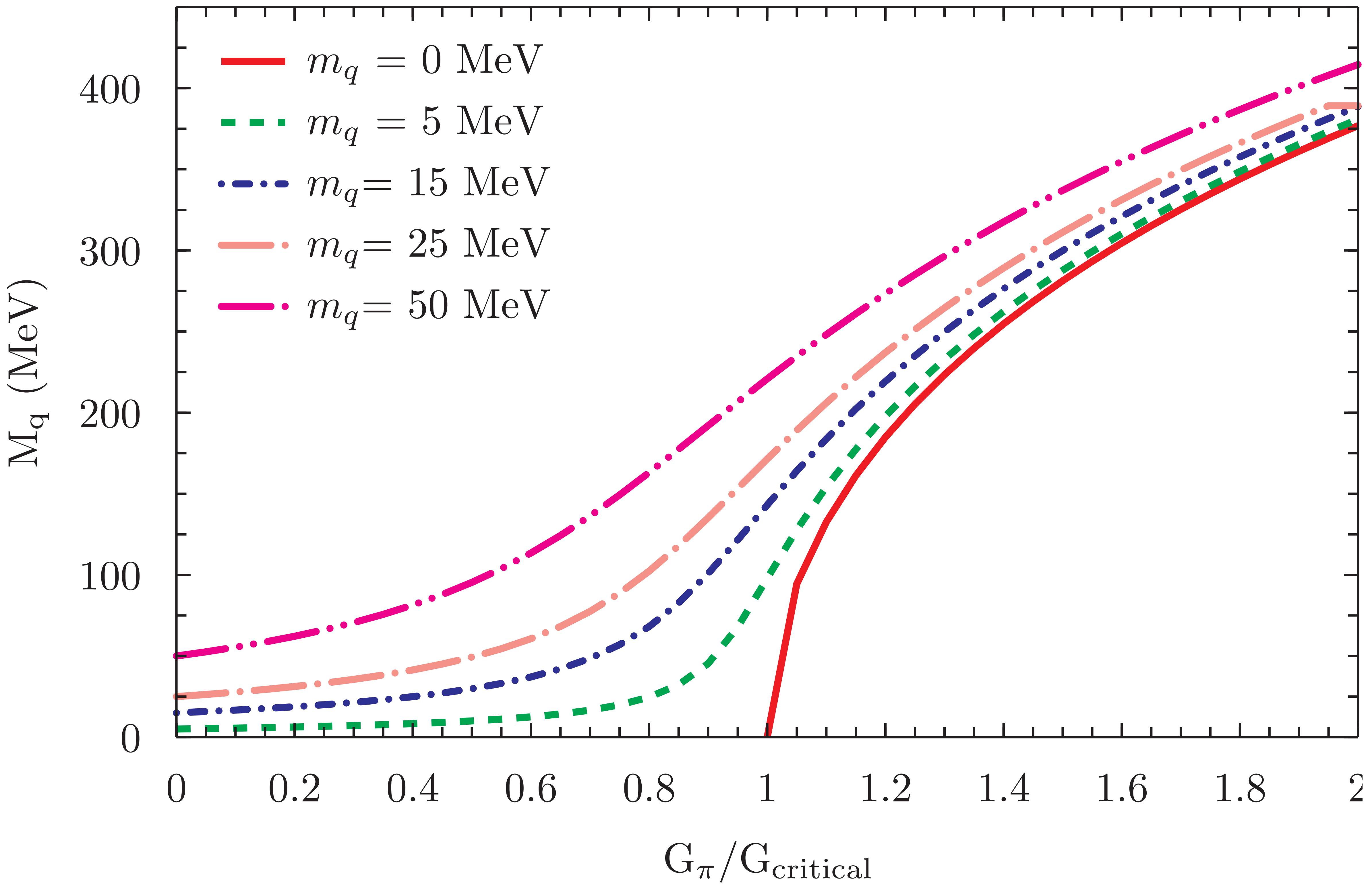


- $\langle \bar{\psi}\psi \rangle \neq 0$ —chiral QCD condensate—order parameter of chiral spontaneously symmetry breaking (CSSB)—generated mass via interaction with vacuum

NJL model

NJL Gap equation — dynamical quark mass

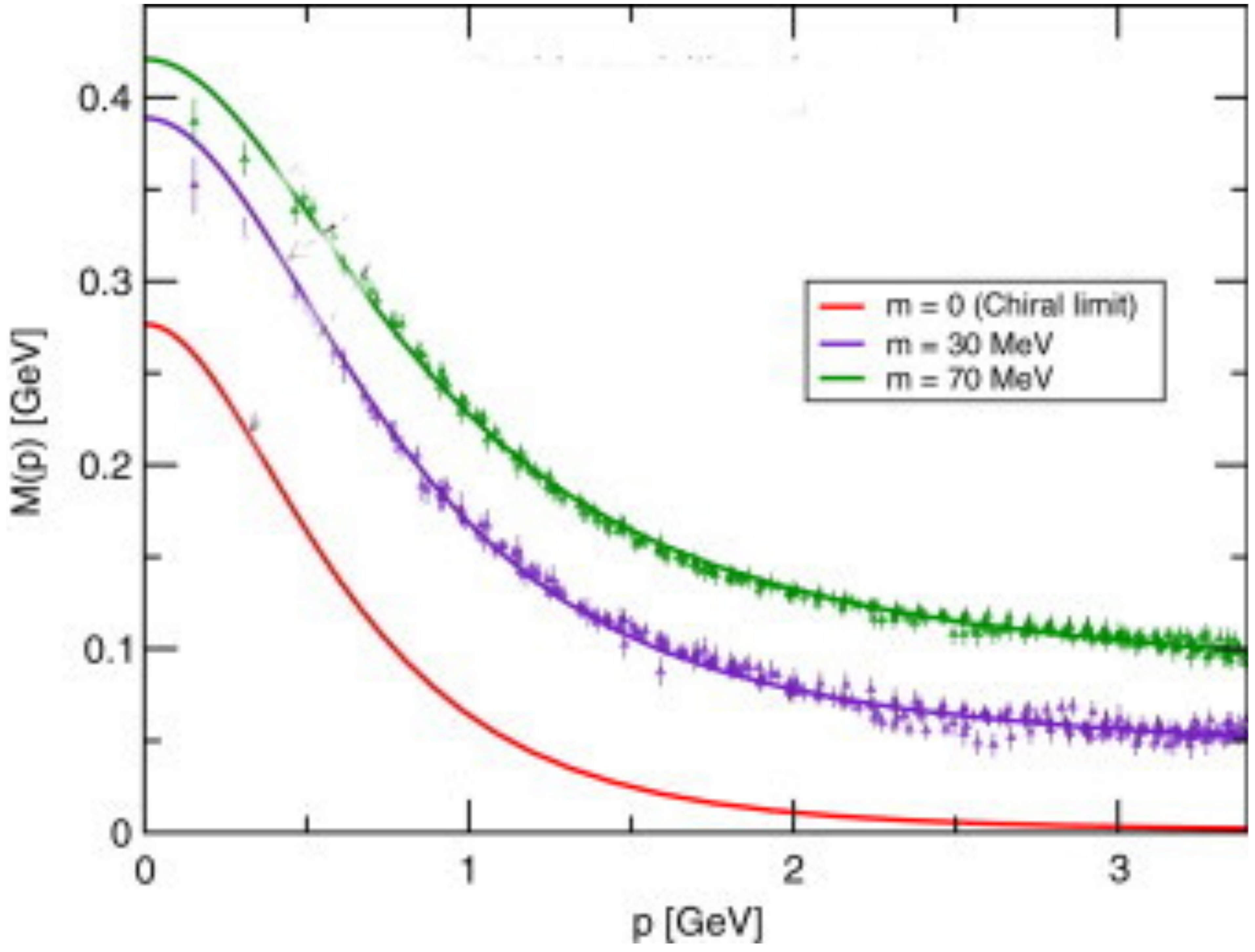
- Result for the NJL dynamical quark mass—without momentum dependent

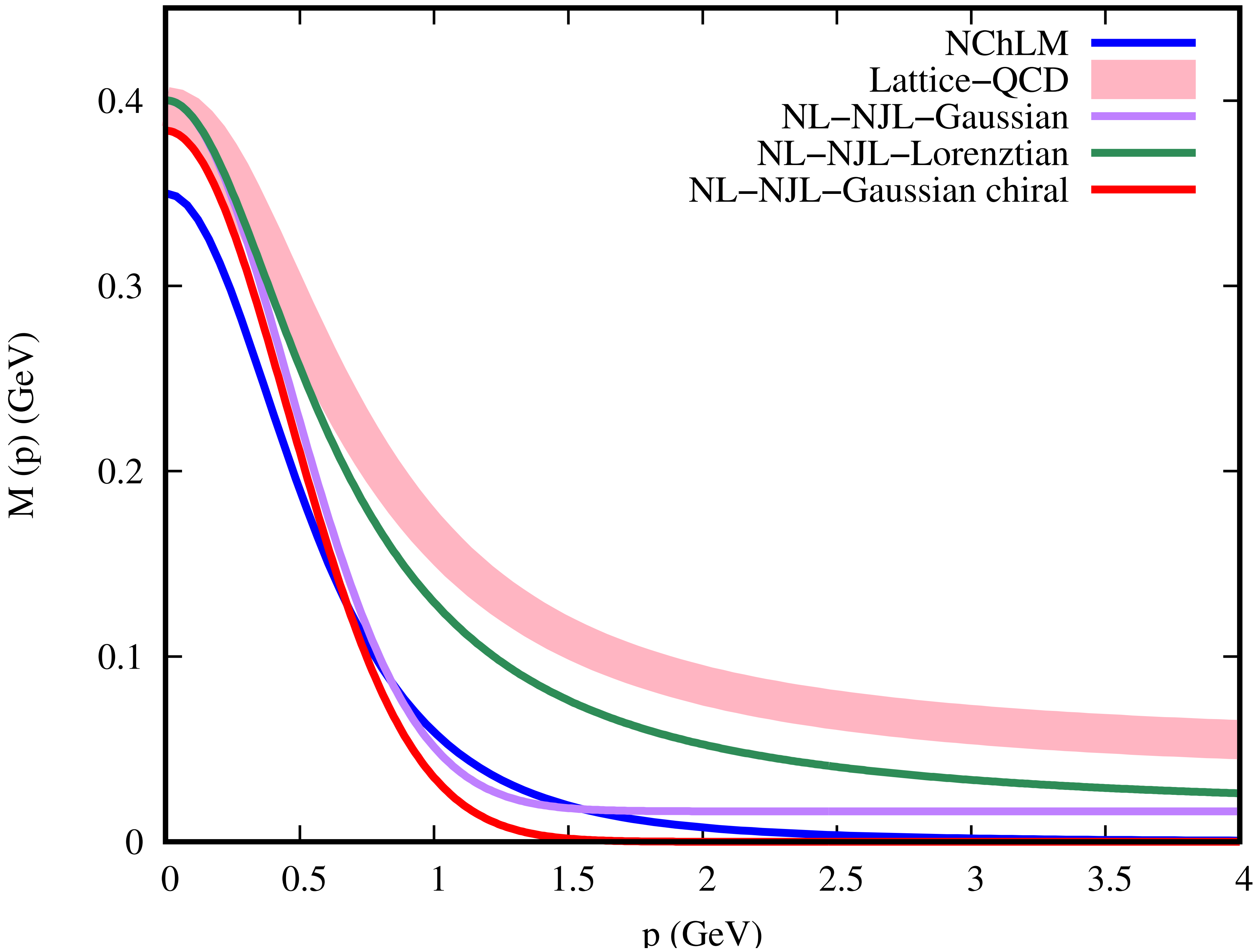


NJL model

DSE model—comparison with the BSE—NJL model

- Dynamical quark mass in the DSE model





GPD NUCLEON IN THE NJL MODEL

GPD NUCLEON IN THE NJL MODEL

- Inclusive DIS of leptons from nucleon – Universal Parton distribution functions (PDFs)
- Hard Exclusive – generalized parton distributions (GPDs), where the high energy virtual photon with momentum q^μ is absorbed by a quark in a nucleon, producing a real photon or a meson without breaking up the nucleon
- Averaged nucleon four-momentum $P = \frac{\left(p + p'\right)}{2}$ and q^μ are collinear along the z-axis

GPD NUCLEON IN THE NJL MODEL

- At the leading twist (twist-2), the GPDs are formally written

$$\frac{P_-}{2}\int dy_+ e^{ixP_- y^+}\langle p'\lambda' \mid \bar{\psi}_q(-y/2)\gamma^+\psi_q(y/2) \mid p\lambda\rangle_{y_-=\vec{y}_\perp=0}$$

$$= \bar{u}_N(p',\lambda')\left[H^q(x,\xi,\Delta^2)\gamma^+ + E^q(x,\xi,\Delta^2)\frac{i\sigma^{+\nu}\Delta^\nu}{2M_N}\right]u_N(p,\lambda) + \cdots$$

Where $\Delta = p' - p$, and $\mid p\lambda\rangle$ is a nucleon state with momentum p and helicity λ. The ellipsis $\left(\cdots\right)$ stands for the higher twist contributions

GPD NUCLEON IN THE NJL MODEL

- In the momentum space, the GPDs can rewritten as

$$\int \frac{d^4K}{(2\pi)^4}\delta\Big(x - \frac{K^+}{P^+}\Big)\mathrm{Tr}\Big[\gamma^+\chi_{\mathrm{qN}}(\mathrm{p}, \mathrm{p}', \mathrm{K})\Big]$$

$$= \bar{u}_N(p', \lambda')\Big[H^q(x, \xi, \Delta^2)\gamma^+ + E^q(x, \xi, \Delta^2)\frac{i\sigma^{+\nu}\Delta^\nu}{2M_N}\Big]u_N(p, \lambda) + \cdots$$

Where $k = (x + \xi)P^+$, and $k' = (x - \xi)P^+$ are respectively the initial and final quark momenta. $K = \dfrac{(k + k')}{2}$, and $\chi_{qN}(p, p', K)_{ji} = \int d^4y e^{iK\cdot y}\langle p'\lambda' \mid \bar{\psi}_i(-y/2)\psi_j(y/2) \mid p\lambda\rangle$ is the quark-nucleon scattering amplitude

GPD NUCLEON IN THE NJL MODEL

- In the light cone (LC) momentum fraction x and skewness ξ are given by

$$x \equiv \frac{K^+}{P^+} \text{ and } \xi \equiv \frac{-\Delta^+}{2P^+} \text{ with } 0 < \xi < \sqrt{\frac{-\Delta^2}{4M_N^2 - \Delta^2}} < 1,$$

In the on-shell conditions, $p^2 = p'^2 = M_N^2$, we then have

$$\Delta^2 = -\frac{4\xi^2 M_N^2 + \vec{\Delta}_\perp^2}{1 - \xi^2}$$

GPD emitting a parton with momentum fractions $x + \xi$ and reabsorbing with momentum fractions $x - \xi$

GPD NUCLEON IN THE NJL MODEL

- For $x > \xi$, both the emitted and absorbed parsons are quarks
- For $x < -\xi$, both are antiquarks
- If $|x| < \xi$, the two partons involved are quark-antiquark pair
- In the forward scattering limit, $\xi = 0$, GPD reduces back to parton distributions

$$q(x) = H^q(x,0,0)$$

GPD NUCLEON IN THE NJL MODEL

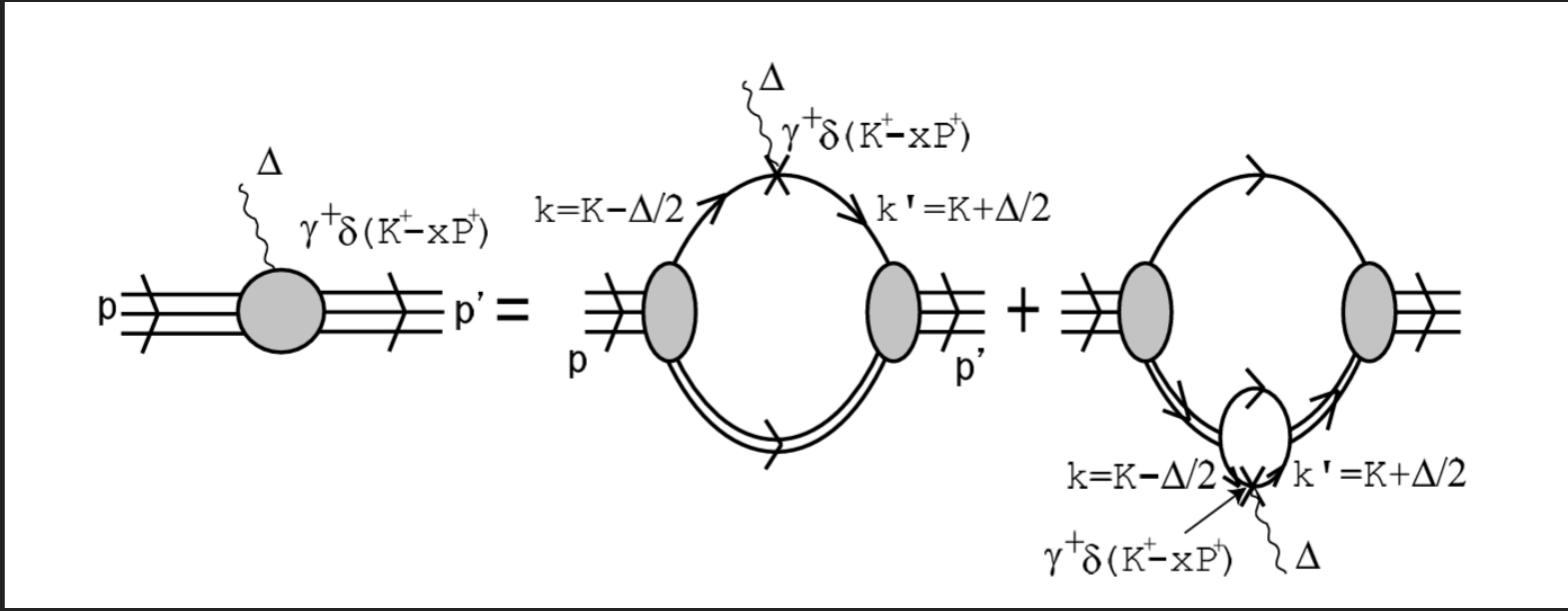
- The nucleon elastic form factor can be obtained by integrating $H^q(x, \xi, \Delta^2)$ over x

$$\int_{-1}^{1} dx H^q(x, \xi, \Delta^2) = F_1^q(\Delta^2)$$

$$\int_{-1}^{1} dx E^q(x, \xi, \Delta^2) = F_2^q(\Delta^2)$$

GPD NUCLEON IN THE NJL MODEL

- In the NJL model, GPDs can be computed from the quark and diquark currents



GPD NUCLEON IN THE NJL MODEL

- In the NJL model, GPDs can be computed from the quark $J^{Q}_{\lambda',\lambda}(x,\xi,\Delta^2)$ and diquark $J^{D}_{\lambda',\lambda}(x,\xi,\Delta^2)$ currents

$$J^{u}_{\lambda',\lambda}(x,\xi,\Delta^2) = J_{\lambda',\lambda}(x,\xi,\Delta^2) + J^{D}_{\lambda',\lambda}(x,\xi,\Delta^2)$$

$$J^{d}_{\lambda',\lambda}(x,\xi,\Delta^2) = J^{D}_{\lambda',\lambda}(x,\xi,\Delta^2)$$

Where diquark consists of isoscalar diquark and ~~isovector diquark~~

Q(D) stands for the quark (diquark) current contribution

GPD NUCLEON IN THE NJL MODEL

- Quark

$$J^{Q}_{\lambda',\lambda}(x,\xi,\Delta^2) \equiv \bar{u}(p',\lambda')\left[H^{Q}(x,\xi,\Delta^2)\gamma^{+} + E^{Q}(x,\xi,\Delta)\frac{i\sigma^{+\nu}\Delta_{\nu}}{2M_N}\right]u_N(p,\lambda)$$

- Diquark

$$J^{D}_{\lambda',\lambda}(x,\xi,\Delta^2) \equiv \bar{u}(p',\lambda')\left[H^{D}(x,\xi,\Delta^2)\gamma^{+} + E^{D}(x,\xi,\Delta)\frac{i\sigma^{+\nu}\Delta_{\nu}}{2M_N}\right]u_N(p,\lambda)$$

GPD NUCLEON IN THE NJL MODEL

- Matrix element of Dirac spinors $\bar{u}_N(p',\lambda')\mathscr{M}u_N(p,\lambda)$

$\mathcal{M}$	$\frac{\delta_{\lambda',\lambda}}{\sqrt{p'^+p^+}}\bar{u}_N(p',\lambda')\mathcal{M}u_N(p,\lambda)$	$\frac{\delta_{\lambda',-\lambda}}{\sqrt{p^+p'^+}}\bar{u}_N(p',\lambda')\mathcal{M}u_N(p,\lambda)$
1	$\frac{M_N}{p'^+}+\frac{M_N}{p^+}$	$\frac{p'_\perp(\lambda)}{p'^+}-\frac{p_\perp(\lambda)}{p^+}$
γ^+	2	0
γ^-	$\frac{1}{p'^+p^+}(\vec{p'}_\perp\cdot\vec{p}_\perp+M_N^2+i\lambda p'_\perp\wedge p_\perp)$	$\frac{M_N}{p'^+p^+}(p'_\perp(\lambda)-p_\perp(\lambda))$
$\vec{\gamma}_\perp\cdot\vec{a}_\perp$	$\vec{a}_\perp\cdot\left(\frac{p'_\perp}{p'^+}+\frac{p_\perp}{p^+}\right)-i\lambda a_\perp\wedge\left(\frac{p'_\perp}{p'^+}-\frac{p_\perp}{p^+}\right)$	$-a_\perp(\lambda)\left(\frac{M_N}{p'^+}-\frac{M_N}{p^+}\right)$
$\gamma^-\gamma^+$	$\frac{2}{p'^+}M_N$	$\frac{2}{p'^+}p'_\perp(\lambda)$
$\vec{\gamma}_\perp\cdot\vec{a}_\perp\gamma^+$	0	$2a_\perp(\lambda)$
$\gamma^-\gamma^+\gamma^-$	$\frac{2}{p'^+p^+}(\vec{p'}_\perp\cdot\vec{p}_\perp+M_N^2+i\lambda p'_\perp\wedge p_\perp)$	$\frac{2}{p'^+p^+}(p'_\perp(\lambda)-p_\perp(\lambda))$
$\gamma^-\gamma^+\vec{\gamma}_\perp\cdot\vec{a}_\perp$	$\frac{2}{p'^+}(\vec{a}_\perp\cdot\vec{p'}_\perp-i\lambda a_\perp\wedge p'_\perp)$	$-\frac{2M_N}{p'^+}a_\perp(\lambda)$
$\gamma^+\gamma^-\vec{\gamma}_\perp\cdot\vec{a}_\perp$	$\frac{2}{p^+}(\vec{a}_\perp\cdot\vec{p}_\perp+i\lambda a_\perp\wedge p_\perp)$	$\frac{2M_N}{p^+}a_\perp(\lambda)$
$\vec{a}_\perp\cdot\vec{\gamma}_\perp\gamma^+\vec{b}_\perp\cdot\vec{\gamma}_\perp$	$2(\vec{a}_\perp\cdot\vec{b}_\perp+i\lambda a_\perp\wedge b_\perp)$	0

GPD NUCLEON IN THE NJL MODEL

- Using matrix elements in the Table, the helicity conserving and flipping can be separated:

$$J^{Q}_{\lambda',\lambda}(x,\xi,\Delta^2) = \frac{P^+}{M_N}\bar{u}(p',\lambda')u_N(p,\lambda)\Big[\delta_{\lambda',\lambda}\Big((1-\xi)H^Q(x,\xi,\Delta^2) - \xi^2 E^Q(x,\xi,\Delta^2)\Big) - \delta_{\lambda',-\lambda}E^Q(x,\xi,\Delta^2)\Big]$$

- Diquark

$$J^{D}_{\lambda',\lambda}(x,\xi,\Delta^2) = \frac{P^+}{M_N}\bar{u}(p',\lambda')u_N(p,\lambda)\Big[\delta_{\lambda',\lambda}\Big((1-\xi)H^D(x,\xi,\Delta^2) - \xi^2 E^D(x,\xi,\Delta^2)\Big) - \delta_{\lambda',-\lambda}E^Q(x,\xi,\Delta^2)\Big]$$

GPD NUCLEON IN THE NJL MODEL

- Based on Feynman diagram, the quark current contribution can be written:

$$J^{Q}_{\lambda',\lambda}(x,\xi,\Delta^2) = -Z_N \bar{u}(p',\lambda') \int \frac{d^4K}{(2\pi)^4} \delta\left(x - \frac{K^+}{P^+}\right) S(k')\gamma^+ S(k) \tau_D(p-k) u_N(p,\lambda)$$

- $\tau_D(p-k) \to 4iG_s - \frac{ig_D^2}{k^2 - M_D^2}$ is the reduced t-matrix of the diquark.

$$\tau_D = \tau_D^C + \tau_D^P$$

Where τ_D^C and τ_D^P are the contact and pole contribution, respectively

GPD NUCLEON IN THE NJL MODEL

- The quark current contribution can also be separated into two parts:

$$J^{Q}_{\lambda',\lambda}(x,\xi,\Delta^2) = \Theta(-\xi < x < \xi) J^{Q,C}_{\lambda',\lambda} + \Theta(-\xi < x < 1) J^{Q,P}_{\lambda',\lambda}(x,\xi,\Delta^2)$$

- $J^{Q,C}(x,\xi,\Delta^2)$ contributes only in the region $-\xi < x < \xi$
- $J^{Q,P}(x,\xi,\Delta^2)$ contributes only in the region $-\xi < x < 1$

$$J^{D}_{\lambda',\lambda}(x,\xi,\Delta^2) \equiv \bar{u}(p',\lambda')\left[H^D(x,\xi,\Delta^2)\gamma^+ + E^D(x,\xi,\Delta)\frac{i\sigma^{+\nu}\Delta_\nu}{2M_N}\right]u_N(p,\lambda)$$

GPD NUCLEON IN THE NJL MODEL

- Similarly, based on Feynman diagram, the diquark current contribution can be written as

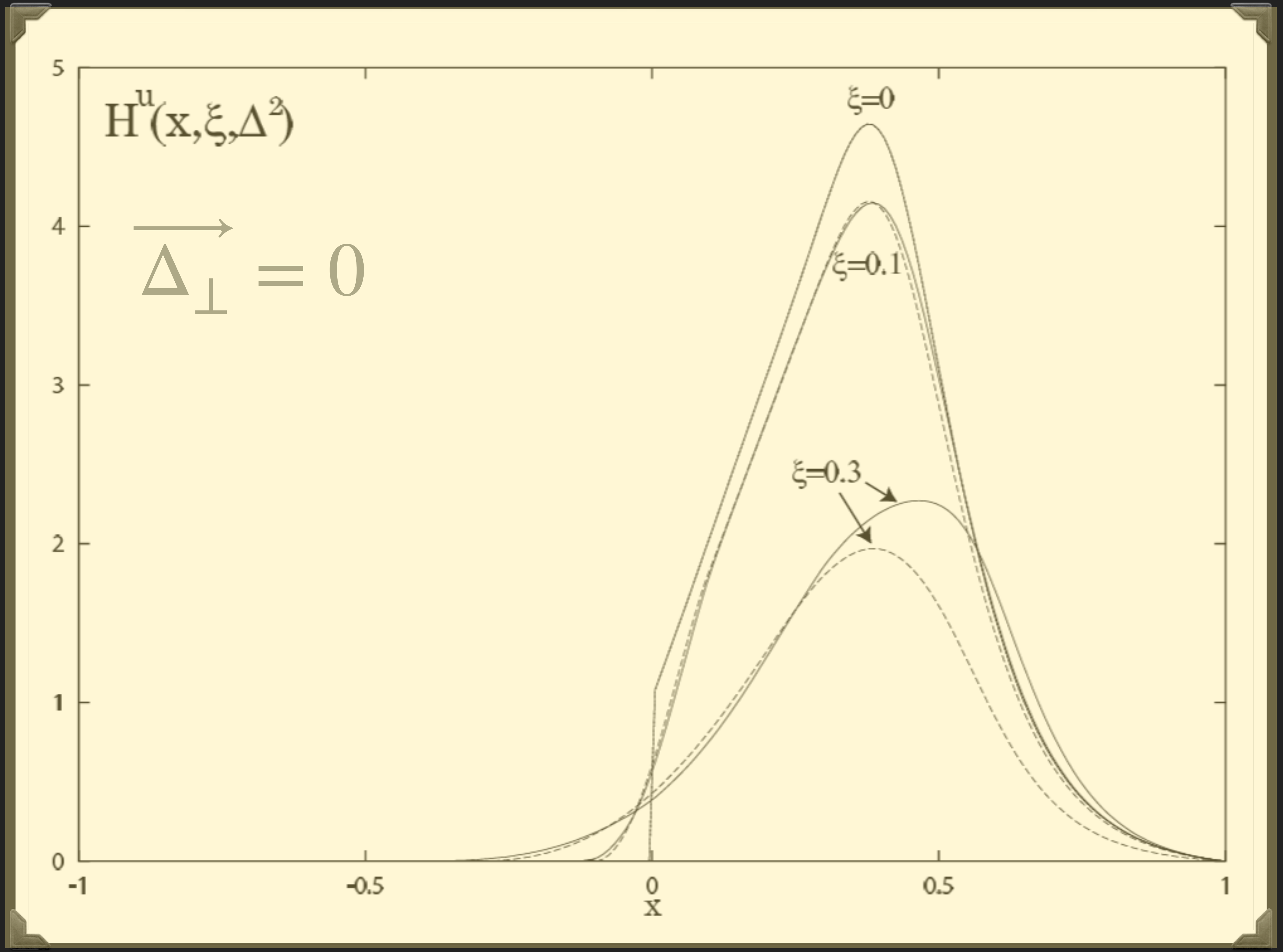
$$J^{D}_{\lambda',\lambda}(x,\xi,\Delta^2) = -Z_N \bar{u}(p',\lambda') \int \frac{d^4T}{(2\pi)^4} iS(P-T)\tau_D(t')\tau_D(t)$$

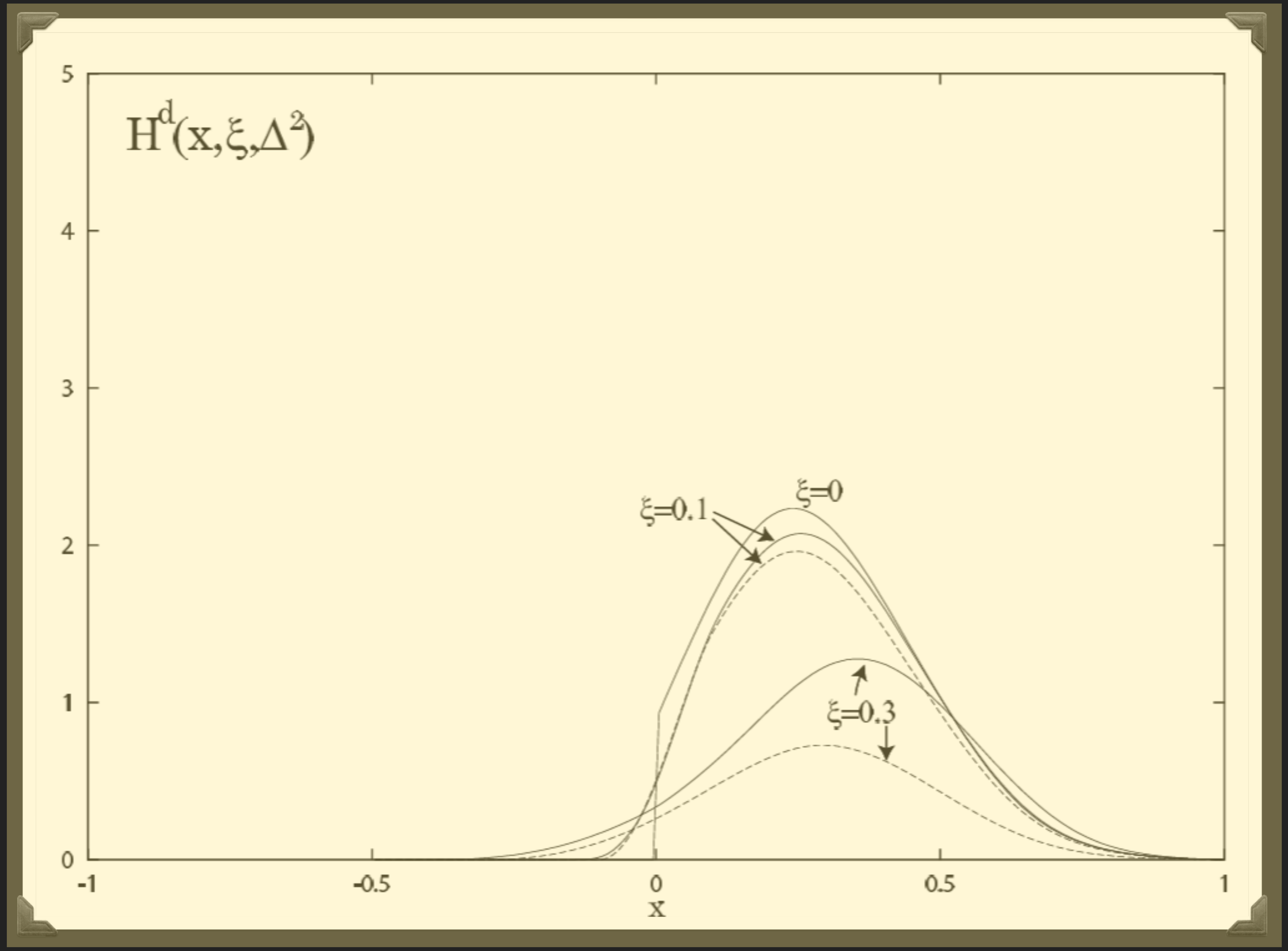
$$\times i \int \frac{d^4K}{(2\pi)^4} \mathrm{Tr}\left[\gamma^5 C\tau_2\beta_A S(k')\gamma^+ S(k) C^{-1}\gamma^5\tau_2\beta_A S(t-K)^T\right]\delta\left(x-\frac{K_-}{P_-}\right)u(p,\lambda)$$

Where $t = T - \frac{\Delta}{2}$ and $t' = T + \frac{\Delta}{2}$ are the diquark momenta

GPD NUCLEON IN THE NJL MODEL

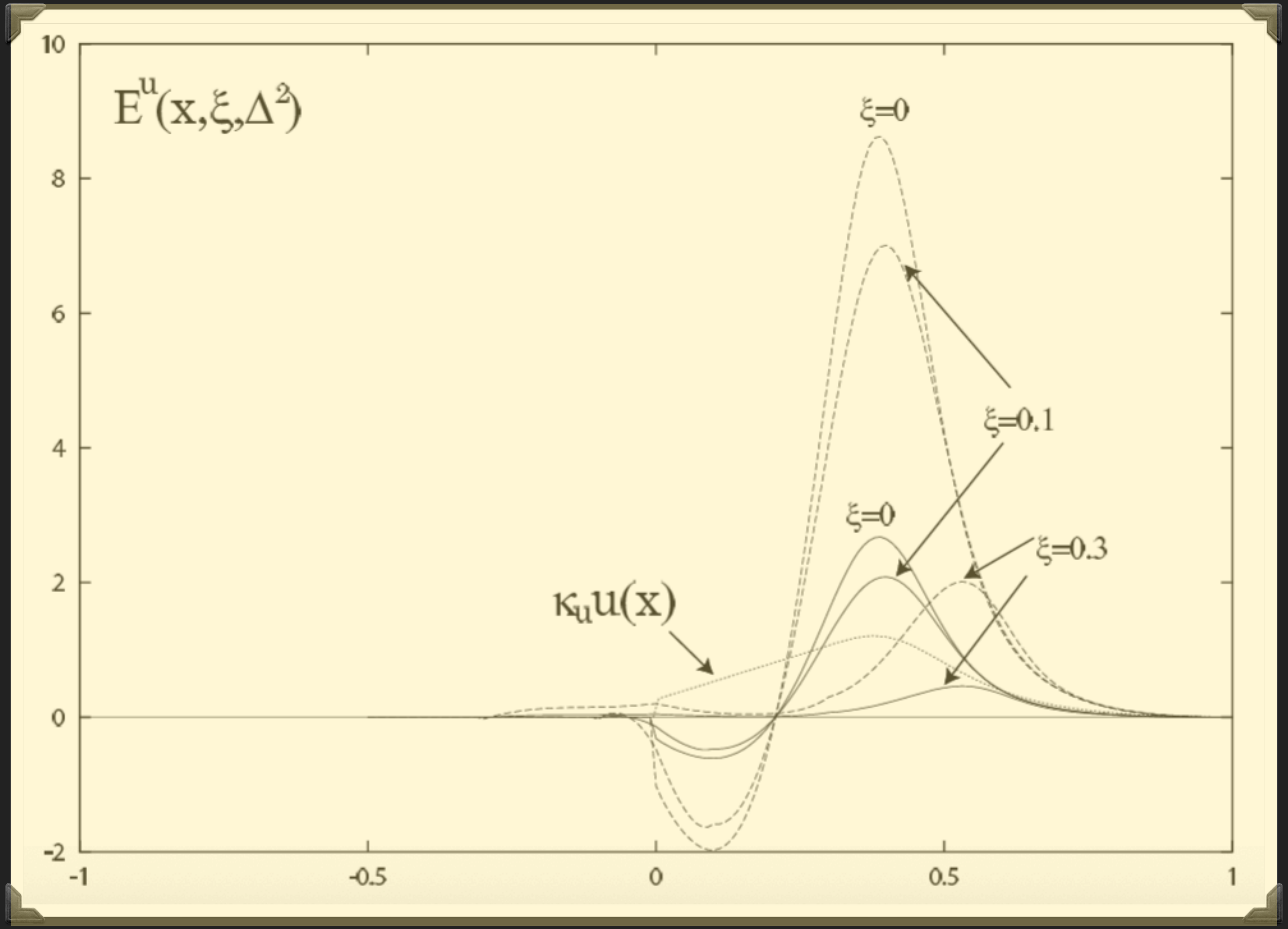
- Results for $H^{Q=u,d}(x,\xi,\Delta^2)$ for $\xi = 0{,}0.1{,}0.3$ in the NJL model (solid line) and

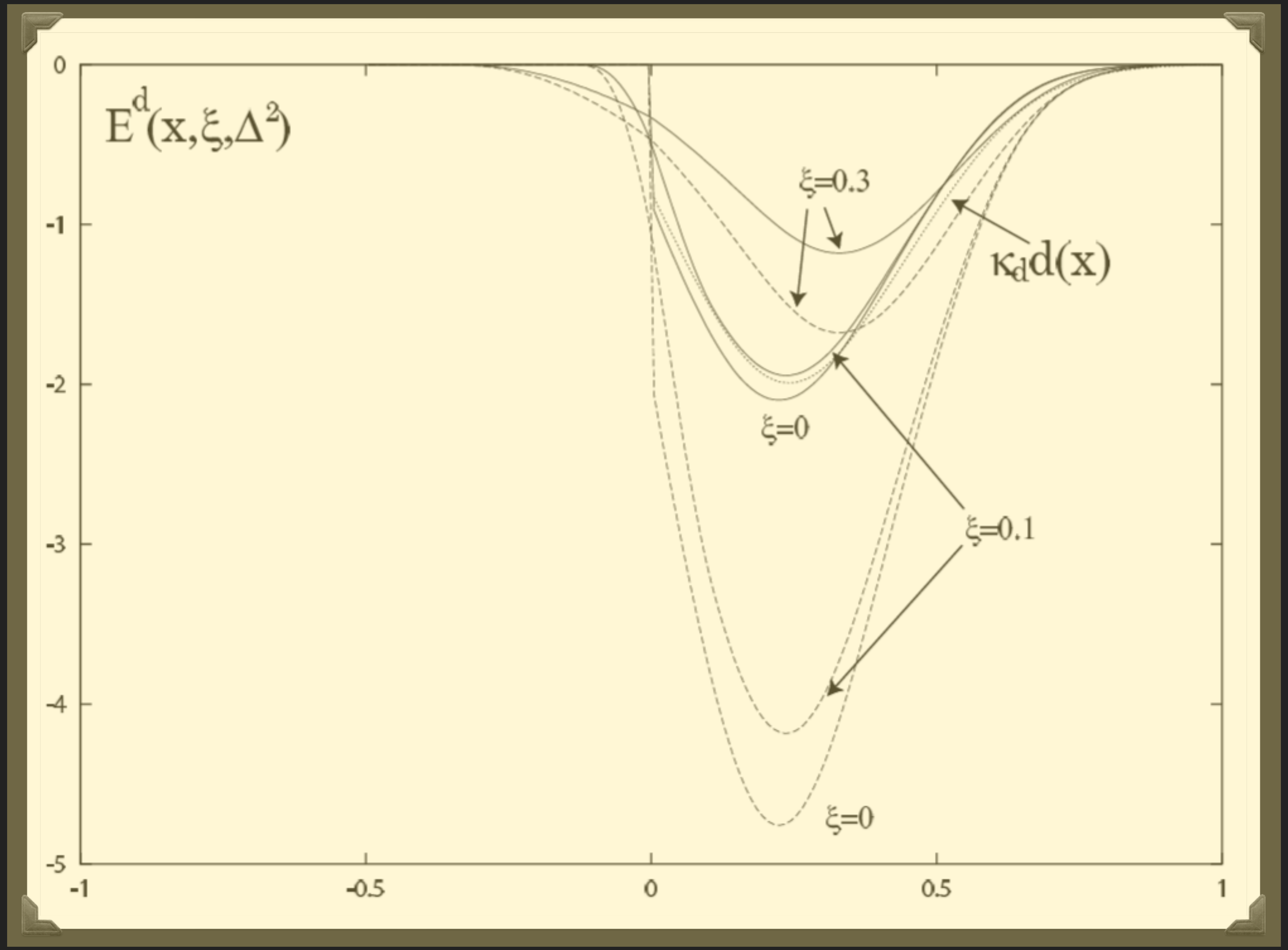




GPD NUCLEON IN THE NJL MODEL

- Results for $E^{Q=u,d}(x,\xi,\Delta^2)$ for $\xi = 0{,}0.1{,}0.3$ in the NJL model (solid line) and



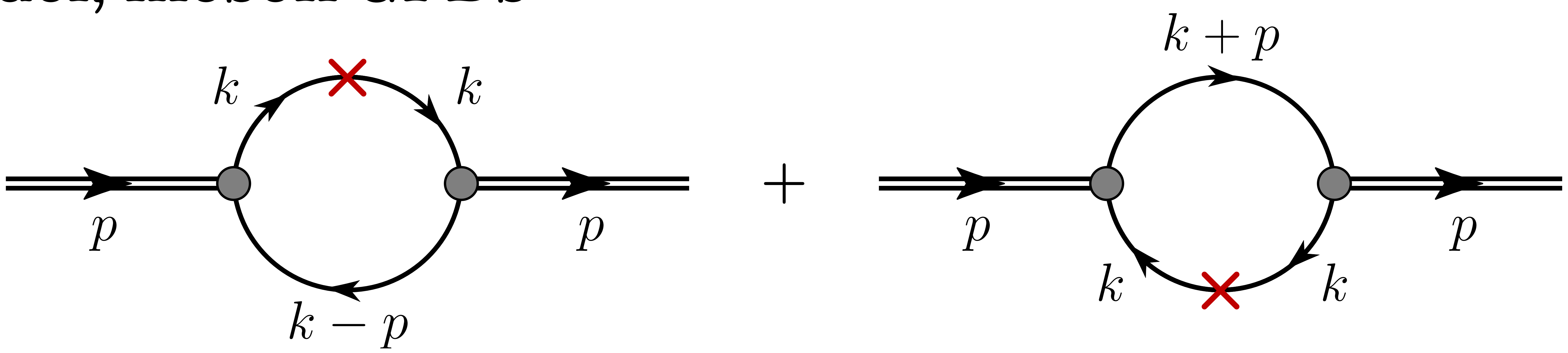


GPD MESONS IN THE NJL MODEL

BSE-NJL model

Generalized Parton distributions (GPDs)

- In the NJL model, meson GPDs



- where the initial and final meson momentum are respectively given by p and p'

$$p^2 = p'^2 = m^2_{(\pi,K)}, \qquad t = q^2 = -Q^2 = (p' - p)^2, \qquad P = \frac{p + p'}{2}, \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+}$$

- With ξ stands for **the skewness parameter** and the light-cone four-vector is given as $n = (1,0,0,-1)$

BSE-NJL model

The vector and tensor quark GPDS of the meson — General definition

- The vector (**no spin flip**) and tensor (**spin flip**) quark GPDs of the meson are given by

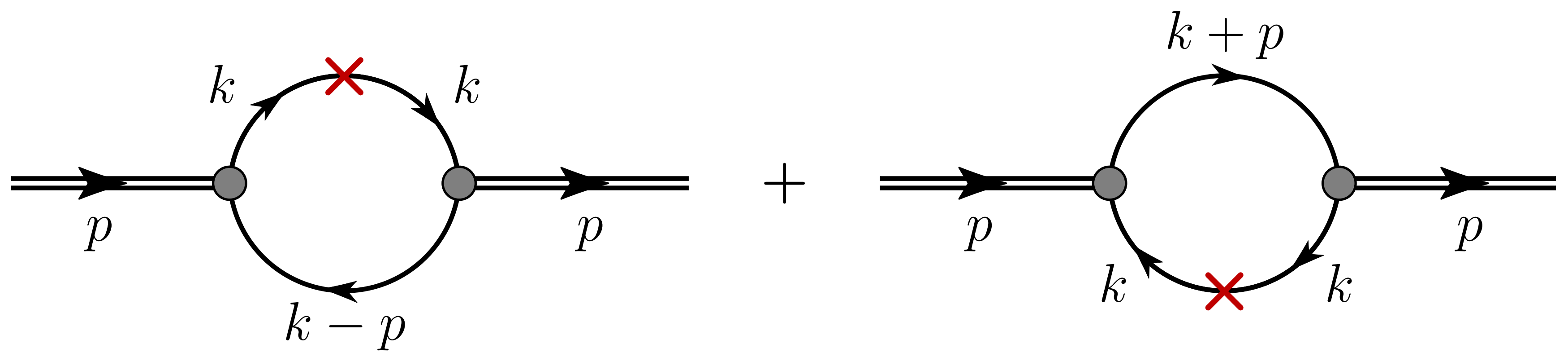
$$H^q(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} \exp[ixP^+z^-]\langle p' \mid \bar{\psi}_q\left(-\frac{1}{2}z\right)\gamma^+\bar{\psi}_q\left(\frac{1}{2}z\right) \mid p\rangle\Big|_{z^+=0,\mathbf{z}=0}$$

$$E^q(x,\xi,t) = \frac{P^+m_{(\pi,K)}}{2(P^+q^j - P^jq^+)}\int \frac{dz^-}{2\pi} \exp[ixP^+z^-]\langle p' \mid \bar{\psi}_q\left(-\frac{1}{2}z\right)i\sigma^{+j}\psi_q\left(\frac{1}{2}z\right) \mid p\rangle\Big|_{z^+=0,\mathbf{z}=0}$$

- Where x is the longitudinal momentum

BSE-NJL model

Up-quark vector and tensor GPDs for the kaon



- In the NJL model, up-quark vector and tensor GPDs for the kaon are given by

$$H^u(x,\xi,t) = 2iN_c g_{Kq\bar{q}}^2 \int \frac{d^4k}{(2\pi)^4} \delta\left(xP^+ - k^+\right) \mathrm{Tr}[\gamma_5 S_u(k+\frac{q}{2})\gamma^+ S_u(k-\frac{q}{2})\gamma_5 S_s(k-P)]$$

$$E^u(x,\xi,t) = 2iN_c g_{Kq\bar{q}}^2 \left(\frac{P^+ m_K}{(P^+q^j - P^j q^+)}\right) \int \frac{d^4k}{(2\pi)^4} \delta\left(xP^+ - k^+\right) \mathrm{Tr}[\gamma_5 S_u(k+\frac{q}{2} i\sigma^{+j} S_u(k-\frac{q}{2})\gamma_5 S_s(k-P)]$$

- Performing the Feynman parametrization, WTI-like, and the proper-time regularization scheme

- Finally, the up-quark vector and tensor GPDs for the kaon are obtained by

BSE-NJL model

NJL up-quark vector and tensor GPDs for the kaon — final expressions

- Vector GPDs for the kaon in the proper-time regularization scheme

$$H^u(x,\xi,t) = \frac{N_c g_{Kq\bar{q}}^2}{8\pi^2}\left[\Theta_{\bar{\xi}_1}\bar{C}_1(\sigma_3) + \Theta_{\xi_1}\bar{C}_1(\sigma_4) + \frac{\Theta_{\bar{\xi}\xi}}{\xi}x\bar{C}_1(\sigma_5)\right] + \frac{N_c g_{Kq\bar{q}}^2}{8\pi^2}\int_0^1 dx\frac{\Theta_{x\xi}}{\xi}\frac{1}{\sigma_6}\bar{C}_2(\sigma_6)((1-x)t + 2x(m_K^2 - (M_u - M_s)^2))$$

- Tensor GPDs for the kaon in the proper-time regularization scheme

$$E^u(x,\xi,t) = \frac{N_c g_{Kq\bar{q}}^2}{4\pi^2}\int_0^1 dx\frac{\Theta_{x\xi}}{\xi}m_K((M_s - M_u)x + M_u)\frac{1}{\sigma_6}\bar{C}_2(\sigma_6)$$

- The Θ is the step function

BSE-NJL model
Properties of the GPDs

1. Forward limit – $\xi = 0$, and $t = 0$, the vector GPDs can be reduced into the kaon PDFs

2. Symmetries properties

$$H^{[I=0]}(x, \xi, t) = H^u(x, \xi, t) - H^u(-x, \xi, t)$$

$$H^{[I=1]}(x, \xi, t) = H^u(x, \xi, t) + H^u(-x, \xi, t)$$

3. The NJL results preserve the time reversal invariance property of GPDs

$$H^u(x, \xi, t) = H^u(x, -\xi, t) \qquad E^u(x, \xi, t) = E^u(x, -\xi, t)$$

BSE-NJL model

Properties of the GPDs

4. Condition of the Polynomiality

$$\int_{-1}^{1} x^n dx H^q(x, \xi, t) = \sum_{i=0}^{((n+1)/2)} \xi^{2i} \mathscr{A}^q_{(n+1),2i}(t)$$

$$\int_{-1}^{1} dx E^q(x, \xi, t) = \sum_{i=0}^{((n+1)/2)} \xi^{2i} \mathscr{B}^q_{(n+1),2i}(t)$$

5. For n = 0, we simply obtain the u-quark vector FFs ($F^u_K(Q^2)$) and tensor FFs ($F^u_T(Q^2)$)

$$\int_{-1}^{1} H^u(x, \xi, t) dx = \mathscr{A}^u_{1,0}(t) = F^u_K(Q^2)$$

$$\int_{-1}^{1} E^u(x, \xi, t) dx = \mathscr{B}^u_{1,0}(t) = F^u_T(Q^2)$$

BSE-NJL model

Properties of the GPDs

6. For n= 1, the GPDs will preserve the sum rule:

$$\int_{-1}^{1} xH^u(x,\xi,t)dx = \mathscr{A}^u_{2,0}(t) + \xi^2\mathscr{A}^u_{2,2}(t) = \Theta^u_2(t) - \xi^2\Theta^u_1(t)$$

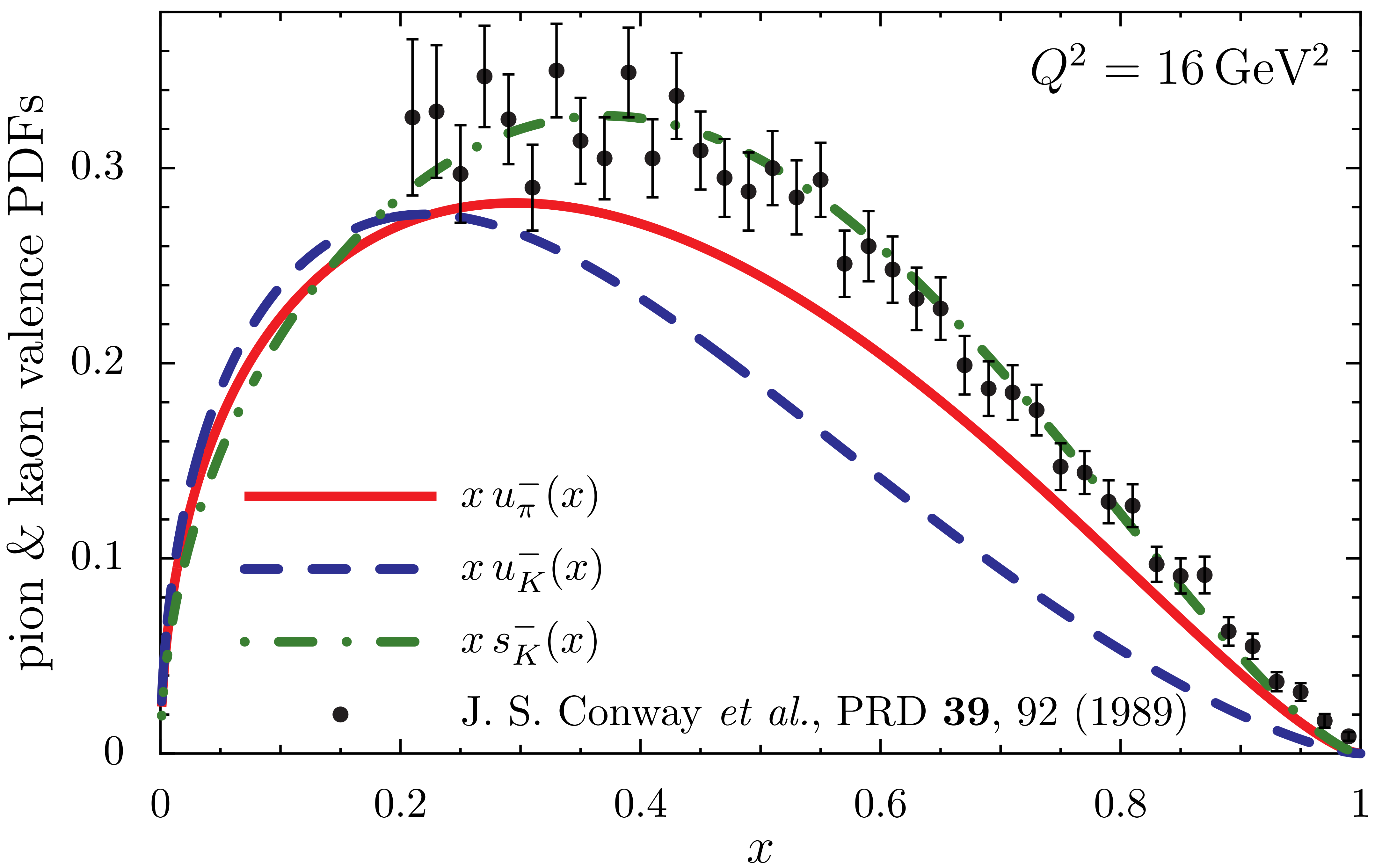
- $\Theta^u_2(t)$ and $\Theta^u_1(t)$ – the u-quark distribution for the kaon and pressure distribution
- $\mathscr{A}^u_{2,0}(Q^2)$ and $\mathscr{A}^u_{2,2}(Q^2)$ are the generalized FFs for n=1 in the BSE-NJL model
- The first derivation of $\mathscr{A}^u_{2,0}(Q^2)$ in respect with Q^2 at around $Q^2 = 0$ will give the light-cone energy radius
- $\mathscr{B}^u_{2,0}(Q^2)$ and $\mathscr{B}^u_{2,2}(Q^2) = 0$ are the u-quark tensor GPD for the kaon in the BSE-NJL model

$$\int_{-1}^{1} xE^u(x,\xi,t)dx = \mathscr{B}^u_{2,0}(t) + \xi^2\mathscr{B}^u_{2,2}(t)$$

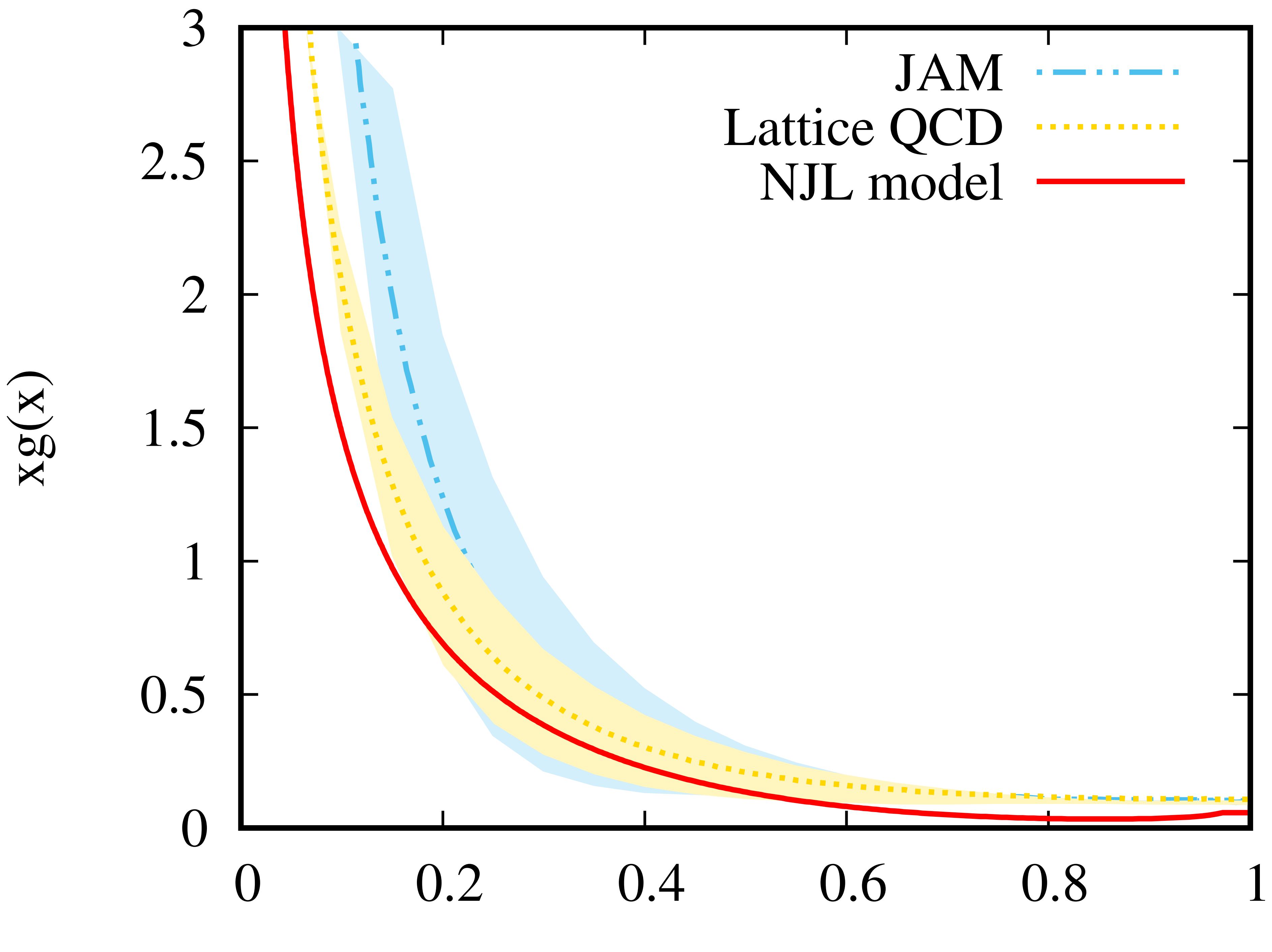
BSE-NJL model

Parton distribution functions for the meson—Forward limit $\xi = 0$ and $t = 0$

- Parton distribution functions for the pion and kaon after evolving at Q^2 =16 GeV² using NLO—DGLAP QCD evolution



PTPH, Ian Cloet & Anthony Thomas, PRC94(2016)

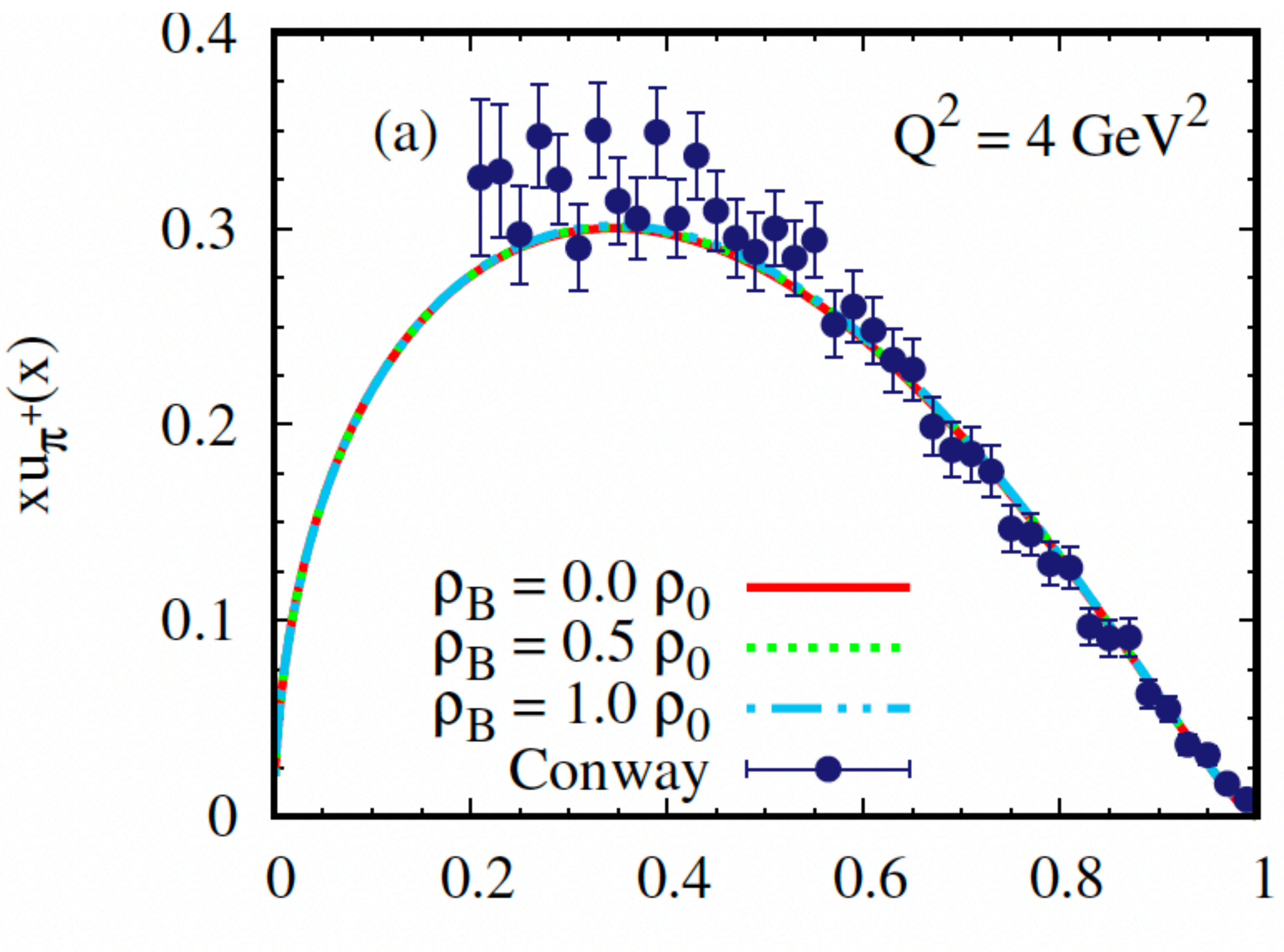


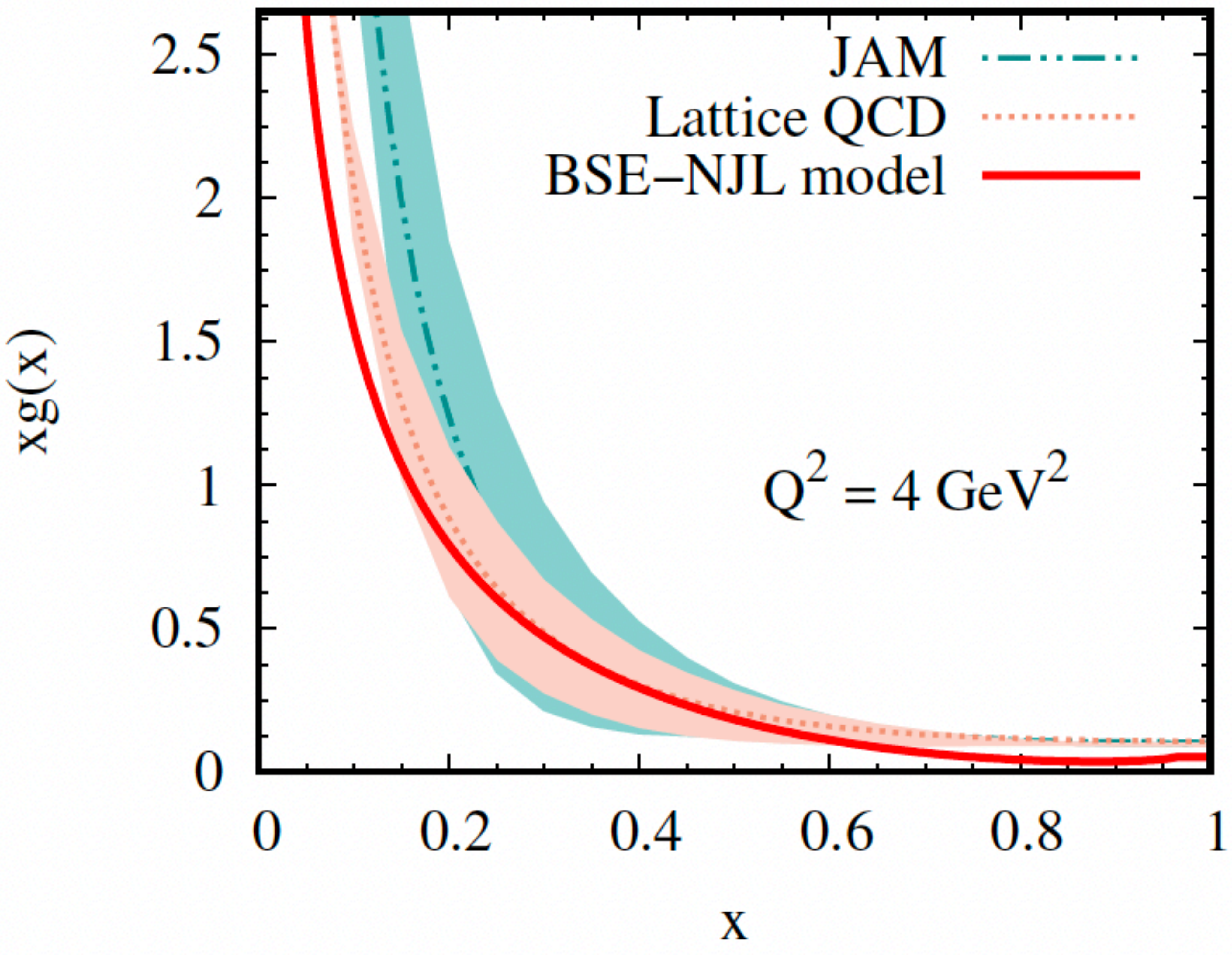
PTPH & Seung-il Nam, PRD105(2022)

BSE-NJL model

Parton distribution functions for the meson—Forward limit $\xi = 0$ and $t = 0$

- Valence and gluon distributions for the pion at Q^2 = 4 GeV²



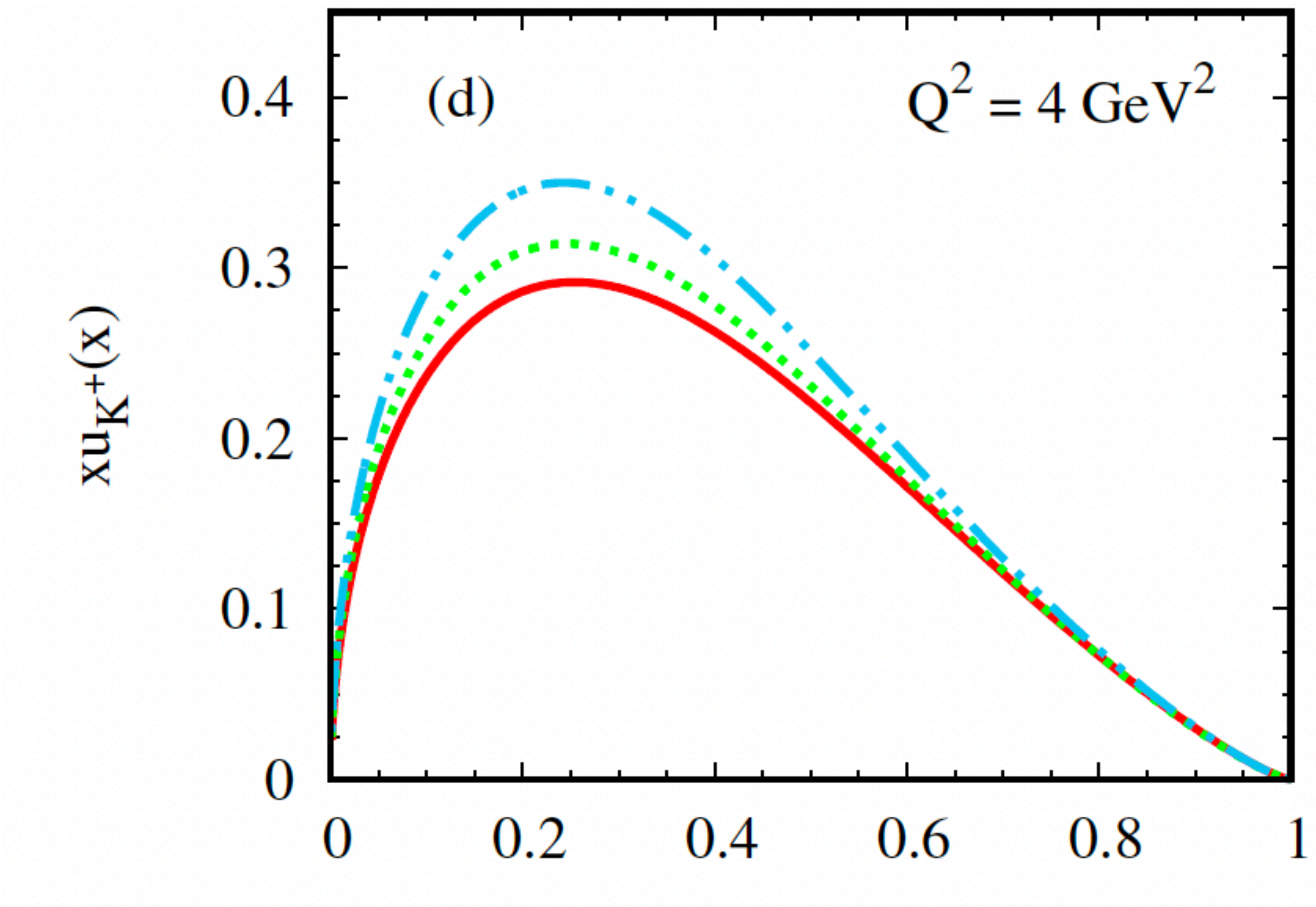


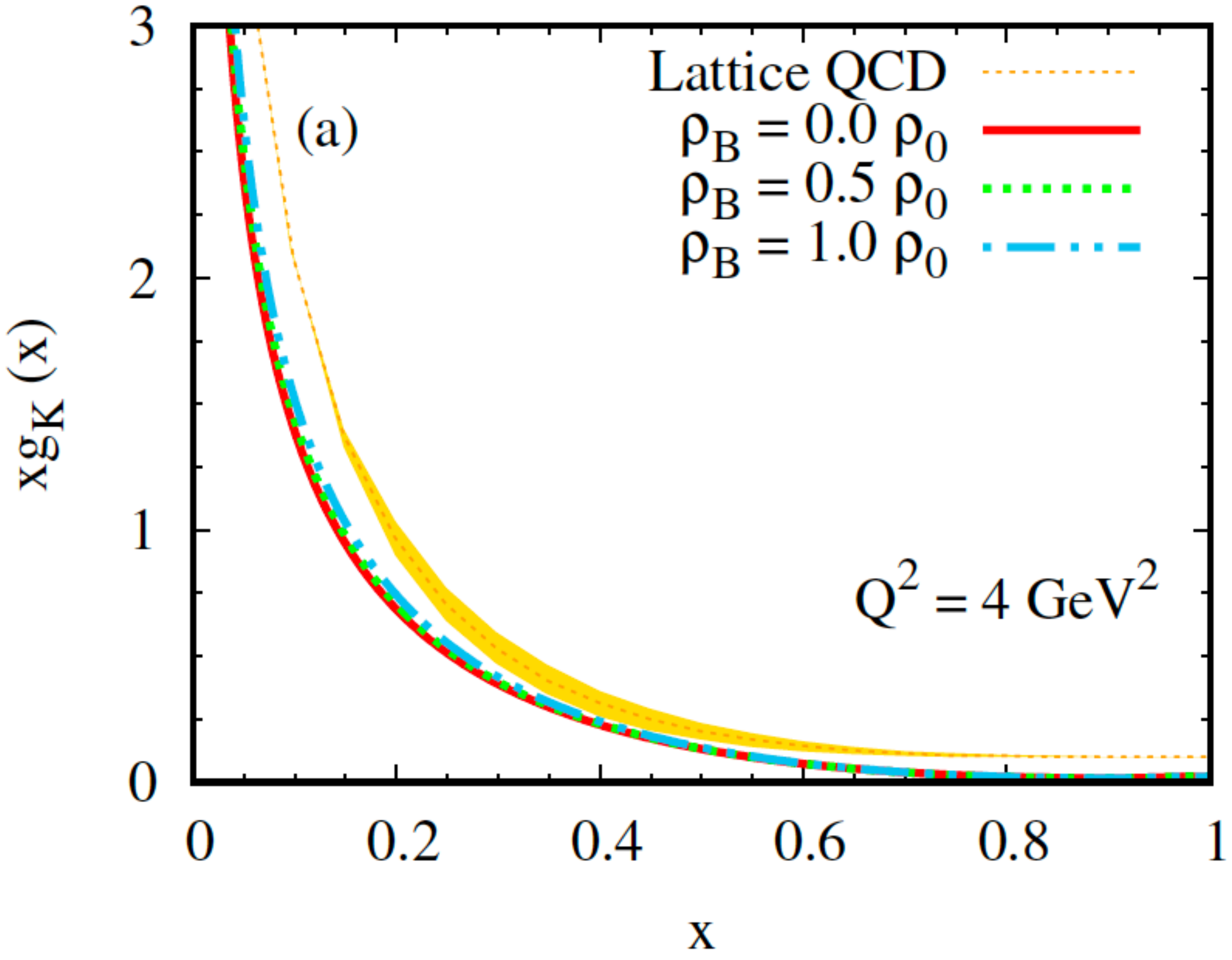
PTPH, EPJA(2022) submitted

BSE-NJL model

Parton distribution functions of the meson—Forward limit $\xi = 0$ and $t = 0$

- The valence and gluon distributions for the kaon at Q^2 = 4 GeV²



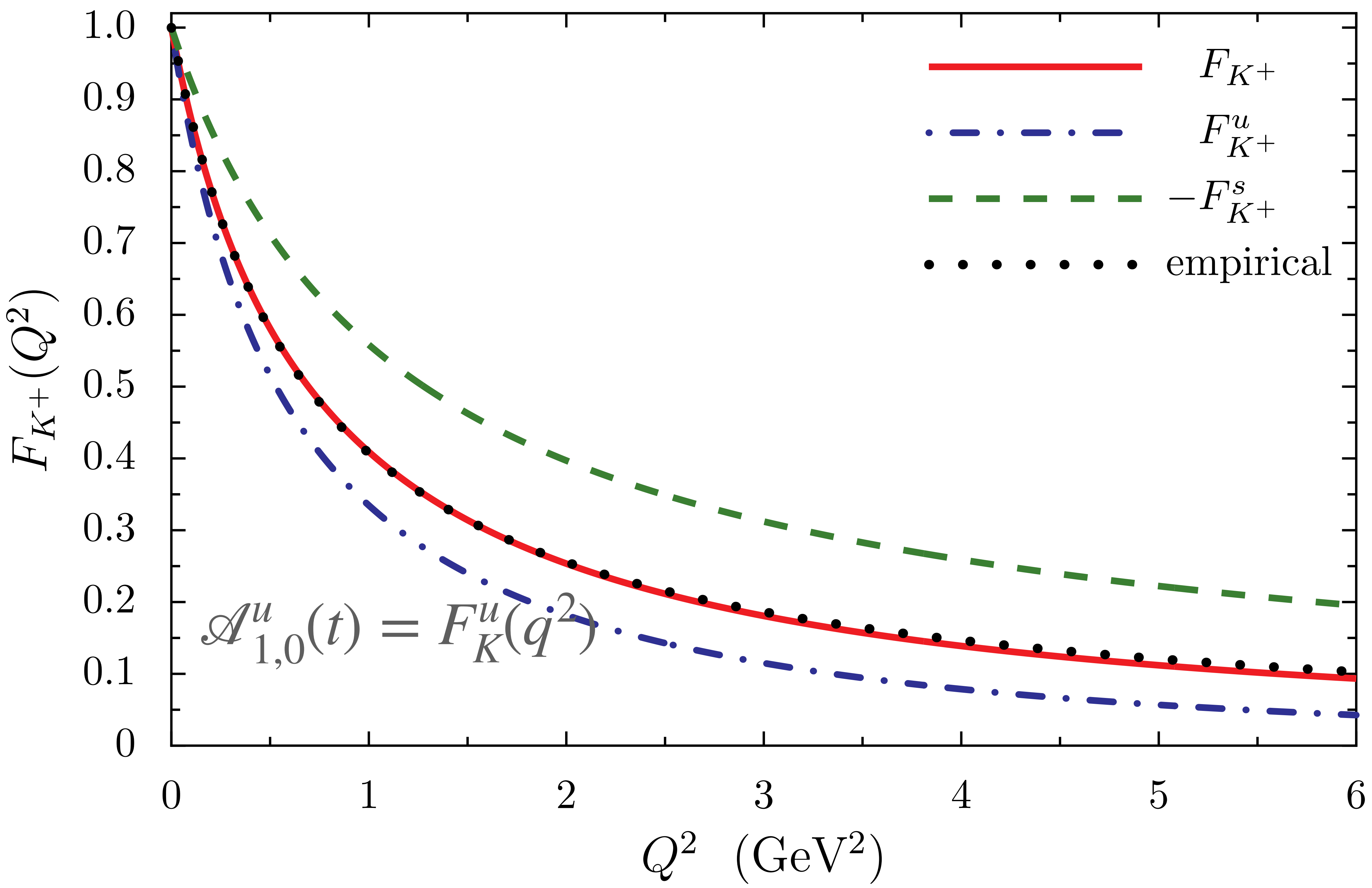


PTPH, EPJC(2022) submitted

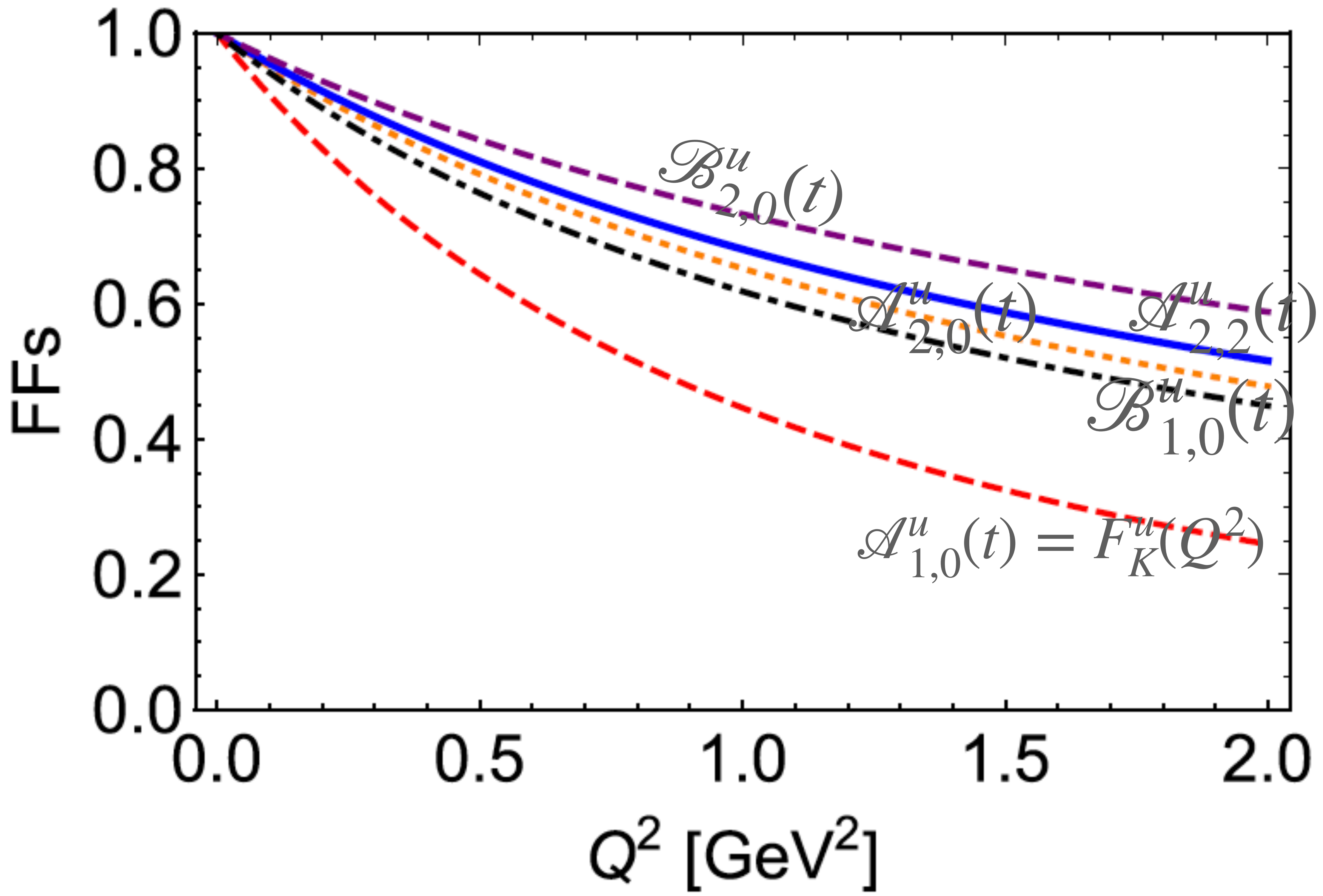
BSE-NJL model

Form Factors for the meson

- Form factor for the pion and kaon $\mathscr{A}^u_{1,0}(t) = F^u_K(Q^2)$ and $\mathscr{B}^u_{1,0}(t) = F^u_T(Q^2)$



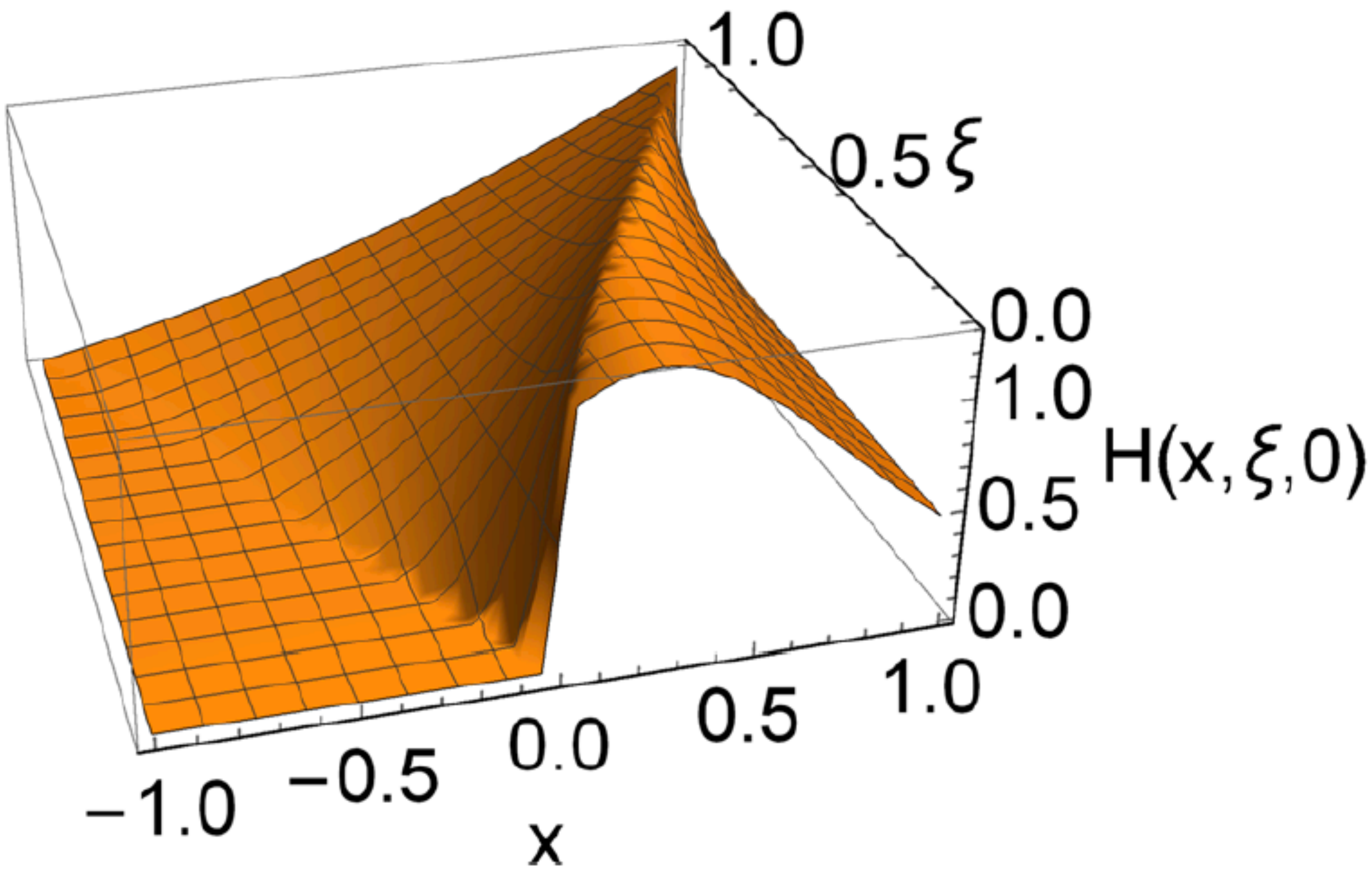
PTPH, Ian Cloet & Anthony Thomas, PRC94(2016)

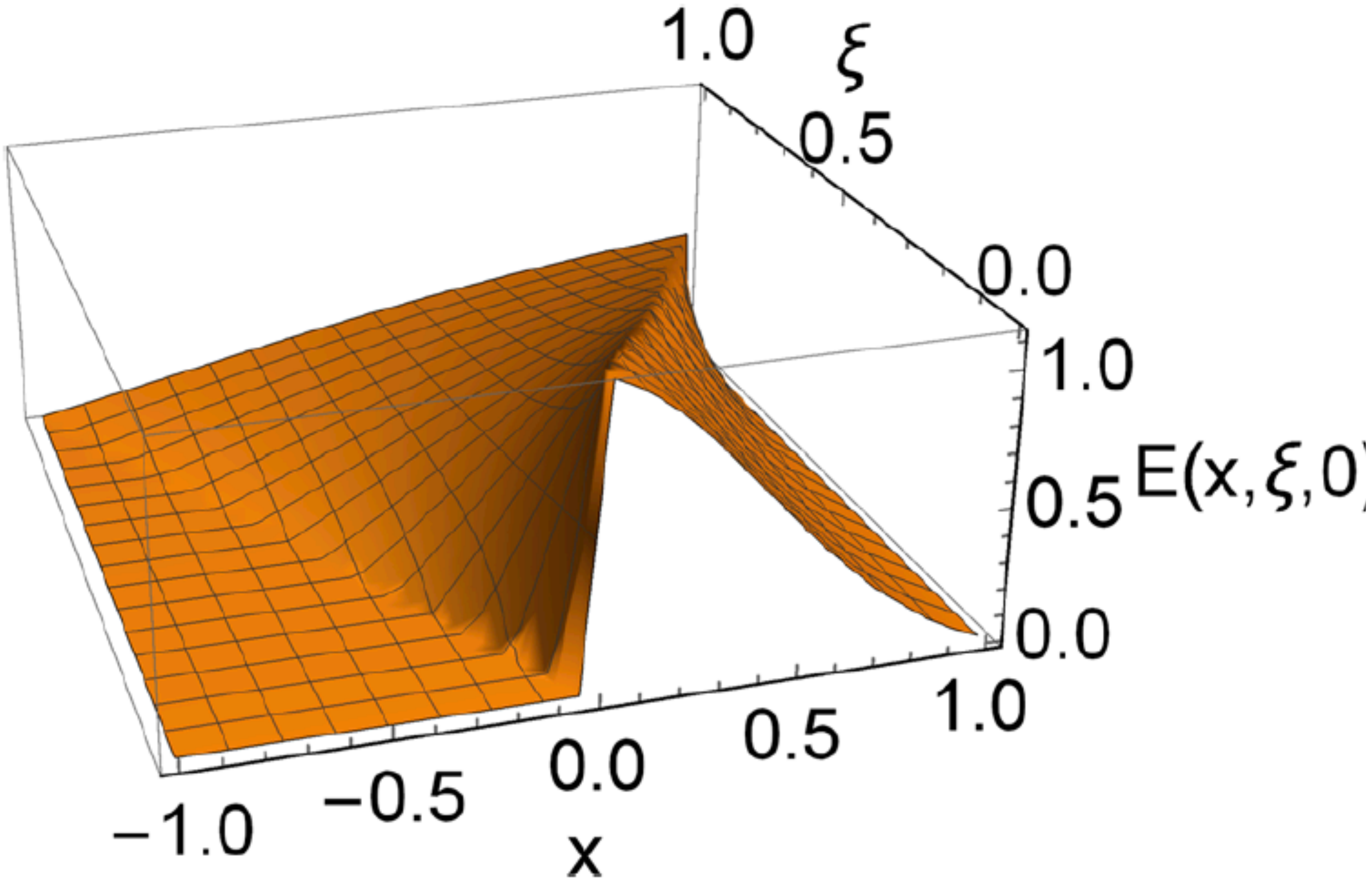


BSE-NJL model

Kaon vector GPD—$H^u(x, \xi, 0)$ and tensor GPD—$E^u(x, \xi, 0)$

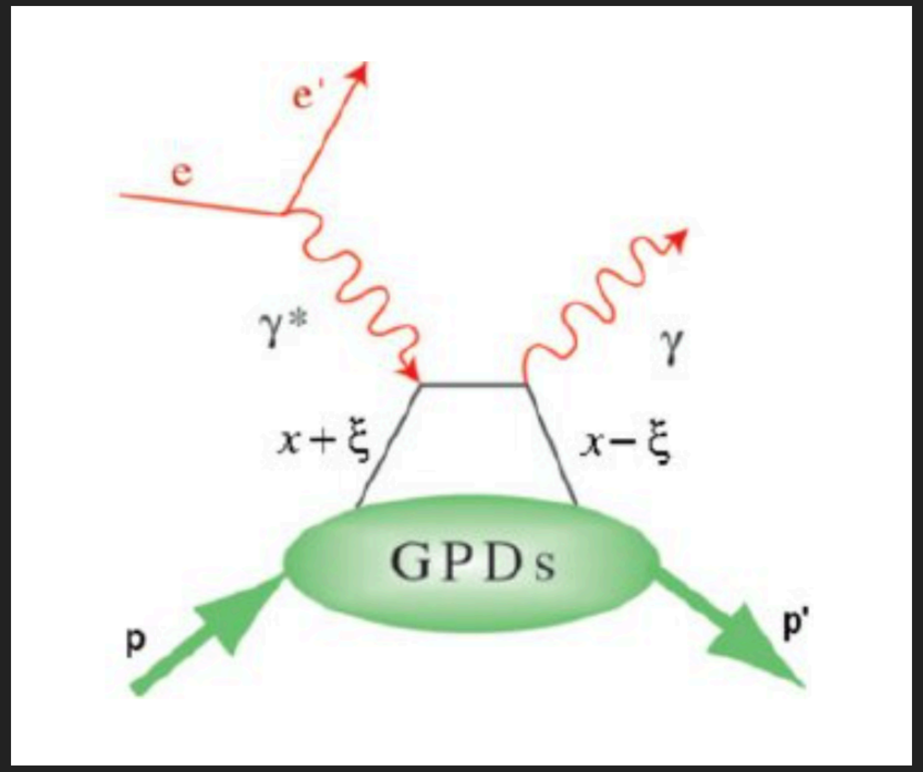
- Kaon vector and tensor GPDs for the kaon for $\xi > 0$

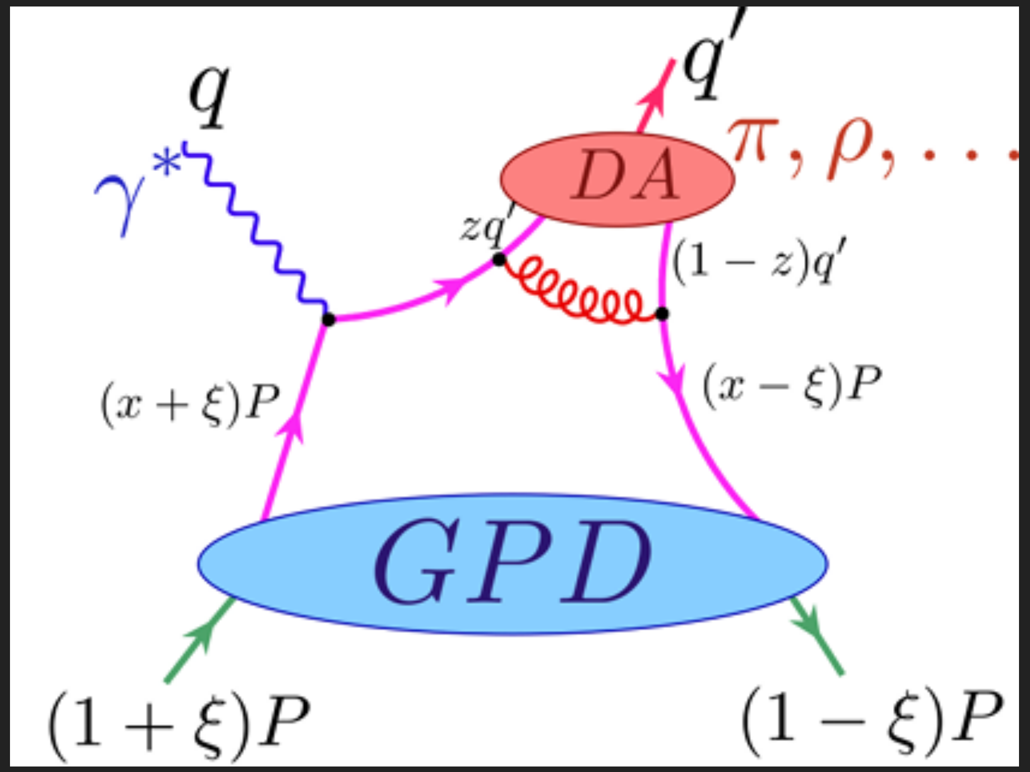


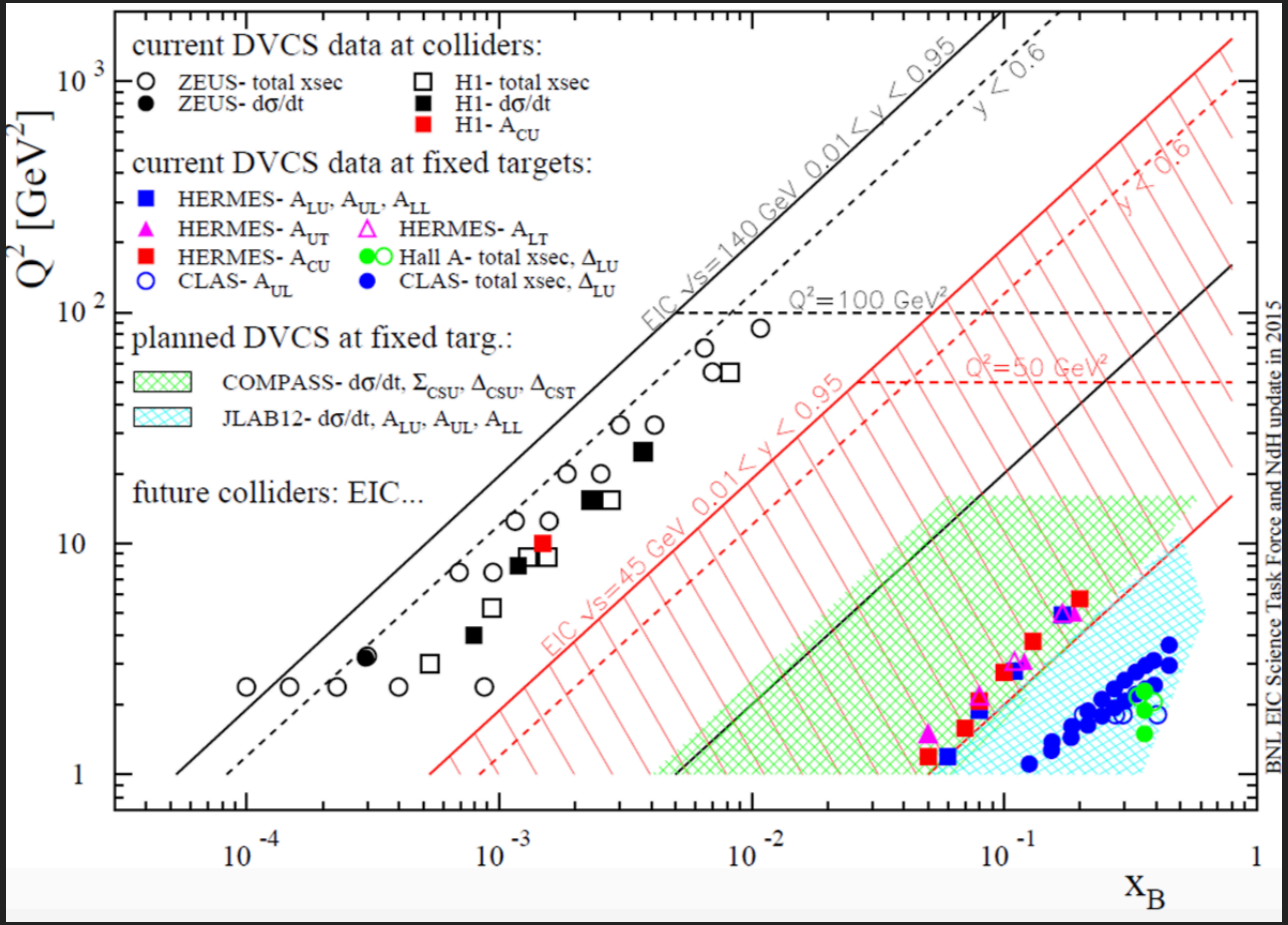


GPD EXTRACTION: MACHINE LEARNING

- Extraction GPD is more challenging
- More data with kinematic range
- DVCS and DVMP

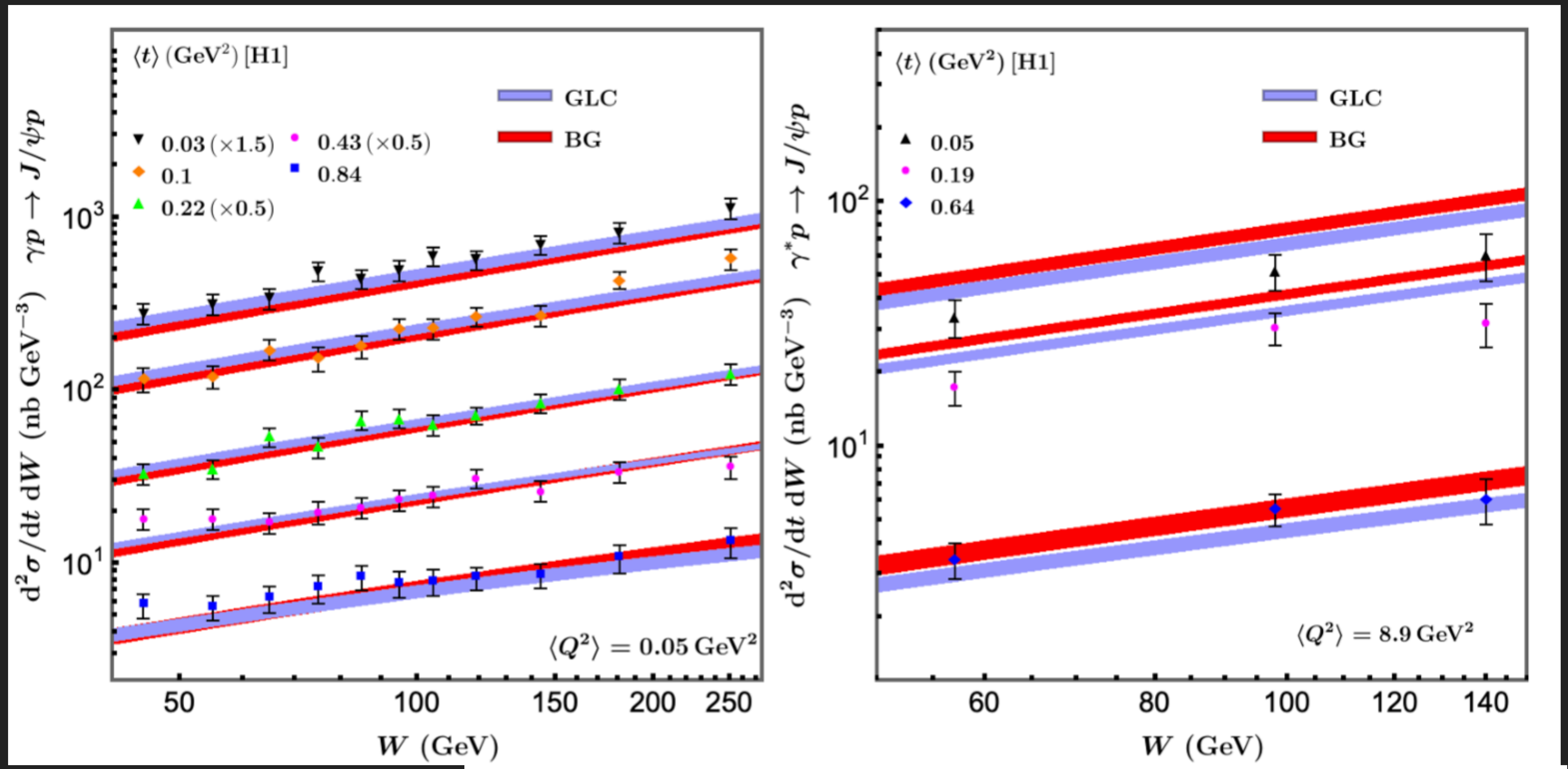


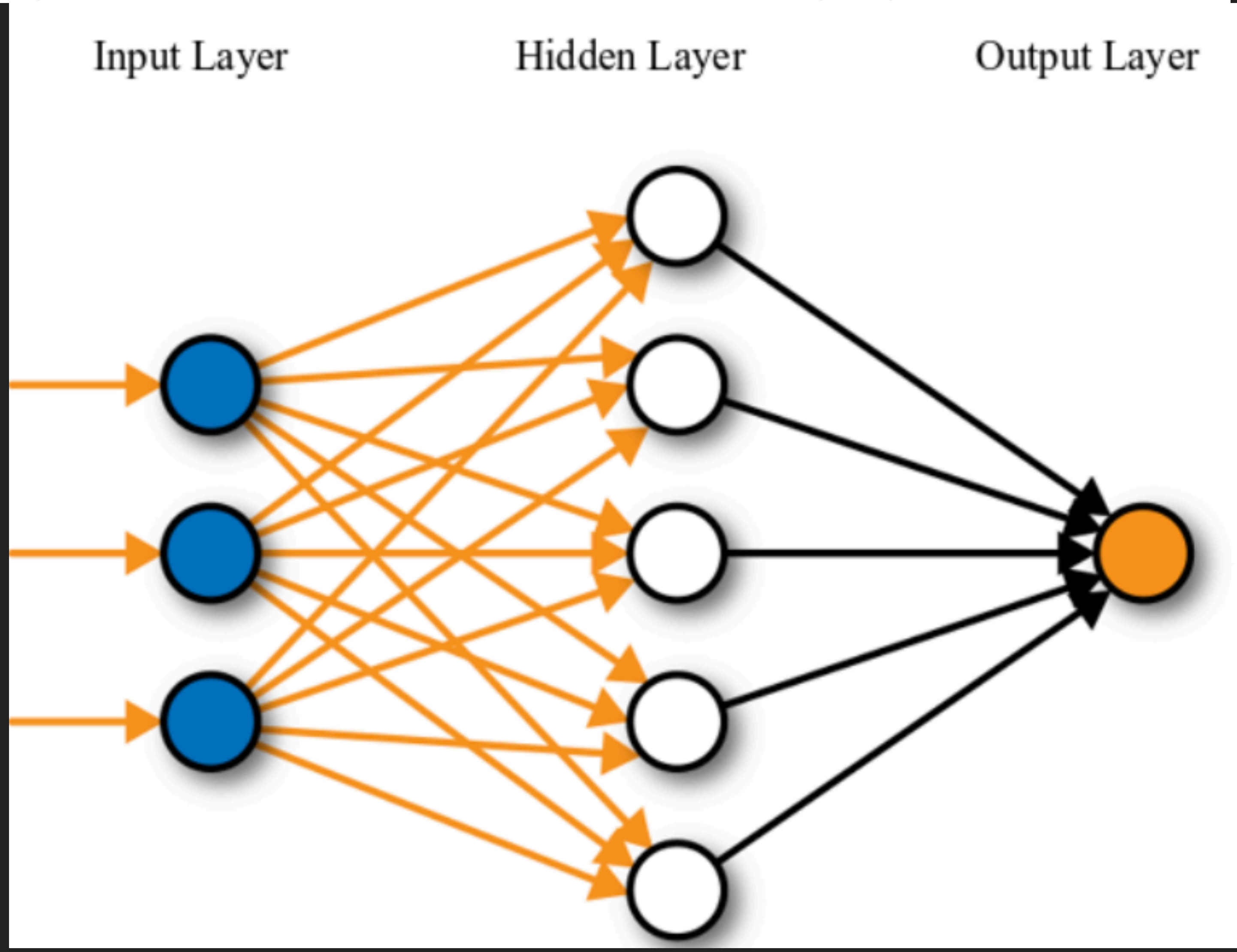


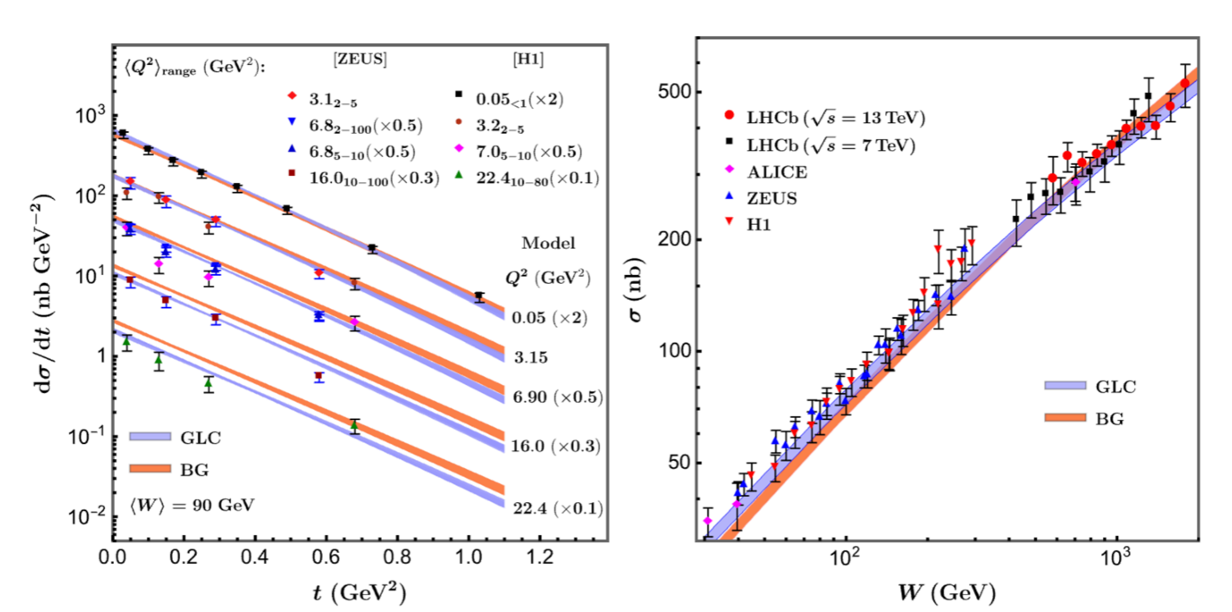


GPD EXTRACTION: MACHINE LEARNING

- Input layer: Q^2, t, and ξ
- Output: cross-section
- We are evaluating exclusive coherent diffractive J/ψ production [H1, Zeus, HERA] and photoproduction J/ψ [H1]
- Unfortunately, No result yet been obtained
- Please stay tuned!!







- Collaboration with Dr. S. Chalis, Dr. Apriadi Adam, and Dr. Zulkaida Akbar

Summary and outlook

- We have calculated the GPDs of the nucleon and meson in the effective quark theory and the prediction results are shown
- New data from the EIC, EIcC, AMBER COMPASS, and upgrade JLab-22 are very important to compare with theory predictions
- Future theory improvement: Higher twist, nonlocal,....
- GPD nucleon and meson in the nonlocal NJL model or nonlocal chiral quark model [in progress.... Collaboration with Dr. H. Son]
- Development of GPD extraction using machine learning [in progress ... Collaboration with Dr. S. Chalis, Dr. Apriadi Adam, and Dr. Zulkaida Akbar]

Thank you for attention!

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