Preambles on Nucleon GPDs

Chueng-Ryong Ji
North Carolina State University

3D Structure of the Nucleon via Generalized Parton Distributions

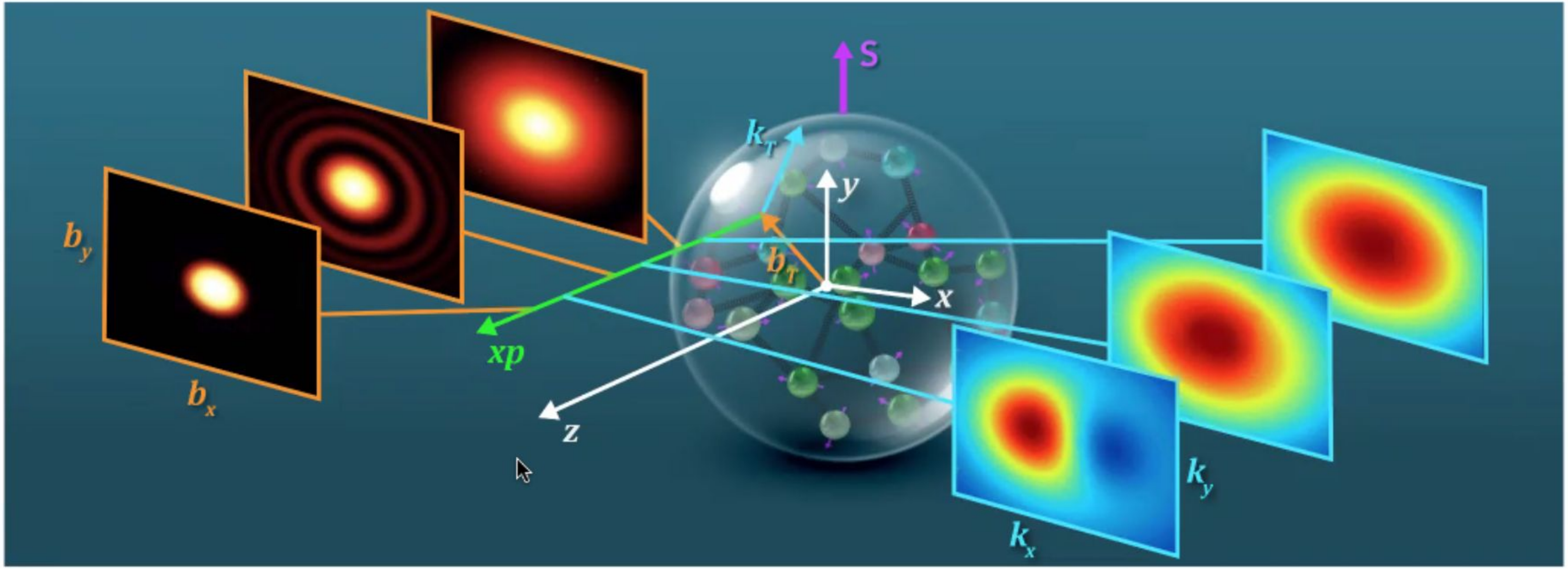
Incheon, June 25, 2024

Outline

- What are we looking for? How do we do it?
- Looking forward to EIC
- Looking back efforts up to now
  - Experiments
  - Parametrizations
- Lattice QCD
- Various Models
- Chiral Dynamics
  - Link to QCD
  - Non-analytic behavior
- Summary & Outlook

What are we looking for?

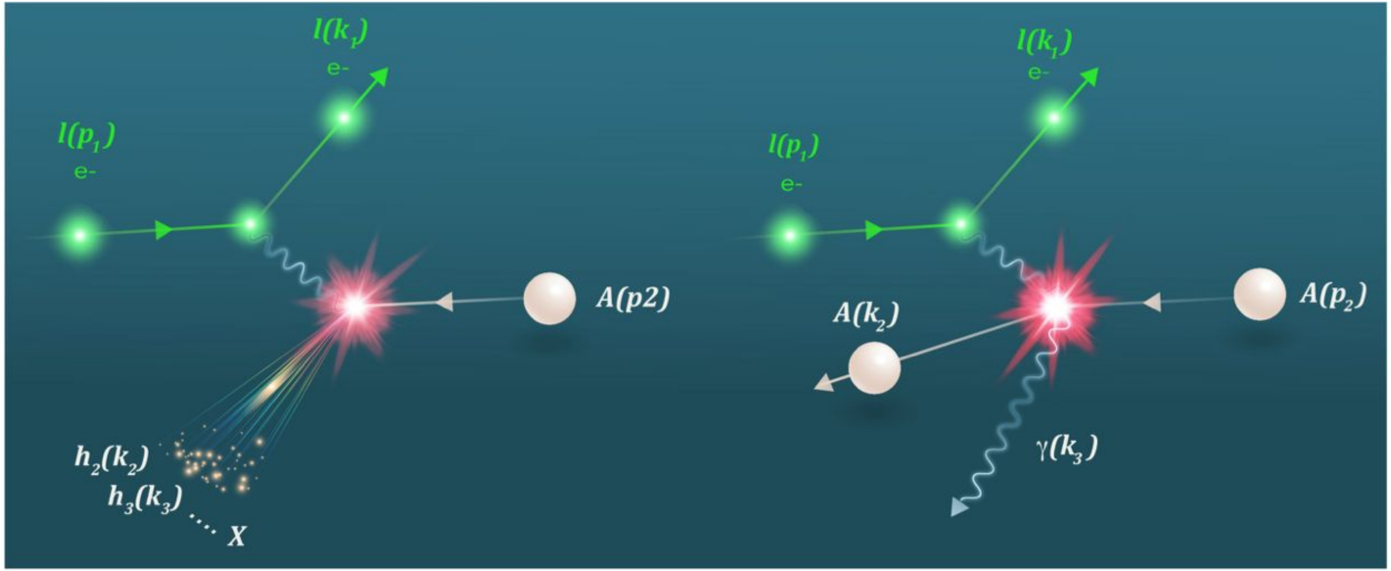
- Images of protons in coordinate and momentum space



P.Achenbach et al., The present and future of QCD,NPA1047(2024)122874

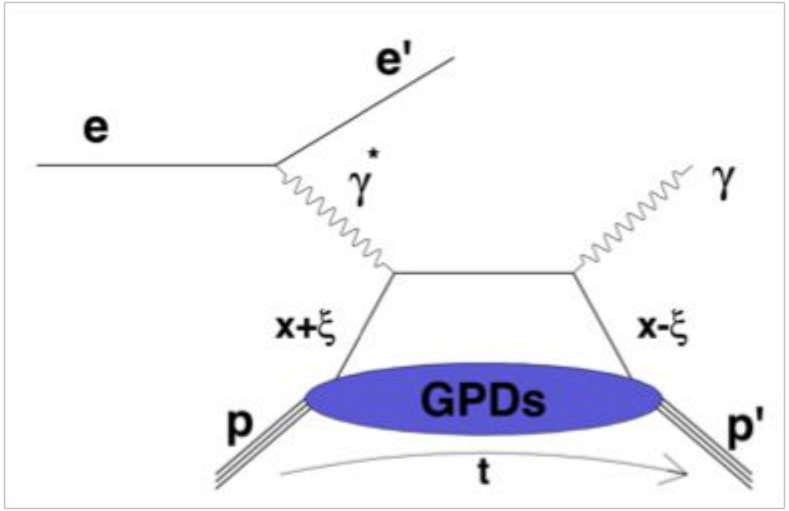
How do we do it?

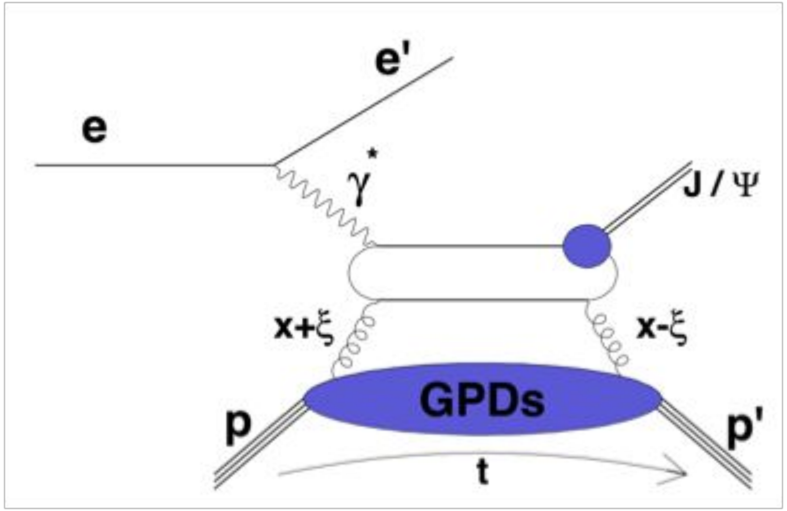
- Through experimental data in factorizable processes
- Focus on electron-induced reactions that can be realized at EIC and JLab



SIDIS -- TMDs

DVCS -- GPDs





$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{M_N} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} + M_N\,\bar{c}^a(t)g_{\mu\nu}\right]u$$

$a = g, Q$ (gluon or quark parts)

$$A^a(t) \updownarrow \delta g^{00} \downarrow \text{Mass}$$

$$\sum_a A^a(0) = 1$$

$$J^a(t) \updownarrow \delta g^{0i} \downarrow \text{Spin}$$

$$\sum_a J^a(0) = \frac{1}{2}$$

$$D^a(t) \updownarrow \delta g^{ij} \downarrow$$

deformation of space = elastic properties of N

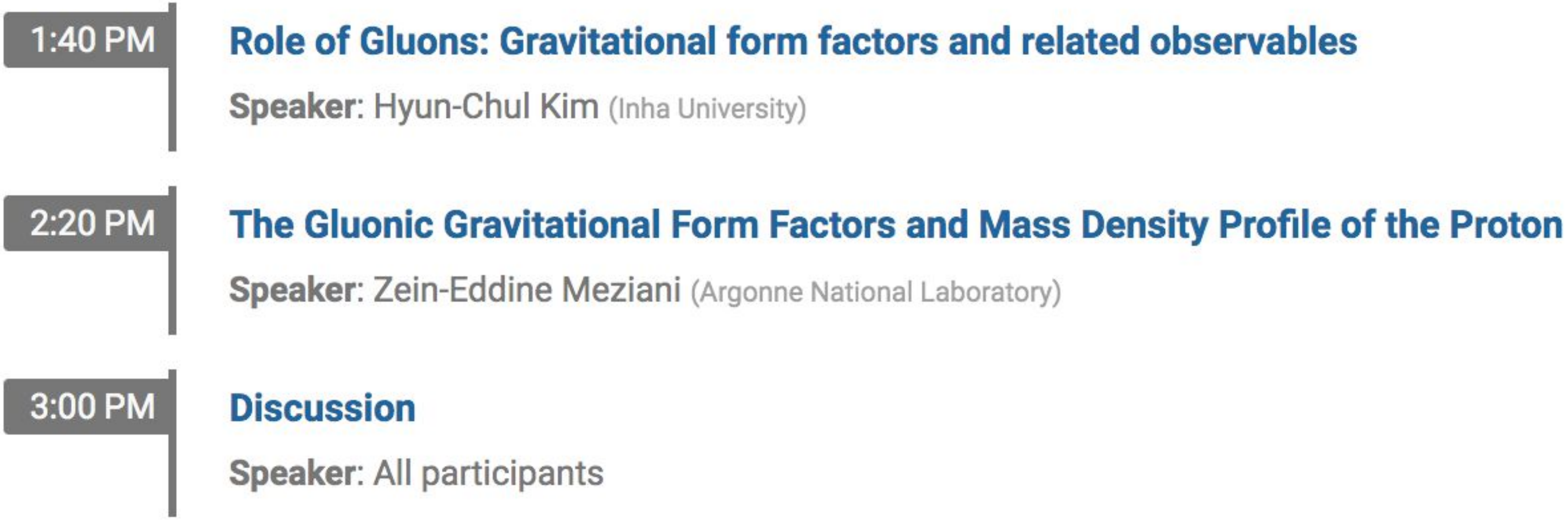
$$\bar{c}^a(t) \updownarrow$$

non − conservation of EMT pieces

$$\downarrow \sum_a \bar{c}^a(t) = 0$$

Session: 5

WEDNESDAY, JUNE 26



1:40 PM **Role of Gluons: Gravitational form factors and related observables**

Speaker: Hyun-Chul Kim (Inha University)

2:20 PM **The Gluonic Gravitational Form Factors and Mass Density Profile of the Proton**

Speaker: Zein-Eddine Meziani (Argonne National Laboratory)

3:00 PM **Discussion**

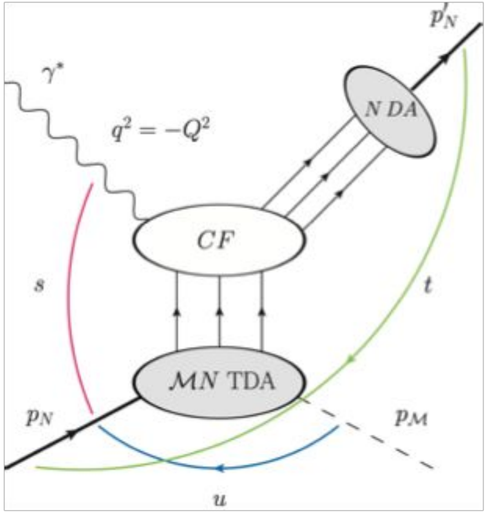
Speaker: All participants

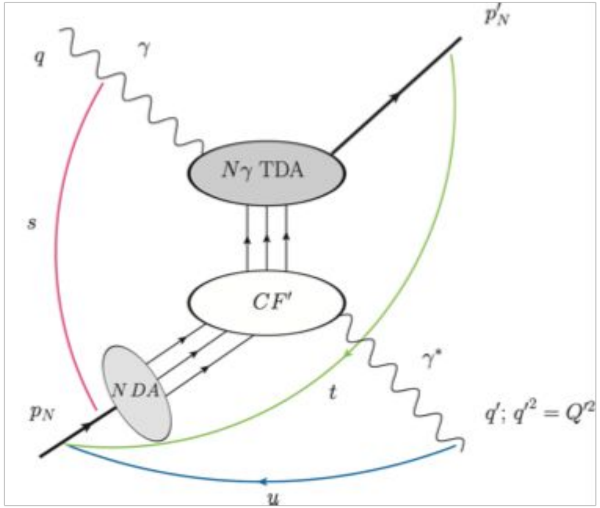
Session: 10

THURSDAY, JUNE 27

4:50 PM **Gravitational form factors, equivalence principle and shear viscosity**

Speaker: Oleg Teryaev (JINR)





Session: 8

THURSDAY, JUNE 27

10:50 AM

N->N* Transition GPDs

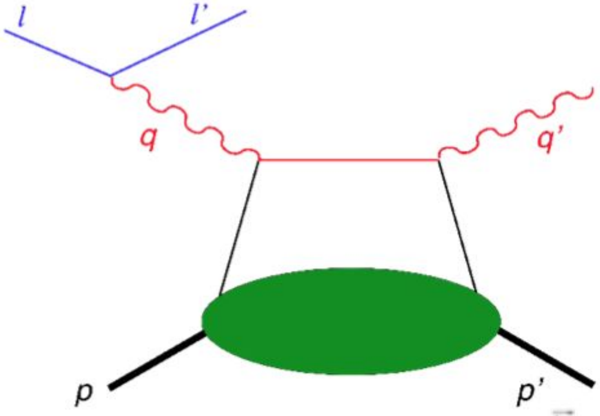
Speaker: Kyungseon Joo (University of Connecticut)

11:30 AM

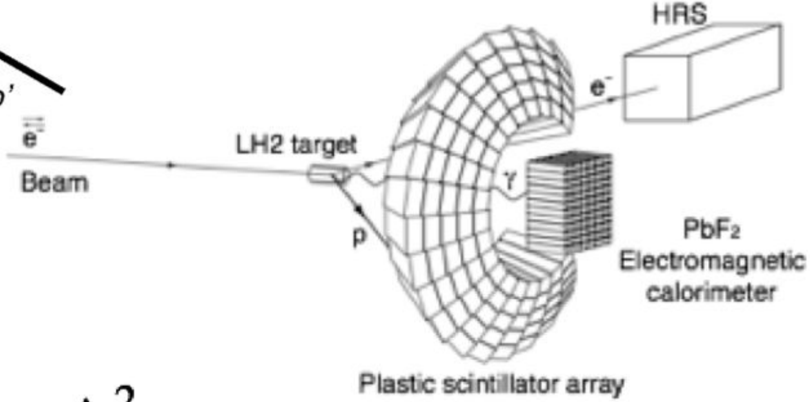
Non-diagonal GPDs and the structure of hadrons

Speaker: Kirill Semenov-Tyan-Shanskiy (Kyungpook National University)

Better Work in Forward Direction



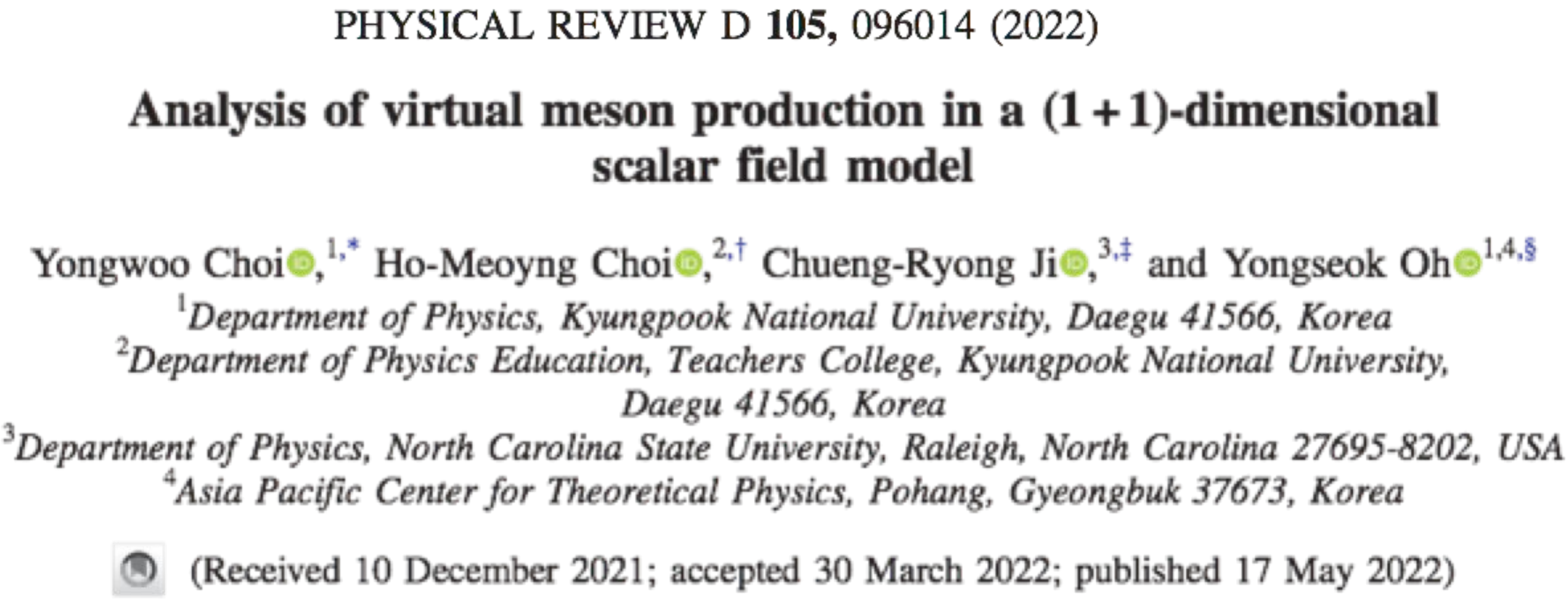
GPD



LFD

$$t = \Delta^2 = -\frac{\zeta^2 M^2 + \Delta_\perp^2}{1-\zeta}; \Delta^+(\equiv \Delta^0 + \Delta^3) = \zeta P^+; \Delta_\perp^2 > \Delta_{\perp\min}^2 \neq 0$$

Theoretical Simulation of the Virtual Meson Production in the Forward Direction



PHYSICAL REVIEW D **105,** 096014 (2022)

Analysis of virtual meson production in a (1 + 1)-dimensional scalar field model

Yongwoo Choi,[1,*] Ho-Meoyng Choi,[2,†] Chueng-Ryong Ji,[3,‡] and Yongseok Oh[1,4,§]

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[2]*Department of Physics Education, Teachers College, Kyungpook National University, Daegu 41566, Korea*
[3]*Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202, USA*
[4]*Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 37673, Korea*

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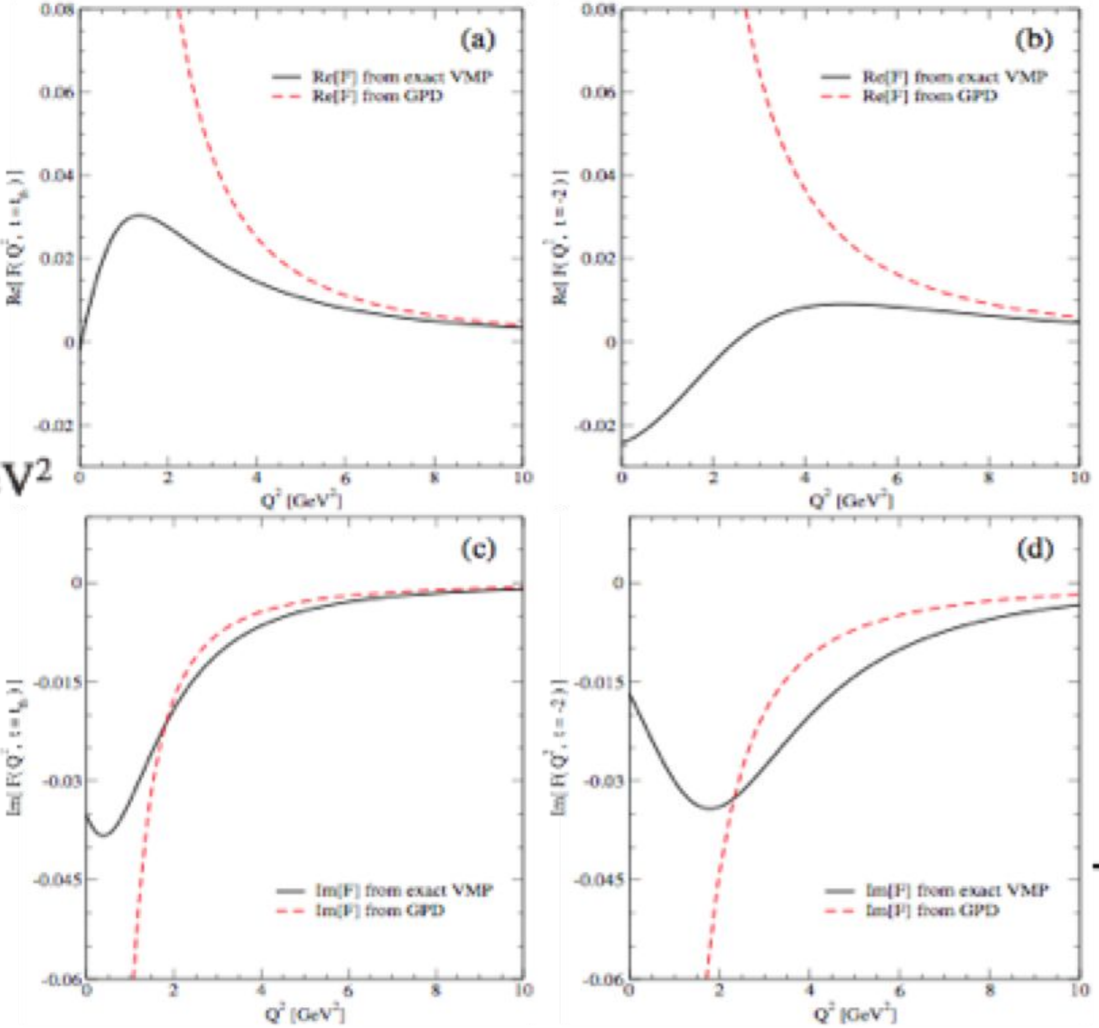
FRIDAY, JUNE 28 **Session: 11** 9:00 AM

Beam-spin asymmetry in DVMP on helium-4

Speaker: Yongwoo Choi (Inha University)

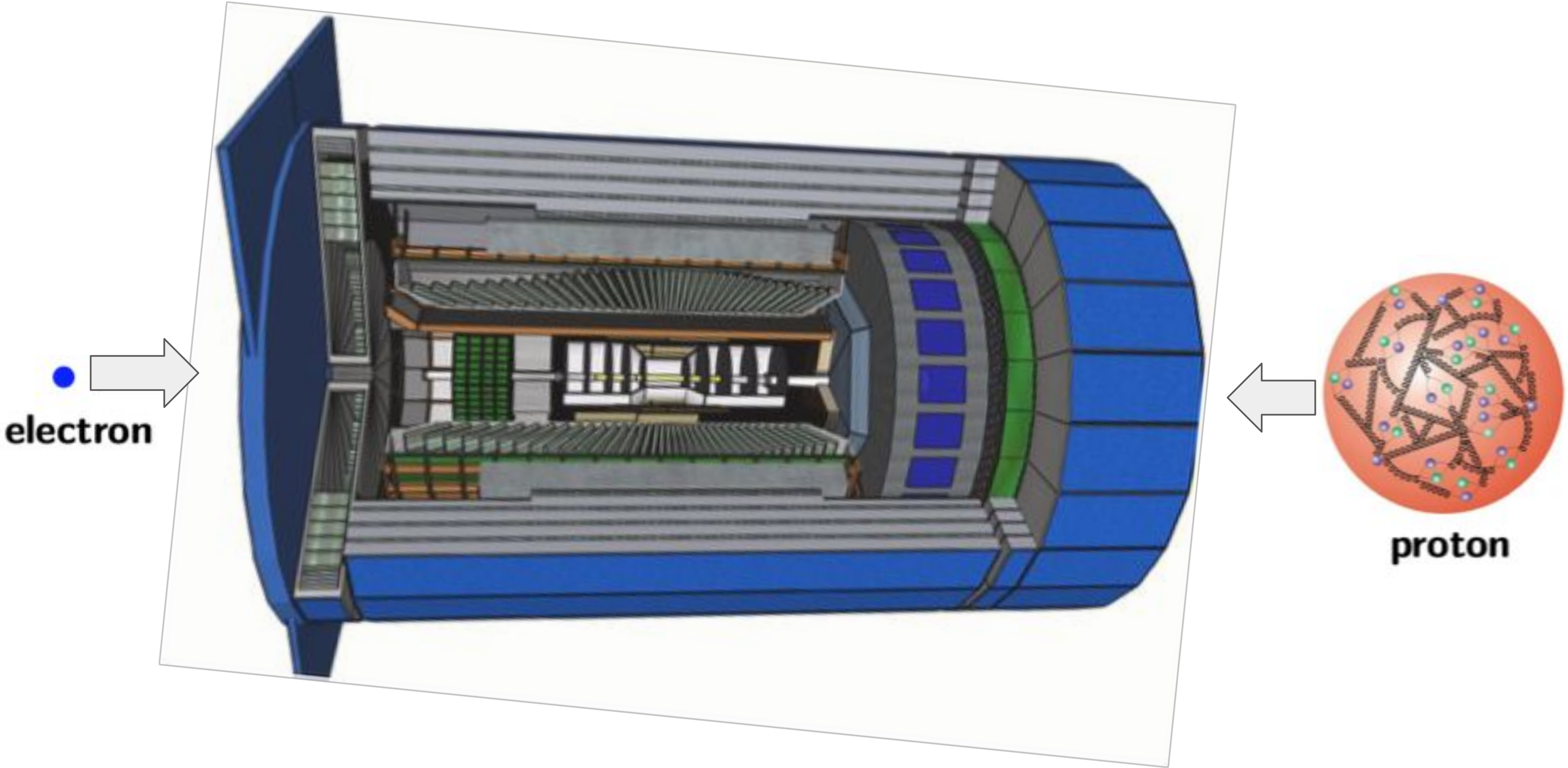


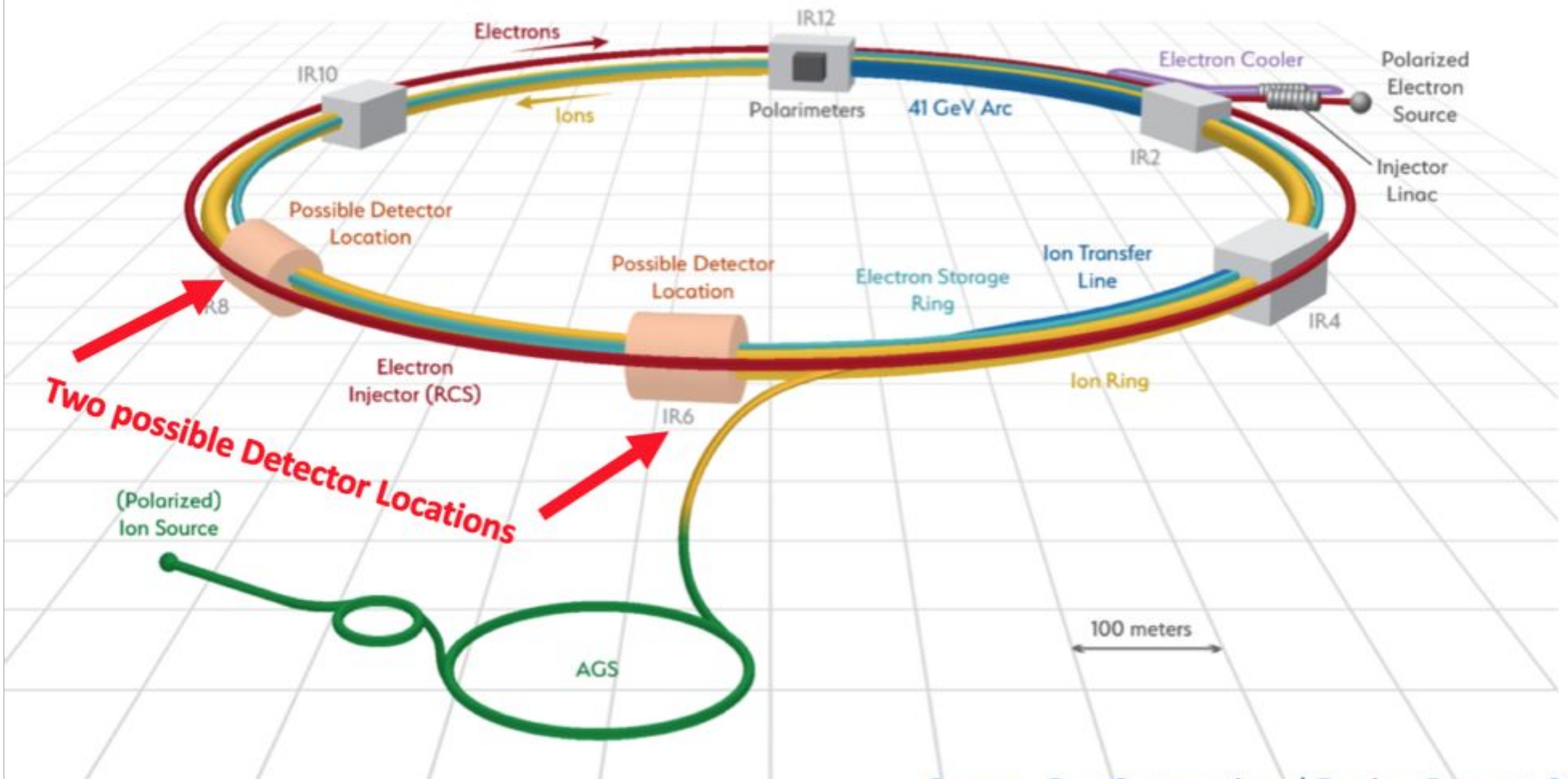
$t = t_{\text{th}} \simeq -0.7593\,\text{GeV}^2$

$t = -2\,\text{GeV}^2$

$-t/Q^2 \lesssim 0.1$

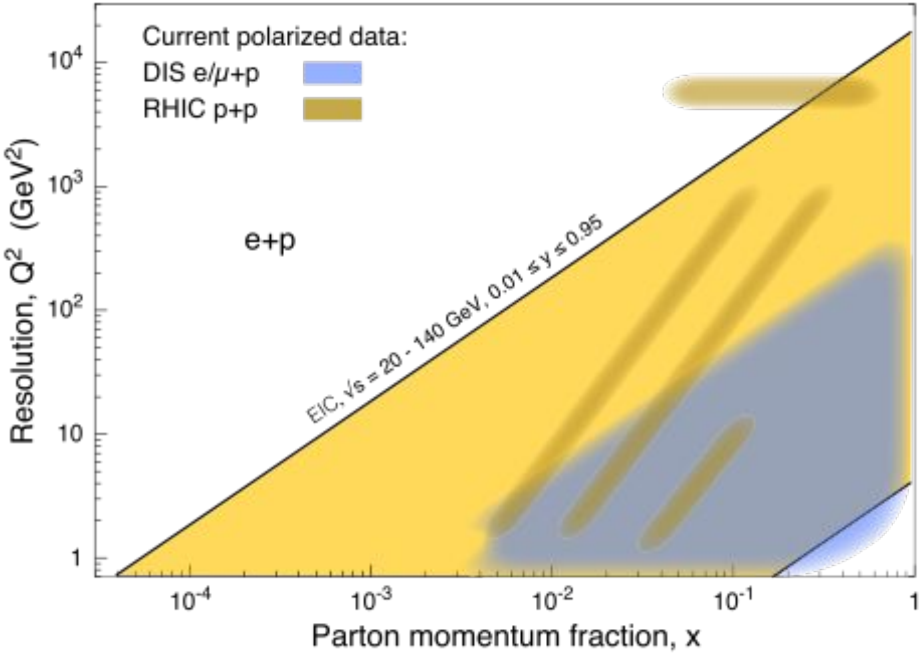
Electron-Proton/Ion Collider (ePIC)

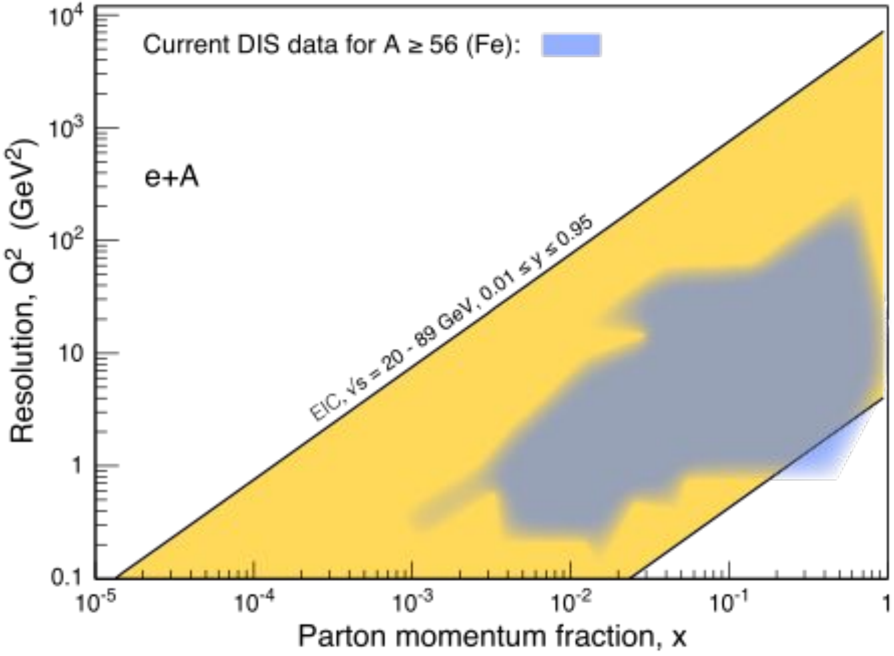




Two possible Detector Locations

Source: Pre-Conceptional Design Report, 2019

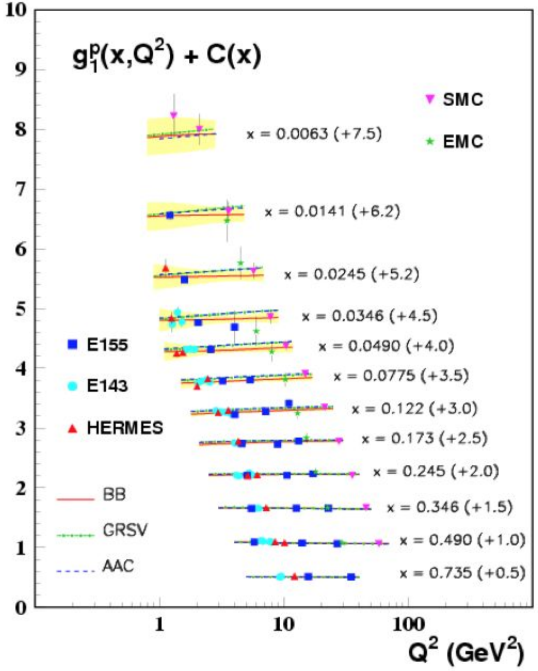




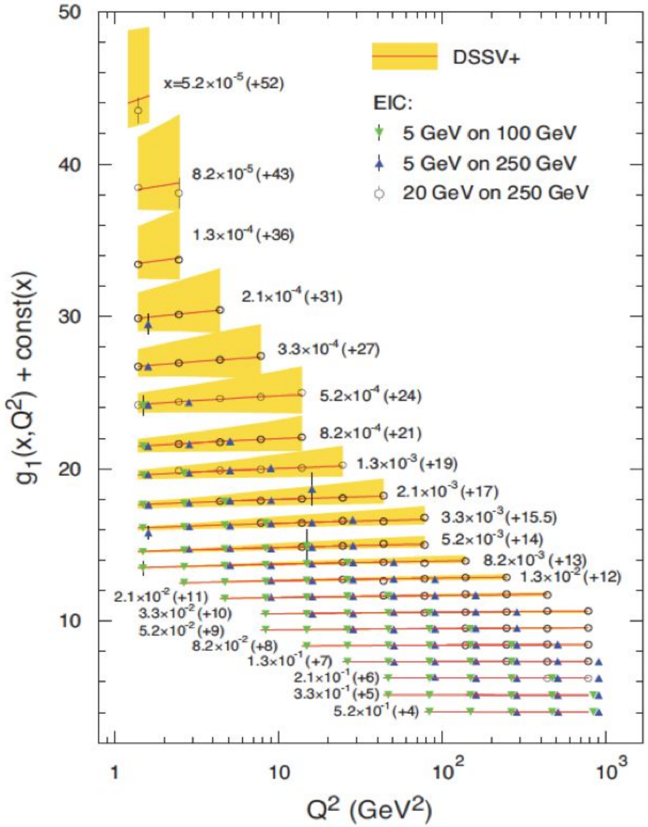
- Highly polarized electron (~70%) and proton (~70%) beams;
- Ion beams from deuterons to heavy nuclei such as gold, lead, or uranium;
- Variable *e+p* center-of-mass energies from 28–100 GeV, upgradable to 28–140 GeV;
- High collision electron-nucleon luminosity 10^{33}–10^{34} cm^{-2} s^{-1};
- The possibility of more than one interaction region.

The Future: Challenges & opportunities

☐ The power & precision of EIC:



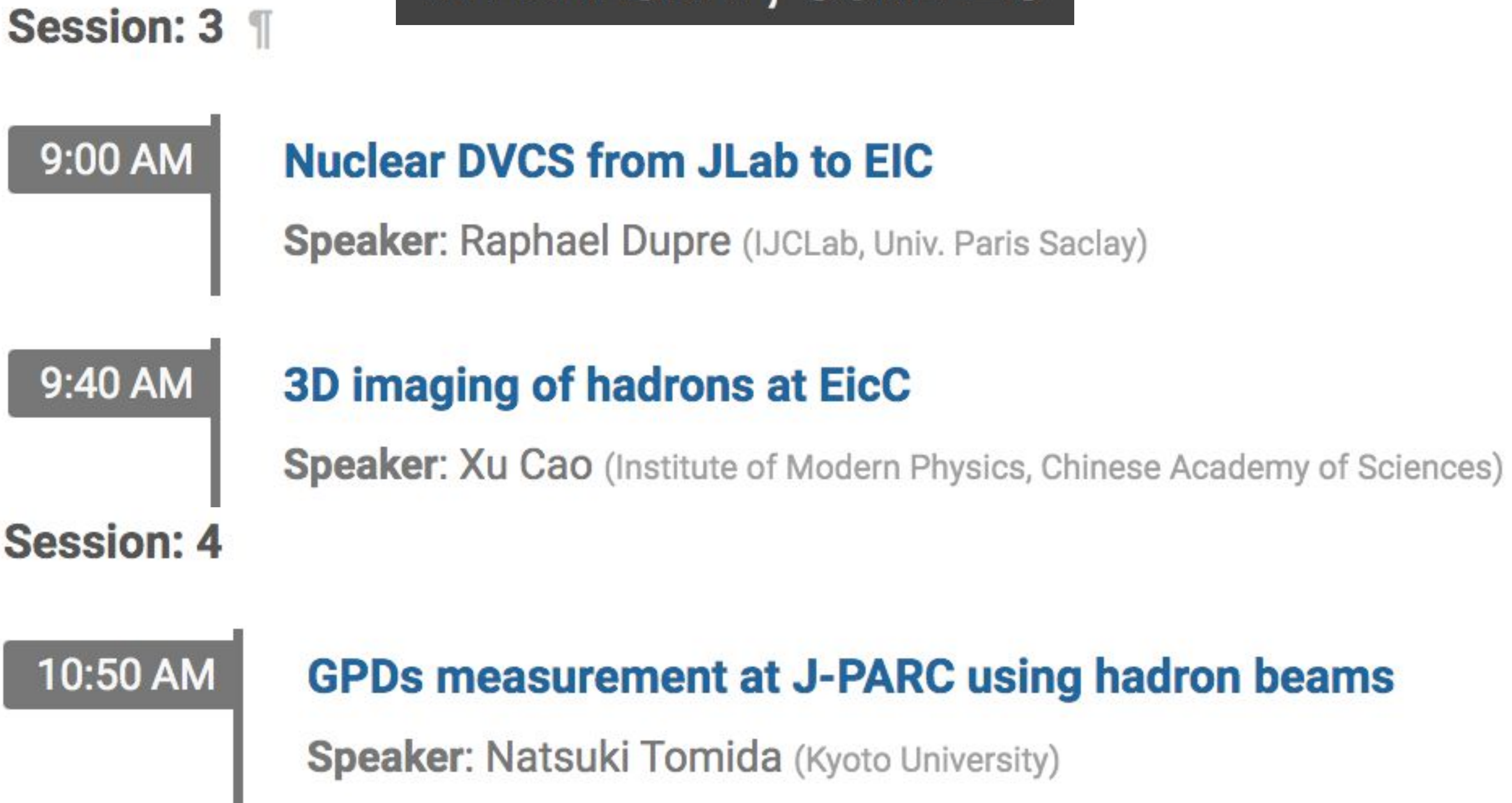
at EIC



☐ Reach out the glue:

$$\frac{dg_1(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}P_{qg}\otimes\Delta g(x,Q^2)+\cdots$$

WEDNESDAY, JUNE 26



Session: 3 ¶

9:00 AM

Nuclear DVCS from JLab to EIC

Speaker: Raphael Dupre (IJCLab, Univ. Paris Saclay)

9:40 AM

3D imaging of hadrons at EicC

Speaker: Xu Cao (Institute of Modern Physics, Chinese Academy of Sciences)

Session: 4

10:50 AM

GPDs measurement at J-PARC using hadron beams

Speaker: Natsuki Tomida (Kyoto University)

Session: 1

TUESDAY, JUNE 25

2:20 PM

GPD studies at Jefferson Lab Hall A/C

Speaker: Alexandre Camsonne (Jefferson Laboratory)

Session: 2

4:50 PM

Deeply virtual Compton scattering with CLAS12 at Jefferson Laboratory

Speaker: Adam Hobart (IJCLab CNRS-IN2P3)

Session: 10

THURSDAY, JUNE 27

4:10 PM

GPD Studies at the COMPASS Experiment

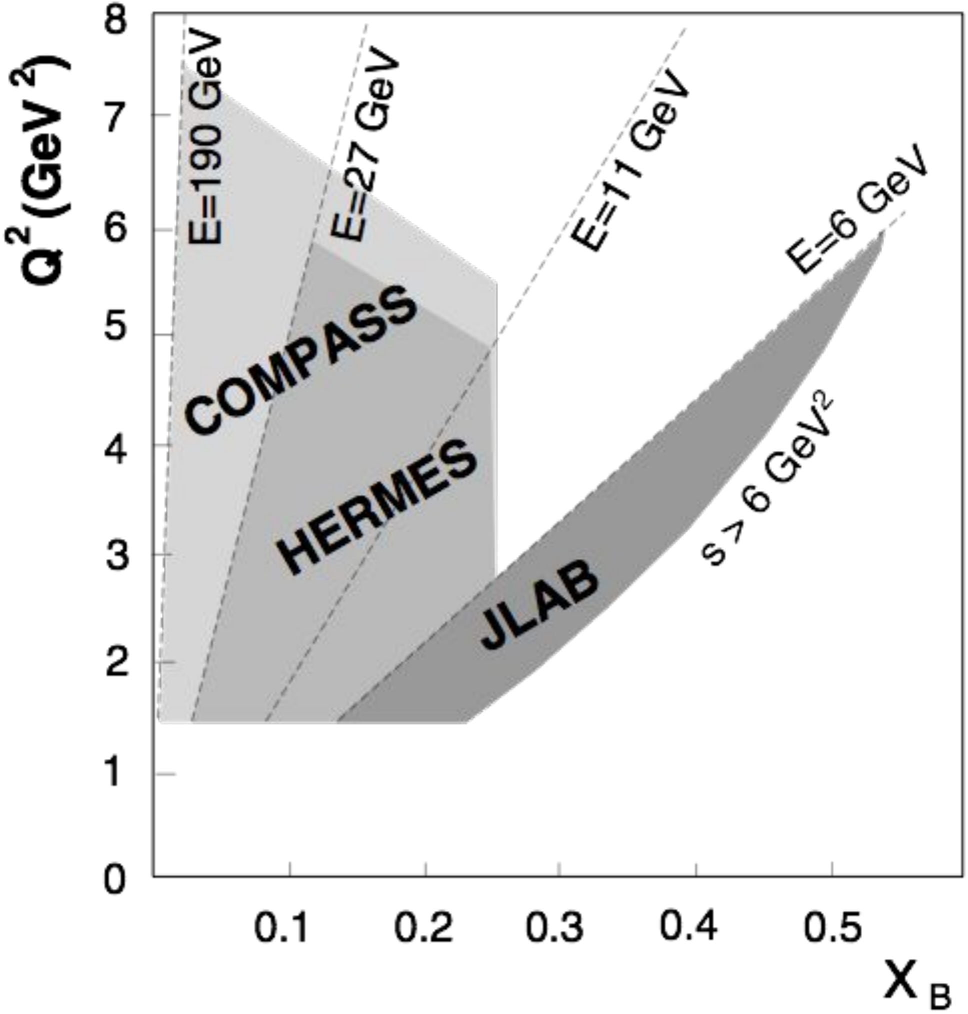
Speaker: Po-Ju Lin (National Central University)

HERA @ DESY and COMPASS @ CERN

- **H1 :** C. Adloff *et al.*, Phys. Lett. B **517**, 47 (2001)
  A. Aktas *et al.*, Eur. Phys. J. C **44**, 1 (2005)
  F. D. Aaron *et al.*, Phys. Lett. B **681**, 391 (2009)
- **ZEUS :** J. Breitweg *et al.*, Eur. Phys. J. C **6**, 603 (1999)
  S. Chekanov *et al.*, Phys. Lett. B **573**, 46 (2003)
- **HERMES :** A. Airapetian *et al.*, Phys. Rev. Lett. **87**, 182001 (2001)
  A. Airapetian *et al.*, Phys. Lett. B **704**, 15 (2011)
  A. Airapetian *et al.*, J. High Energy Phys. **07**, 032 (2012)
- **COMPASS :** N. d'Hose *et al.*, Eur. Phys. J. A **19S1**, 47 (2004)
  E. Fuchey, PoS **QCDEV2015**, 048 (2015)

CLAS and Hall A @ JLab

- **CLAS :** S. Stepanyan *et al.*, Phys. Rev. Lett. **87**, 182002 (2001)
  I. Bedlinskiy *et al.*, Phys. Rev. Lett. **109**, 112001 (2012)
  E. Seder *et al.*, Phys. Rev. Lett. **114**, 032001 (2015)
  H. S. Jo *et al.*, Phys. Rev. Lett. **115**, 212003 (2015)
  M. Hattawy *et al.*, Phys. Rev. Lett. **123**, 032502 (2019)
  V. Burkert *et al.*, Eur. Phys. J. A **57**, 186 (2021)
- **Hall A :** C. Muñoz Camacho *et al*, Phys. Rev. Lett. **97**, 262002 (2006)
  M. Defurne *et al.*, Phys. Rev. C **92**, 055202 (2015)
  M. Dlamini *et al.* Phys. Rev. Lett. **127**, 152301 (2021).
  F. Georges *et al.*, Phys. Rev. Lett. **128**, 252002 (2022)



H1 $2 < Q^2 < 20\,\mathrm{GeV}^2$ $|t| < 1\,\mathrm{GeV}^2$

$$W^2 = \frac{Q^2}{x}(1-x) \quad 30 < W < 120\,\mathrm{GeV}$$

ZEUS $5 < Q^2 < 100$ GeV^2

$40 < W < 140$ GeV

production of ρ^0

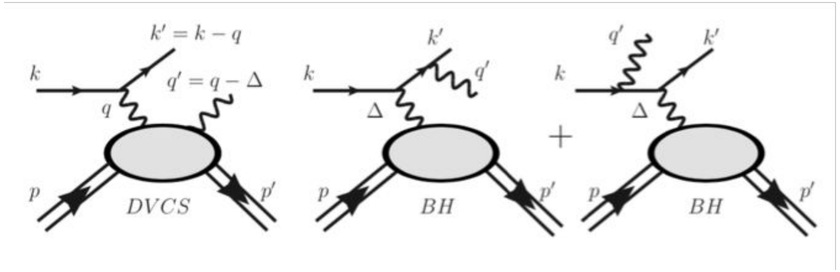
$0.25 < Q^2 < 50$ GeV^2

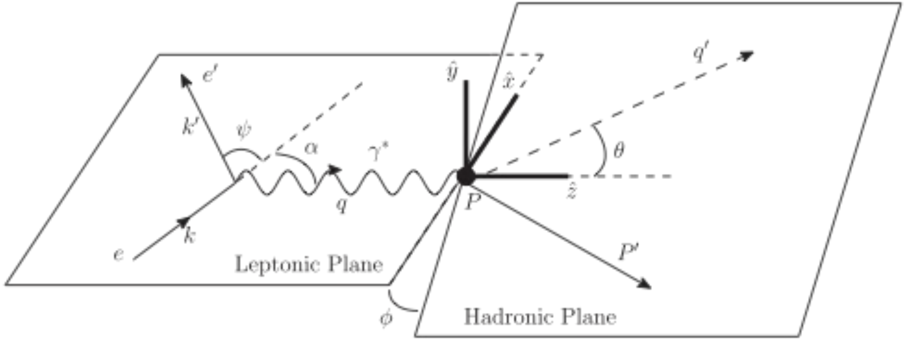
$20 < W < 167$ GeV

production of J/ψ

$2 < Q^2 < 40$ GeV^2

$50 < W < 150$ GeV



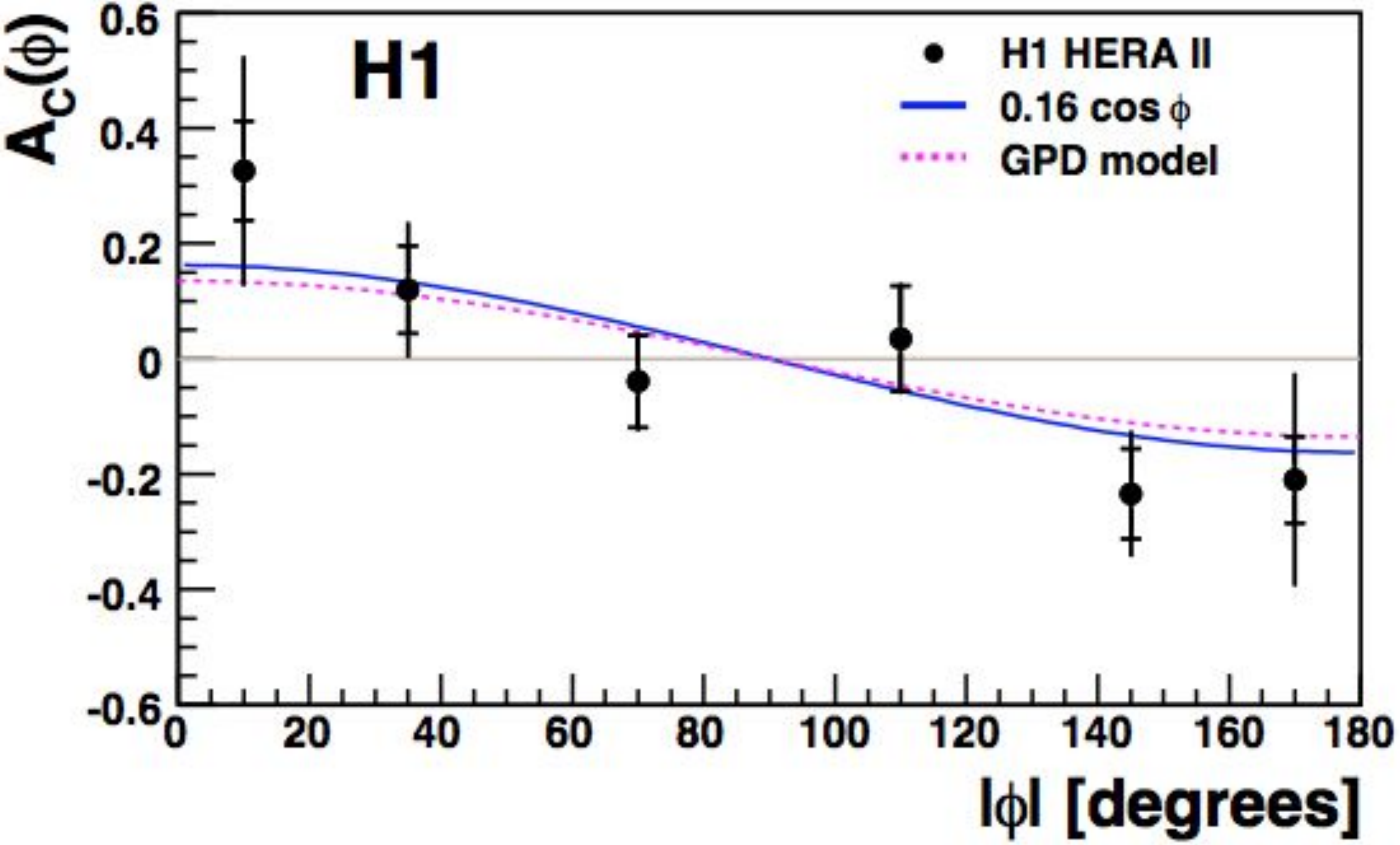


$$|A|^2 = |A_{BH}|^2 + |A_{DVCS}|^2 + \underbrace{A_{DVCS}\,A^*_{BH} + A^*_{DVCS}\,A_{BH}}_{I}$$

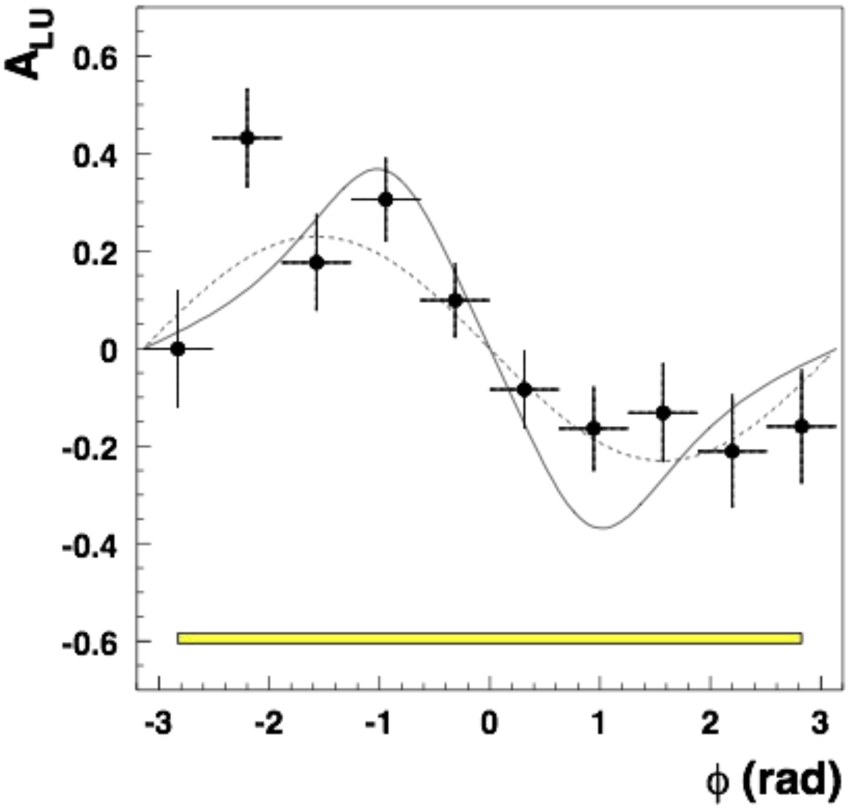
$$I \propto -C\left[a_1 \cos\phi\,\mathrm{Re}A_{DVCS} + a_2 P_l \sin\phi\,\mathrm{Im}A_{DVCS}\right]$$

$$A_C(\phi) = \frac{d\sigma^+/d\phi - d\sigma^-/d\phi}{d\sigma^+/d\phi + d\sigma^-/d\phi} = 2A_{BH}\frac{\mathrm{Re}A_{DVCS}}{|A_{DVCS}|^2 + |A_{BH}|^2}\cos\phi$$

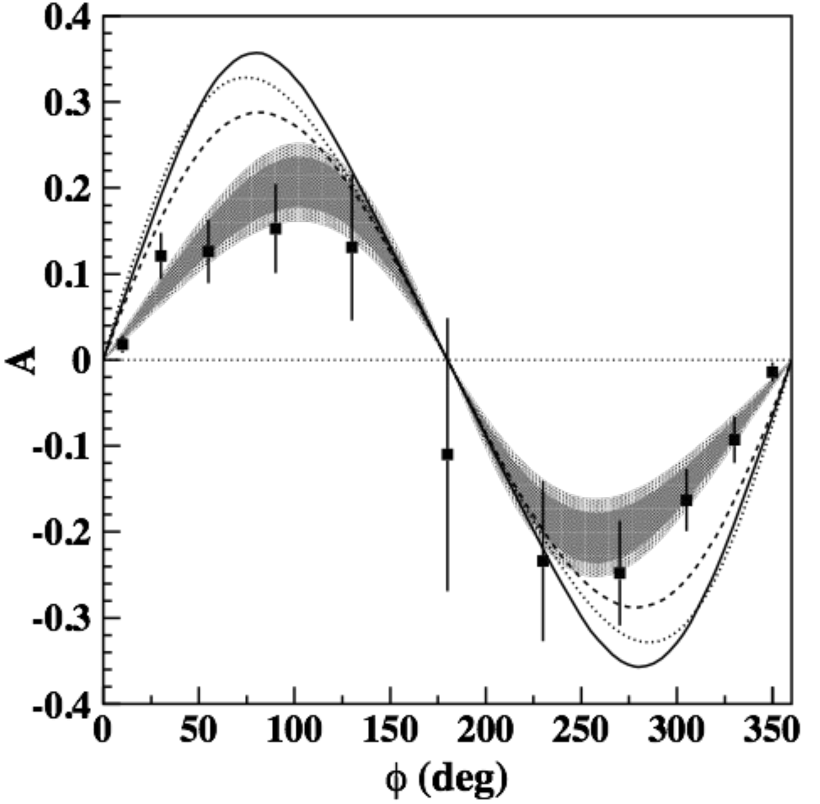
$$\mathcal{A}_{\mathrm{LU}}(\phi) = \frac{\mathrm{d}\sigma^{\dashrightarrow}(\phi) - \mathrm{d}\sigma^{\dashleftarrow}(\phi)}{\mathrm{d}\sigma^{\dashrightarrow}(\phi) + \mathrm{d}\sigma^{\dashleftarrow}(\phi)} \propto \mathrm{Im}\widehat{M}_{\mathrm{U}}\sin\phi$$


H1
H1 HERA II
0.16 cos ϕ
GPD model
A_C(ϕ)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
20
40
60
80
100
120
140
160
180
|ϕ| [degrees]

HERMES 27.6 GeV positrons



CLAS 4.25 GeV electrons



- **The parameterization of valence quark GPDs in the proton**

M. Diehl, T. Feldmann , R. Jakob and P. Kroll, EPJC39 (2015)

Profile function

$$H^q(x,t) = q_v(x)\,\exp\left[t\, f_q(x)\right],$$

$$E^q(x,t) = \underline{e_q(x)}\,\exp\left[t\, f_q(x)\right],$$

$$\widetilde{H}^q(x,t) = \underline{\Delta q_v(x)}\,\exp\left[t\, \tilde{f}_q(x)\right].$$

Unpolarized PDF

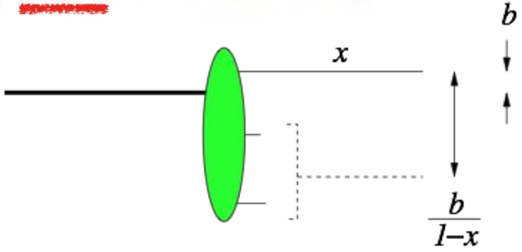
Spin flip PDF

Polarized PDF

Fourier transformation

$$q_v(x,\mathbf{b}) = \int \frac{\mathrm{d}^2\mathbf{\Delta}}{(2\pi)^2}\, e^{-i\mathbf{b}\mathbf{\Delta}}\, H_v^q(x, t=-\mathbf{\Delta}^2)$$

$$\langle \mathbf{b}^2\rangle_x^q = \frac{\int \mathrm{d}^2\mathbf{b}\,\mathbf{b}^2\, q_v(x,\mathbf{b})}{\int \mathrm{d}^2\mathbf{b}\, q_v(x,\mathbf{b})} = 4 f_q(x)$$



M. Vanderhaeghen, P. A. M. Guichon, and M. Guidal, Phys. Rev. Lett. **80**, 5064 (1998).

K. Kumerički and D. Müller, Nucl. Phys. **B841**, 1 (2010)

G. R. Goldstein, J. O. Hernandez, and S. Liuti, Phys. Rev. D **84**, 034007 (2011)

Y. Guo, X. Ji, and K. Shiells, J. High Energy Phys. **09**, 215 (2022)

Y. Guo, X. Ji, M. G. Santiago, K. Shiells, and J. Yang, (2023), arXiv:2302.07279 [hep-ph]

K. A. Mamo and I. Zahed, (2024), arXiv:2404.13245 [hep-ph]

TUESDAY, JUNE 25

Session: 1

3:00 PM **GPD and development of its extraction technique**

Speaker: Parada Tobel Paraduan Hutauruk (Pukyong National University (PKNU))

Session: 2

4:10 PM **Phenomenology of virtual Compton scattering processes in the era of new experiments**

Speaker: Paweł Sznajder (National Centre for Nuclear Research, Poland)

WEDNESDAY, JUNE 26

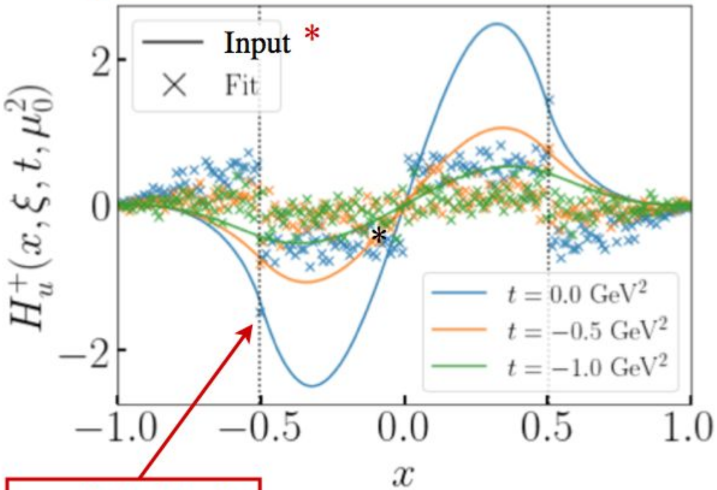
Session: 4

11:30 AM **Photoproduction of J/psi meson off nucleon and nuclei**

Speaker: Sangho Kim (Soongsil University)

Are GPDs reconstructed from CFFs unique?

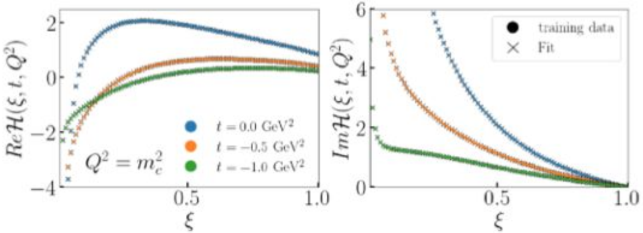
Num pixels (dd space): 2970300



$x = \xi$ "ridge"

* "toy analysis": no evolution, only u-quark flavor, CFFs only sensitive to charge-even combination, no uncertainty quantifications

→ both "input" and "fit" GPDs give same CFFs!



→ pixel-based reconstruction shows clear demonstration of <u>shadow GPDs</u>

Bertone *et al.*, Phys. Rev. D **103**, 114019 (2021)

→ more inputs needed (models and/or lattice and/or experiment) to reconstruct x dependence of GPDs

Moffat *et al.*, Phys. Rev. D **108**, 036027 (2023)

→ future of GPD reconstruction

— use lattice QCD input (QGT)

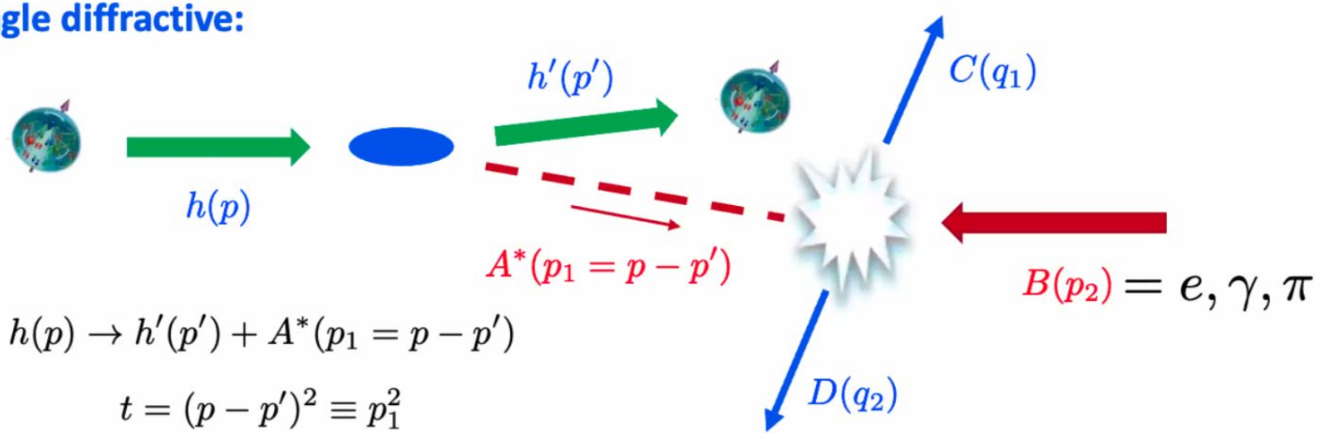
— use double DVCS, SDHEPS data (LDRD)

Single-Diffractive Hard Exclusive Processes (SDHEP)

Qiu & Yu, JHEP 08 (2022) 103,
PRD 107 (2023) 1, in preparation

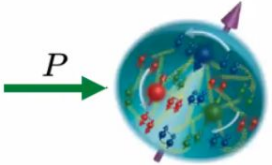
☐ Two-stage diffractive $2 \to 3$ hard exclusive processes:

- **Single diffractive:**



$$h(p) \to h'(p') + A^*(p_1 = p - p')$$

$$t = (p - p')^2 \equiv p_1^2$$



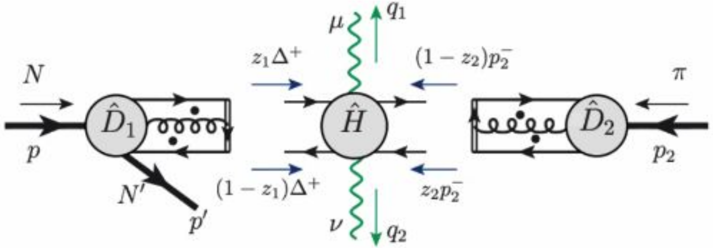
Probing its structure without breaking it!

- **Hard probe:** $2 \to 2$ **high** q_T **exclusive process**

$$A^*(p_1) + B(p_2) \to C(q_1) + D(q_2)$$

$$(p - p') \cdot n \gg \sqrt{|t|} \quad \Longleftrightarrow \quad |q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$$

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$



Lattice QCD Development

- **Nucleon Electromagetic Form Factors,** C. Alexandrou et.al.,PRD74,034508(2006)
- **LaMET & Quasi-PDF**, X.Ji, PRL110,262002(2013)
- **Pseudo-PDF**, K.Orginos,A.Radyushkin et al., PRD96,094503(2017)
- **Lattice Good Cross Sections**, Y.-Q Ma and J.-W Qiu, PRD98,074021(2018); PRL120,022003(2018)

Session: 7

THURSDAY, JUNE 27

9:00 AM **New developments on proton Generalized Parton Distributions from lattice QCD**

Speaker: Martha Constantinou (Temple University)

9:40 AM **Generalized parton distributions from lattice QCD** ¶

Speaker: Huey-Wen Lin (Michigan State University)

- **Unpolarized and Helicity GPDs**,
  C. Alexandrou et.al.,PRL125,262001(2020)
- **Nucleon Tomography and GPD at Physical Pion Mass**,
  H.-W.Lin, PRL127,182001(2021)
- **GPDs from LQCD with asymmetric momentum transfer**,
  S.Bhattacharya et al.,
  PRD106,114512(2022)
- **…**

Various Model Computations are useful to explore insights on characteristics of GPDs.

- **Chiral Quark Soliton Model - Large Nc picture of nucleon**
- **Light-Front Quark Model - Light-Front Dynamics**

Session: 11

FRIDAY, JUNE 28

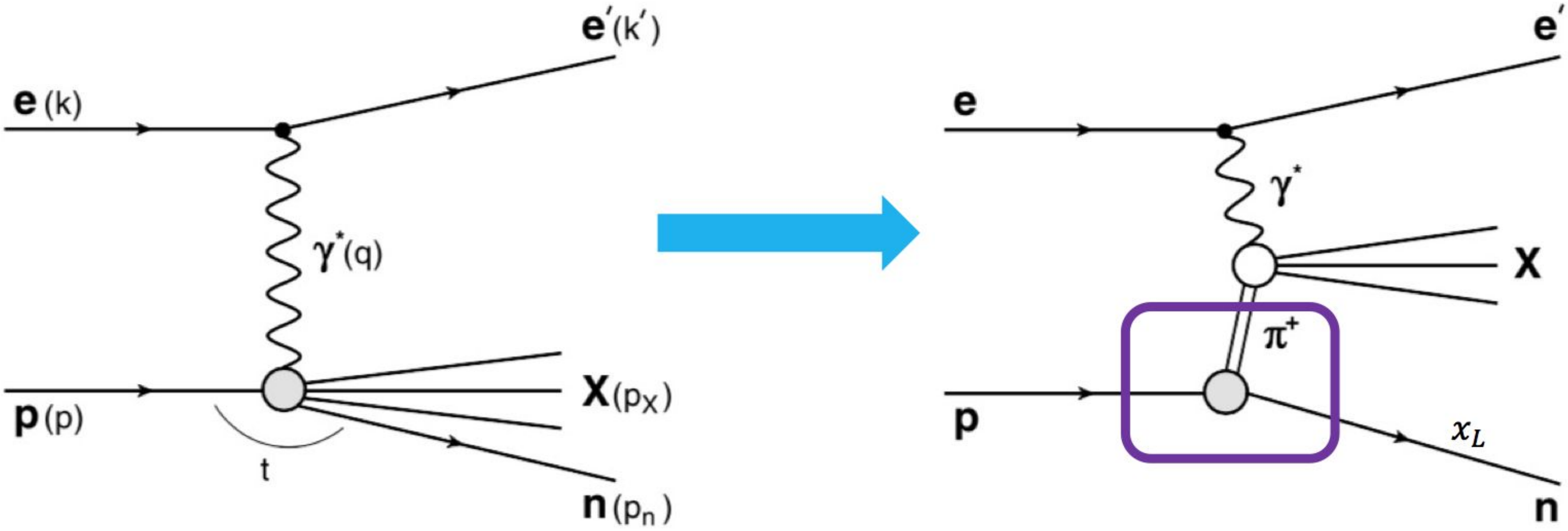
9:40 AM

Consistency of the pion form factor and unpolarized transverse momentum dependent parton distributions beyond leading twist in the light-front quark model

Speaker: Ho-Meoyng Choi (Kyungpook National University)

- **Nambu–Jona-Lasinio Model**
- **Color Glass Condensate Model**
- **Models in Covariant Bethe-Salpeter Approach, etc.**

Incorporating Chiral Effective Theory in hadron structure study



Denotes the splitting of *e.g.*, $p \rightarrow \pi^{+} n$
Notationally: $f_{\pi}(\bar{x}_L = 1 - x_L)$

Connection with QCD

- $$(\bar d - \bar u)(x) = \frac{2}{3}\int_x^1 \frac{dy}{y}\, f_\pi(y)\ \bar q^\pi(x/y)$$

$$\boxed{f_\pi(y) = \frac{3g^2_{\pi NN}}{16\pi^2}\, y \int dt\, \frac{-t\ \mathcal{F}^2_{\pi NN}(t)}{(t-m_\pi^2)^2}}$$

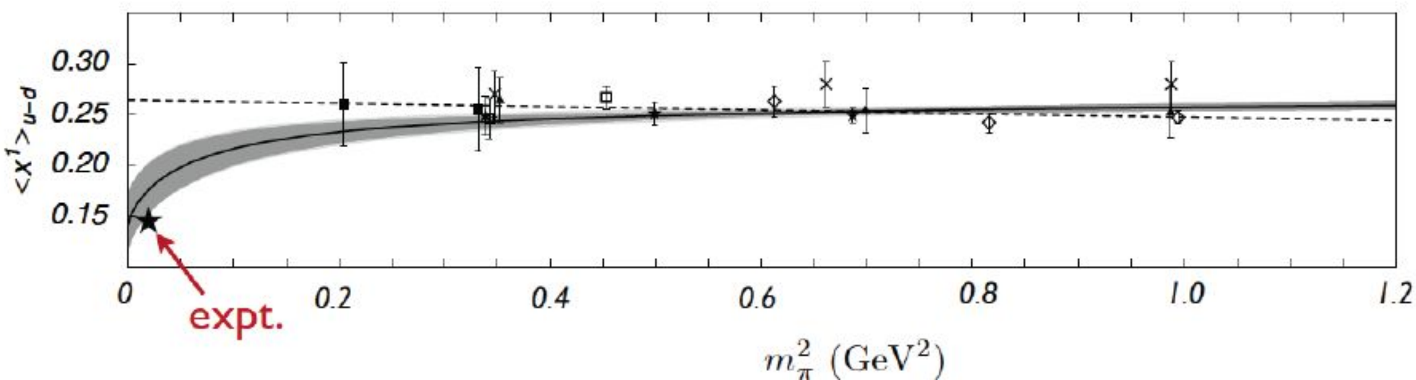
→ *model-independent* leading nonanalytic (LNA) behavior *consistent with Chiral Symmetry of QCD.*

$$\langle x^0\rangle_{\bar d-\bar u} \equiv \int_0^1 dx(\bar d - \bar u)$$

$$\boxed{m_\pi^2 f_\pi^2 = -2m_q < \bar q q >}$$

$$= \frac{2}{3}\int_0^1 dy\, f_\pi(y) = \frac{2g_A^2}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2)\ +\ \text{analytic terms}$$

- Nonanalytic behavior vital for chiral extrapolation of lattice data



*Thomas, Melnitchouk, Steffens, PRL **85**, 2892 (2000)*

Matrix element for GPDs in quark level:

$$V_q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix(Pz)} < p'|O_q|p > |_{z=\lambda n}, \quad \textbf{where} \quad O_q = \bar{q}(-\frac{1}{2}z)\gamma^+ q(\frac{1}{2}z)$$

Match to the hadron level

Convolution formulas for GPD (zero-skewness)

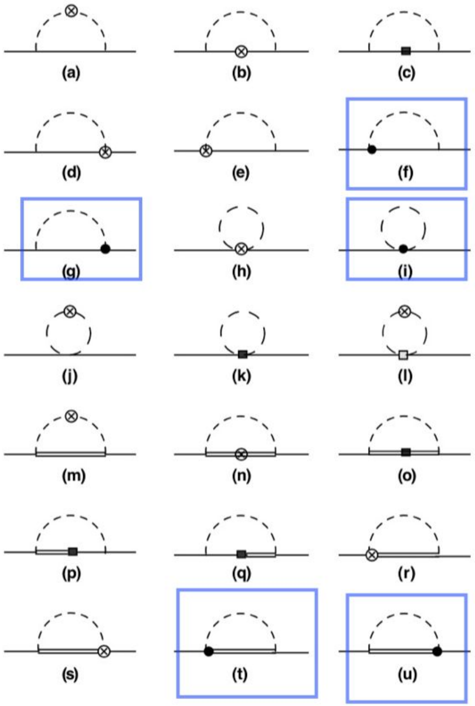
$$V_q = \frac{1}{2}\sum_H \int_0^1 dy\, \theta(0 \le \frac{x}{y} \le 1) \times q_H^v(\frac{x}{y},0,t) \times \boxed{\int \frac{dz^-}{2\pi} e^{iy(Pz)} < p'|O_H|p >}$$

Valence GPD in the intermediate hadron state H

Splitting function

Where O_H is bilocal hadron operator, $q_H^v(\frac{x}{y},0,t)$ is quark valance GPD in hadron state H.

Splitting functions



⊗ electric vector current
■ magnetic vector current
● additional vertex
— Baryon octet
═ Baryon decuplet
- - - - meson

Generalized parton distribution

$$\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-ix\lambda}\,\langle\, p'|\bar{\psi}_q(\tfrac{1}{2}\lambda n)\not{n}\,\psi_q(-\tfrac{1}{2}\lambda n)|p\rangle = \bar{u}(p')\Big[\not{n}H_p^q(x,\xi,t)+\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M}\,E_p^q(x,\xi,t)\Big]u(p),$$

Splitting function

The distribution of Σ^+ in the proton

$$\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-ix\lambda}\,\langle\, p'|\bar{\Sigma}^+(\tfrac{1}{2}\lambda n)\not{n}\,\Sigma^+(-\tfrac{1}{2}\lambda n)|p\rangle = \bar{u}(p')\Big[\not{n}f_p^{\Sigma^+}(x,\xi,t)+\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M}\,g_p^{\Sigma^+}(x,\xi,t)\Big]u(p),$$

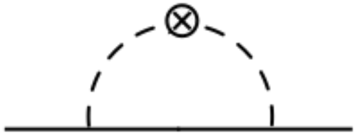
Form factor

$$\langle N(p')|J^\mu|N(p)\rangle = \bar{u}(p')\Big[\gamma^\mu F_1^N(t)+\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}F_2^N(t)\Big]u(p) \equiv \int \mathrm{d}^4k\,\widetilde{\Gamma}^\mu(k),$$

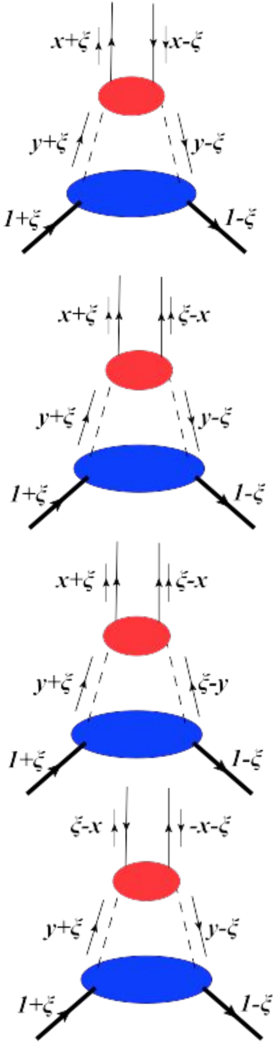
Splitting function

$$\bar{u}(p')\Big[\gamma^+ f(y,\xi,t)+\frac{i\sigma^{+\nu}\Delta_\nu}{2M}\,g(y,\xi,t)\Big]u(p) = \int \mathrm{d}^4k\,\widetilde{\Gamma}^+(k)\,\delta\Big(y-\frac{k^+}{P^+}\Big) \equiv \Gamma^+.$$

Chiral Effective Theory for GPDs



$$H_q^{(\mathrm{rbw})}(x,\xi,t) = \begin{cases} \displaystyle\int_x^1 \frac{\mathrm{d}y}{y} f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t)\, H_{q/\phi}\Big(\frac{x}{y},\frac{\xi}{y},t\Big), & [\xi < x < y] \\ \displaystyle\int_\xi^1 \frac{\mathrm{d}y}{y} f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t)\, H_{q/\phi}\Big(\frac{x}{y},\frac{\xi}{y},t\Big), & [x < \xi < y] \\ \displaystyle\int_{-\xi}^{\xi} \frac{\mathrm{d}y}{2y} f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t)\frac{1}{\pi}\int_{s_0}^{\infty}\mathrm{d}s\, \frac{\mathrm{Im}\Phi_{q/\phi}\big(\frac{1}{2}(1+\frac{x}{\xi}),\frac{1}{2}(1+\frac{y}{\xi}),s\big)}{s-t+i\epsilon}, & [|x|,|y| < \xi] \\ \displaystyle\int_{-x}^1 \frac{\mathrm{d}y}{y} f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t)\, H_{q/\phi}\Big(\frac{x}{y},\frac{\xi}{y},t\Big), & [\xi < -x < y < 1] \end{cases}$$



`Z.Gao,F.He,C.Ji,W.Melnitchouk,Y.Salamu,P.Wang,[arXiv:2406.03412 [hep-ph]]`

Regularization in ChPT

- **Dimensional regularization scheme**

Heavy baryon method

V. Bernard, N. Kaiser, J. Kambor, U. Meissner, NPB 388 (1992)

Infrared Regularization

T. Becher, H. Leutwyler, EPJC9(1999)

Extended on mass shell renormalization schemes (EOMS)

T. Fuchs, J. Gegelia, G. Japaridze, S. Scherer, PRD68(2003)

Rescue the power counting broken problem caused by the baryon propagator.

- **However, the upper limit of integral for the loop momentum is infinity in the dimensional regularization scheme, thus the short distance physics will be overestimated. This overestimated short distance is absorbed into the low energy constants (LECs), it will lead the values of LECs are large and chiral convergence is poor.**

J. Donoghue, B. Holstein, B. Borasoy, Phys.Rev.D 59 (1999)

Review

Progress in Particle and Nuclear Physics 129 (2023) 104017

Nucleon form factors and parton distributions in nonlocal chiral effective theory

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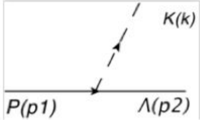
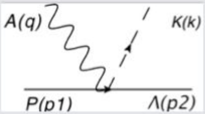
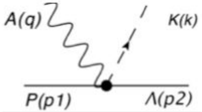
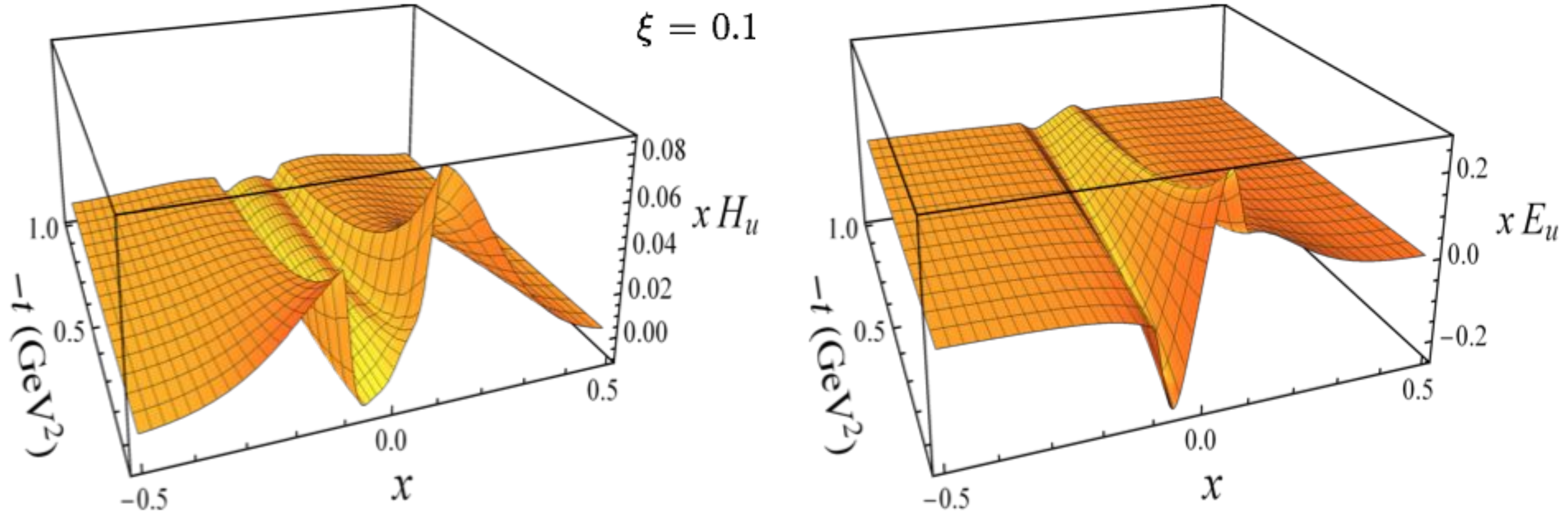
We present a review of recent applications of nonlocal chiral effective theory to hadron structure studies. Starting from a nonlocal meson–baryon effective chiral Lagrangian, we show how the introduction of a correlation function representing the finite extent of hadrons regularizes the meson loop integrals and introduces momentum dependence in vertex form factors in a gauge invariant manner. We apply the framework to the calculation of nucleon electromagnetic form factors, unpolarized and polarized parton distributions, as well as transverse momentum dependent distributions and generalized parton distributions.

- **Expand the nonlocal Lagrangian**

$$
\begin{aligned}
\mathcal{L}_K{}^{nl} &= -\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)(\partial_\mu + \boxed{i\, e_s \mathscr{A}_\mu^s(x)})\Big(\boxed{\exp[ie_s \int_x^y dz^\nu\, \mathscr{A}_\nu^s(z)]}\, K^+(y)F(x-y)\Big), \\
&= -\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)\partial_{\mu,x}\Big(K^+(y)F(x-y)\Big) \\
&\quad -\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)\boxed{(i\, e_s \mathscr{A}_\mu^s(x))}\Big(K^+(y)F(x-y)\Big) \\
&\quad -\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)\partial_{\mu,x}\Big(\boxed{ie_s \int_x^y dz^\nu\, \mathscr{A}_\nu^s(z)}\, K^+(y)F(x-y)\Big),
\end{aligned}
$$

Vertexes	Feynman rules in local case	Feynman rules in nonlocal case
 	$\frac{D+3F}{\sqrt{12}f}k_\mu\gamma^\mu\gamma^5$	$\frac{D+3F}{\sqrt{12}f}k_\mu\gamma^\mu\gamma^5\underline{F(k)}$ $F(k)=\frac{(\Lambda^2-M_K^2)^2}{(\Lambda^2-k^2)^2}$ F(k) is the Fourier transformation of F(x-y)
 	$\frac{D+3F}{\sqrt{12}f}\gamma^\mu\gamma^5$	$\frac{D+3F}{\sqrt{12}f}\gamma^\mu\gamma^5\underline{F(k)}$
 	Nonexistence	$\frac{D+3F}{\sqrt{12}f}k_\nu\gamma^\nu\gamma^5\frac{[F(k+q)-F(k)](2k+q)^\mu}{2k\,.\,q+q^2}$ **(Additional vertex)**



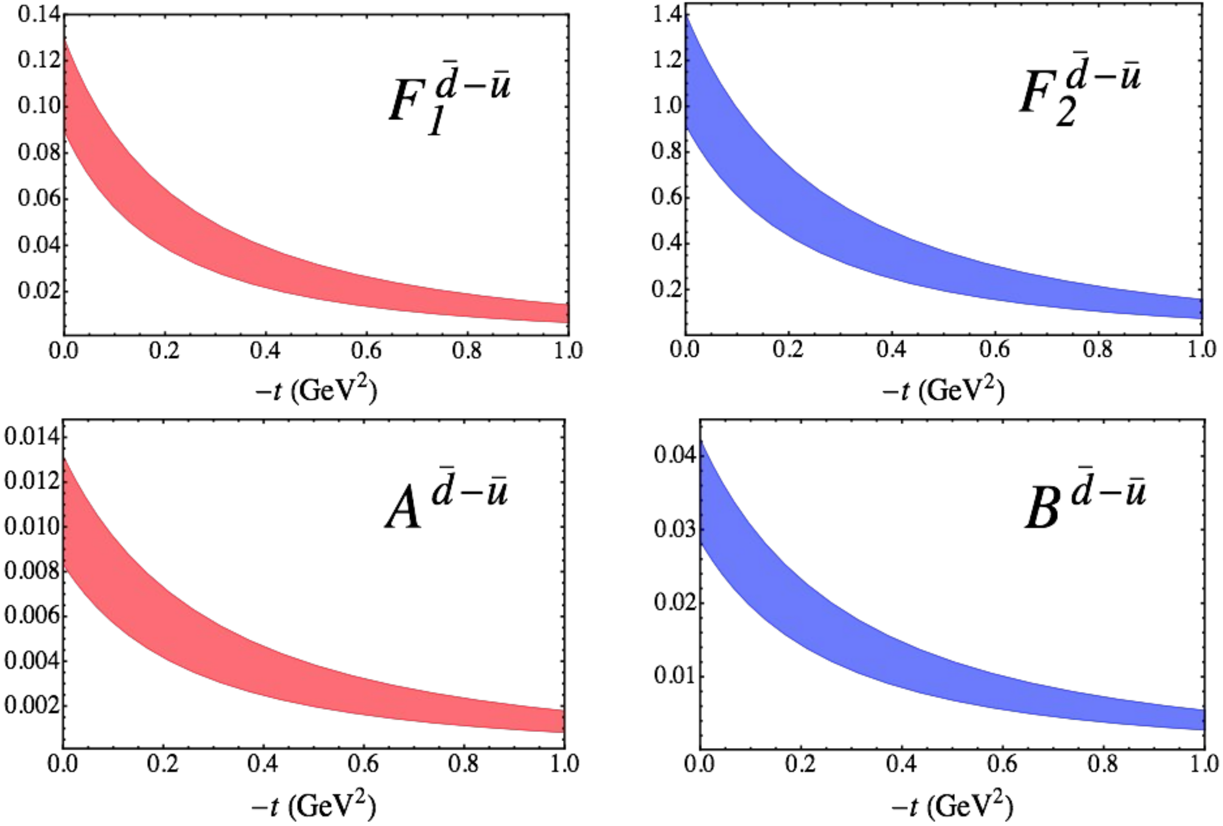
$$F_1^{\bar{d}-\bar{u}}(t) = \int_0^1 \mathrm{d}x\,\big(H_u(x,0,t) + H_u(-x,0,t)\big) \qquad F_2^{\bar{d}-\bar{u}}(t) = \int_0^1 \mathrm{d}x\,\big(E_u(x,0,t) + E_u(-x,0,t)\big)$$

$$A^{\bar{d}-\bar{u}}(t) = \int_0^1 \mathrm{d}x\; x\big(H_u(x,0,t) + H_u(-x,0,t)\big) \qquad B^{\bar{d}-\bar{u}}(t) = \int_0^1 \mathrm{d}x\; x\big(E_u(x,0,t) + E_u(-x,0,t)\big)$$

$$F_1^{\bar{d}-\bar{u}}(0) = \int_0^1 \mathrm{d}x\,\left(\bar{d}(x) - \bar{u}(x)\right) = 0.11(2)$$

E866 (Fermilab), PRD ***64****, 052002 (2001)*

$$\int_{0.015}^{0.35} \mathrm{d}x\,(\bar{d} - \bar{u}) = 0.0803(11)$$



F. He,C.Ji,W.Melnitchouk,
A.Thomas and P.Wang,
PRD106,054006(2022)

$$F_2^{\bar{d}-\bar{u}}(0) = 1.15(25)$$

$$A^{\bar{d}-\bar{u}}(0) = 0.01^{+0.003}_{-0.002}$$

$$B^{\bar{d}-\bar{u}}(0) = 0.035(7)$$

Summary and Outlook

- We summarized the present state of nucleon GPDs and identified the theory progress needed for maximizing the impact on the study of nucleon GPDs.
- There are many theoretical challenges that have to be addressed.
- Examples include the incorporation of lattice QCD data into global analysis increasing utilization of AI/ML tools to meet complexity challenge.
- Useful tools to study the nucleon structures include the meson structure studies incorporating the chiral effective theory.
- We need to pay attention to the importance of strong theory support alongside the experimental program to realize the full discovery potential of the 3D nucleon structure via GPDs.