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Quantum Simulation of Light-Front Quantum Field Theory

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ILCAC Seminar

Outline

1. What quantum computers are?
2. How can QCs accelerate solving LF QFT?
3. Peculiarities of simulating LF QFT on QCs.
4. Past, Current, & Future work.

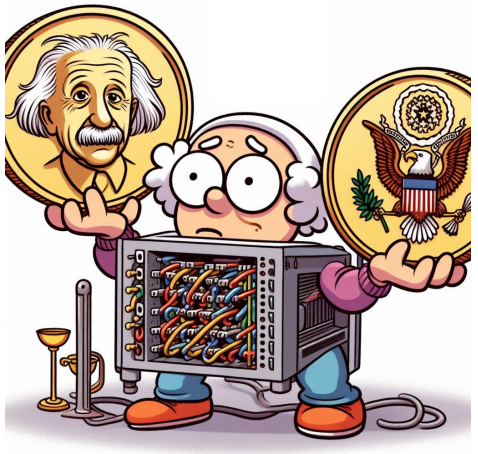
1. What is a QC?

What is a QC?

- Wiki:
  "A quantum computer is a computer that takes advantage of quantum mechanical phenomena."

Google Gemini AI

- What I see:
  "A system capable of naturally storing superpositions of states which can be manipulated and measured with sufficient precision to enable its utility as a computer."



Microsoft Designer AI

What is a QC?
User perspective

- Wiki:
  "A classical computer can solve the same computational problems as a quantum computer, given enough time. Quantum advantage comes in the form of time complexity rather than computability."

- What I see:
  "Quantum computers are (potentially?) powerful calculators for (highly?) specific tasks."

What is a QC?
Physicist perspective

Why would a physicist be interested in QC?

- Running previously inaccessible calculations / simulations 
- Thinking differently 
  - Quantum-inspired classical algorithms (lin. alg.; q. chem; etc.);
  - Hamiltonian approaches to QFT*
    (ET Hamiltonian QFT offers many lessons for LF QFT);
  - Investigations of computational complexity.
- Scientific cross-pollination 

Digital QCs

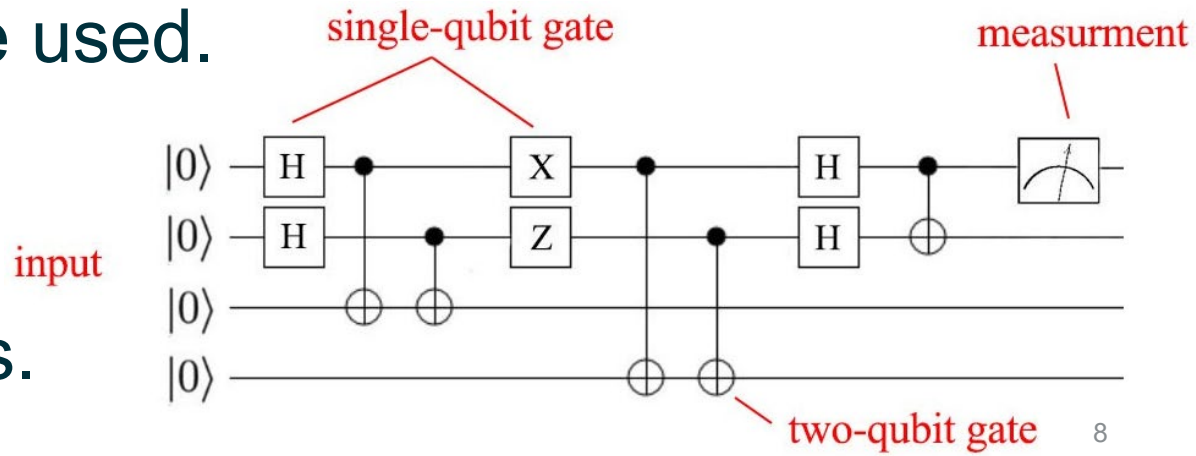
- States of (deterministic) classical and quantum computers:

Type	Initial State	Intermediate State	Final State
Classical	0	$0, 1$	$0, 1$
	00	$00, 01, 10, 11$	$00, 01, 10, 11$
	…	$\{0,1\}^{\times n}$	…
Quantum	$\|0\rangle$	$c_0\|0\rangle + c_1\|1\rangle$	$0, 1$
	$\|00\rangle$	$\|\psi\rangle = c_{00}\|00\rangle + c_{01}\|01\rangle + c_{10}\|10\rangle + c_{11}\|11\rangle$	$00, 01, 10, 11$
	…	$\{\|0\rangle, \|1\rangle\}^{\otimes n}$	…

- QM measurements have probabilistic nature.

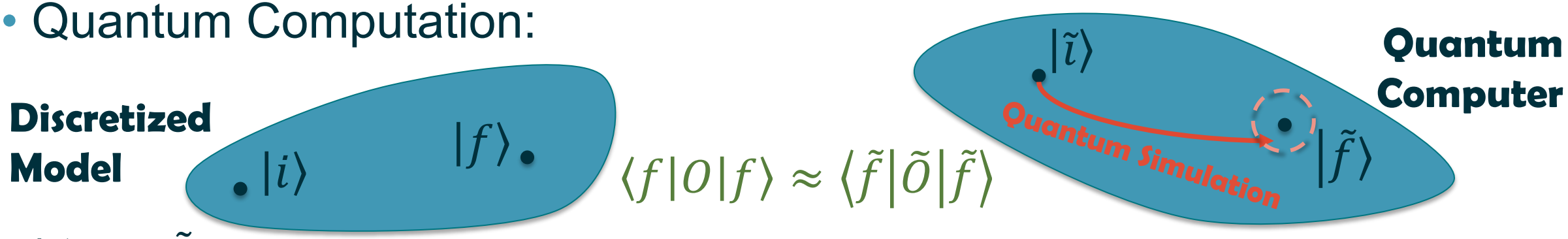
Quantum Computation

- Life story of a state in quantum computer:
  - Initialization $|\psi_i\rangle = |0\rangle^{\otimes n}$
  - Unitary evolution $|\psi_f\rangle = \ldots U_3\, U_2 U_1 |\psi_i\rangle$
  - Measurement $|\psi\rangle \mapsto \{0,1\}^{\otimes n}$
- These stages can be conveniently represented by a circuit diagram:
- 1-, 2-, or many-qubit gates can be used.
- Efficient algorithm:
  #{qubits, gates, circuit runs} are polynomial in problem parameters.



Quantum Simulation

- Choose a discretized model for the system of interest. Contingent on:
  - Observables of interest,
  - Choice of computational tools.
- Mapping physical DOFs onto qubits, e.g., $c_0|\uparrow\rangle + c_1|\downarrow\rangle \rightarrow c_0|0\rangle + c_1|1\rangle$.
- Quantum Computation:



- $|\tilde{\imath}\rangle \rightarrow |\tilde{f}\rangle$ *can* but <u>does not have to</u> resemble some physical process. (Such as discretized time evolution or adiabatic interaction turn-on.)

Remarks

- “Fault-tolerance”:
  Errors in QCs accumulate at an exp rate $\Longrightarrow$ long circuits require error correction via quantum analogs of repetition codes, e.g. $|0\rangle \mapsto |000\rangle$.

- We are at an early stage (not even close to punch cards):
  Current quantum algorithms address operations on individual qubits.

- In the context of studying quantum physics, exponential advantage is likely to result from simulating **many-body** systems.

Quantum Simulation: Details

Manipulations with the state of a QC can be split into several stages:

- **State preparation** (eigenstates, thermal states, etc.);
- Time evolution (if necessary);
- Measurement (of particular observables—not of all the amplitudes!);

Algorithms achieving these tasks are based on a variety of methods:

- Product formulas ($e^{A+B+C+\cdots} \approx e^{\frac{A}{2}} e^{\frac{B}{2}} e^{\frac{C}{2}} \ldots e^{\frac{C}{2}} e^{\frac{B}{2}} e^{\frac{A}{2}}$);
- Hybrid quantum-classical methods (variational, subspace, etc.);
- Quantum Signal Processing (implement $|\psi\rangle \mapsto f(H)|\psi\rangle$ for some H).

Preparing Hamiltonian Eigenstates

- **Variational**: $E_0 \approx \min_{\vec{\theta}} \langle\psi(\vec{\theta})|H|\psi(\vec{\theta})\rangle$, where $|\psi(\vec{\theta})\rangle = U(\vec{\theta})|\psi_{\text{init}}\rangle$.
- Subspace methods (≈Lanczos): $\{|\psi_{\text{init}}\rangle, H|\psi_{\text{init}}\rangle, H^2|\psi_{\text{init}}\rangle, \dots\}$
- **Adiabatic interaction turn-on**.
- **Filtering via Projection:**

  $$\mathrm{P}_0|\psi_{\text{init}}\rangle = |\psi_0\rangle,$$

  where

  $$\mathrm{P}_0 = |\psi_0\rangle\langle\psi_0|.$$

- Thermalization.

	$\|\psi_{\text{init}}\rangle$ optimization	Provably converges	Near-term	Fault-tolerant
Variational	✓	✗	✓	✗
Subspace	✓	✓	✓	✓
Adiabatic	✗	?	✗	✓
Filtering	✓	✓	✗	✓
Thermalization	✗	?	✗	✓

2. How can QCs accelerate solving LF QCD?

Hamiltonian operator

- Assume the following form of the LF Hamiltonian operator:

$$H = \sum_{\xi_1,\xi_2,\ldots}^{N} c_{\xi_1\xi_2\ldots} q_{\xi_1} q_{\xi_2} \,,$$

where

- $\xi = \{n, \vec{n}_\perp, s, \ldots\}$ are the single-particle mode indices,
- q_ξ are boson/fermion creation/annihilation operators,
- $c_{\xi_1\xi_2\ldots}$ are coefficients provided in the form of a lookup table or functional dependence (the latter may give extra advantage).

Exp vs Poly

- The number of states of the form $|\xi_1^{w_1}, \xi_2^{w_2}, \ldots, \xi_J^{w_J}\rangle$ scales as $\sim N^J$ for $J \ll N$, where N is the total number of modes.
- Classical simulation has $\exp(J)$ cost (memory, operations).
- Quantum simulation can be performed with $\mathrm{poly}(J)$ cost:
  - number of qubits: those used for storing the state + “ancillas”,
  - number of quantum gates in a quantum circuit,
  - number of algorithm iterations.

Prerequisites

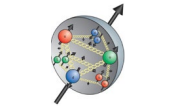
- Providing as input matrix elements of H in the basis of $|\xi_1^{w_1}, \xi_2^{w_2}, \ldots\rangle$ eliminates the possibility of exponential advantage.

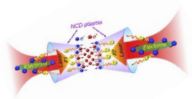
- For an efficient simulation, it is crucial that the input is provided in the form of $c_{\xi_1\xi_2\ldots}$ in $H = \sum_{\xi_1,\xi_2,\ldots}^{N} c_{\xi_1\xi_2\ldots} q_{\xi_1} q_{\xi_2}$.

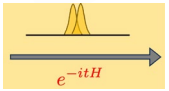
- Therefore, a (properly renormalized) Hamiltonian operator acting on multi-particle states is required.

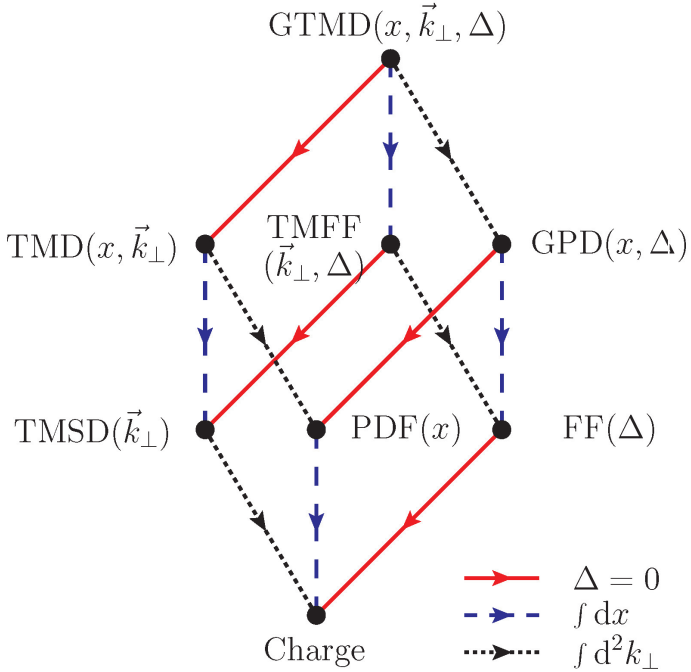
Scenarios

- **Spectra and static observables,**
- Thermodynamics,
- Background fields,
- Dynamics.
- …
- Anything that can be calculated as $\langle\psi_2|\hat{A}|\psi_1\rangle$, where
  - $\hat{A}$ is a known operator,
  - $|\psi_{1,2}\rangle$ are states which one knows* how to prepare.









Pasquini, Lorcé 2012

3. Peculiarities of simulating LF QFT on QCs.

What makes LF QCD an appealing application of QC?

- Resources: (hadron structure from LF) < (*ab initio* lattice dynamics):
  - Box size argument: fewer DOFs (N),
  - Harmonic resolution > 0: fewer occupied modes (J),
  - Fewer fermionic DOFs per spinor.
- Similarity with non-relativistic many-body physics:
  - Trivial vacuum*,
  - Straightforward treatment of fermions and bosons,
  - Easy to get observables from the wavefunction.
- Simple structure of Hamiltonian operator coefficients*.

Toy example: ϕ^4 theory
Mapping onto qubits

- Binary encoding of an integer
  (e.g., occupancy or momentum of a single bosonic DOF):
  $|0\rangle \mapsto |\ldots 00\rangle$, $|1\rangle \mapsto |\ldots 01\rangle$, $|2\rangle \mapsto |\ldots 10\rangle$, $|3\rangle \mapsto |\ldots 11\rangle$, etc.
- Fock state encoding in ϕ^4 theory in 1+1D:

Encoding	Qubit Representation of $\left\|1^5; 2^3; 3\right\rangle \equiv \|1^5; 2^3; 3^1; 4^0; 5^0; \ldots\rangle$	Qubit # Scaling	Qubit # Scaling for QCD
Direct	$\|\underbrace{101}_{w_1=5}; \underbrace{011}_{w_2=3}; \underbrace{001}_{w_3=1}; \underbrace{000}_{w_4=0}; \underbrace{000}_{w_5=0}; \ldots; \underbrace{000}_{w_K=0}\rangle$	$O(K \log K)$	$O(K\Lambda_\perp^2 \log(K+\Lambda_\perp))$
Compact	$\|\{\underbrace{001}_{n=1}, \underbrace{101}_{w_1=5}\}; \{\underbrace{010}_{n=2}, \underbrace{011}_{w_2=3}\}; \{\underbrace{010}_{n=3}, \underbrace{001}_{w_3=1}\}; 000\ldots\rangle$	$O(\sqrt{K} \log K)$	$O(K \log(K+\Lambda_\perp))$

- Not all the simulation techniques are compatible with Compact.

Toy example: ϕ^4 theory
Measurement

- PDF measurement operator:

$$f(n/K) = \langle\psi|a_n^\dagger a_n|\psi\rangle\,.$$

- Corresponds to measuring the expectation value of the $a_n^\dagger a_n$ operator for the corresponding qubit register $|\ldots;\underbrace{\ldots}_{n};\ldots\rangle$.

- Generalizable to GPDs, GTMDs, etc.

What are the complications in QSim of LF QCD?

- What defines simulation cost:
  - Number of elementary operator terms in the Hamiltonian,
  - Locality of Hamiltonian,
  - Hamiltonian norm.

- Number of terms scales as N^3 or N^4 as compared to N in ET LGT.
- LF Hamiltonian is non-local
  (which is true for almost any basis other than real space lattice).
- A **tight** bound on $||H_{LF}||$ is not known (to me).

4. Past, Current, & Future work.

Existing work

- State preparation, fault-tolerant algorithms:
  - 2002.04016, 2105.10941, 2211.07826.
- State preparation, near-term algorithms (variational):
  - 2009.07885, 2011.13443.
- Scattering with background field:
  - 2307.09987, 2310.13742, 2404.00819, 2205.07902, 2311.18209.

SFQED!

- Scattering of dynamical observables:
  - 2310.13742, 2401.04496.

Ongoing work: resource estimation for state preparation

- Motivation: estimating the size of a QC required for *ab initio* state preparation of the Sexaquark (*uuddss*) state, a SM DM candidate.
- DLCQ: $H = \cdots + \sum_{\xi_1\xi_2\xi_3\xi_4}^{N} h_{\xi_1\xi_2\xi_3\xi_4} b^{\dagger}_{\xi_1} b_{\xi_2} a^{\dagger}_{\xi_3} a_{\xi_4} \delta(p_1 + p_3 - p_3 - p_4) + \cdots$
- *State preparation algorithm* implements $|\psi\rangle \rightarrow \mathrm{P}_0|\psi\rangle$.
  - P_0 is constructed using calls to the *Block Encoding* subroutine U_H.
  - U_H provides info about H by implementing $|\psi\rangle \rightarrow H|\psi\rangle$.
- State prep. cost = $\underbrace{\mathbf{Cost}(\boldsymbol{U_H})}_{\sim \boldsymbol{N}\ \checkmark} \times \underbrace{\mathrm{Cost}(||H||)}_{\sim N^3 ???} \times \frac{1}{\Delta} \times \frac{1}{\underbrace{\gamma}_{???}} \times \log\frac{1}{\epsilon}$.

$\Delta = \mathrm{E}_1 - \mathrm{E}_0$ & $\gamma = |\langle\psi_{\mathrm{init}}|\psi_0\rangle|$ & $\epsilon = 1 - |\langle\psi_{\mathrm{init}}|\psi_0\rangle|^2$

Future work

- Deriving renormalized Hamiltonian operators.
- Reference state choice: utilizing best classically obtained solutions.
- Systematic analysis: uncertainties from truncation, $||H||$, etc.
- Applying state-of-the-art simulation algorithms, e.g.:
  - Subspace methods; state preparation via thermalization; etc.
- Transverse LF lattice:
  - Taking advantage of locality in the transverse directions,
  - Suitable for scattering of dynamical composite particles?

Thank You!