Quantum stress within hadrons

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Acknowledgements

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Based on:
Cao, YL, Vary, PRD 108, 056026 (2023),
YL, Vary, PRD 109, L051501 (2024),
Xu, Cao, Hu, YL, Zhao, Vary, arXiv:2404.06259 [hep-ph],
YL, Wang, Vary, arXiv:2405.06892 [hep-ph],
Cao, Xu, YL, Chen, Zhao, Karmanov, Vary, arXiv:2405.06896 [hep-ph]

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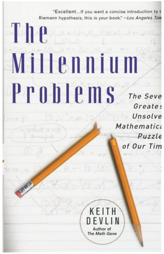
Big puzzles in QCD

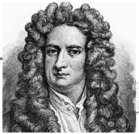
Strong force inside matter:

- Confinement of quarks and gluons
- Origin of >99% nucleon mass
- Origin of nucleon spin

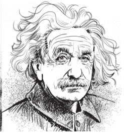


Gross and Klempt et al., 50 Years of quantum chromodynamics, Eur. Phys. J. C, 83 (2023)

Hadronic energy-momentum tensor



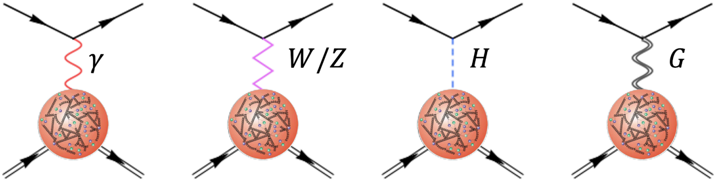
Everything gravitates



They gravitate through energy-momentum tensor

$$H = \int \mathrm{d}^3x\, T^{00}(x) \quad \Rightarrow \quad t^{\alpha\beta}(x) = \langle \Psi | T^{\alpha\beta}(x) | \Psi \rangle$$

Hadronic energy-momentum tensor encodes the energy-stress densities inside hadrons

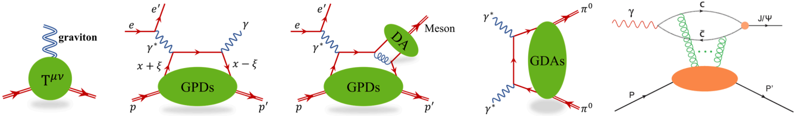


Hadronic matrix elements and gravitational form factors (GFFs): [Kobzarev:1962wt, Pagels:1966zza]

$$\langle p', s' | T^{\mu\nu}(0) | p, s\rangle = \frac{1}{M}\overline{u}_{s'}(p')\Big[P^\mu P^\nu A(q^2) + \frac{1}{2} i P^{\{\mu}\sigma^{\nu\}\rho} q_\rho J(q^2) + \frac{1}{4}(q^\mu q^\nu - g^{\mu\nu} q^2) D(q^2)\Big] u_s(p)$$

Last global unknown

[Polyakov:2018zvc, Burkert:2023wzr]



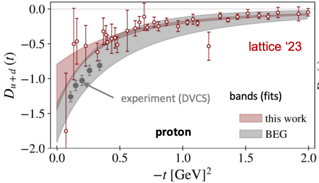
- Ji's sum rules: second Melin moments of the GPDs, e.g., [Ji:1996nm, Polyakov:2002yz]

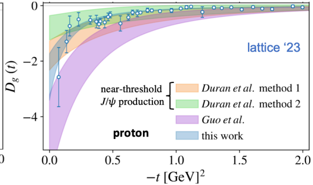
$$\int_{-1}^{1} \mathrm{d}x\, x H^{q,g}(x,\xi,t) = A^{q,g}(t) + \xi^2 D^{q,g}(t)$$

  - Deeply virtual Compton scattering & deeply virtual meson production [Burkert:2018bqq, Burkert:2021ith]
- Di-photon pair production [Kumano:2017lhr]
- Near threshold vector meson production [Kharzeev:2021qkd, Duran:2022xag]
- Large uncertainties from both the theory and experiments → Electron-Ion Colliders

[Lattice '23: Hackett:2023nkr]

electromagnetic	$G_E(0) = Q = 1.602176487(40) \times 10^{-19}\mathrm{C}$ $G_M(0) = \mu = 2.792847356(23)\mu_N$
weak	$G_A(0) = g_A = 1.2694(28)$ $G_P(0) = g_p = 8.06(55)$
gravitational	$mA(0) = m = 938.272013(23)\ \mathrm{MeV}/c^2$ $J(0) = J = \frac{1}{2}$ $D(0) = D = ?$





Mechanical stability of hadrons

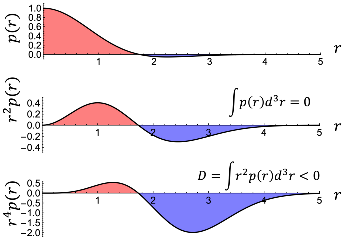
- Energy-momentum conservations imply: [Cotogno:2019xcl, Lorce:2019sbq]

$$A(0) = 1, \quad J(0) = \frac{1}{2}, \quad \lim_{Q^2 \to 0} Q^2 D(Q^2) = 0 \quad \Rightarrow \quad \int \mathrm{d}^3 r\, \mathcal{P}(r) = 0$$

  the von Laue condition implies hadrons are in mechanical equilibrium [Laue:1911lrk]

- Polyakov et al. conjectured that $D < 0$ for mechanically stable systems [Polyakov:2018zvc]

$$D = \int \mathrm{d}^3 r\, r^2 \mathcal{P}(r) \overset{???}{<} 0$$



Trace anomaly

- Trace anomaly in QCD:

$$S \equiv T^{\mu}_{\mu} = \frac{\beta(g_s)}{2g_s} G^{\mu\nu a} G^{a}_{\mu\nu} + O(m_q).$$

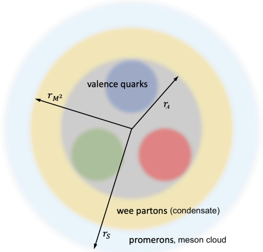
- D is related to the trace anomaly

$$\langle p', s' | S | p, s \rangle = M \bar{u}_{s'}(p') \Big[(1 + \frac{Q^2}{4M^2}) A(Q^2) + \frac{Q^2}{4M^2} (3D(Q^2) - J(Q^2)) \Big] u_s(p)$$

- $D < 0$ implies a layered structure within the proton,

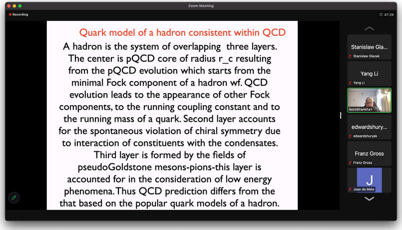
$$r_A < r_{M^2} < r_S$$

where, $r_A^2 = -6A'(0)$, $r_{M^2}^2 = -6(M^2)'(0) = r_A^2 - 3\lambda_C^2 D$, $r_S^2 = -6S'(0) = r_A^2 - \frac{9}{2}\lambda_C^2 D$



pQCD core: $r_c = 0.4 - 0.5$ fm
condensate: $r_N = 0.85$ fm
meson cloud: $r_\pi = 1.0$ fm

[Frankfurt:2022cyk]



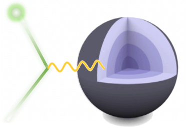
Part II: Macroscopic interpretation of D

Sachs/Breit-frame densities

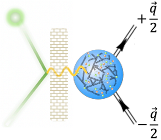
The Sachs densities are defined as the F.T. of the hadronic matrix elements within the Breit frame ($\vec{p}' = -\vec{p} = +\frac{1}{2}\vec{q}$, aka. the brick-wall frame), [Sachs:1962zzc, Polyakov:2018zvc]

$$\mathcal{T}^{\alpha\beta}_{\text{BF}}(\vec{r}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3 2E_q} e^{-i\vec{q}\cdot\vec{r}} \langle +\tfrac{1}{2}\vec{q}|T^{\alpha\beta}(0)|-\tfrac{1}{2}\vec{q}\rangle, \qquad (E_q = \sqrt{M^2 + \tfrac{1}{4}\vec{q}^2})$$

- Frame dependence: the proton is not at rest in the Breit frame. Densities in other frames?
- Lack of local probabilistic interpretation $T^{00} \sim \sum_i \bar{q}_i \gamma^0 i\partial_t q_i \neq \sum_i \omega_i N_i$ [Miller:2018ybm]
- Ambiguities in physical densities, e.g. A vs T^{00} vs $T^{00}/\sqrt{1+\tau}$ [Lorce:2020onh]
- Underlying assumption: proton as a rigid ball -- in contradiction with relativity [Jaffe:2020ebz]



$\lambda_\gamma \sim r_{\text{nucl}} \gg \lambda_{\text{Comp}} = M_{\text{nucl}}^{-1}$



$\lambda_\gamma \sim r_N \sim \lambda_{\text{Comp}} = M_N^{-1}$

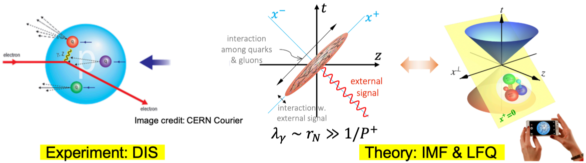
Light-front densities

[Miller:2018ybm]

$$\mathcal{T}^{\alpha\beta}(\vec{r}_\perp; P) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3 2P^+} e^{\frac{i}{2}q^+x^- - i\vec{q}_\perp\cdot\vec{r}_\perp} \langle P + \tfrac{1}{2}q | \frac{1}{2} \int \mathrm{d}x^- T^{\alpha\beta}(x^-; x_\perp = 0) | P - \tfrac{1}{2}q \rangle,$$

$$= \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2 2P^+} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} \langle P + \tfrac{1}{2}q | T^{\alpha\beta}(0) | P - \tfrac{1}{2}q \rangle \Big|_{q^+=0}$$

- Frame independent: boost invariance in light-front dynamics
- Local probabilistic interpretation: $T^{++} \sim \sum_i \bar{q}_i \gamma^+ i\partial^+ q_i \sim \sum_i p_i^+ N_i$
- Intrinsically relativistic and related to the forward generalized parton density $q(x, \vec{b}_\perp)$, i.e. what the probes "see" in high-energy collision experiments [Burkardt:2000za]



light-cone coordinates:

$$x^\pm = x^0 \pm x^3,$$

$$\vec{x}_\perp = (x^1, x^2)$$

Physical densities: energy and momentum [Xu:2024cfa, cf. Freese:2021czn, Freese:2021mzg]

$$\int \mathrm{d}^3x\, T^{+\mu}(x) = P^\mu$$

$$\mathcal{P}^+(r_\perp) \equiv \mathcal{T}^{++}(r_\perp; P) = P^+ \mathcal{A}(r_\perp),$$

$$\mathcal{P}^i_\perp(r_\perp) \equiv \mathcal{T}^{+i}_{ss}(r_\perp; P) = P^i_\perp \mathcal{A}(r_\perp) + \big(\nabla \times \vec{\mathcal{S}}\big)^i, \qquad (i = 1, 2),$$

$$\mathcal{P}^-(r_\perp) \equiv \mathcal{T}^{+-}_{ss}(r_\perp; P) = \frac{P^2_\perp \mathcal{A}(r_\perp) + \vec{P}_\perp \cdot (\nabla \times \vec{\mathcal{S}}) + \mathcal{M}^2(r_\perp)}{P^+}$$

- $\mathcal{A}(r_\perp)$ can be interpreted as the (convective) momentum/matter density
- $\mathcal{M}^2(r_\perp)$ can be interpreted as the invariant mass squared density
- $\vec{\mathcal{S}}(r_\perp)$ can be interpreted as the spin current density

$$P^- = \frac{P^2_\perp + M^2}{P^+}$$

$$\mathcal{A}(r_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} A(q^2_\perp),$$

$$\mathcal{M}^2(r_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} \Big[(M^2 + \frac{1}{4}q^2_\perp)A(q^2_\perp) + \frac{1}{2}q^2_\perp D(q_\perp)\Big],$$

$$\vec{\mathcal{S}}(r_\perp) = 2\vec{s} \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} J(q^2_\perp)$$

A moment of Zen

- Physical densities associated with "bad" components $\mathcal{T}^{-\mu}$ are not well understood
- Light-front densities are 2D $\overset{?}{\to}$ 3D [Panteleeva:2021iip]
- Light-front densities can be understood as equal-time densities in the infinite momentum frame which could be counter-intuitive [Lorce:2020onh]
- Amplitude vs. quantum expectation value: what is truly probed by gravity is the quantum expectation value $t^{\alpha\beta}(x) = \langle\Psi|T^{\alpha\beta}(x)|\Psi\rangle$ where $|\Psi\rangle$ is a generic hadronic state

Pertinent question: What are the proper 3D energy and stress densities within the proton?

$$g_{\alpha\beta}t^{\alpha\beta} = g_{\alpha\beta}\langle\Psi|T^{\alpha\beta}(x)|\Psi\rangle$$

where, $t^{\alpha\beta} = eu^\alpha u^\beta - p\Delta^{\alpha\beta} + \pi^{\alpha\beta}$ for continuum body

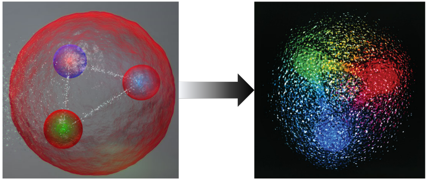
A hadron is a de Broglie wave

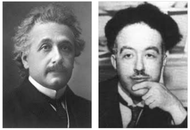
[Li:2022ldb]

resolving a non-relativistic particle: $r_{\text{hadron}} \gg \lambda_\gamma \gg \lambda_{\text{hadron}} \geq \lambda_{\text{C}}$

resolving a relativistic hadron: $\lambda_{\text{hadron}} \gtrsim r_{\text{hadron}} \sim \lambda_{\text{C}} \gg \lambda_\gamma$

where $\lambda_{\text{C}} = M^{-1}$ is the Compton wavelength, $\lambda_\gamma = Q^{-1}$ is the wavelength of the probe, e.g. a photon. λ_{hadron} is the de Broglie wavelength. r_{hadron} is the hadron radius.





- The probe, e.g. the photon, ``sees'' a de Broglie wave! Namely, the proton as a whole is a relativistic continuum -- hydro for hadron!
- The hydrodynamics view of the proton has interesting consequences. For example, the mass decomposition can be viewed as the multi-fluid description of the wave [Lorce:2017xzd]

Relativistic spin medium (continuum)

The energy-momentum tensor of a relativistic spin medium

$$t^{\alpha\beta} = eu^\alpha u^\beta - p\Delta^{\alpha\beta} + \tfrac{1}{2}\partial_\sigma(u^{\{\alpha}s^{\beta\}\sigma}) + \pi^{\alpha\beta} + \text{dissipative terms}$$

where, u^α is the medium velocity with $u_\alpha u^\alpha = 1$, $\Delta^{\alpha\beta} = g^{\alpha\beta} - u^\alpha u^\beta$ is the spatial metric tensor. $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$.

- $e(x)$ is the proper energy density, i.e. energy density measured in local rest frame (LRF)
- Cauchy stress tensor $c^{\alpha\beta} = \pi^{\alpha\beta} - p\Delta^{\alpha\beta}$ can be decomposed into a traceless shear tensor and a normal pressure $p(x)$.
- Shear tensor $\pi^{\alpha\beta}(x)$ is dissipative in fluids but non-dissipative in solids
- There is a new contribution from the spin tensor $s^{\alpha\beta}(x)$, which is recently proposed by Fukushima et. al. in relativistic spin hydrodynamics [Fukushima:2020ucl, cf. Li:2020eon]

Hadronic EMT

[Li:2024vgv]

- It can be shown that the quantum expectation value of the EMT tensor can be written as,

$$\langle \Psi | T^{\alpha\beta}(x) | \Psi \rangle = \langle \mathcal{E} \mathcal{U}^\alpha \mathcal{U}^\beta - \mathcal{P} \Delta^{\alpha\beta} + \tfrac{1}{2} \partial_\rho (\mathcal{U}^{\{\alpha} \mathcal{S}^{\beta\}\rho}) + \mathit{\Pi}^{\alpha\beta} \rangle_\Psi$$

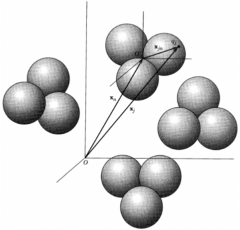
where,

$$\langle \mathcal{O}(x) \rangle_\Psi = \int \mathrm{d}^3 z \, \overline{\Psi}(z) \mathcal{O}(x-z) \Psi(z) \Big|_{x^0=z^0},$$

is a convolution with the wavepacket $\Psi(x)$.

- The convolution is consistent with the classical treatment of a medium of composite particles, e.g. molecules [J. D. Jackson, Classical electrodynamics, Wiley]

$$\rho(x) = \int \mathrm{d}^3 r \, \underbrace{\mathcal{D}(\vec{r})}_{\text{intrinsic}} \, \underbrace{\rho_f(\vec{x} - \vec{r}, t)}_{\text{c.m.}} \equiv \langle \mathcal{D}(\vec{r}) \rangle$$



where $\mathcal{D}(\vec{x})$ is the intrinsic density, i.e. density within the molecule.
ρ_f is the distribution of the molecules, i.e. molecular density.

Quantum wave kinematics

[Li:2022ldb]

- Hadronic wavepacket:

$$\Psi(x) = \sum_s \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2p^0} e^{-ip\cdot x} u_s(p) \langle p, s|\Psi\rangle$$

which satisfies the Dirac equation (hence not a true probabilistic wave function)

- Conserved number current: $(f\overleftrightarrow{\partial}g \equiv f\partial g - \partial f g)$

$$n^\mu(x) \equiv \frac{1}{2M} \overline{\Psi}(x) i\overleftrightarrow{\partial}^\mu \Psi(x) \quad \Rightarrow \quad \partial_\mu n^\mu = 0$$

$$= n u^\alpha, \quad (u_\alpha u^\alpha = 1)$$

$n = n_\alpha u^\alpha$ is the proper number density and u^α the wave velocity.

- Quantum wave velocity:

$$u^\alpha(x) \equiv \overline{\Psi}(x)\mathcal{U}^\alpha \Psi(x) = \frac{[\overline{\Psi} i\overleftrightarrow{\partial}^\alpha \Psi]}{\sqrt{4M^2 + \partial^2}} = n^\alpha - \frac{1}{8M^2}\partial^2 n^\alpha + \frac{3}{16M^4}\partial^4 n^\alpha - \cdots$$

- More wave kinematics:

$$\partial_\alpha u_\beta = u_\alpha \underbrace{a_\beta}_{\text{acceleration}} + \underbrace{\Omega_{\alpha\beta}}_{\text{vorticity}} + \underbrace{\Sigma_{\alpha\beta}}_{\text{shear}}$$

Hadronic EMT

[Li:2024vgv]

- We identify the hadronic EMT as,

$$\mathcal{T}^{\alpha\beta} \equiv \mathcal{E}\mathcal{U}^\alpha\mathcal{U}^\beta - \mathcal{P}\Delta^{\alpha\beta} + \tfrac{1}{2}\partial_\rho(\mathcal{U}^{\{\alpha}\mathcal{S}^{\beta\}\rho}) + \mathit{\Pi}^{\alpha\beta}$$

- $\mathcal{E}, \mathcal{P}, \mathcal{S}^{\alpha\beta}, \mathit{\Pi}$ can be uniquely identified as the hadronic energy density, pressure, spin tensor and shear, respectively:

$$\mathcal{E}(x) = M\int\frac{\mathrm{d}^3q}{(2\pi)^3}e^{iq\cdot x}\Big\{\Big(1-\frac{q^2}{4M^2}\Big)A(q^2) \; + \frac{q^2}{4M^2}\Big[2J(q^2)-D(q^2)\Big]\Big\},$$

$$\mathcal{P}(x) = \frac{1}{6M}\int\frac{\mathrm{d}^3q}{(2\pi)^3}e^{iq\cdot x}q^2D(q^2),$$

$$\mathcal{S}^{\alpha\beta}(x) = \int\frac{\mathrm{d}^3q}{(2\pi)^3}e^{iq\cdot x}\Big\{i\sigma^{\alpha\beta}\sqrt{1-\frac{q^2}{4M^2}}-\frac{U^{[\alpha}q^{\beta]}}{2M}\Big\}J(q^2),$$

$$\mathit{\Pi}^{\alpha\beta}(x) = \frac{1}{4M}\int\frac{\mathrm{d}^3q}{(2\pi)^3}e^{iq\cdot x}\Big(q^\alpha q^\beta-\frac{q^2}{3}\Delta^{\alpha\beta}\Big)D(q^2)\,,$$

Factorization of the hadronic density

[Li:2022ldb, cf. Freese:2022fat]

$$e(x) = \int \mathrm{d}^3 z\, \overline{\Psi}(x-z)\Big\{ M \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{iq\cdot x}\Big\{\Big(1-\frac{q^2}{4M^2}\Big)A(q^2) + \frac{q^2}{4M^2}\Big[2J(q^2)-D(q^2)\Big]\Big\}\Big\}\Psi(x-z)\Big|_{z^0=0}$$

The hadronic part is *not factorizable* due to the dependence of $\vec{P}=(-i/2)\overleftrightarrow{\nabla}_x$ in $q^2=(q^0)^2-\vec{q}^2$, where $q^0=\sqrt{(\vec{P}+\frac{1}{2}\vec{q})^2+M^2}-\sqrt{(\vec{P}-\frac{1}{2}\vec{q})^2+M^2}$

- Taylor expansion around $\vec{P}=0$: multipole series,

$$\mathcal{E}(\vec{r}) = \sum_{n=0}^{\infty}\frac{(-i)^n}{2^n n!}\mathcal{E}_n^{i_1 i_2\cdots i_n}(\vec{r})\overleftrightarrow{\nabla}^{i_1}\overleftrightarrow{\nabla}^{i_2}\ldots\overleftrightarrow{\nabla}^{i_n}$$

- Monopole density gives the Breit-frame distribution (Sachs distribution)

$$\mathcal{E}_0(\vec{r}) = M\int\frac{\mathrm{d}^3 q}{(2\pi)^3}e^{-i\vec{q}\cdot\vec{r}}\Big\{\Big(1+\frac{\vec{q}^2}{4M^2}\Big)A(-\vec{q}^2)-\frac{\vec{q}^2}{4M^2}\Big[2J(-\vec{q}^2)-D(-\vec{q}^2)\Big]\Big\}$$

- High-multipole moments exist due to Lorentz distortion
- No special frame (such as $q^0=0$) or non-relativistic approximation is taken
- The expansion is based on the separation of intrinsic hadronic scale $l_{\text{had}}=\max\{r_p,\lambda_{\text{Comp}}\}$ from the scale of the wavepacket $l_\Psi=\lambda_{\text{deBrog}}$. For example, $q^2=-\vec{q}^2+O(l^2_{\text{had}}/l^2_\Psi)$

Light-front distribution

[Li:2022ldb]

Is the multipole expansion unique? No! $\to$ Alternative: Taylor (Laurent) expansion around $1/|\vec{P}| = 0$

- Sufficient to take $P_z \to \infty \;\Rightarrow\; |\vec{P}| = \sqrt{\vec{P}_\perp^2 + P_z^2} \to \infty$
- Monopole density gives the 2D light-front distribution

$$\mathcal{E}_0(x) = \delta(x_\parallel) M \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{x}_\perp} \Big[\Big(1 + \frac{\vec{q}_\perp^2}{4M^2}\Big) A(-\vec{q}_\perp^2) - \frac{\vec{q}_\perp^2}{4M^2}\Big(2J(-\vec{q}_\perp^2) - D(-\vec{q}_\perp^2)\Big)\Big].$$

- No special frame (e.g. Drell-Yan $q^+ = 0$ frame) is chosen
- Relativistic, suitable also for massless hadrons (in contrast to the Sachs distribution)
- Convergence of the multipole series: $|\vec{P}| \gg \lambda_{\text{hadron}}^{-1} \gg \{M, r_{\text{hadron}}^{-1}\}$ -- sufficiently *localized* z-direction
- In the infinite momentum frame (IMF), components of the EMT form a hierarchy:

$$\underbrace{\mathcal{T}^{++} \sim P_z^2}_{\text{best}}, \quad \underbrace{\mathcal{T}^{+i} \sim P_z^1}_{\text{good}}, \quad \underbrace{\mathcal{T}^{+-} \sim \mathcal{T}^{ij} \sim P_z^0}_{\text{bad}}, \quad \underbrace{\mathcal{T}^{-i} \sim P_z^{-1}}_{\text{worse}}, \quad \underbrace{\mathcal{T}^{--} \sim P_z^{-2}}_{\text{worst}}$$

Part III: Microscopic interpretation of D

Light-front wave function representation

- Drell-Yan-West formula: [Drell:1969km]

$$\rho_{\text{ch}}(r_\perp) = \sum_n \int [\mathrm{d}x_i \mathrm{d}^2 r_{i\perp}]_n \left|\widetilde{\psi}_n(\{x_i, \vec{r}_{i\perp}\})\right|^2 \sum_j e_j \delta^2(r_\perp - r_{j\perp}) \equiv \Big\langle \sum_j e_j \delta^2(r_\perp - r_{j\perp}) \Big\rangle$$

- Brodsky-Hwang-Ma-Schmidt formula: [Brodsky:2000ii]

$$\mathcal{A}(r_\perp) = \Big\langle \sum_j x_j \delta^2(r_\perp - r_{j\perp}) \Big\rangle$$

Matter density $\mathcal{A}(r_\perp)$ mainly samples the valence partons $x_j \sim O(1)$; wee parton $x_j \ll 1$ contributions suppressed

- What about a LFWF representation of D?

International Journal of Modern Physics A | Vol. 33, No. 26, 1830025 (2018)

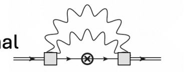
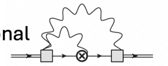
| Reviews

Forces inside hadrons: Pressure, surface tension, mechanical radius, and all that

Maxim V. Polyakov and Peter Schweitzer

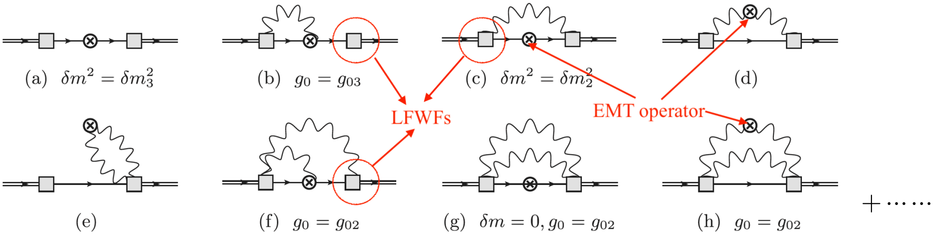
https://doi.org/10.1142/S0217751X18300259 | **Cited by:** 212 **(Source: Crossref)**

$\hat{T}_{++}$ of the EMT. Being related to the stress tensor $\hat{T}_{ij}$ the form factor $D(t)$ naturally "mixes" good and bad light-front components and is described in terms of transitions between different Fock state components in overlap representation. As a quantity intrinsically nondiagonal in a Fock space, it is difficult to study the D-term in approaches based on light-front wave functions. This is due to the rela-

diagonal  non-diagonal 

Strongly coupled scalar theory as an example

[Cao:2023ohj]



- Adopt a strongly coupled scalar theory as an example to derive the LFWF representation

$$\mathcal{L} = -g|\chi|^2\varphi$$

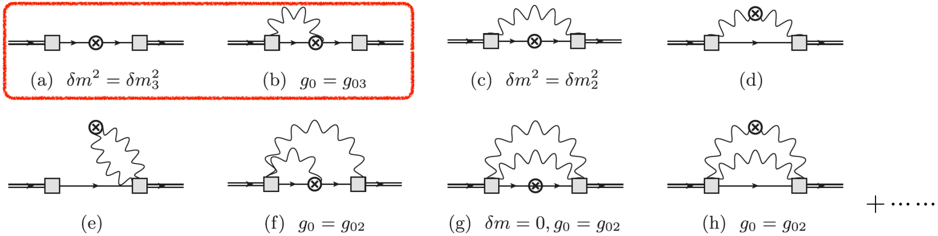
- Quenched theory: excluding nucleon-antinucleon d.o.f. to avoid vacuum instability
- Systematic Fock sector expansion and sector dependent renormalization [Li:2015iaw, Karmanov:2016yzu]
- All divergence cancels out with the sector dependent counterterms, e.g. $(a) + (b)$

$$t_a^{\alpha\beta} = Z[(\frac{1}{2}q^2 - \delta m_3^2)g^{\alpha\beta} + p^{\{\alpha}p'^{\beta\}}] = Z[2P^\alpha P^\beta + (\frac{1}{2}q^2 - \delta m_3^2)g^{\alpha\beta} - \frac{1}{2}q^\alpha q^\beta]$$

$$t_b^{\alpha\beta} = -\sqrt{Z}g^{\alpha\beta}\int\frac{\mathrm{d}x}{2x(1-x)}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}g_{03}\psi_2(x,k_\perp) = g^{\alpha\beta}Z\delta m_3^2$$

Strongly coupled scalar theory as an example

[Cao:2023ohj]



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Light-front energy density

- Hadronic matrix elements within Drell-Yan-Breit frame ($q^+ = 0, \vec{P}_\perp = 0$):

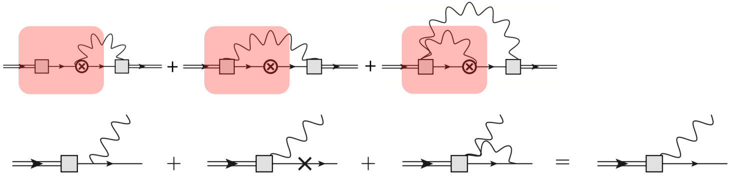
$$
\begin{aligned}
t^{++} &= 2(P^+)^2 A(-q_\perp^2) \\
t^{ij} &= \frac{1}{2}(q^i q^j - \delta^{ij} q_\perp^2) D(-q_\perp^2) \\
t^{+-} &= 2(m^2 + \frac{1}{4} q_\perp^2) A(-q_\perp^2) + q_\perp^2 D(-q_\perp^2) \\
t^{--} &= 8\Big(\frac{m^2 + \frac{1}{4}q_\perp^2}{P^+}\Big)^2 A(-q_\perp^2) \\
t^{-i} &= t^{+i} = 0
\end{aligned}
$$

$$P^\mu = \int \mathrm{d}^3 x \, T^{+\mu}(x)$$

- $A(Q^2)$ can be extracted from t^{++}
- $D(Q^2)$ can be extracted from either t^{+-} and t^{ij}, which are equally ``bad" currents
- We adopt t^{+-} because it is constrained by energy conservation in the forward limit

$$P^\mu |p\rangle = p^\mu |p\rangle, \quad \Rightarrow \quad t^{+\mu}(q_\perp^2 \to 0) = p^\mu$$

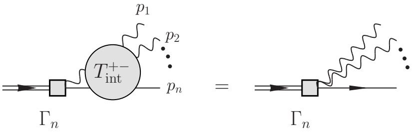
Light-front energy density



- There are indeed non-diagonal contributions. However, all non-diagonal contributions add up to a diagonal contribution
- Indeed, T^{+-}_{int} only involves diagonal overlaps in the forward limit ($q = 0$)

$$\langle \{x_i p^+, \vec{k}_{i\perp} + x_i \vec{p}_\perp\}_n | T^{+-}_{\text{int}}(0) | \psi(p) \rangle = 2(M^2 - s_n)\psi_n(\{x_i, \vec{k}_{i\perp}\}).$$

Here, $s_n = \sum_i (k^2_{i\perp} + m^2_i)/x_i$ is the n-body light-front kinetic energy



$$\Gamma_n = (s_n - M^2)\psi_n$$

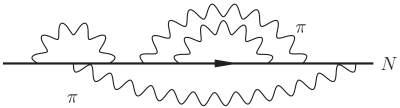
Light-front wave function representation

$$t^{+-} = 2\Big\langle \underbrace{\sum_j e^{i\vec{r}_{j\perp}\cdot\vec{q}_\perp} \frac{-\nabla_{j\perp}^2 + m_j^2 - \frac{1}{4}q_\perp^2}{x_j}}_{\text{kinetic part}} + \underbrace{\Big[M^2 - \sum_j \frac{-\nabla_{j\perp}^2 + m_j^2}{x_j}\Big] e^{i\vec{r}_{N\perp}\cdot\vec{q}_\perp}}_{\text{potential part}} \Big\rangle$$

- Recall the quantum average is defined as,

$$\langle O \rangle \equiv \sum_n \int \left[\mathrm{d}x_i \mathrm{d}^2 r_{i\perp}\right]_n \widetilde{\psi}_n^*(\{x_i, \vec{r}_{i\perp}\}) O_n \widetilde{\psi}_n(\{x_i, \vec{r}_{i\perp}\})$$

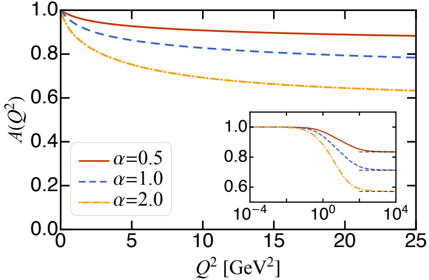
- The off-shell factors $e^{i\vec{r}_{j\perp}\cdot\vec{q}_\perp} \xrightarrow{\text{F.T.}} \delta^2(r_\perp - r_{j\perp})$ indicate the location of the graviton coupling
- $\vec{r}_{N\perp}$ is the location of the nucleon -- in the quenched approximation, all interaction happens at $r_{N\perp}$

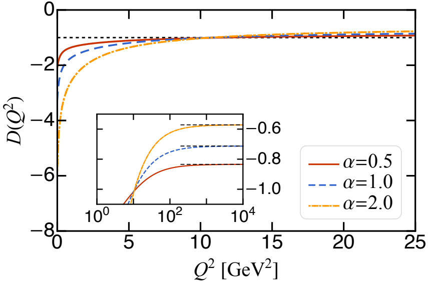


pion cloud in the "quenched" scalar theory

Numerical results of the scalar theory

[Cao:2023ohj]





$$\alpha = \frac{g^2}{16\pi m^2}$$

- For small α, $D(Q^2)$ is close to -1, the result of the free scalar theory
- For small Q^2 (forward limit):

$$\lim_{Q^2\to 0} A(Q^2) = 1, \qquad \lim_{Q^2\to 0} D(Q^2) = D = \text{finite} \qquad \lim_{Q^2\to 0} Q^2 D(Q^2) = 0$$

  all conservation laws are preserved
- For large Q^2,

$$\lim_{Q^2\to\infty} A(Q^2) = Z, \qquad \lim_{Q^2\to\infty} D(Q^2) = -Z,$$

  revealing a pointlike core, consistent with the physical picture of the model

Quantum stress within charmonium

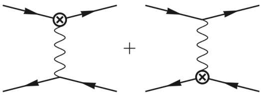
[Xu:2024cfa]

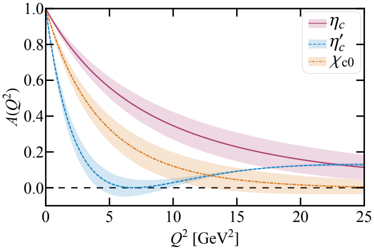
- The above LFWF representation can be generalized to two-body $q\bar{q}$ bound states by identifying the correct location of the graviton coupling:

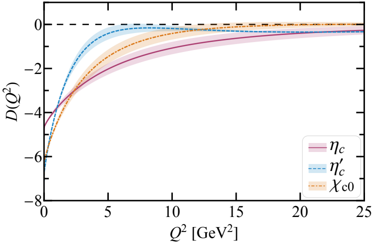
$$t^{+-}_{\text{int}}(q_\perp^2) = 2\sum_{s,\bar{s}}\int\frac{\mathrm{d}x}{4\pi x(1-x)}\int \mathrm{d}^2 r_\perp \widetilde{\psi}^*_{s\bar{s}}(x,\vec{r}_\perp)\frac{1}{2}\Big[e^{i\vec{q}_\perp\cdot\vec{r}_{1\perp}} + e^{i\vec{q}_\perp\cdot\vec{r}_{2\perp}}\Big]v(x,\vec{r}_\perp,-i\nabla_\perp)\widetilde{\psi}_{s\bar{s}}(x,\vec{r}_\perp)$$

where, $v(x,\vec{r}_\perp,-i\nabla_\perp) = M^2 - \frac{-\nabla_\perp^2+m_q^2}{x} - \frac{-\nabla_\perp^2+m_{\bar{q}}^2}{1-x}$.

- As an application, we consider the charmonium and adopt charmonium wave functions from previous basis light-front quantization (BLFQ) calculations [Li:2017mlw]







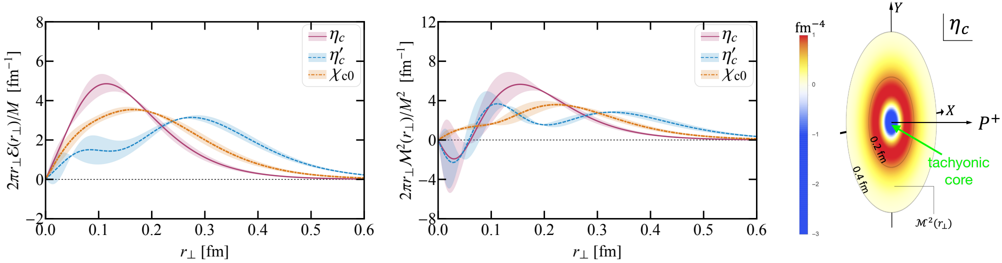
Energy density vs invariant mass squared density

Energy density $\mathcal{E}(r_\perp)$ vs the invariant mass squared density $\mathcal{M}^2(r_\perp)$:

$$\mathcal{E}(r_\perp) = M \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{r}_\perp} \Big\{ \Big(1 + \frac{q_\perp^2}{4M^2}\Big) A(q_\perp^2) + \frac{q_\perp^2}{4M^2} D(q_\perp^2) \Big\},$$

$$\mathcal{M}^2(r_\perp) = M^2 \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{r}_\perp} \Big\{ \Big(1 + \frac{q_\perp^2}{4M^2}\Big) A(q_\perp^2) + \frac{q_\perp^2}{2M^2} D(q_\perp^2) \Big\} = M \Big[\mathcal{E}(r_\perp) - \frac{3}{2} \mathcal{P}(r_\perp) \Big]$$

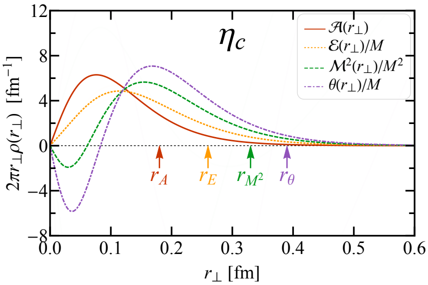
- Energy density is positive
- Invariant mass squared density becomes negative at small $r_\perp$: tachyonic core within charmonium?

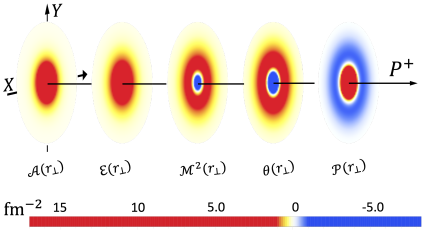


Physical densities

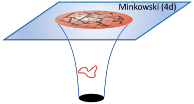
Matter density $\mathcal{A}(r_\perp)$, energy density $\mathcal{E}(r_\perp)$, invariant mass squared density $\mathcal{M}^2(r_\perp)$ and scalar density $\theta(r_\perp)$:

$$\mathcal{A}(r_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} A(q_\perp^2),$$
$$\theta(r_\perp) = \mathcal{E}(r_\perp) - 3\mathcal{P}(r_\perp).$$





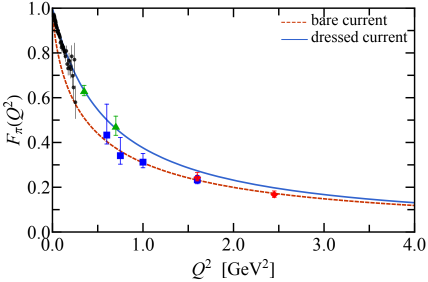
Application to the pion in AdS/QCD

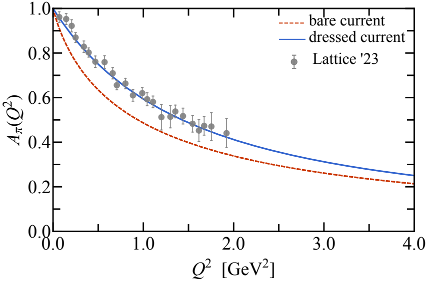


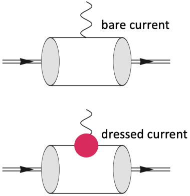
- AdS/QCD is a bottom-up approach to QCD based on string-gauge duality
- Form factors $F_{\pi,K,N}(Q^2)$ and $A_{\pi,K,N}(Q^2)$ in AdS/QCD [Abidin:2009hr]

$$F_\pi(Q^2) = \int \mathrm{d}^2 z\, \big|\varphi_\pi(z)\big|^2 V(Q^2, z), \qquad A_\pi(Q^2) = \int \mathrm{d}^2 z\, \big|\varphi_\pi(z)\big|^2 H(Q^2, z),$$

where, $V(Q^2, z)$ and $H(Q^2, z)$ are vector and tensor bulk-to-boundary propagators







Application to the pion in AdS/QCD

[Li:2023izn]

- Unfortunately, the gravitational wave in AdS_5 can only couples to the traceless part of the EMT $\to D(Q^2)$ is not fully constrained [Abidin:2008ku, cf. Mamo:2019mka, Mamo:2021tzd, Fujita:2022jus]
- Light-front holography: correspondence between AdS/QCD and LFQCD [Brodsky:2014]

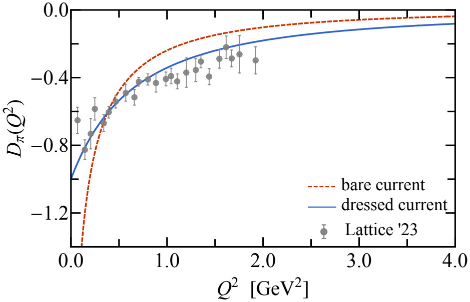
$$z \leftrightarrow \zeta_\perp = \sqrt{x(1-x)}r_\perp$$

- Effective $q\bar{q}$ interaction from soft-wall AdS/QCD: $U_{q\bar{q}} = \kappa^4\zeta_\perp^2 + 2\kappa^2(J-1)$

$$D_\pi^{\text{bare}}(Q^2) = \int \mathrm{d}^2 z |\varphi_\pi(z)|^2 \Big\{ \overbrace{\frac{z^2Q^2}{4}K_2(zQ)}^{\text{tensor}} - \overbrace{2K_0(zQ)}^{\text{scalar}} - \frac{2U(z)}{Q^2}\Big[\underbrace{zQK_1(zQ) - \frac{1}{2}z^2Q^2K_2(zQ)}_{\text{vector - tensor = scalar}}\Big]\Big\}$$

Current dressing:

$$zQK_1(zQ) \to V(Q^2, z),$$
$$\frac{1}{2}z^2Q^2K_2(zQ) \to H(Q^2, z),$$
$$2K_0(zQ) \to S(Q^2, z)$$



Summary

- Hadronic energy-momentum tensor and the gravitational form factor D
- Macroscopic picture of hadrons as a relativistic continuum
- Microscopic picture of hadrons from light-front wave functions

Thank you!