Two Approaches to Adjoint QCD_2

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Based on:

- arXiv:2101.05432 with Ross Dempsey and Igor Klebanov
- arXiv:2210.10895 with Ross Dempsey, Igor Klebanov, and Loki Lin
- arXiv:2311.09334 with Ross Dempsey, Igor Klebanov, and Benjamin Søgaard

February 21, 2024

Introduction: two-dimensional gauge theories

- **Two-dimensional gauge theories** are useful toy models for:
  - confinement and mass gap; generalized symms, anomalies (HET)
  - numerical algorithms (cond-mat)
  - quantum simulators (atomic physics)
- Simpler, b/c gluons are non-dynamical
- Naively: Coulomb potential is linear, so all gauge theories w/o (fundamental) quarks should be confining.
- Pure $SU(N_c)$ gauge theory
  - confining, fundamental string tension $\sim g_{\text{YM}}^2 N_c$, no particles
- 't Hooft model: $SU(N_c)$ gauge theory $+$ 1 fundamental Dirac fermion (quark) of mass m_{fund}.
  - particles: mesons. (Large N_c single-trace spectrum: discrete, one Regge trajectory, solved using light-cone quantization ['t Hooft '74])
  - maybe a bit too simple b/c it lacks adjoint d.o.f.'s

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$SU(N_c)$ + adjoint Majorana fermion

- **This talk:** $SU(N_c)$ gauge theory
  $+$ 1 adjoint Majorana fermion (gluino) of mass m

$$L = \mathrm{tr}\left(-\frac{1}{2g_{\mathrm{YM}}^2}F_{\mu\nu}^2 + i\bar{\Psi}\not{D}\Psi - m\bar{\Psi}\Psi\right) .$$

  - studied at large N_c using discretized light-cone quantization (DLCQ) [Dalley, Klebanov '92; Kutasov '93; Bhanot, Demeterfi, Klebanov '93; Dempsey, Klebanov, SSP '21; Dempsey, Klebanov, Lin, SSP '22] or using light-cone Hamiltonian truncation [Katz, Tavares, Xu '13]
  - $m = 0$: mass gap, screening of charges in the fundamental representation (!) [Gross, Klebanov, Matytsin, Smilga '95; Komargodski, Ohmori, Roumpedakis, Seifnashri '20; Dempsey, Klebanov, SSP '21] (see also [Lenz, Shifman, Thies '94; Cherman, Jacobson, Tanizaki, Unsal '20; Cherman, Jacobson, Neuzil '21]). Deep IR: coset $\frac{SO(N_c^2-1)_1}{SU(N_c)_{N_c}}$ w/ $c = 0$.
  - $m > 0$: mass gap, confining
  - particles: gluinoballs

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Outline

1. Adjoint QCD_2 in DLCQ at large N_c.
2. Probe fundamental string tension with additional massive quarks [Dempsey, Klebanov, SSP '21] .
3. Spectrum of adjoint QCD_2 at *finite* N_c using DLCQ [Dempsey, Klebanov, Lin, SSP '22] .
4. A Hamiltonian lattice model for adjoint QCD_2 + numerical results [Dempsey, Klebanov, SSP, Søgaard '23] .

Light-cone quantization

Think of $x^+ = \frac{t+x}{\sqrt{2}}$ as **time** and $x^- = \frac{t-x}{\sqrt{2}}$ as **space** and perform canonical quantization.

- Choose gauge $A_- = 0$ and write $\Psi_{ij} = \begin{pmatrix} \psi_{ij} \\ \chi_{ij} \end{pmatrix}$.
- A_+, χ are not dynamical (no x^+ derivatives) $\to$ can be eliminated.
- To compute mass spectrum, first compute light-cone momentum P^+ and light-cone Hamiltonian P^-, and then $M^2 = 2P^+P^-$:

$$P^+ = \int dx^- \; \mathrm{tr}\left(i\psi\partial_-\psi\right) ,$$

$$P^- = -\int dx^- \; \mathrm{tr}\left(g_{\mathrm{YM}}^2 J^+ \frac{1}{\partial_-^2} J^+ + im^2 \psi \frac{1}{\partial_-}\psi\right) .$$

where $J^+_{ij} = \psi_{ik}\psi_{kj} - \frac{1}{N_c}\delta_{ij}\psi_{kl}\psi_{lk}$ is the $SU(N_c)$ current.

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Discretized light-cone quantization

- Discretization: compactify x^- into a circle of radius L with **anti-periodic** BC's for fermions [Brodsky, Hornbostel, Pauli '88]

$$\psi_{ij}(x^-) = \frac{1}{\sqrt{4\pi L}} \sum_{\text{odd } n>0} \left(B_{ij}(n) e^{-inx^-/2L} + B_{ji}^\dagger(n) e^{inx^-/2L} \right)$$

- Creation ops $B_{ij}^\dagger(n)$, annihilation ops $B_{ij}(n)$, for $n = 1, 3, 5, \ldots$.
- **States:** act w/ $B^\dagger(n_i)$'s on $|0\rangle$, contract all $SU(N_c)$ indices.

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Discretized light-cone quantization

Procedure to obtain **mass spectrum**:

- Write all states (finitely many) with $\sum_i n_i = K$ and $P^+ = \frac{K}{2L}$.
- We have
  - bosons for even K
  - fermions for odd K
- Compute P^- and diagonalize $M^2 = 2P^+P^-$.
- Extrapolate to $K \to \infty$.
- Charge conjugation symmetry $\mathcal{C}\psi_{ij}\mathcal{C}^{-1} = \psi_{ji}$ splits the states into $\mathbb{Z}_2$-even (bosons & fermions) and $\mathbb{Z}_2$-odd (bosons & fermions).
- **Start with large** N_c b/c it is simpler: single trace sector decouples.

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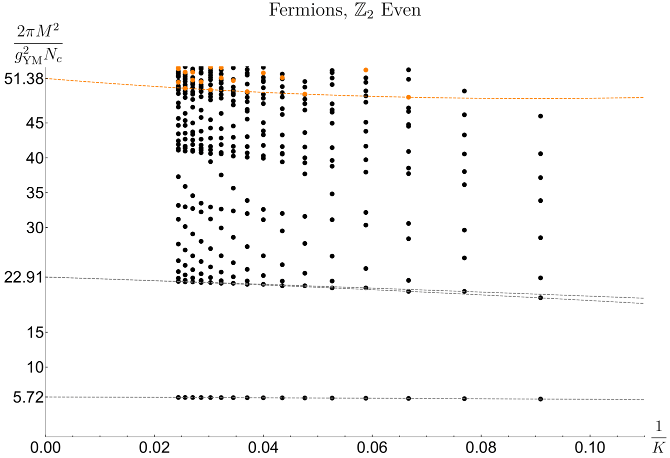
Numerical results for gluinoballs for $m = 0$

- Gluinoballs: First done in the '90's.
- Lowest state: $\mathbb{Z}_2$-even fermion w/ $M_1^2 \approx 5.72$ (units of $g_{\text{YM}}^2 N_c / 2\pi$), almost entirely 3-bit



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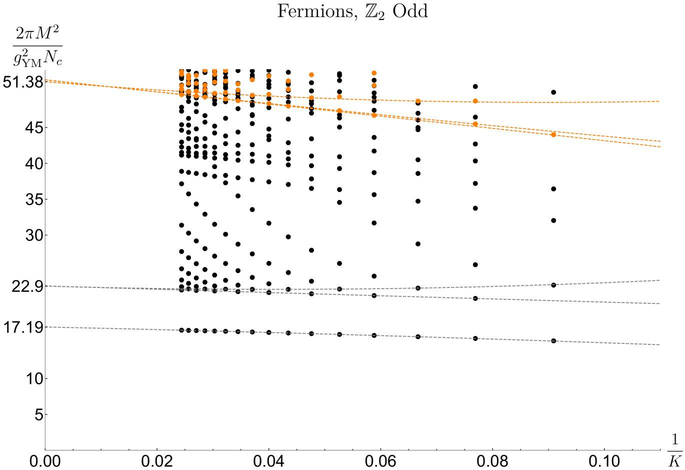
- In each sector, it looks like there's a continuum starting at $4M_1^2$!
  (Very surprising—these are single trace states!)



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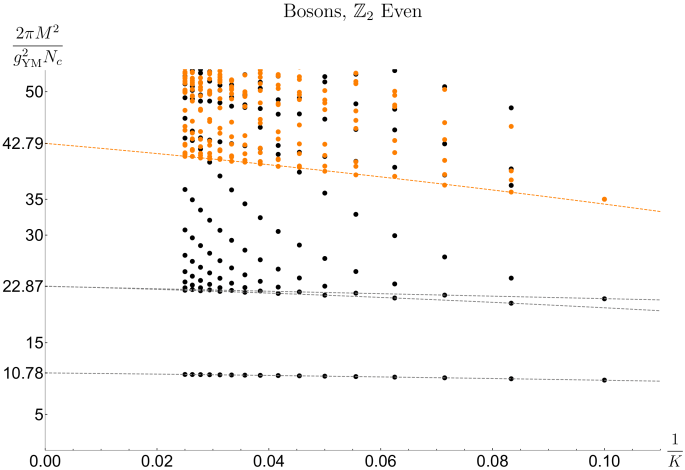

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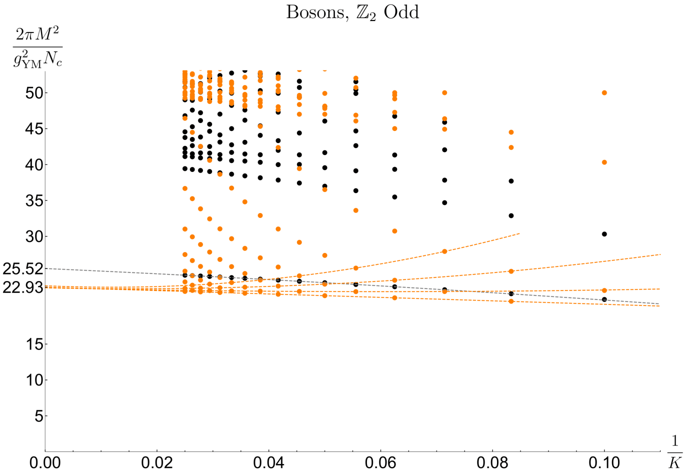
- The largest matrix we diagonalized has dim 9 030 450 at $K = 41$!



Numerical results for gluinoballs for $m = 0$

- Exact degeneracies (in the discretized problem!) of the form [Gross, Hashimoto, Klebanov '97]

$$P^-(K) = \sum_{i=1}^{\mathfrak{n}} P^-(K_i)\,, \qquad K = \sum_{i=1}^{\mathfrak{n}} K_i$$

  for any fermionic gluinoballs with $P^-(K_i)$.

  - The orange points in the plots obey this exact relation.

- $\Longrightarrow$ For any set of fermionic trajectories that asymptote to M_i as $K \to \infty$, there's a continuum starting at $M_{\text{threshold}} = \sum_{i=1}^{\mathfrak{n}} M_i$ in the fermionic/bosonic spectrum if $\mathfrak{n}$ is odd/even.

- If $m > 0$, the continuum disappears.

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Universality of massive spectrum

- "Explanation" for degeneracies: **universality of massive spectrum + large N_c factorization** [Kutasov, Schwimmer '95; Dempsey, Klebanov, SSP '21]
- For 2d QCD w/ $m_{\text{ferm}} = 0$ (in any rep), in the *discretized* problem:

$$P^- = \frac{2g^2 L}{\pi} \sum_{\text{even } n>0} \frac{\text{tr}[J(-n)J(n)]}{n^2}$$

  where $J(n)$ are the Fourier modes of the $SU(N_c)$ current.
- The current obeys a **Kac-Moody (KM) algebra** at level k_{KM}

$$[J_{ij}(n), J_{kl}(m)] = \delta_{kj} J_{il}(n+m) - \delta_{il} J_{kj}(n+m) + k_{\text{KM}} \frac{n\delta_{n,-m}}{2} \left(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl} \right)$$

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Kac-Moody blocks

- States (gauge-invariant and non-gauge invariant) split into **current blocks (KM irreps)**:
  - KM primary $|\chi\rangle_I$ annihilated by all $J_{ij}(n)$ with $n > 0$.
    I labels states in irrep of $SU(N_c)$.
  - KM descendants $J_{ij}(-n_1)J_{kl}(-n_2)\cdots|\chi\rangle_I$
- Physical states are annihilated by $J(0)$.
- P^- e'values depend only on → KM irrep, → KM level k_{KM}

[Kutasov, Schwimmer '95]

$\Longrightarrow$ two (or more) KM irreps (either in the same theory or in different theories with same k_{KM}) give same P^- e'values.

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(massless) adj QCD$_2$ $k_{\text{KM}} = N_c$ $B^\dagger_{ij}$	$\longleftrightarrow$	(massless) QCD$_2$ $+N_c$ fundamental quarks $k_{\text{KM}} = N_c$ $C^\dagger_{i\alpha}, D^\dagger_{i\alpha}, \alpha = 1, \ldots, N_c$

share some of the same P^- spectrum. Schematically,

- $\mathfrak{n} = 0$: bosonic gluinoballs
  $\text{tr}\,[J(-n_1)\cdots J(-n_p)]\,|0\rangle \longleftrightarrow \text{tr}\,[J(-n_1)\cdots J(-n_p)]\,|0\rangle$.
- $\mathfrak{n} = 1$: fermionic gluinoballs $\longrightarrow$ single string states
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- $\mathfrak{n} = 2$: bosonic gluinoballs $\longrightarrow$ double string states
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  degenerate w/ sums of single string states
- $\mathfrak{n} = 3$: fermionic gluinoballs $\longrightarrow$ triple string states, etc.

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Gluinoball degeneracies

- At large N_c, the P^- e'values of $\mathfrak{n} > 1$ states (orange dots in previous plots) are sums of e'values of $\mathfrak{n} = 1$ states (black dots in previous plots of fermionic e'vals).
- These arguments **prove** the existence of continuum starting at $2M_1$ in bosonic spectrum (where M_1 is the lowest fermionic gluinoball).

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Additional massive quark and meson spectrum

- Continuum was suspected to be related to the fact that the massless theory is screening of fundamental charges [Gross, Hashimoto, Klebanov '97] .
- Can gain more insight by adding a very massive fundamental quark: $SU(N_c)$ gauge theory + 1 Majorana adjoint fermion (gluino) + 1 fundamental fermion (quark)

$$L = \operatorname{tr}\left(-\frac{1}{2g_{\text{YM}}^2}F_{\mu\nu}^2 + \frac{i}{2}\bar{\Psi}\not{D}\Psi - \frac{m}{2}\bar{\Psi}\Psi\right) + \left(i\bar{q}\not{D}q - m_{\text{fund}}\bar{q}q\right).$$

- Use DLCQ to compute the meson spectrum.
- Focus on $m_{\text{fund}} > 0$ (quark is a probe of adj QCD)
- Set $m_{\text{fund}}^2 = 1$ (in units of $g^2 N_c/2\pi$) in the following plots as an example.

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- Set $m_{\mathrm{fund}}^2 = 1$ (in units of $g^2 N_c/2\pi$) in the following plots as an example.

Additional massive quark and meson spectrum

- Continuum was suspected to be related to the fact that the massless theory is screening of fundamental charges [Gross, Hashimoto, Klebanov '97] .
- Can gain more insight by adding a very massive fundamental quark: $SU(N_c)$ gauge theory + 1 Majorana adjoint fermion (gluino) + 1 fundamental fermion (quark)

$$L = \mathrm{tr}\left(-\frac{1}{2g_{\mathrm{YM}}^2}F_{\mu\nu}^2 + \frac{i}{2}\bar{\Psi}\not{D}\Psi - \frac{m}{2}\bar{\Psi}\Psi\right) + \left(i\bar{q}\not{D}q - m_{\mathrm{fund}}\bar{q}q\right) .$$

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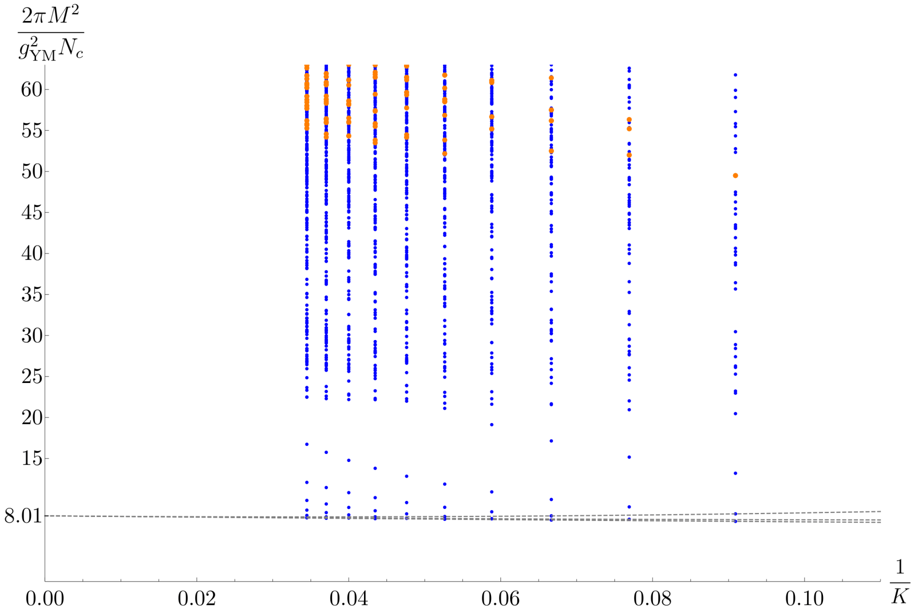
Fermions, $\mathbb{Z}_2$ odd



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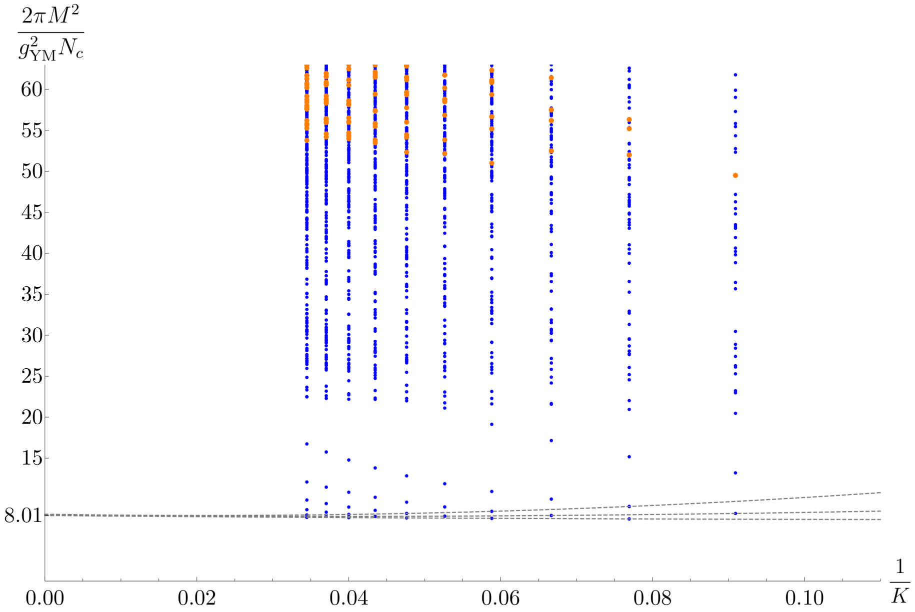
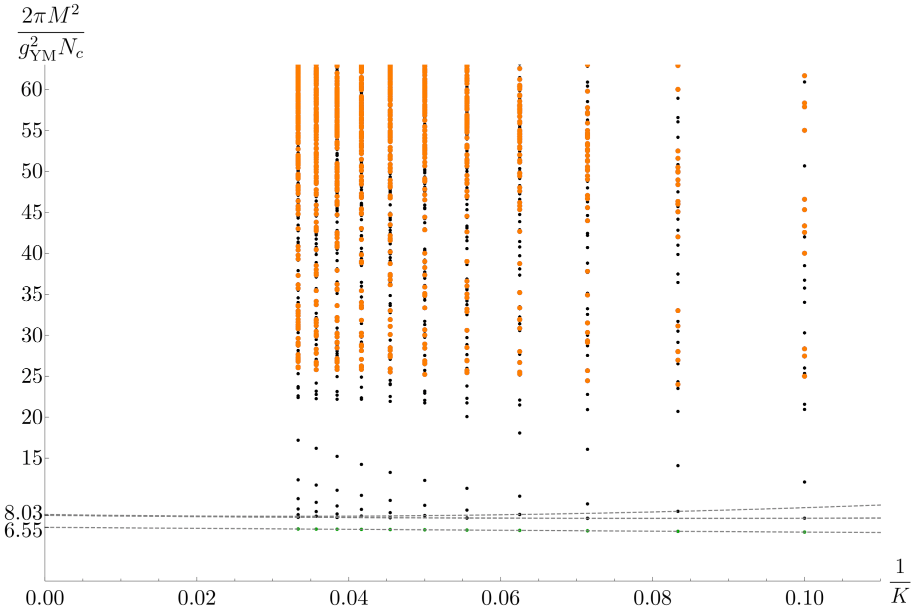


Bosons, $\mathbb{Z}_2$ odd



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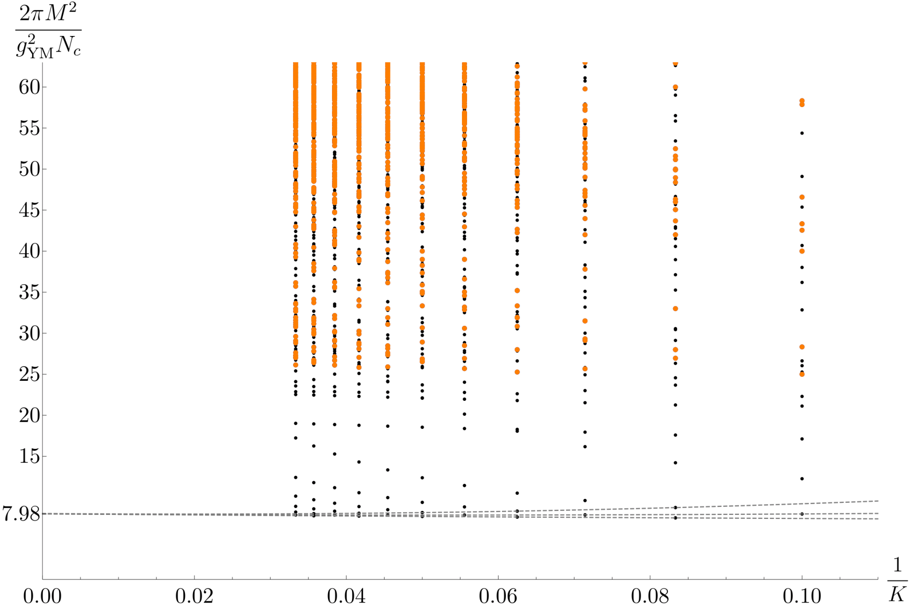

Degeneracies when $m_{\text{fund}} > 0$

- Degeneracies: the following theories share P^- evalues

THEORY $\mathcal{T}$
$SU(N_c)$
$+$ 1 massless adjoint
$+$ 1 massive fundamental

$\longleftrightarrow$

THEORY $\mathcal{T}'$
$SU(N_c)$
$+$ N_c massless fundamentals (L)
$+$ 1 massive fundamental (H)

Mesons in $\mathcal{T} \longleftrightarrow$ states in $\mathcal{T}'$ of the form $[C^\dagger J \cdots JD^\dagger][C^\dagger J \cdots JD^\dagger] \cdots |0\rangle$:

- $\mathfrak{m} = 0$: bosonic mesons in $\mathcal{T} \longleftrightarrow [H - H]$ in $\mathcal{T}'$
- $\mathfrak{m} = 1$: fermionic mesons in $\mathcal{T} \longleftrightarrow [H - L][L - H]$ in $\mathcal{T}'$
- $\mathfrak{m} = 2$: bosonic mesons in $\mathcal{T} \longleftrightarrow [H - L][L - L][L - H]$ in $\mathcal{T}'$
  (same P^- as sum of $\mathfrak{m} = 1$ meson and $\mathfrak{n} = 1$ gluinoball in $\mathcal{T}$)
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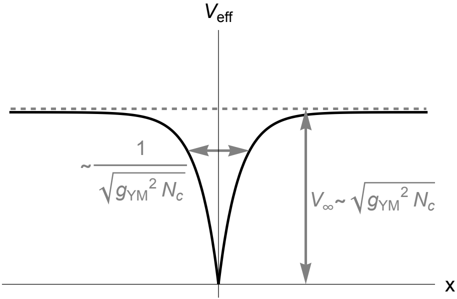
- etc.

- **All** fermionic (and some bosonic) P^- e'values in theory $\mathcal{T}$ are sums of P^- e'values in $\mathcal{T}'$ $\Longrightarrow$ **continuous spectrum in $\mathcal{T}$**
  $\Longrightarrow$ quark-antiquark potential levels off $\Longrightarrow$ screening in adj QCD

- Bound state(s) $\Longrightarrow$ quark-antiquark potential is not exactly flat:



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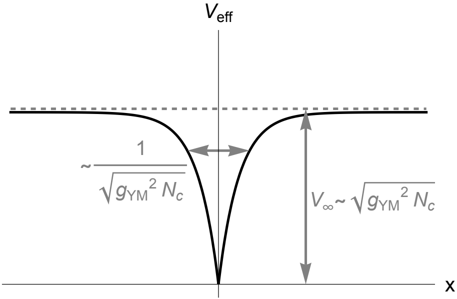

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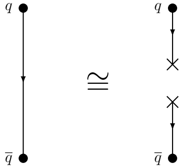

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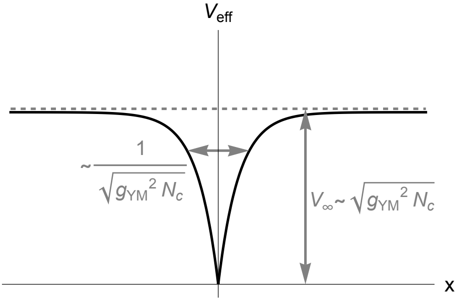

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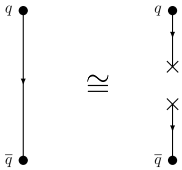

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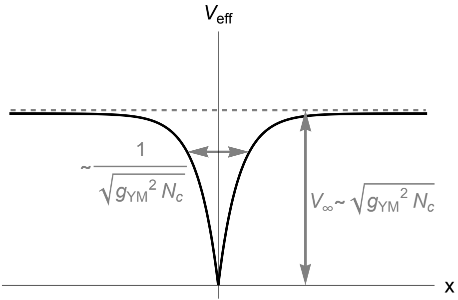


Estimating V_∞

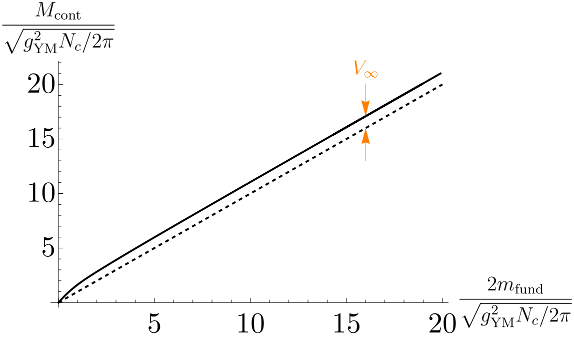
- Can estimate V_∞ from where the continuum starts. At large m_{fund},

$$M_{\text{cont}} \approx 2m_{\text{fund}} + V_\infty \,,$$

where $M_{\text{cont}} = 2M_{[H-L]}$ in $\mathcal{T}'$.
($M_{[H-L]}$ and hence M_{cont} can be found more precisely w/ DLCQ directly in $\mathcal{T}'$.)

We find

$$V_\infty \approx 1.1\sqrt{\frac{g_{\text{YM}}^2 N_c}{2\pi}}$$



Numerics at finite N_c

- **Finite N_c:** Must include multi-trace states, e.g. $(\mathrm{tr}(B^\dagger(1)B^\dagger(3)))^2|0\rangle$ [Antonuccio, Pinsky '98]
- Also: there are $SU(N_c)$ trace relations (null states).
  - E.g. in $SU(2)$: $\mathrm{tr}(B^\dagger(1)B^\dagger(1)B^\dagger(3)B^\dagger(3)) - \left(\mathrm{tr}(B^\dagger(1)B^\dagger(3))\right)^2 = 0$.
  - Main challenge: determine the trace relations!
- For example, when $K = 35$:
  There are 3,421,191 single trace + multi-trace states.
  For $N_c = 2$, only 350 of them are not null.
  For $N_c = 3$, only 19,954 of them are not null, etc.
- Very difficult to remove null states from basis via inner products, but we developed an efficient method that avoids inner products [Dempsey, Klebanov, Lin, SSP '21] .

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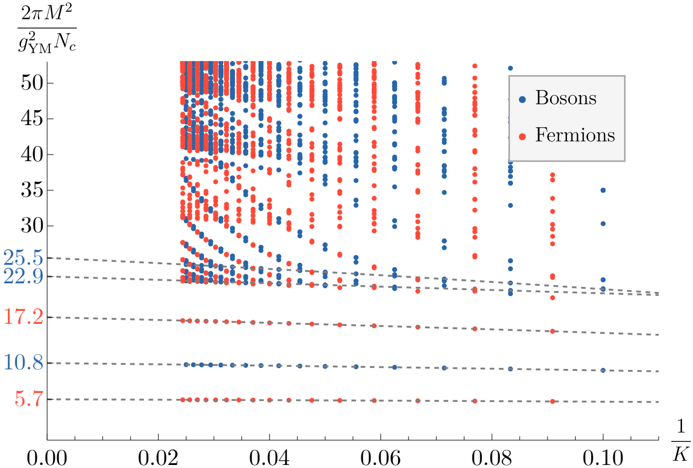
Large N_c spectrum for **massless theory**

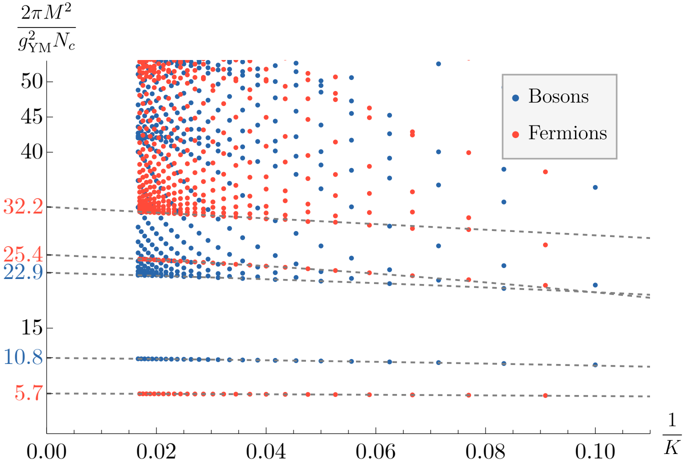
- To get oriented: **Single-trace** + **multi-trace states** at large N_c:
  [Dempsey, Klebanov, SSP '21]

Lightest fermion:
$M_f^2 \approx 5.7 \frac{g_{\rm YM}^2 N_c}{2\pi}$

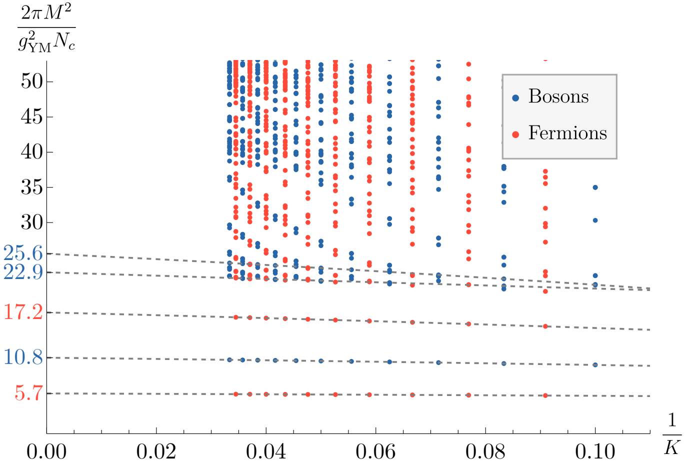
Lightest boson:
$M_b^2 \approx 10.8 \frac{g_{\rm YM}^2 N_c}{2\pi}$



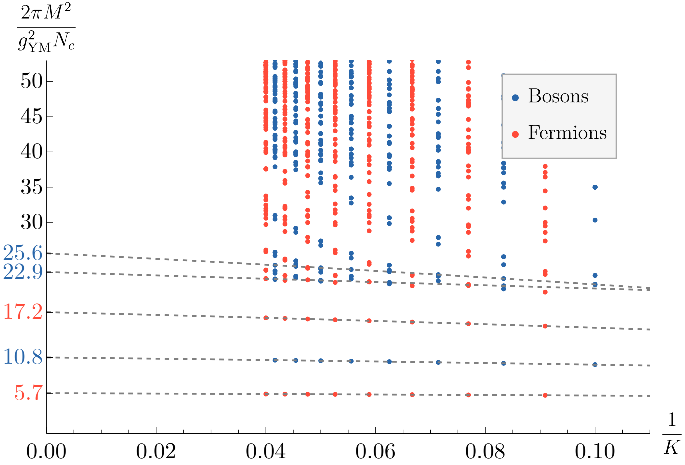
Gluinoball spectrum for $SU(2)$ + adjoint



Gluinoball spectrum for $SU(3)$ + adjoint



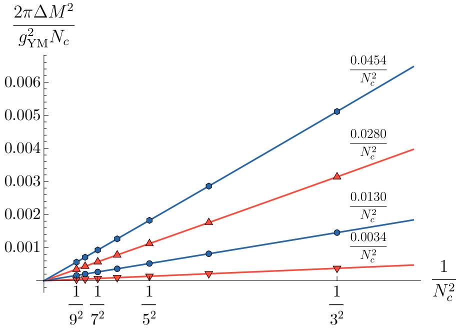
Gluinoball spectrum for $SU(4)$ + adjoint



$1/N_c$ corrections

- Large N_c corrections are very small:

$$M^2 = \frac{g_{\rm YM}^2 N_c}{2\pi}\left(a_0 + a_1/N_c^2 + O(N_c^{-4})\right) .$$

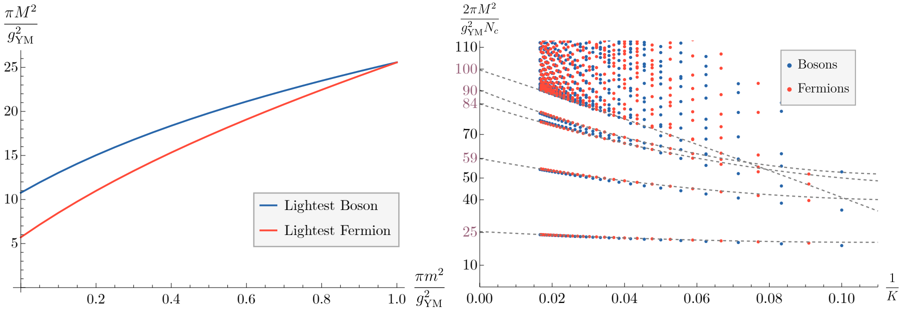


- For lowest state $a_0 \approx 5.72$ and $a_1 \approx 0.0034$.

Turning on mass for adjoint fermion

- Can turn on mass m for the adjoint fermion.
- At $m^2 = g_{\mathrm{YM}}^2 N_c/(2\pi)$: $(1,1)$ **supersymmetry** [Kutasov '93; Boorstein, Kutasov '94; Popov '22]

In $SU(2)$ adjoint QCD$_2$ theory:



LEFT: Lightest fermion and boson mass for varying m for $N_c = 2$.
RIGHT: Spectrum of $N_c = 2$ theory at $m^2 = g_{\mathrm{YM}}^2/\pi$.

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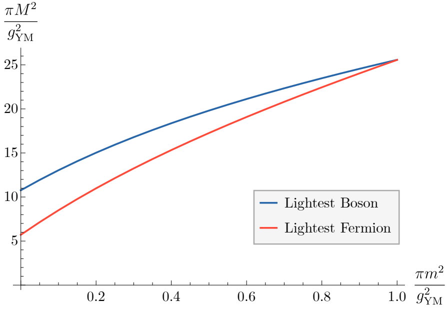

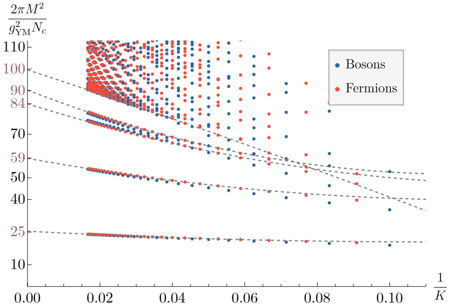

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Beyond DLCQ

- Potential problem: DLCQ does not distinguish b/w $SU(N_c)$ and $SU(N_c)/\mathbb{Z}_{N_c}$ gauge group, but spectra are different.
- For $SU(N_c)$, there are N_c distinct “universes”/flux tube sectors [Witten ’79, …, Komargdoski, Roumpedakis, Seifnashri ’20] .
- For $SU(N_c)/\mathbb{Z}_{N_c}$: choose one of the N_c flux sectors depending on the value of a discrete theta angle.
- For which theory/universe is DLCQ computing the spectrum??

Rest of the talk: Hamiltonian lattice gauge theory model.

Lagrangian LGT $\rightarrow$ discretize both space and Euclidean time
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Hamiltonian lattice model

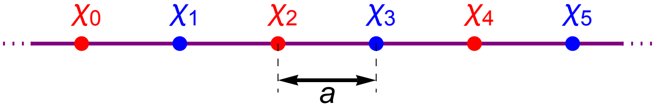
- First, let's consider one Majorana fermion Ψ with

$$L = \frac{i}{2}\bar{\Psi}\partial\!\!\!/\Psi - \frac{m}{2}\bar{\Psi}\Psi\,.$$

- With $\gamma^0 = \sigma_2$, $\gamma^1 = -i\sigma_3$, $\gamma^5 = \gamma^0\gamma^1 = \sigma_1$, write $\Psi(x) = \begin{pmatrix} \psi_u(x) \\ \psi_d(x) \end{pmatrix}$ and discretize ψ_u and ψ_d in a staggered way:

$$\chi_n = \begin{cases} \sqrt{2a}\,\psi_u(x_n)\,, & \text{if } n \text{ is even} \\ \sqrt{2a}\,\psi_d(x_n)\,, & \text{if } n \text{ is odd} \end{cases}$$

where $x_n = na$ and a is the lattice spacing.



Hamiltonian lattice model

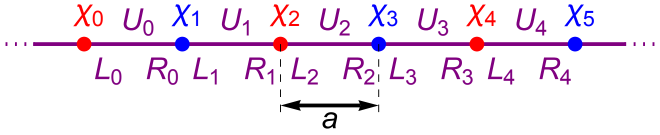
- The discretization gives the Hamiltonian for a **Majorana chain**:

$$H = \sum_{n=0}^{N-1} \left[-\frac{i}{a} \chi_n \chi_{n+1} - im(-1)^n \chi_n \chi_{n+1} \right]$$

where $n \sim n + N$ and the lattice fermions χ_n obey $\{\chi_n, \chi_m\} = \delta_{nm}$.

- For $SU(N_c)$ gauge thy w/ adj. Majorana, start w/ $N_c^2 - 1$ Majorana chains and add gauge d.o.f's.

- $A_1(x) \to$ group elements $U_n \in SU(N_c)$ on links
$E(x) \to$ Lie alg. elements $L_n, R_n \in \mathfrak{su}(N_c)$ acting on left/right of link



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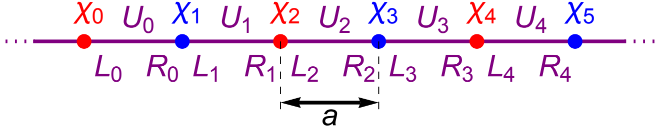

Hamiltonian lattice model

- Lattice Hamiltonian (for $n = 0, \ldots, N-1$ and $A, B = 1, \ldots N_c^2 - 1$)

$$H_m = \sum_{n=0}^{N-1} \left[\frac{g_{\text{YM}}^2 a}{2} L_n^A L_n^A - \frac{i}{a} \chi_n^A U_n^{AB} \chi_{n+1}^B - im(-1)^n \chi_n^A U_n^{AB} \chi_{n+1}^B \right]$$

where $n \sim n + N$ and $U_n^{AB} \equiv 2\,\mathrm{tr}(T^A U_n T^B U_n^{-1})$ performs the parallel transport in the adjoint representation.

- Commutation relations (note $L_n^A = U_n^{AB} R_n^B$):

$$\{\chi_n^A, \chi_m^B\} = \delta_{nm} \delta^{AB}$$

$$[L_n^A, L_m^B] = -i\delta_{nm} f^{ABC} L_n^C\,, \quad [L_n, U_m] = \delta_{nm} T^A U_n\,,$$

$$[R_n^A, R_m^B] = i\delta_{nm} f^{ABC} R_n^C\,, \quad [R_n, U_m] = \delta_{nm} U_n T^A\,, \qquad \text{etc.}$$

- Gauss law:

$$L_n^A - R_{n-1}^A = Q_n^A = -\frac{i}{2} f^{ABC} \chi_n^B \chi_n^C\,.$$

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$$[R_n^A, R_m^B] = i\delta_{nm} f^{ABC} R_n^C \,, \quad [R_n, U_m] = \delta_{nm} U_n T^A \,, \qquad \text{etc.}$$

- Gauss law:

$$L_n^A - R_{n-1}^A = Q_n^A = -\frac{i}{2} f^{ABC} \chi_n^B \chi_n^C \,.$$

Hamiltonian lattice model

- Lattice Hamiltonian (for $n = 0, \ldots, N-1$ and $A, B = 1, \ldots N_c^2 - 1$)

$$H_m = \sum_{n=0}^{N-1} \left[\frac{g_{\text{YM}}^2 a}{2} L_n^A L_n^A - \frac{i}{a} \chi_n^A U_n^{AB} \chi_{n+1}^B - im(-1)^n \chi_n^A U_n^{AB} \chi_{n+1}^B \right]$$

where $n \sim n + N$ and $U_n^{AB} \equiv 2\,\mathrm{tr}(T^A U_n T^B U_n^{-1})$ performs the parallel transport in the adjoint representation.

- Commutation relations (note $L_n^A = U_n^{AB} R_n^B$):

$$\begin{aligned} \{\chi_n^A, \chi_m^B\} &= \delta_{nm}\delta^{AB} \\ [L_n^A, L_m^B] &= -i\delta_{nm} f^{ABC} L_n^C\,, \quad [L_n, U_m] = \delta_{nm} T^A U_n\,, \\ [R_n^A, R_m^B] &= i\delta_{nm} f^{ABC} R_n^C\,, \quad [R_n, U_m] = \delta_{nm} U_n T^A\,, \qquad \text{etc.} \end{aligned}$$

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Symmetries

- Focus on $N_c = 2$ for concreteness and simplicity.
- Spectrum (from diagonalizing H) can be split into:
  - **Bosons and Fermions**.
    Fermion Hilbert sp. of dim $2^{3N/2}$ is a rep. of the $\mathfrak{so}(3N)$ Clifford alg.
    The $\mathfrak{so}(3N)$ chirality matrix = fermion parity operator.
  - **Two "universes"** (trivial flux tube and fundamental flux tube)
    Roughly: gauge field irreps on links are integer / half-integer.
    More precise: Fermionic states are doublets of $\mathfrak{su}(2)$ on each site.

    with $|\ell_n - \ell_{n+1}| = 1/2$.
- **Translation by one lattice site** takes simultaneously $m \to -m$, bosons $\leftrightarrow$ fermions, trivial universe $\leftrightarrow$ non-trivial universe while preserving the energy! (Same as $\mathbb{Z}_2$ axial rotation in continuum.)

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Numerical exact diagonalization

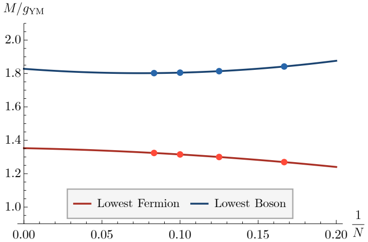
- Numerics: exact diagonalization of H restricted to gauge-invariant sector
- For cutoff $\ell_n \leq \ell_{\max}$, the number of gauge-invariant states is

$\ell_{\max} \backslash N$	4	6	8	10	12
2	40	224	1312	7808	46720
3	64	384	2432	15872	105472
4	88	544	3552	23936	164608

- Total # of states is much larger $\Longrightarrow$ find and work only with gauge-invariant states.

Numerics at $m = 0$

- To extract mass spectrum: diagonalization, then $N \to \infty$, $a \to 0$. (Careful, b/c behavior on small circle is very different.)
- First, $m = 0$. (We did $N = 6, 8, 10, 12$ for a range of a values.)



- Extrapolating lightest fermionic/bosonic states:

$$M_f \approx 1.35 g_{\mathrm{YM}} \quad \Longrightarrow \quad M_f^2 \approx 5.7\, g_{\mathrm{YM}}^2/\pi$$

$$M_b \approx 1.83 g_{\mathrm{YM}} \quad \Longrightarrow \quad M_b^2 \approx 10.5\, g_{\mathrm{YM}}^2/\pi$$

Good agreement w/ DLCQ! ($M_f^2 \approx 5.7\, g_{\mathrm{YM}}^2/\pi$, $M_b^2 \approx 10.8\, g_{\mathrm{YM}}^2/\pi$)

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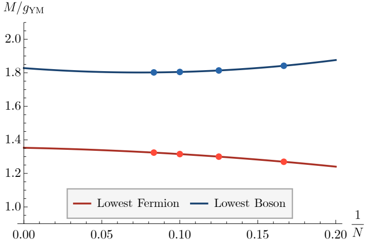

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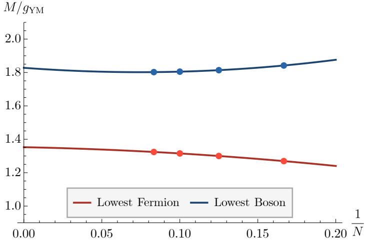

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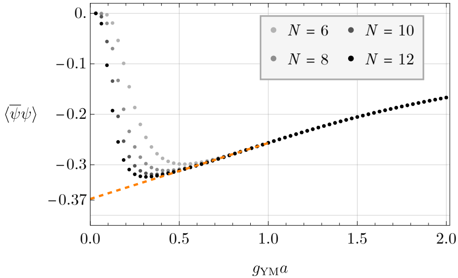
Fermion bilinear condensate

- Can also compute $\langle \bar{\psi}\psi \rangle$.
- Trivial universe: Extrapolation gives $\langle \bar{\psi}\psi \rangle \approx -0.37 g_{\text{YM}}$.



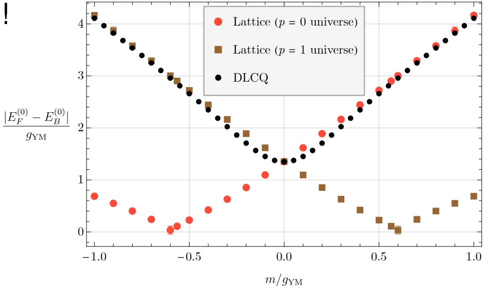
- $\langle \bar{\psi}\psi \rangle$ has opposite sign in the other universe.

Changing the fermion mass

- Plot difference b/w lowest fermionic and bosonic energy levels in each universe.
- Matches well the lowest fermionic gluinoball mass M_f obtained from DLCQ for $m \geq 0$!

Recall:

$$m_{\text{SUSY}} = \frac{1}{\sqrt{\pi}} g_{\text{YM}} \approx 0.56 g_{\text{YM}}$$



- Gap vanishes at $m = -m_{\text{SUSY}}$ (in trivial universe) or at $m = m_{\text{SUSY}}$ (in non-trivial universe) $\Longrightarrow$ massless goldstino due to SUSY breaking by the flux tube.
- Agreement b/w lattice and DLCQ only for $m \geq 0$ in trivial universe and $m \leq 0$ in non-trivial universe.

Conclusion

- 2d gauge theories with adjoint matter are interesting toy models.
- **DLCQ** and **Hamiltonian lattice gauge theory** give good quantitative handles on the bound state spectrum and other observables.

For the future:

- More lattice numerics (for $N_c > 2$ and more precise studies, perhaps using DMRG).
- Add four-fermion interactions to adjoint QCD_2 (Gross-Neveu adjoint QCD_2) as in [Cherman, Jacobson, Tanizaki, Unsal '20; Komargodski, Ohmori, Roumpedakis, Seifnashri '20]
- Determine the precise quark-anti-quark potential in adjoint QCD_2.
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