GPDs from lattice QCD: new developments beyond leading twist

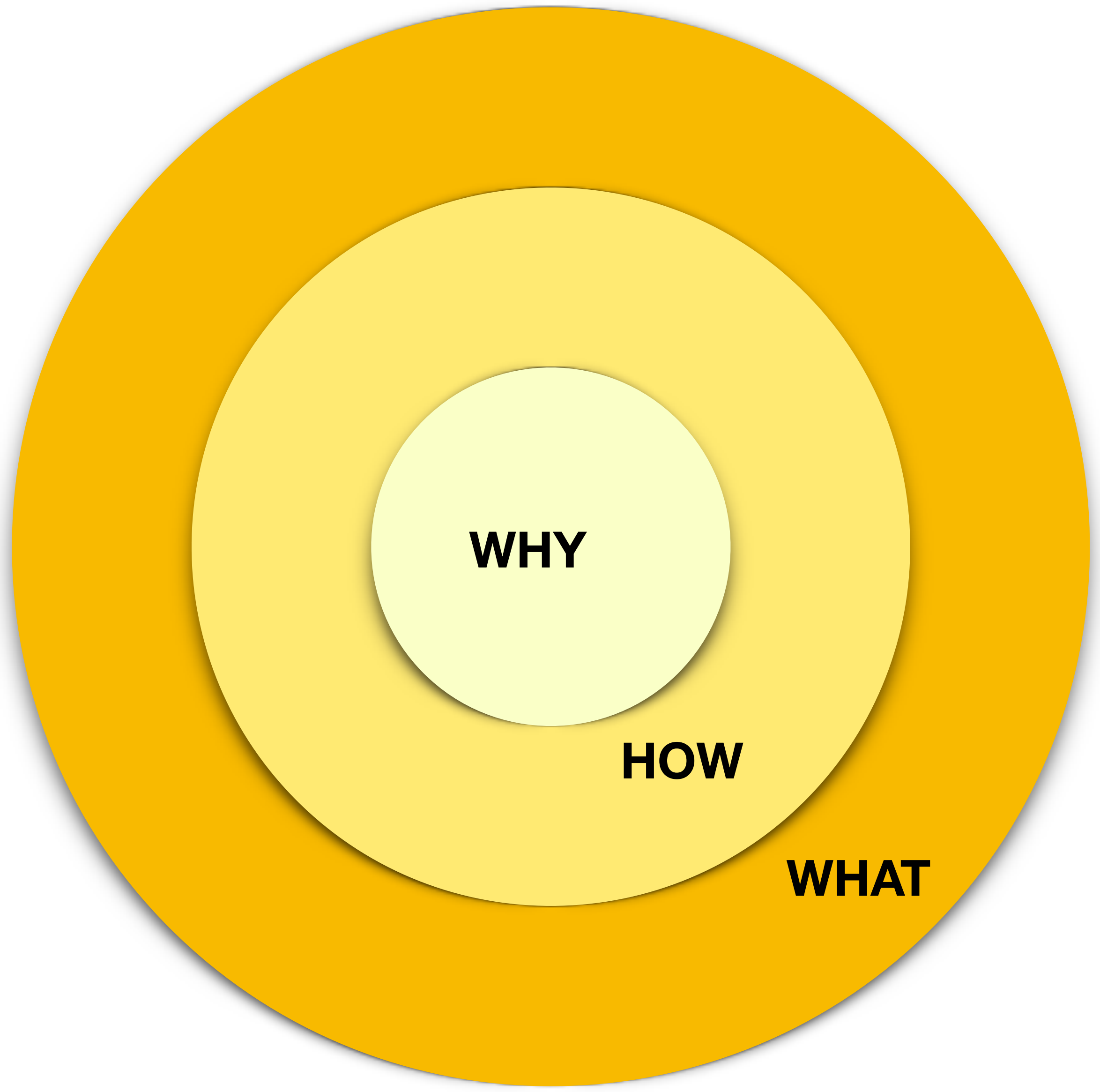
Martha Constantinou

Temple University

The International Light Cone Advisory Committee, Inc.

ILCAC Seminar
November 15, 2023

The Golden Circle

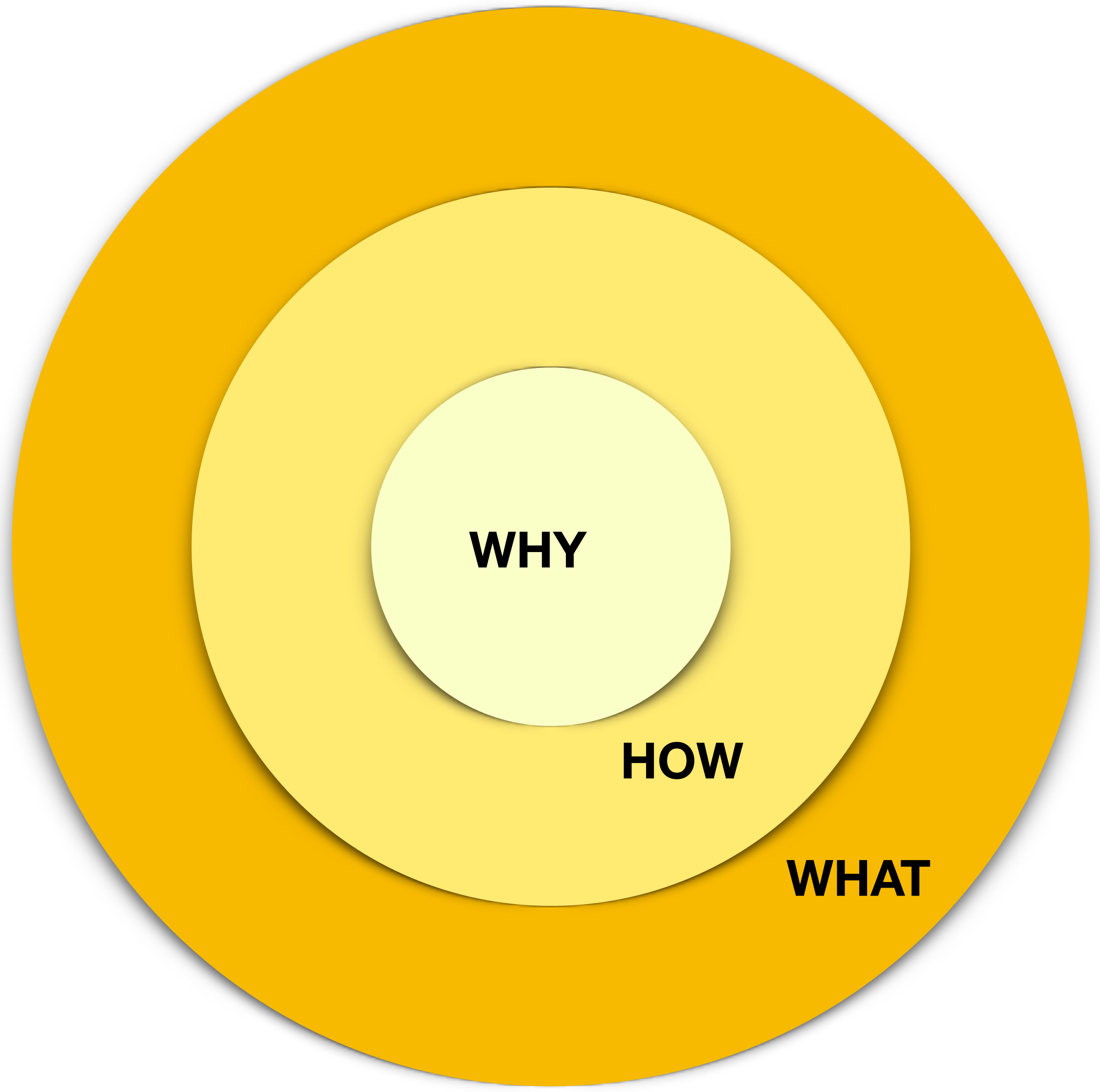


What is the physics we are after?

How can we achieve our goals?

Why is it important?

The Golden Circle



What is the physics we are after?

★ — Map the 3D structure of the proton in terms of their partonic content.
— Characterize hadron structure in new ways

How can we achieve our goals?

★ Numerical simulations of QCD (lattice QCD):
— billions of degrees of freedom
— mathematical & computational challenges

Why is it important?

★ Comprehend and interpret the core of the visible matter

Outline

PHYSICAL REVIEW D **102,** 111501(R) (2020)

Rapid Communications | Editors' Suggestion

Lattice

Insights on proton structure from lattice QCD: The twist-3 parton distribution function $g_T(x)$

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

PHYSICAL REVIEW D **102,** 034005 (2020)

theory

One-loop matching for the twist-3 parton distribution $g_T(x)$

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

PHYSICAL REVIEW D **104,** 114510 (2021)

Lattice

Parton distribution functions beyond leading twist from lattice QCD: The $h_L(x)$ case

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[3]

PHYSICAL REVIEW D **102,** 114025 (2020)

theory

The role of zero-mode contributions in the matching for the twist-3 PDFs $e(x)$ and $h_L(x)$

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

PHYSICAL REVIEW D **108,** 054501 (2023)

Lattice

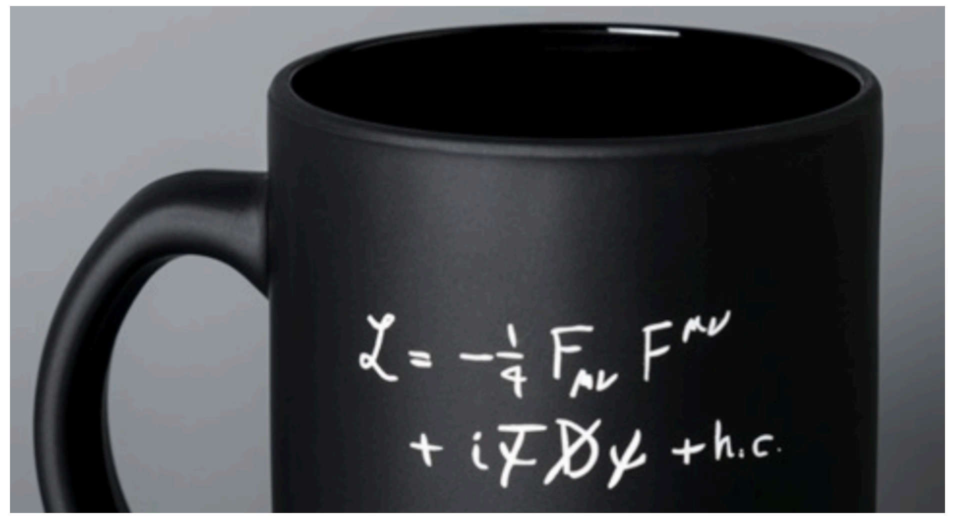
Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[1] Jack Dodson,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[4]

Collaborators

- **S. Bhattacharya**
  Brookhaven National Lab
- **K. Cichy**
  Adam Mickiewicz University
- **J. Dodson**
  Temple University
- **A. Metz**
  Temple University
- **J. Miller**
  Temple University
- **A. Scapellato**
  Temple University
- **F. Steffens**
  University of Bonn

In the quest of solving complex problems



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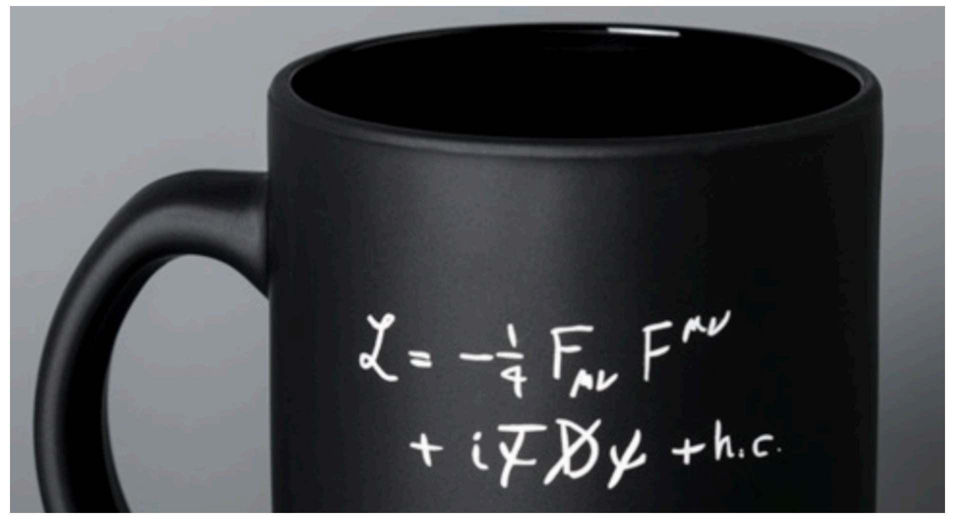

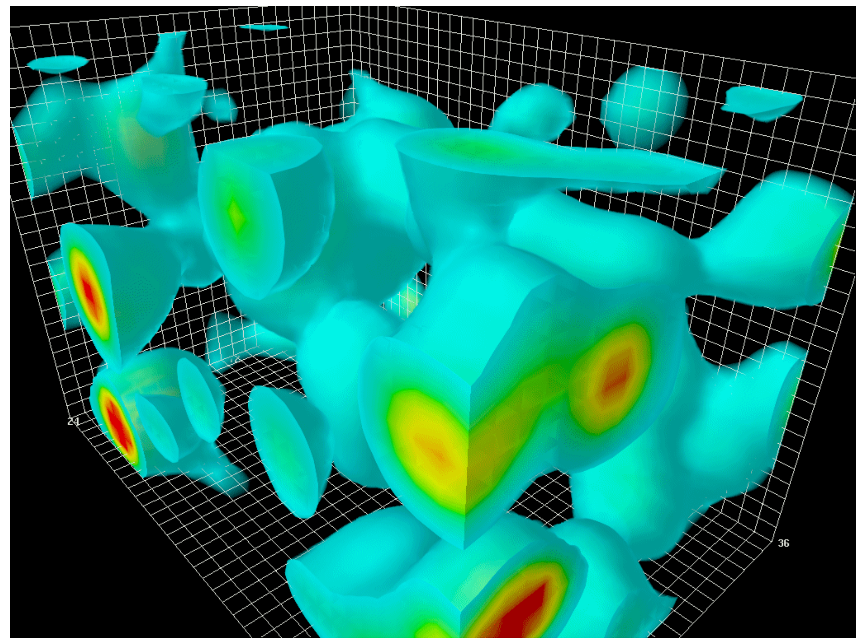

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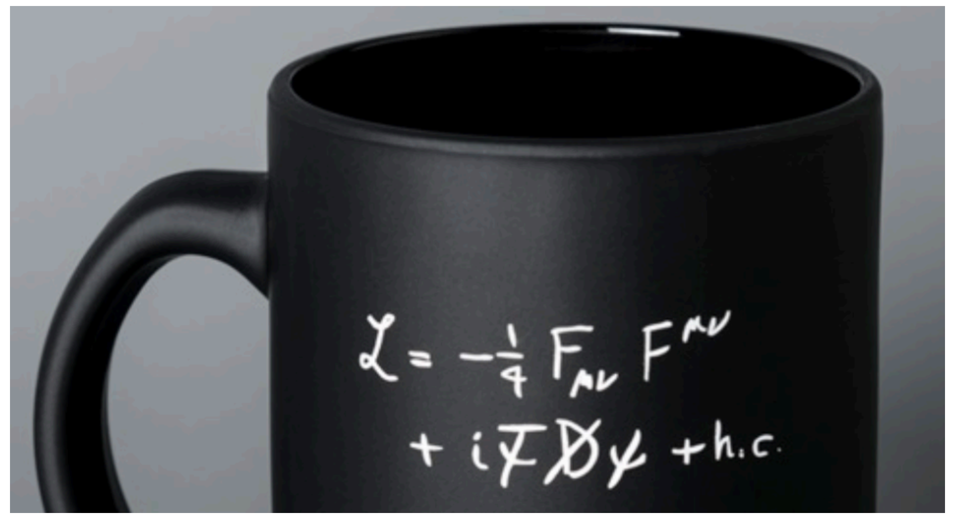

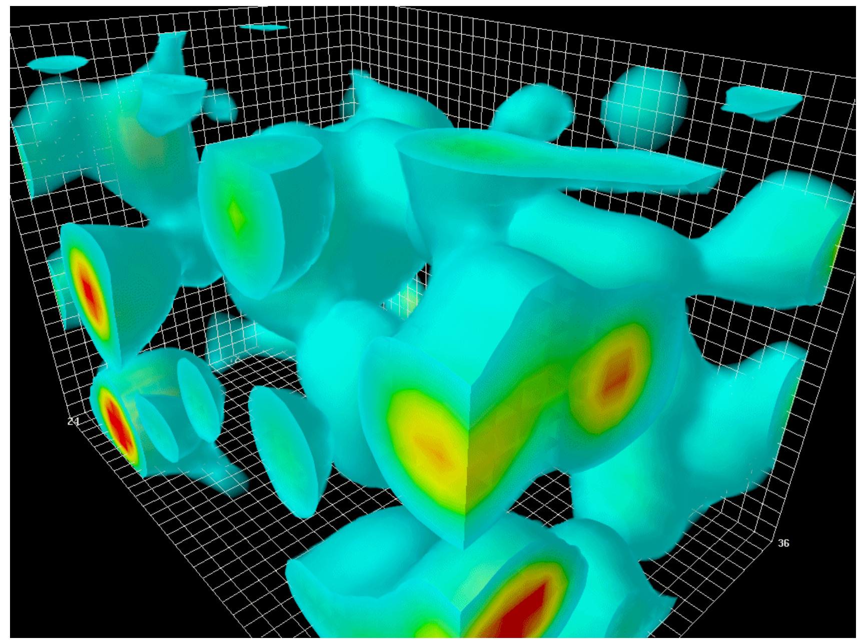

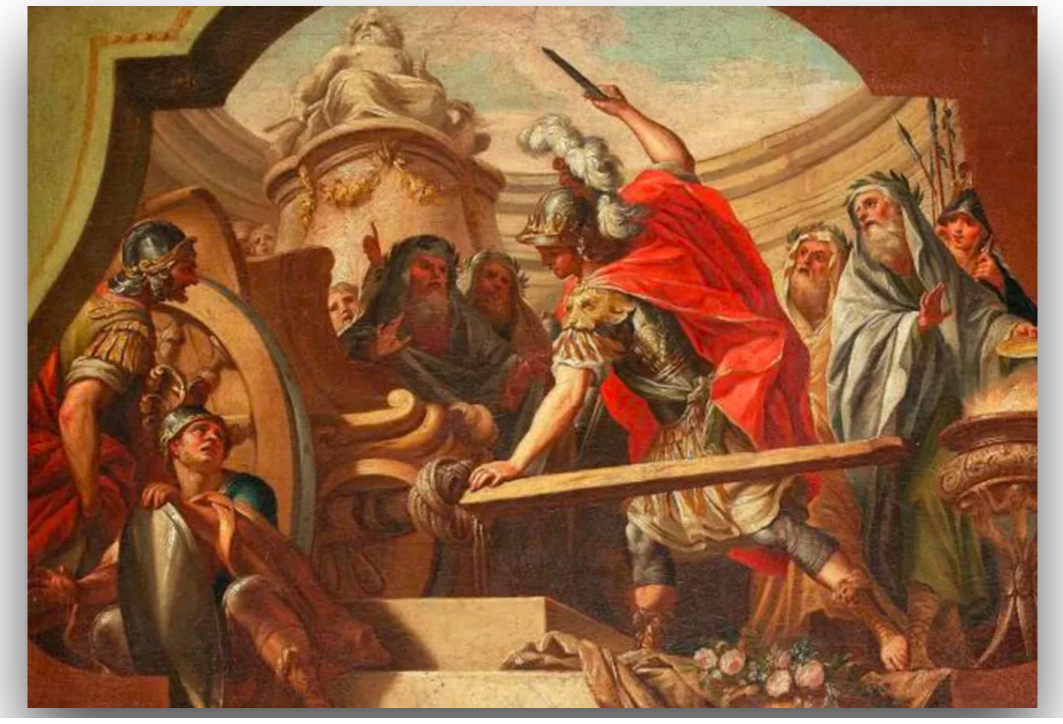

"Οτι δεν λύνεται, κόβεται"

Alexander the Great while
cutting the Gordian knot

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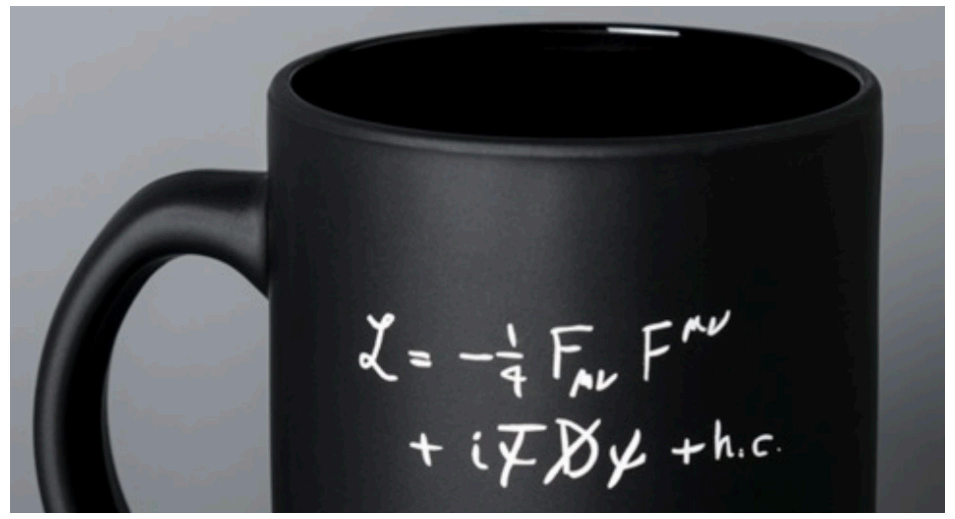

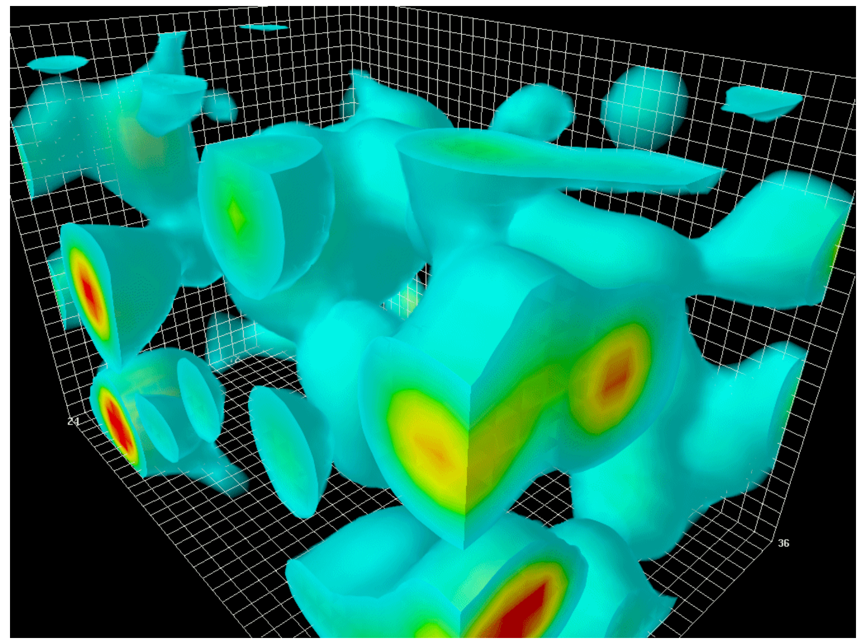

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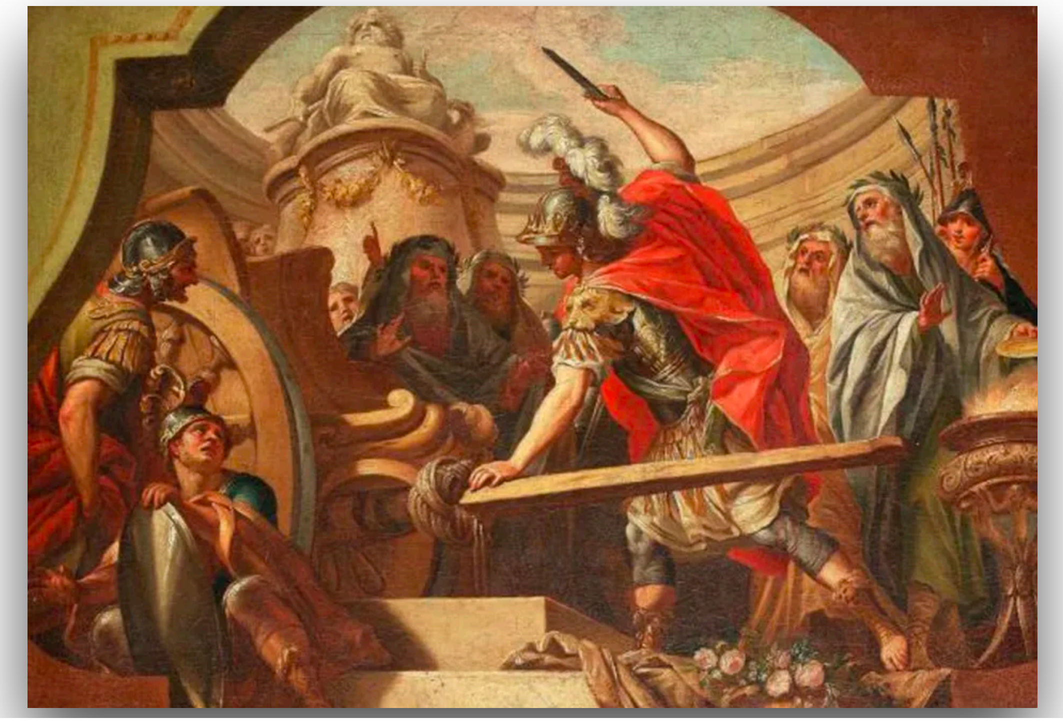

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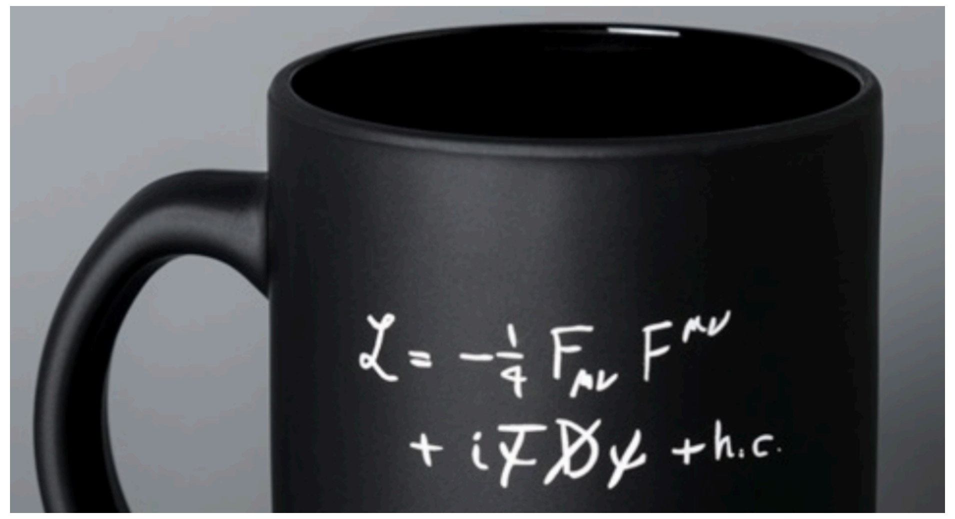

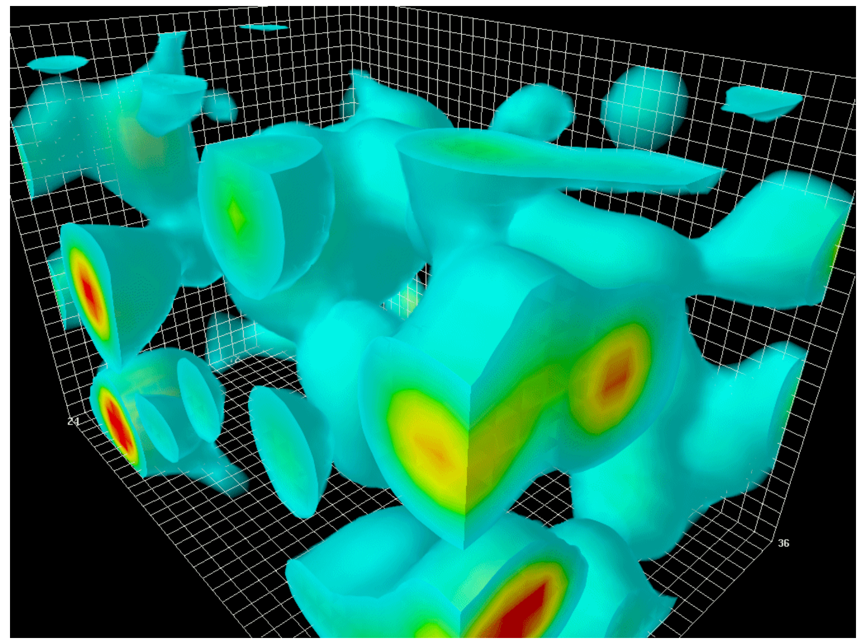

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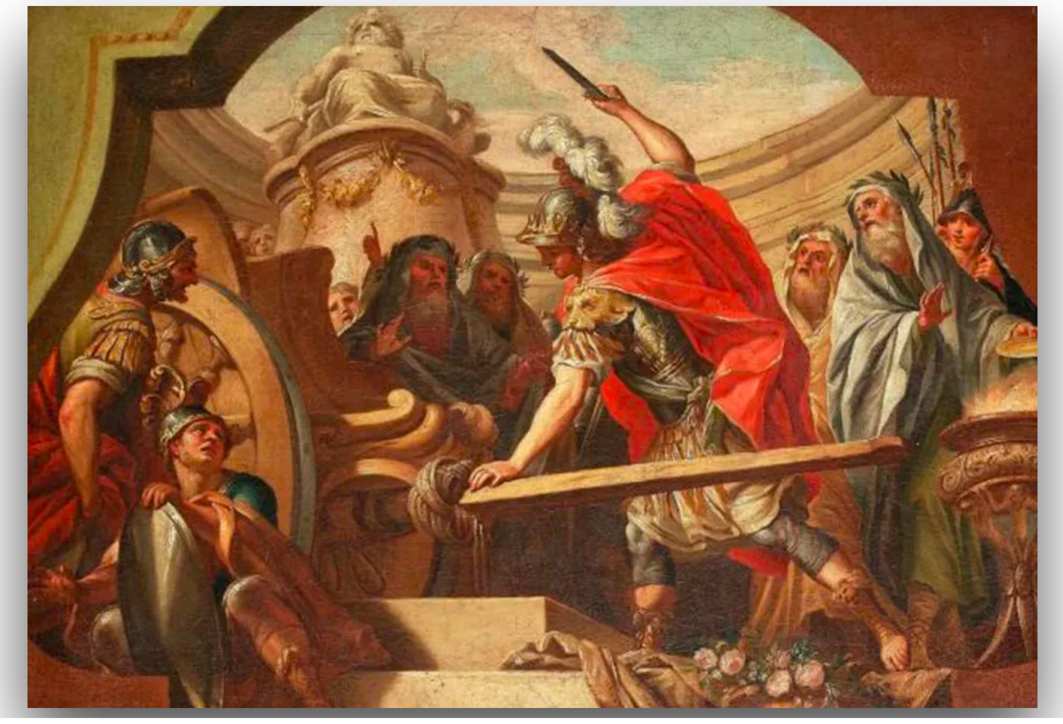

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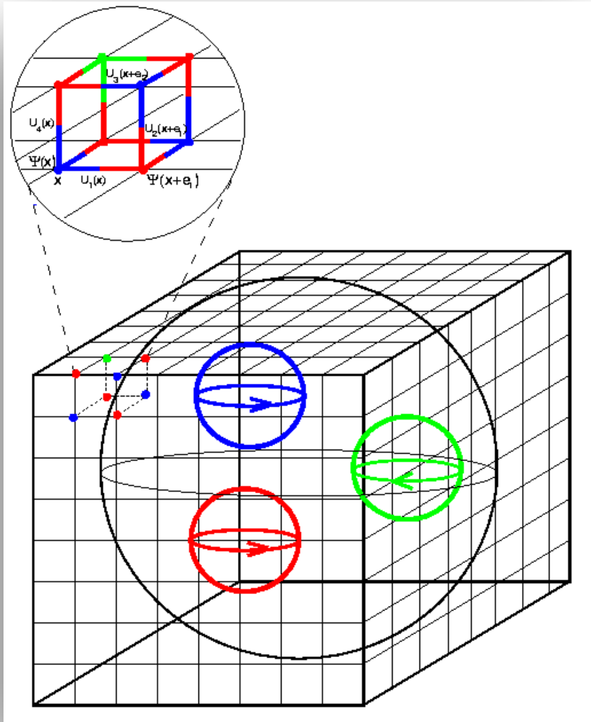
QCD and beyond



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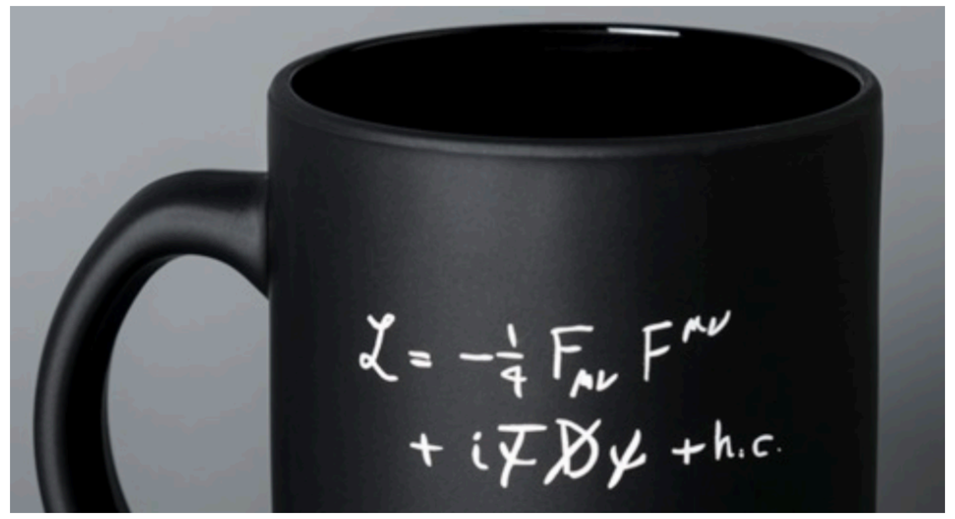

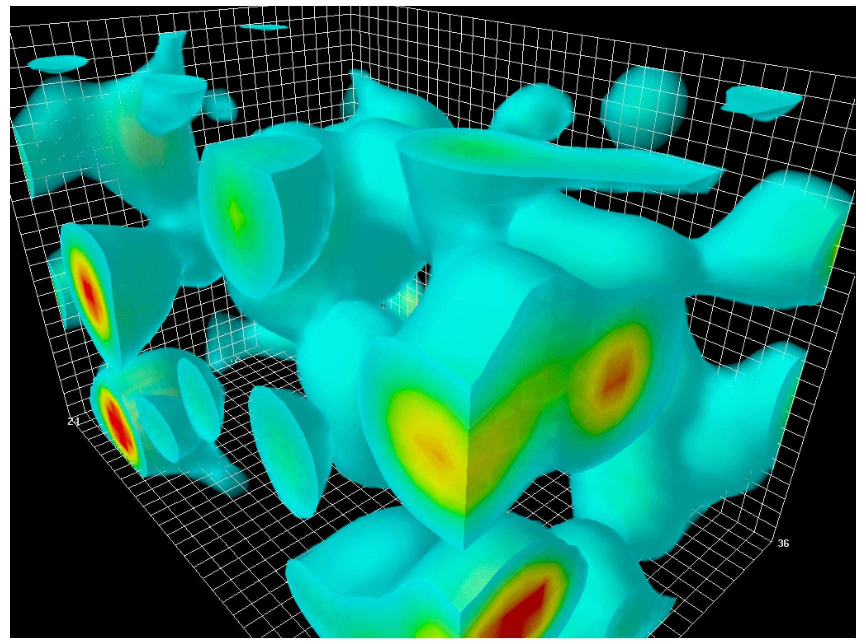

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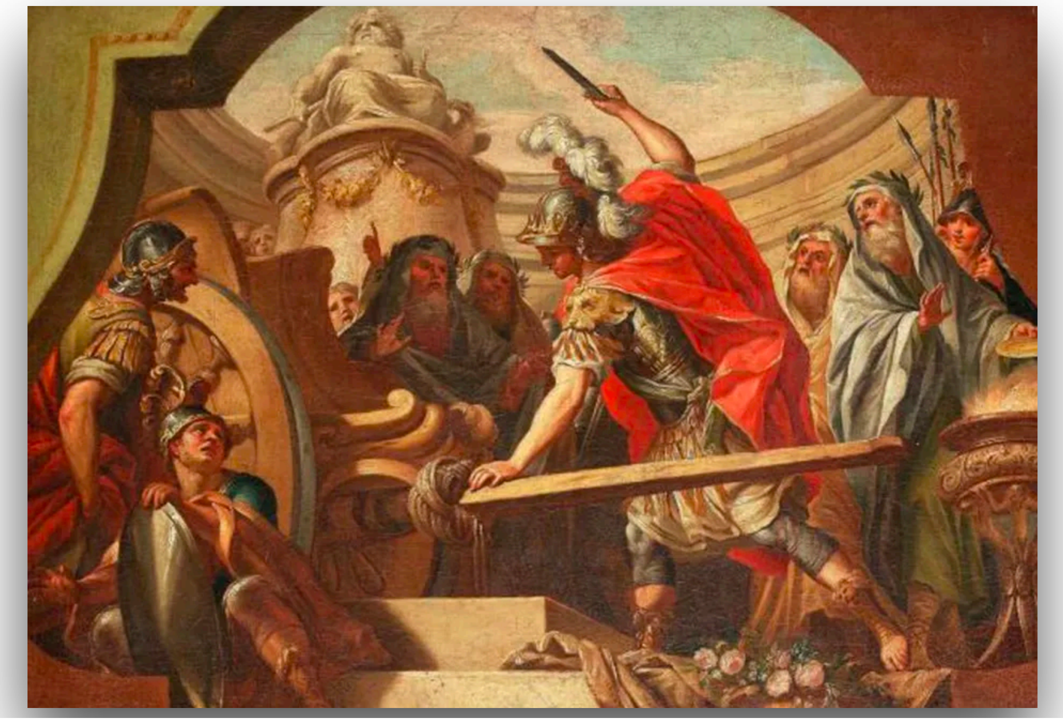

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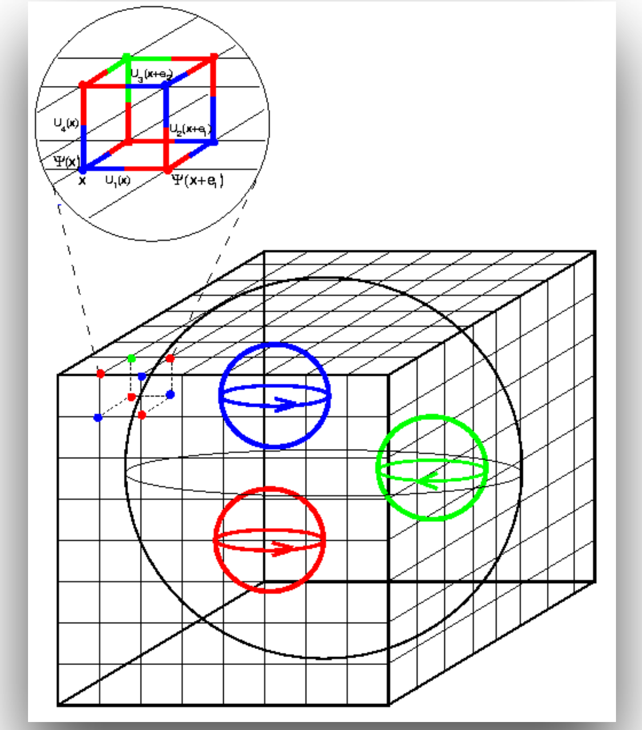


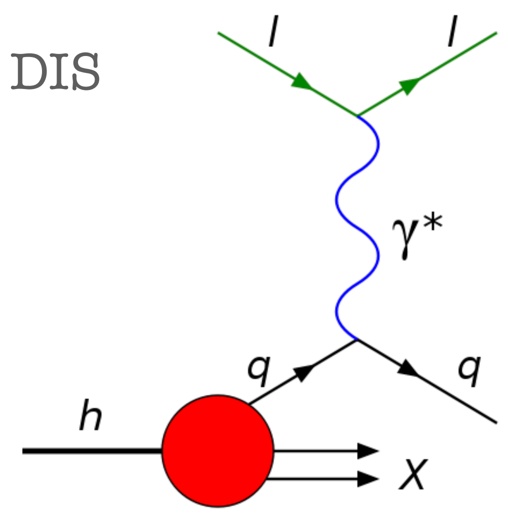
Lattice QCD:

- ★ First principle formulation of QCD
- ★ Space-time discretization of the theory (finite degrees of freedom)
- ★ Same parameters as QCD in continuum
- ★ Discretization is not unique
- ★ Serves as a regulator:
  - UV cut-off: inverse lattice spacing
  - IR cut-off: inverse lattice size
- ★ Removal of regulator:
  - zero lattice spacing
  - infinite volume
- ★ Quantum fluctuations in the vacuum dictate observables
- ★ Statistical mechanics methods may be utilized

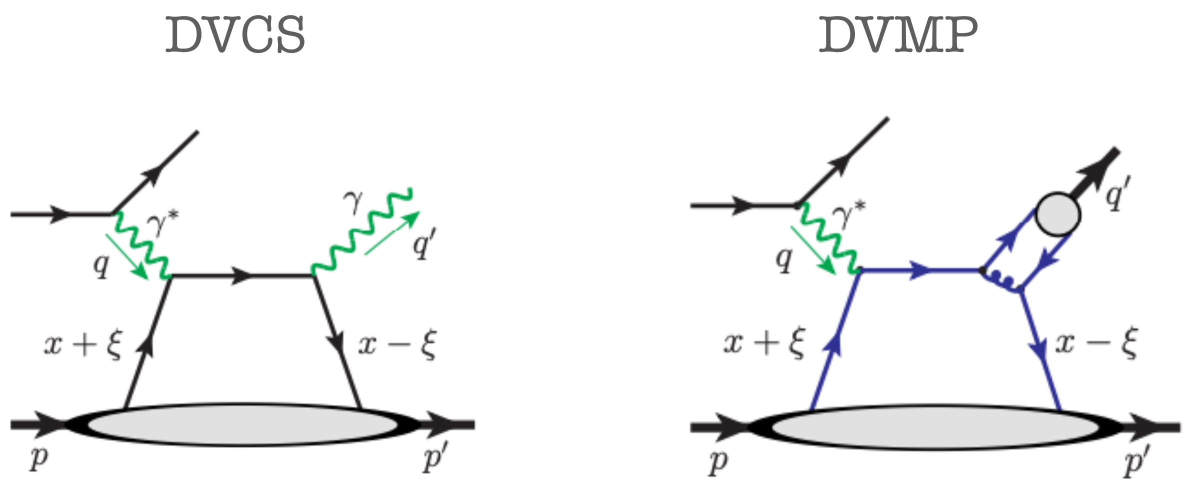
Exploration of hadron structure

★ Structure of hadrons explored in high-energy scattering processes, e.g.,

➡ Inclusive processes

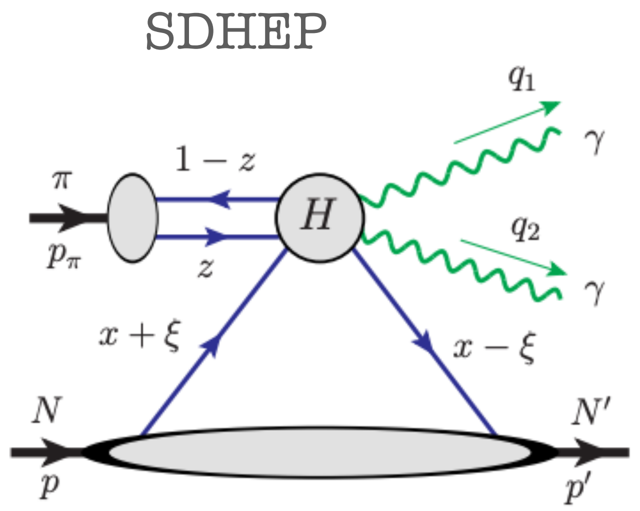


➡ Exclusive reactions



[X.-D. Ji, PRD 55, 7114 (1997)]

➡ Exclusive pion-nucleon diffractive production of a γ pair of high $p_\perp$



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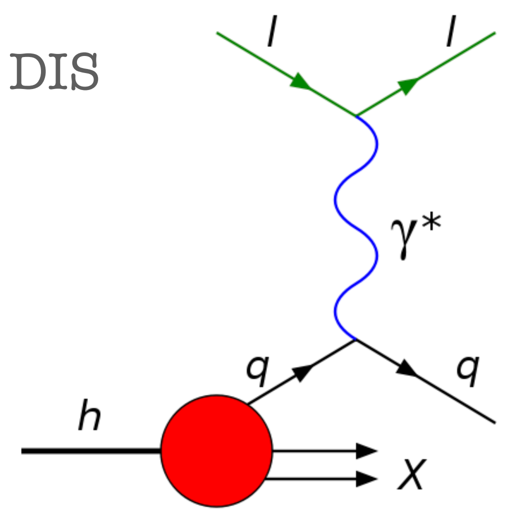

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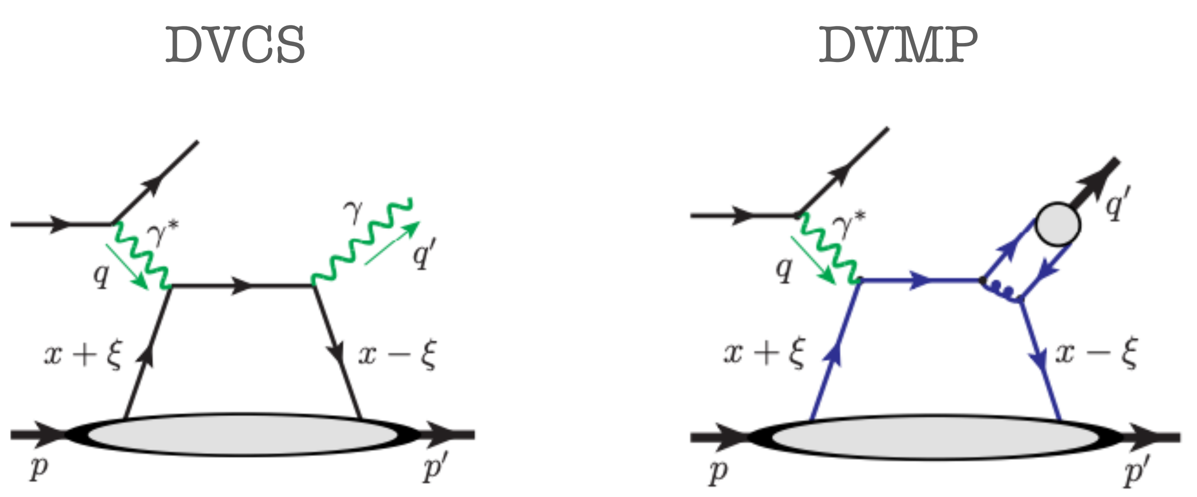

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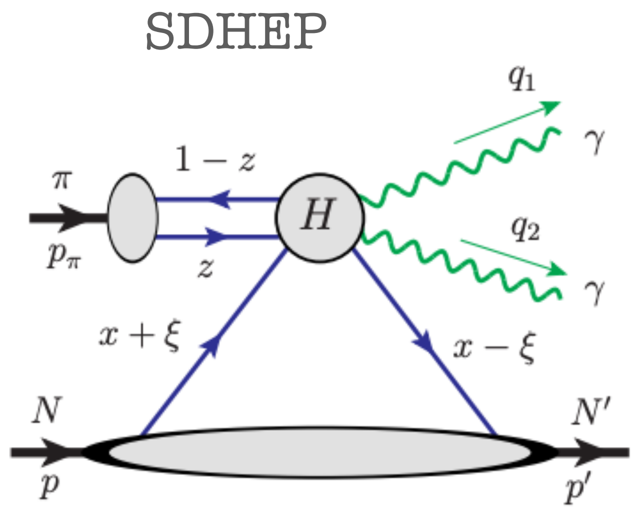

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★ Due to asymptotic freedom, e.g.

$$\sigma_{\rm DIS}(x,Q^2) = \sum_i \left[H^i_{\rm DIS} \otimes f_i\right](x,Q^2) \qquad [a\otimes b](x) \equiv \int_x^1 \frac{d\xi}{\xi}\, a\left(\frac{x}{\xi}\right) b(\xi)$$

Perturb. part (process dependent)

Non-Perturb. part (process “independent”)

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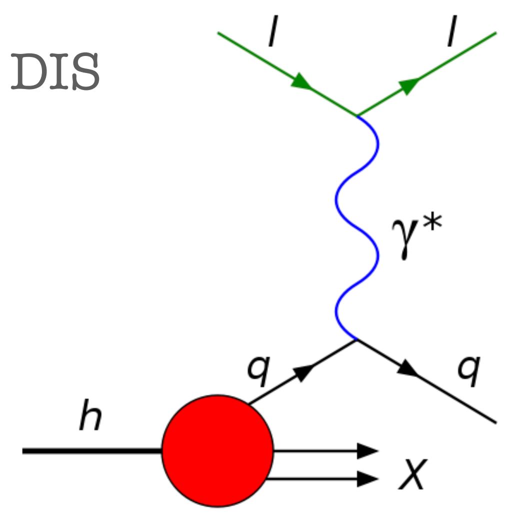

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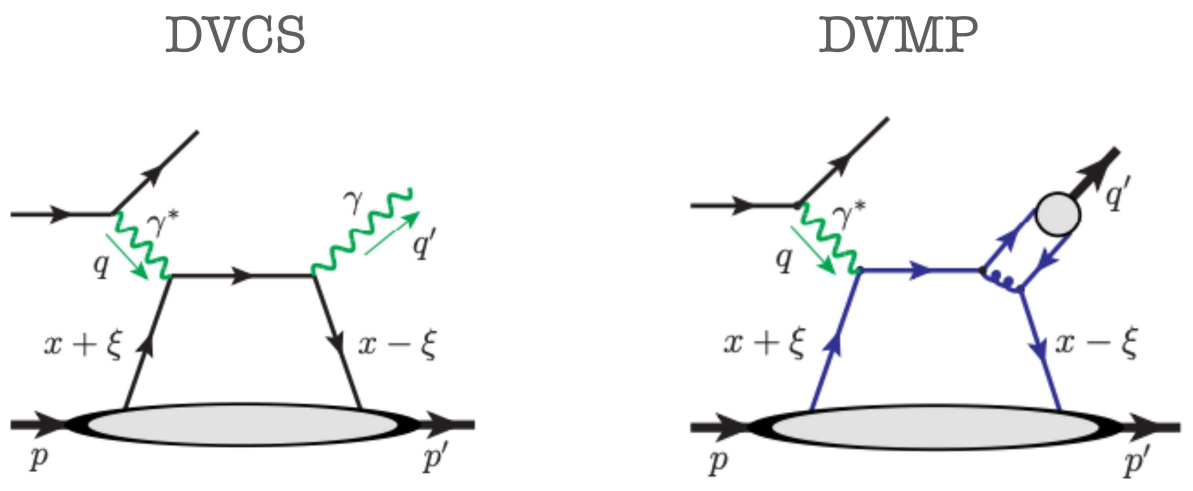

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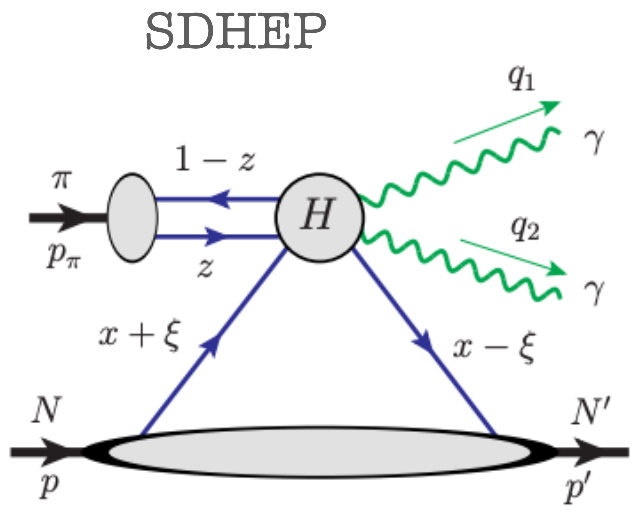

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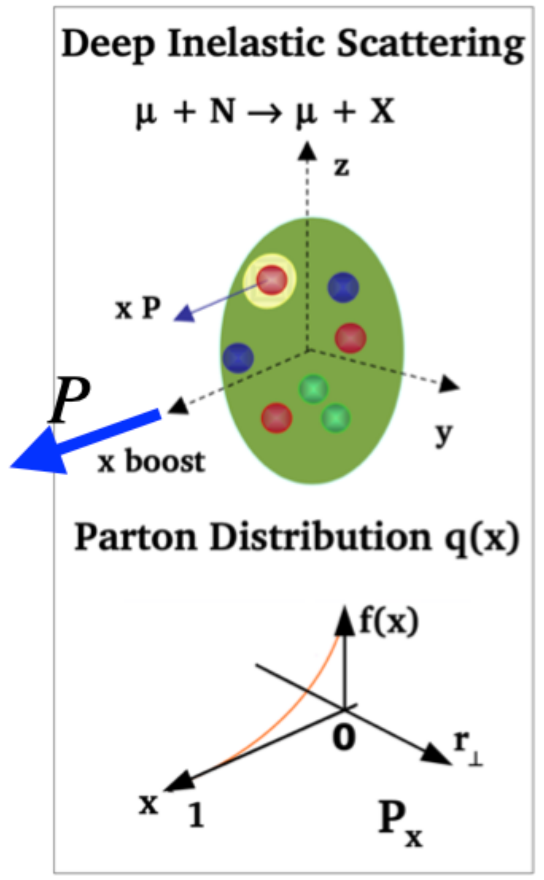
➡ Non-pert. component provides information on, e.g., distribution of partons inside hadron

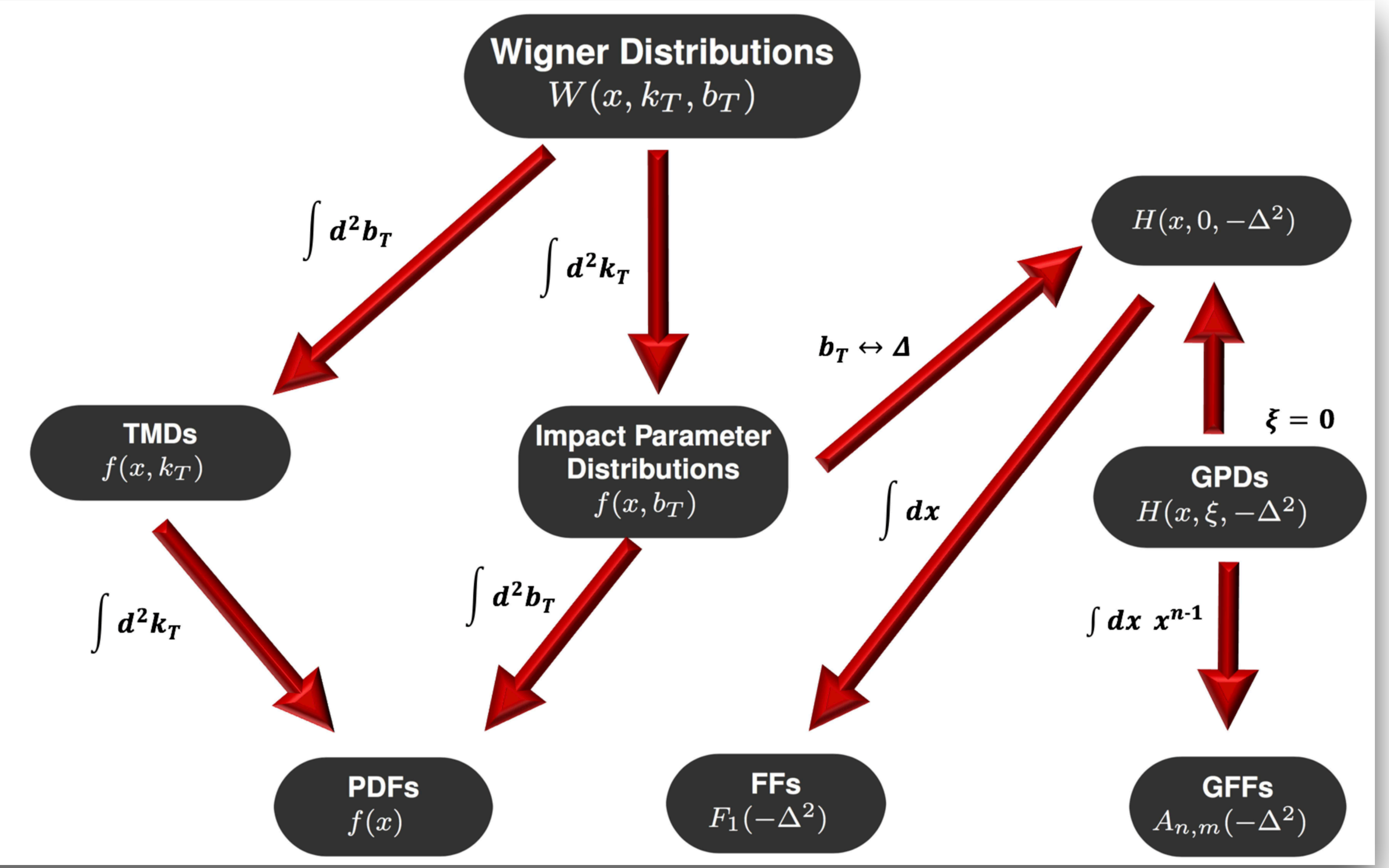


Nucleon Characterization

Wigner distributions

★ provide multi-dimensional images of the parton distributions in phase space

★ encode both TMDs and GPDs in a unified picture



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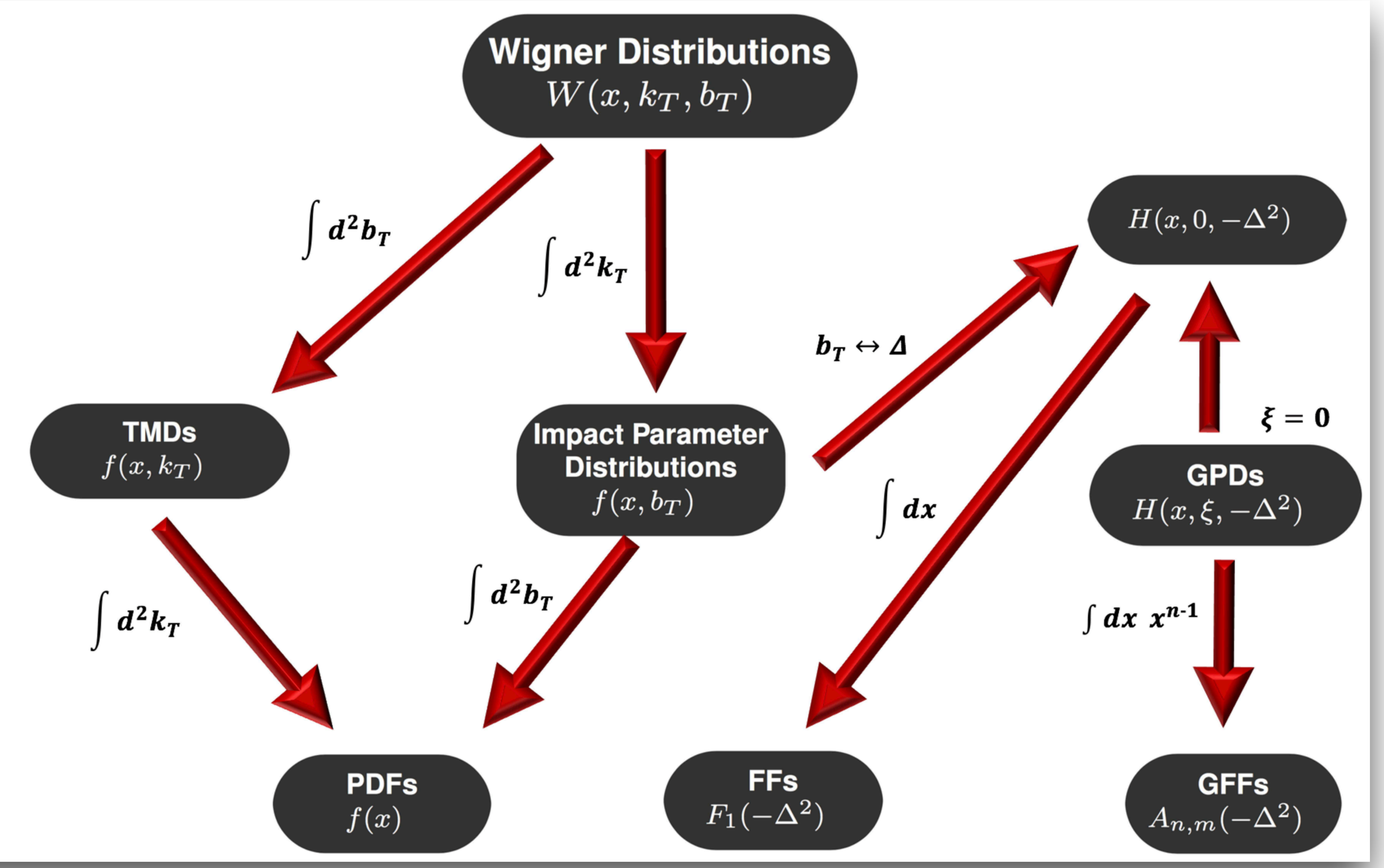

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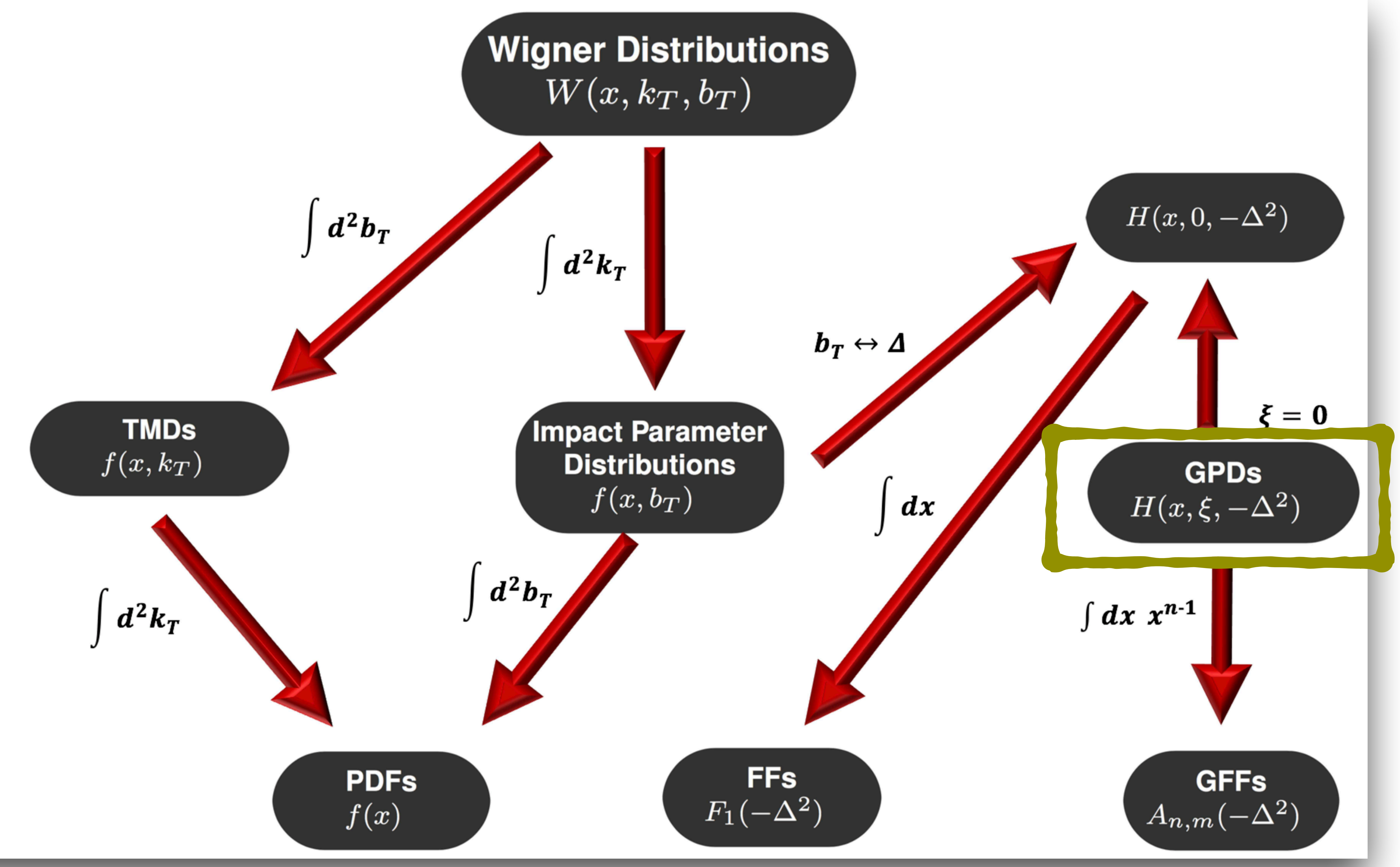

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GPDs

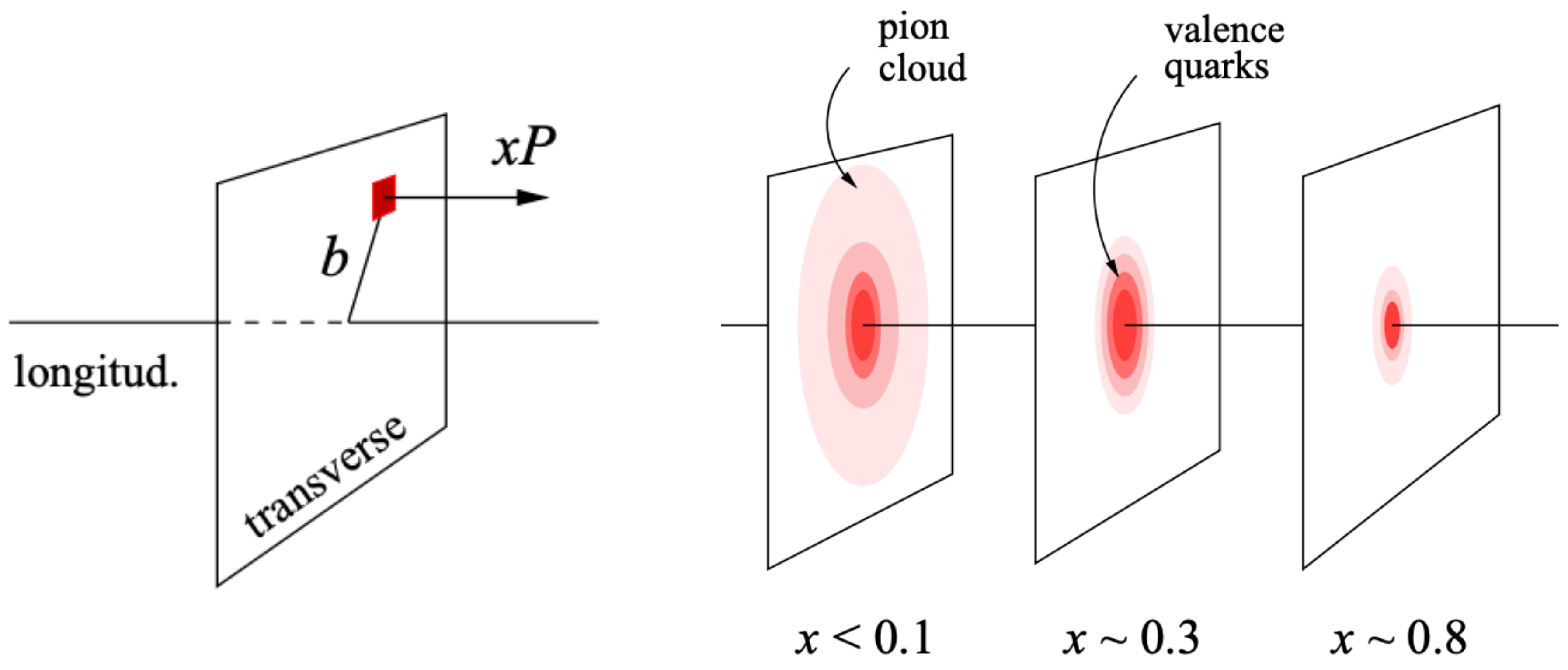
★ "Parent" functions for PDFs, FFs, GFFs

★ Multi-dimensional objects

★ Provide correlation between transverse position and longitudinal momentum of the partons in the hadron

★ Information on the hadron's mechanical properties (OAM, pressure, etc.)

Generalized Parton Distributions



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]

$1_{mom} + 2_{coord}$ tomographic images of quark distribution in nucleon at fixed longitudinal momentum

3-D image from FT of the longitudinal momentum transfer

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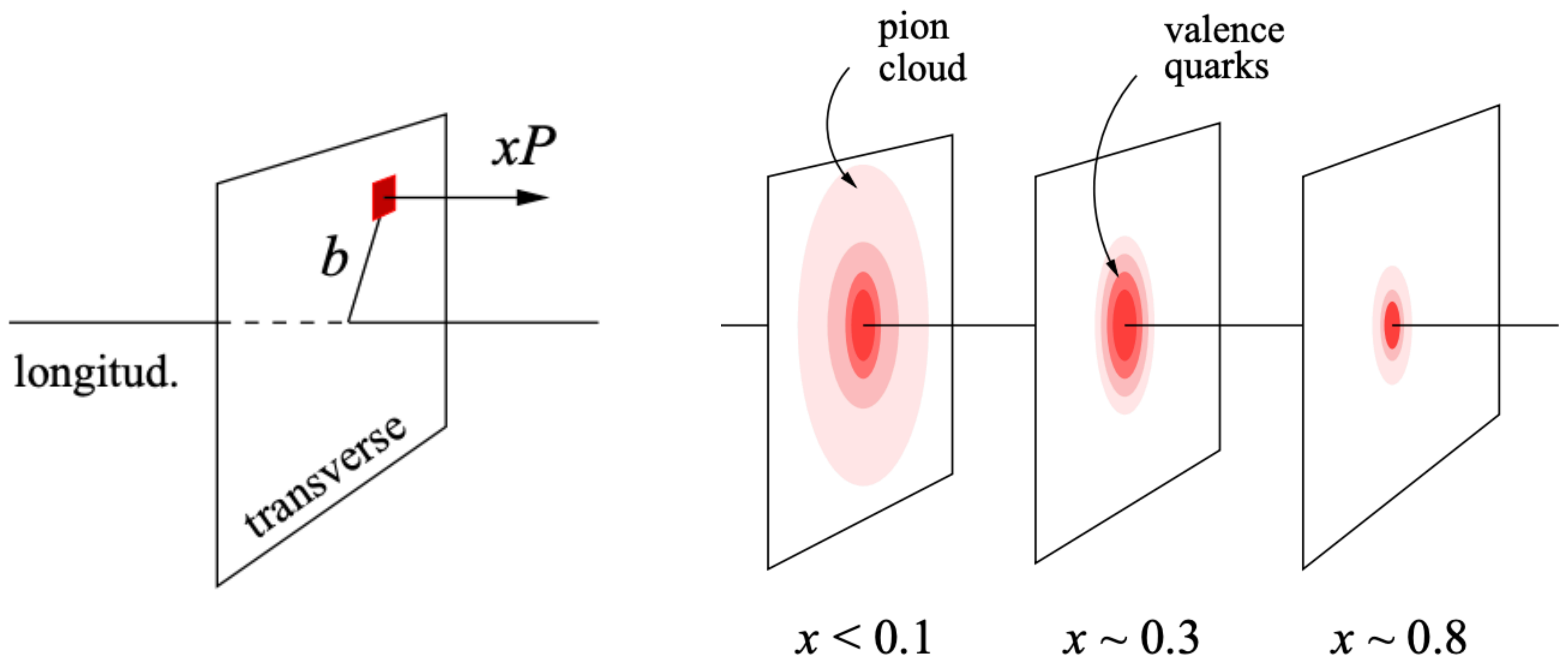

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★ GPDs are not well-constrained experimentally:

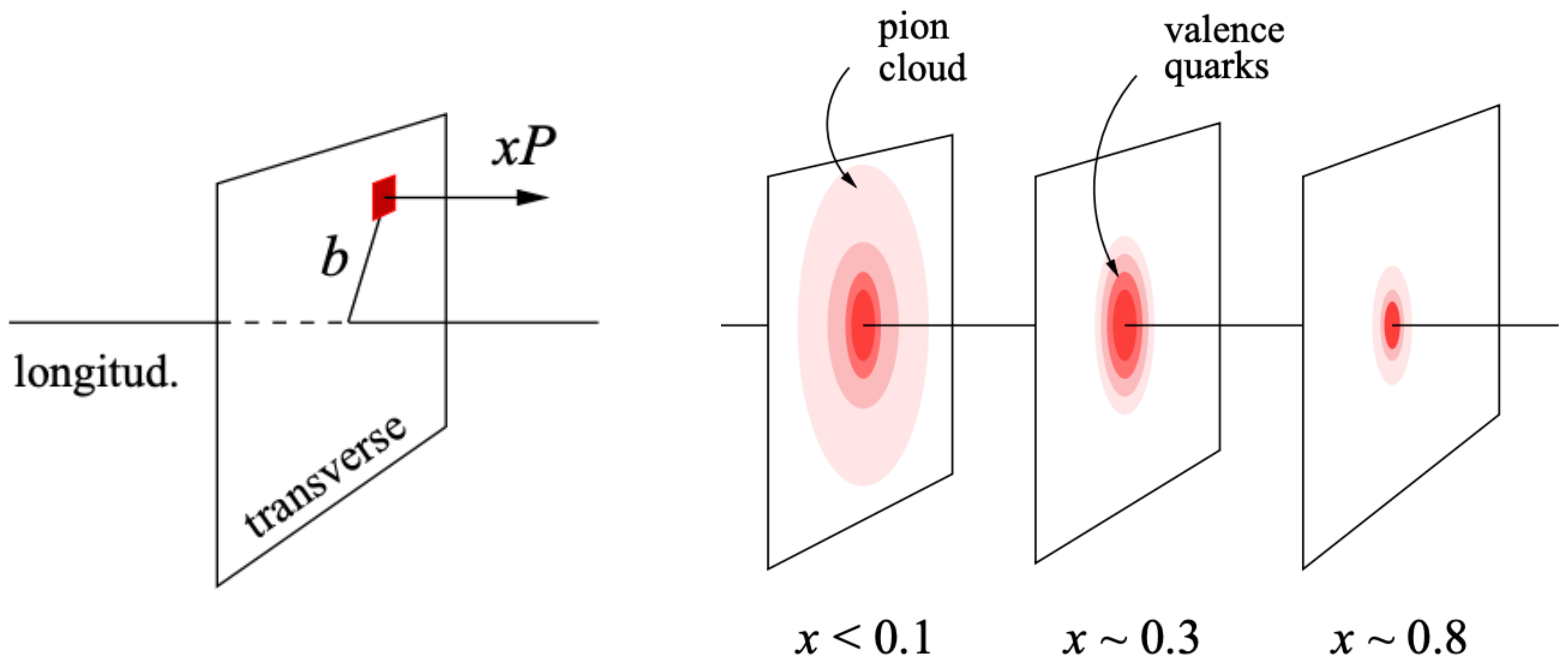
- x-dependence extraction is not direct.

DVCS amplitude: $$\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$$

(SDHEP [J. Qiu et al, JHEP 103 (2022)] gives access to x)

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges …

Generalized Parton Distributions



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Essential to complement the knowledge on GPD from lattice QCD

Twist-classification of GPDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Q : hard scale

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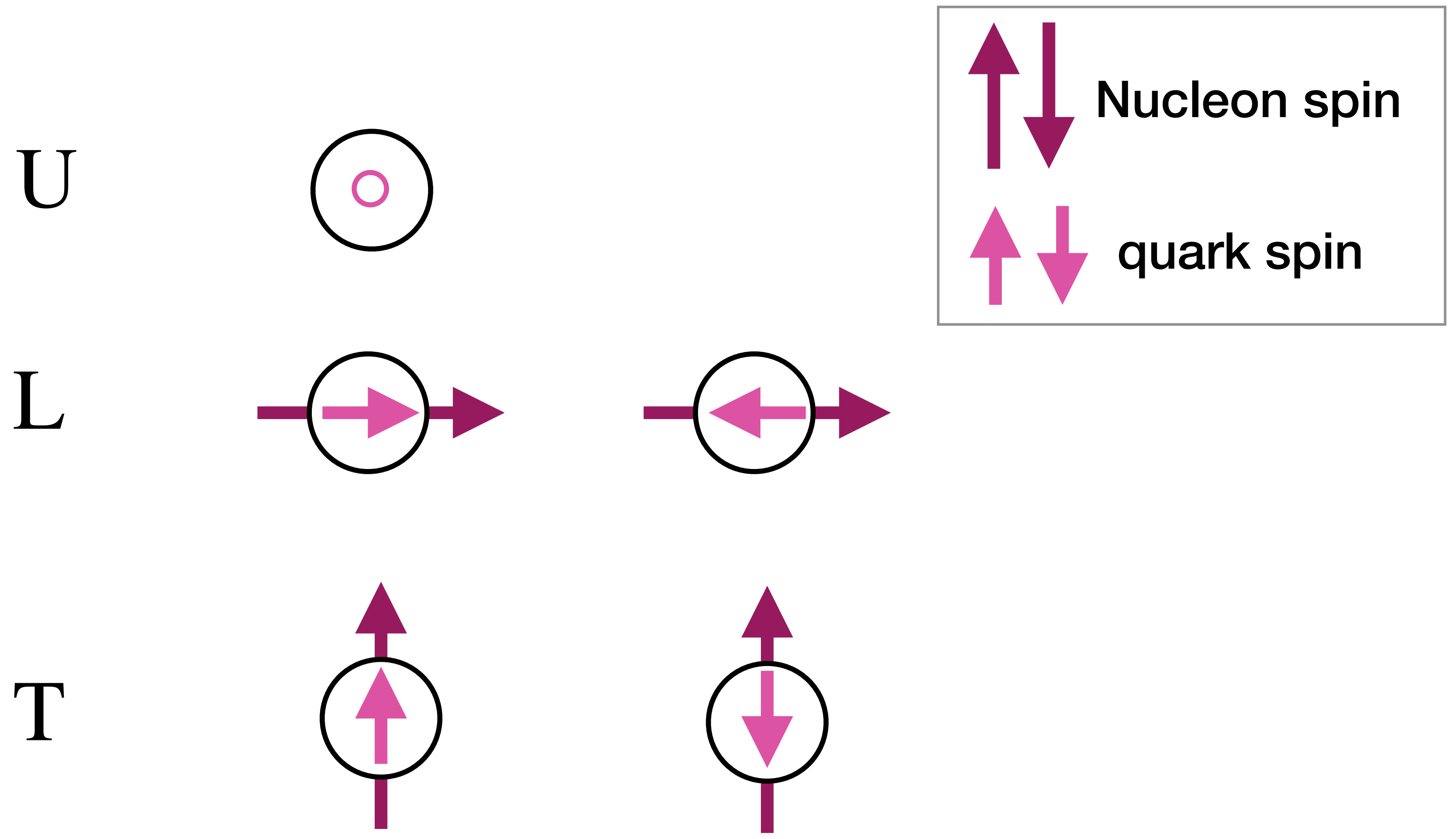
Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

Probabilistic interpretation



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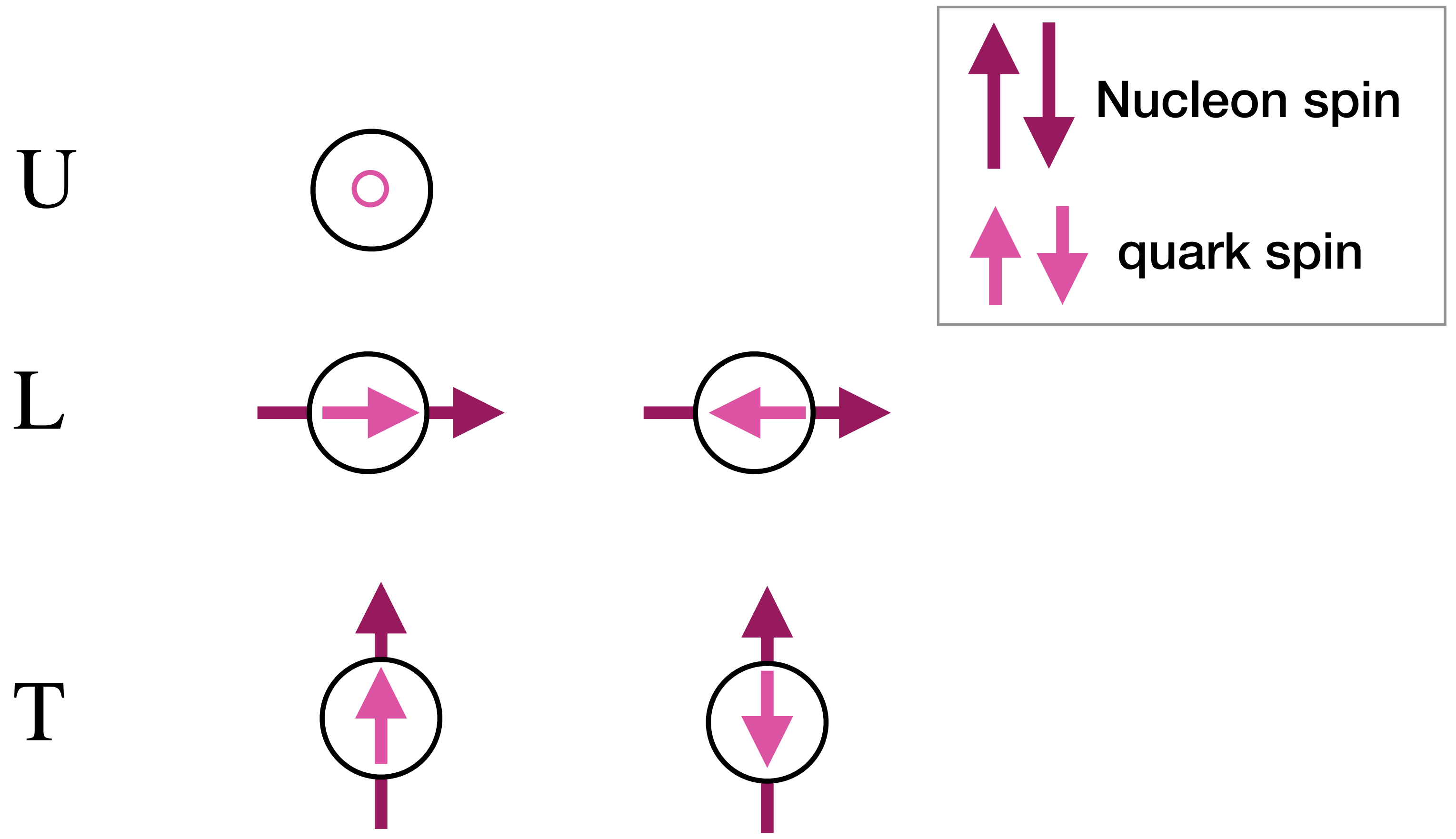
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

Twist-3 $(f_i^{(1)})$ (Selected)

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

Probabilistic interpretation



- ★ Lack density interpretation, but not-negligible
- ★ Contain info on quark-gluon-quark correlators
- ★ Physical interpretation, e.g., transverse force
- ★ Kinematically suppressed
  Difficult to isolate experimentally
- ★ Theoretically: contain $\delta(x)$ singularities

Twist-3 PDFs / GPDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

★ Certain observables require the use of twist-3 correlators

★ Proton collinear twist-3 PDFs: $g_T(x),\ e(x),\ h_L(x)$

- chiral-even $g_T(x)$ couples to inclusive DIS
- $e(x),\ h_L(x)$: chiral-odd (need e.g. chirality flip process)
- $h_L(x)$: double-polarized Drell-Yan process,
  single-inclusive particle production in proton-proton collisions

★ Twist-3 GPDs practically unknown; several challenges

- inverse problem - shadow GPDs [Phys.Rev.D 103 (2021) 11, 114019, Phys.Rev.D 108 (2023) 3, 036027]

★ Twist-3 GPDs contain physical information

- $\widetilde{H} + \widetilde{G}_2$ related to tomography of F⊥ acting on the active q in DIS off a transversely polarized N right after the virtual photon absorbing [Phys.Rev.D 88 (2013) 114502, Phys.Rev.D 100 (2019) 9, 096021]
- Related to certain spin-orbit correlations [Phys.Lett.B 735 (2014) 344, Phys.Lett.B 774 (2017) 435]
- $G_2(x, \xi, t)$ related to $L_q^{\rm kin}$ [Phys.Lett.B 491 (2000) 96]

$$L_q^{\rm kin} = -\int_{-1}^{1} dx\, x\, G_2^q(x, \xi, t=0)$$

GPDs

From Lattice QCD

Accessing information on GPDs

$$\sigma_{\text{DIS}}(x, Q^2) = \sum_i \left[H^i_{\text{DIS}} \otimes f_i\right](x, Q^2)$$

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Calculable in lattice QCD

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Calculable in lattice QCD

★ Mellin moments
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\dots z_{\alpha_n}\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\dots\overleftrightarrow{D}^{\alpha_n} q\right]$$

Accessing information on GPDs

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local operators

Accessing information on GPDs

$$\sigma_{\text{DIS}}(x, Q^2) = \sum_i \left[H^i_{\text{DIS}} \otimes f_i\right](x, Q^2)$$

Calculable in lattice QCD

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z, \tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty} \frac{1}{n!}\, z_{\alpha_1} \ldots z_{\alpha_n} \left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \ldots \overleftrightarrow{D}^{\alpha_n} q\right]$$

local operators

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\\text{even}}}^{n-1} \left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^\mu\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\text{ even}}\Big)\Big\} + \frac{\Delta^\mu\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\text{ even}}\Big]U(P)$$

Accessing information on GPDs

$$\sigma_{\rm DIS}(x,Q^2) = \sum_i \left[H^i_{\rm DIS} \otimes f_i\right](x,Q^2)$$

Calculable in lattice QCD

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★ Matrix elements of non-local operators
(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar\Psi(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

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★ Advantages

- Frame independence
- Several values of momentum transfer with same computational cost
- Form factors extracted with controlled statistical uncertainties

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- Geometrical twist classification (coincides with dynamical twist of scattering processes only at leading order)
- Signal-to-noise ratio decays with the addition of covariant derivatives
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Form Factors & Generalizations

★ Ultra-local operators (FFS)

$$\langle N(P')|\overline{q}(0)\gamma^\mu q(0)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}F_2(t)\right\}U(P)\,,$$

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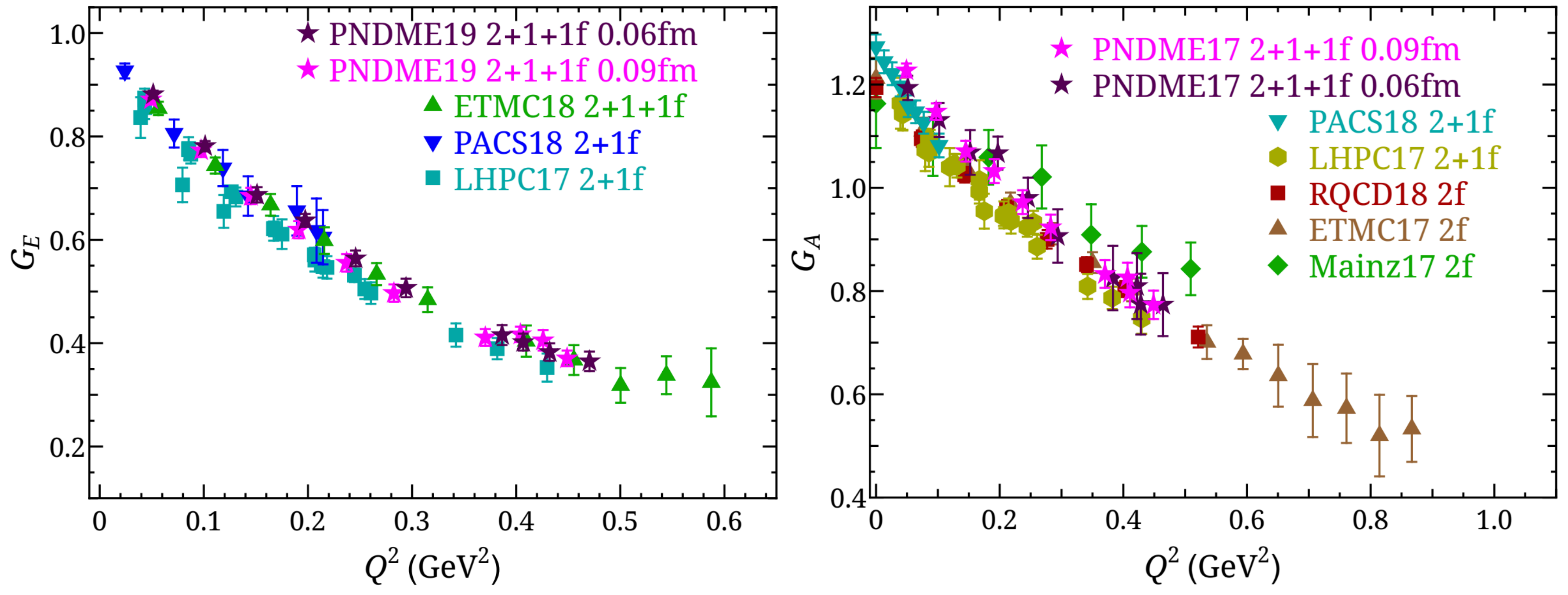

[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908, arXiv:2006.08636]

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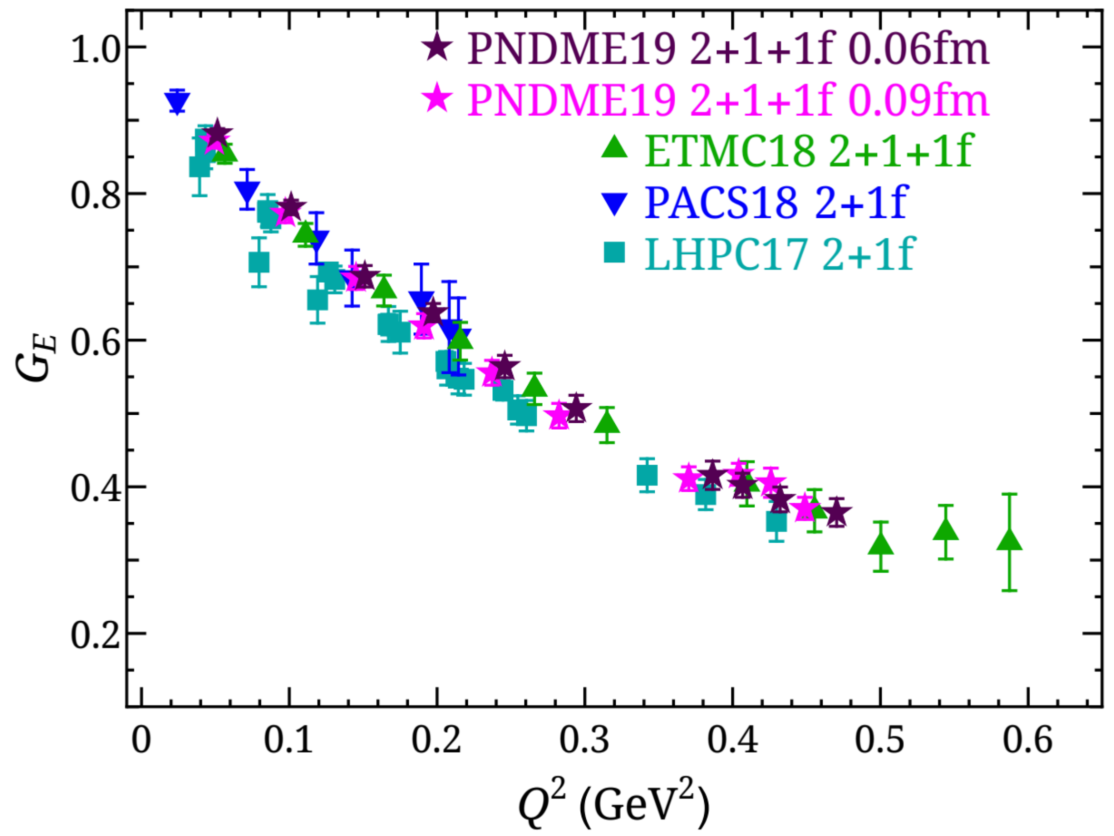

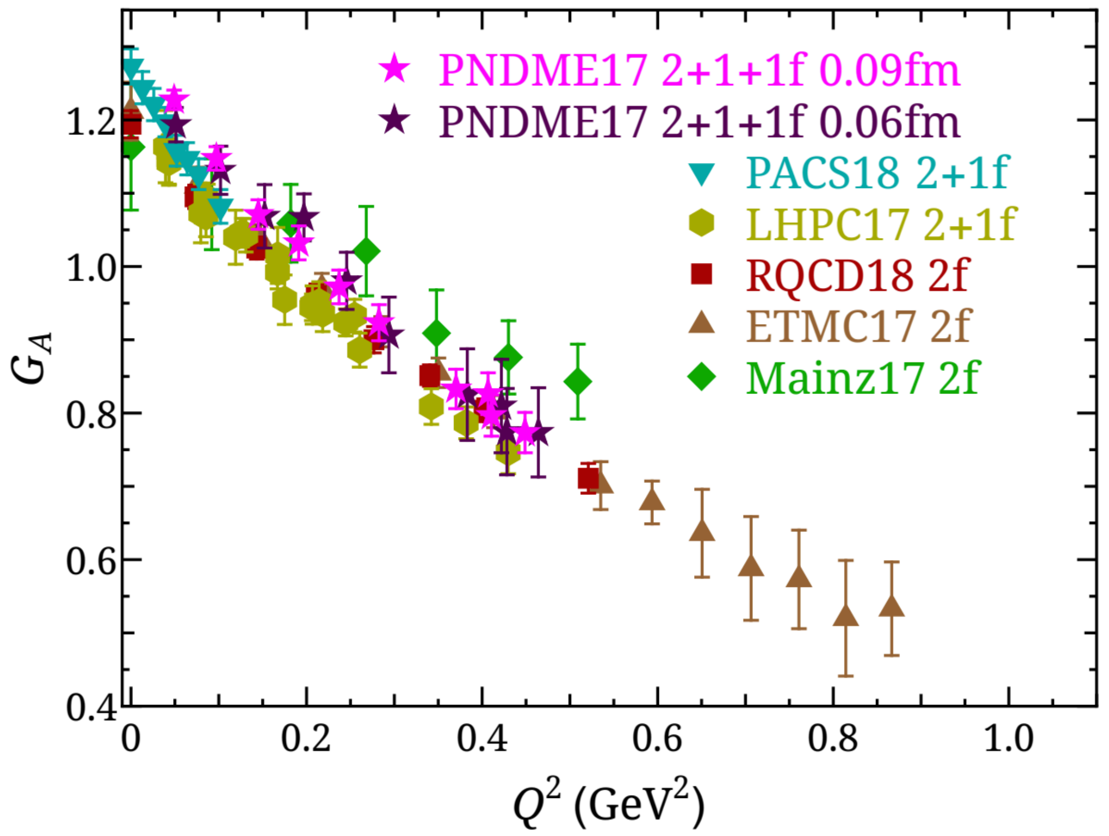

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- Precision data era
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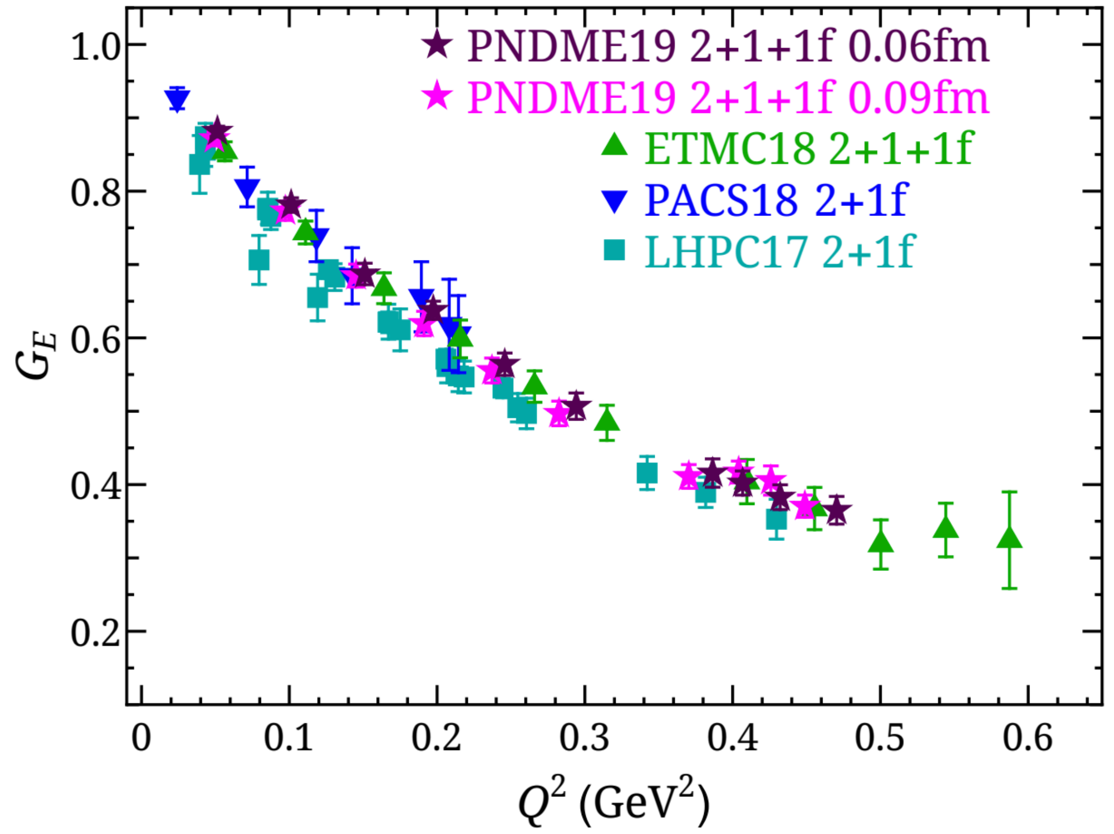

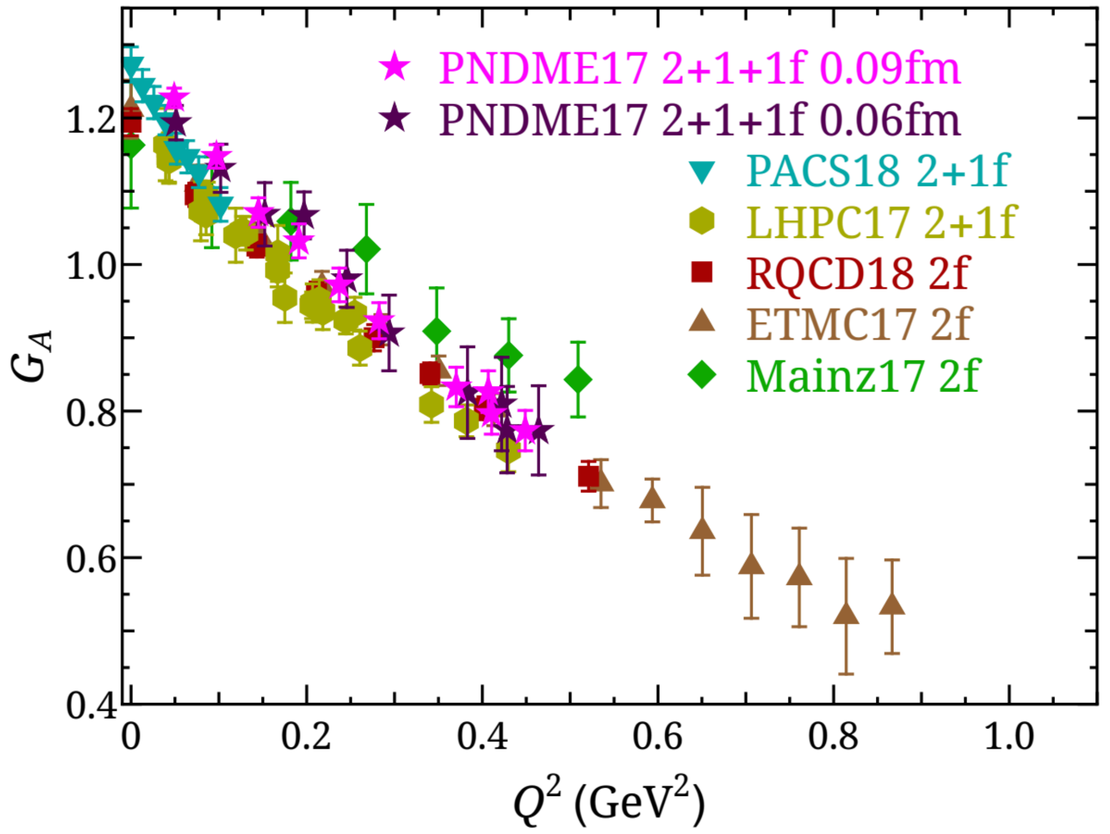

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★ 1-derivative operators (GFFs)

$$\langle N(p',s')|\mathcal{O}_V^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{1}{2}\Big[A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m_N} + C_{20}(q^2)\,\frac{1}{m_N}q^{\{\mu}q^{\nu\}}\Big]u_N(p,s),$$

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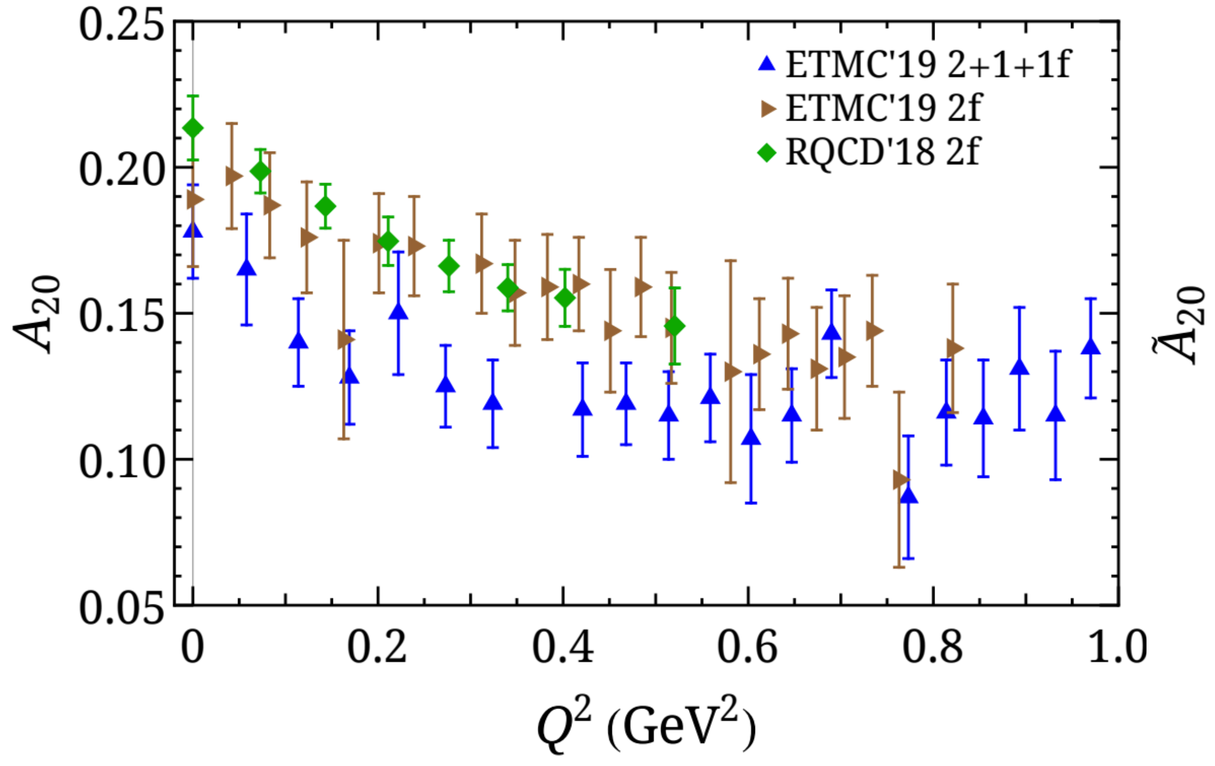

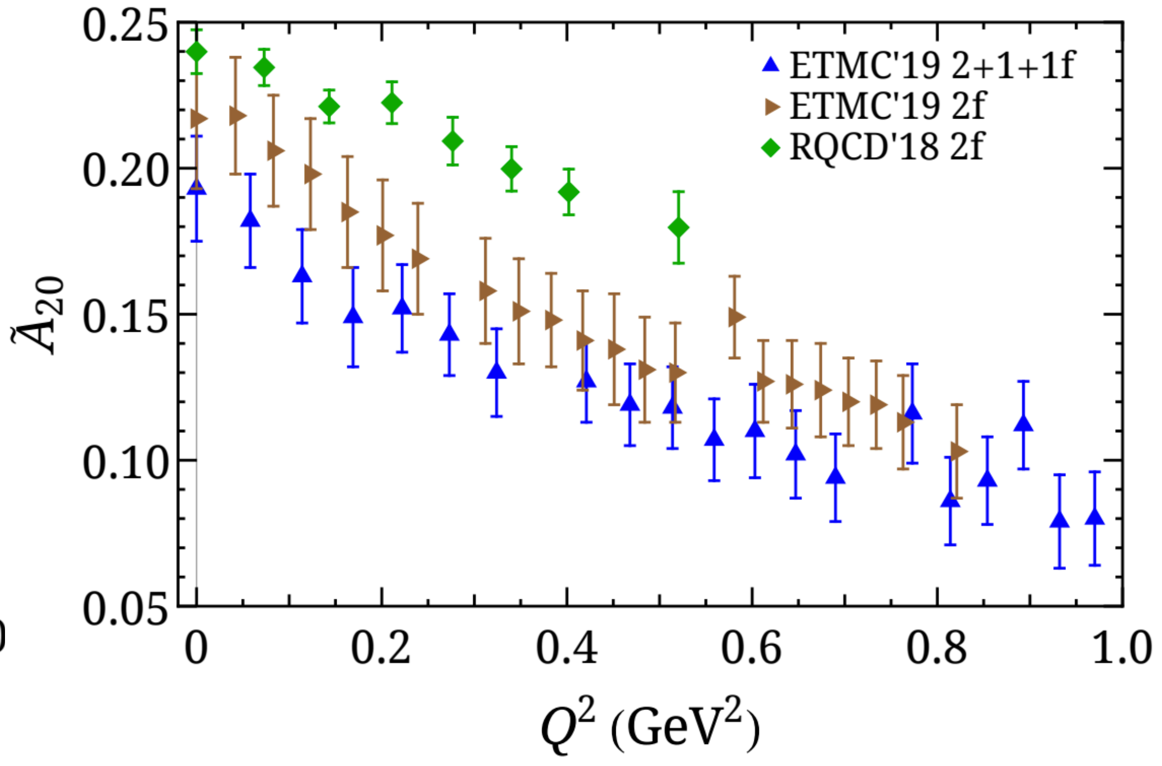

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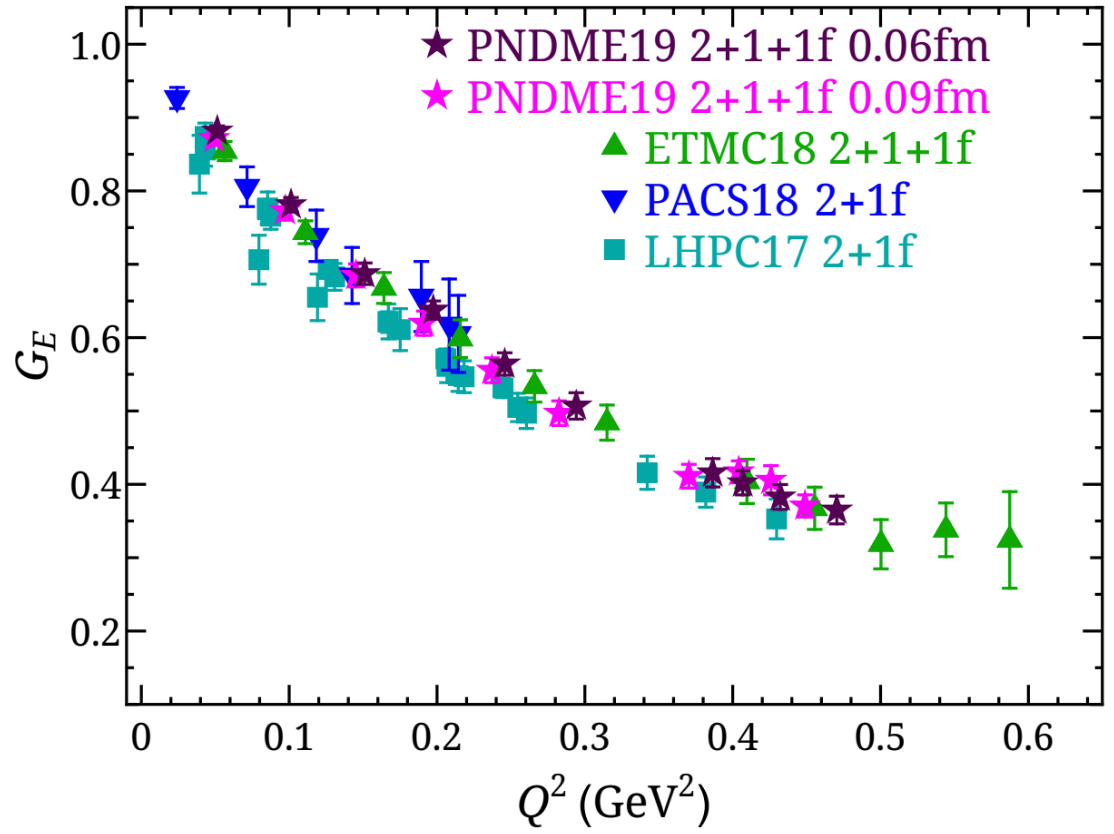

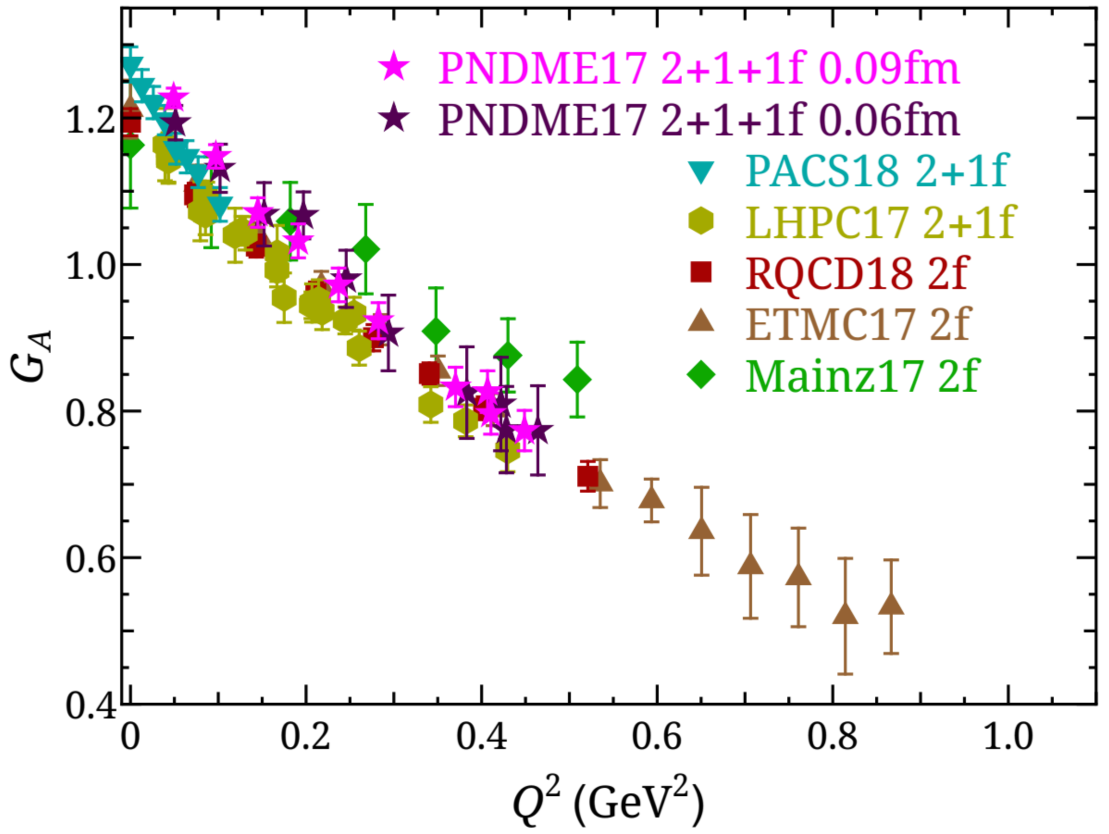

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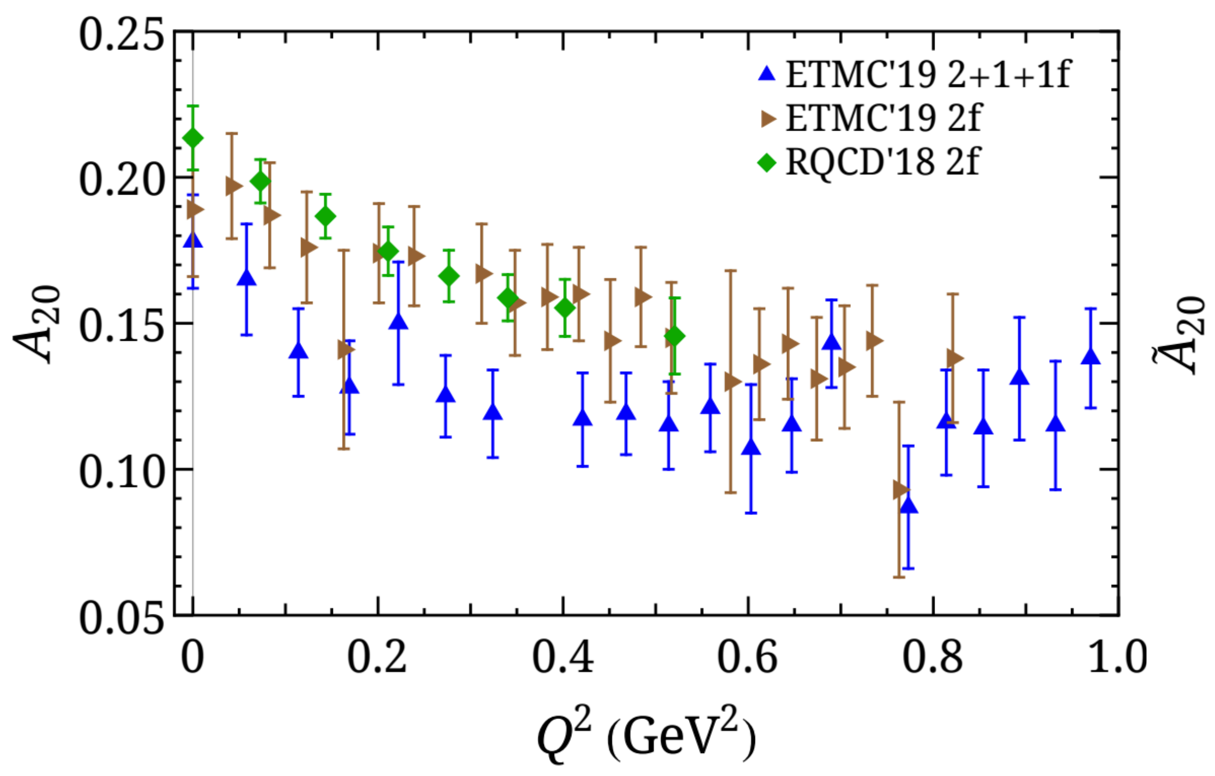

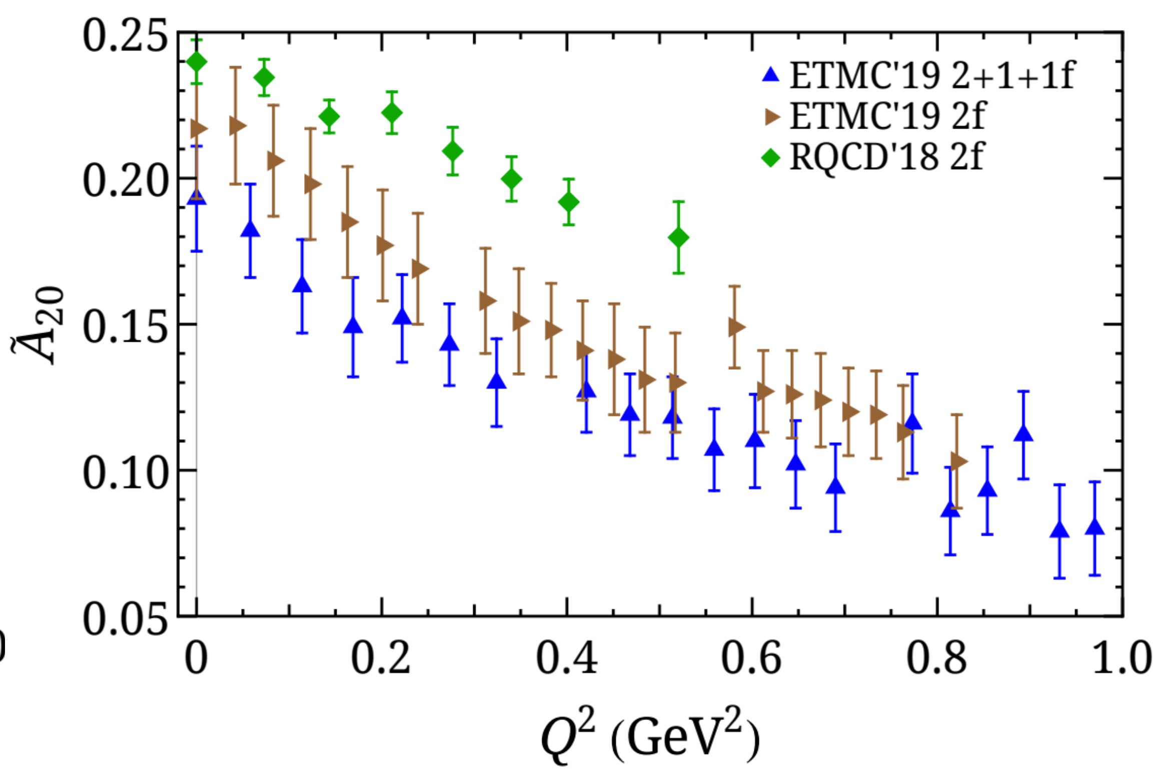

- Lesser studied compared to FFs at physical point
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[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908, arXiv:2006.08636]

GPDs

Through non-local matrix elements

of fast-moving hadrons

GPDs on the lattice

★ GPDs: off-forward matrix elements of non-local light-cone operators

★ Off-forward correlators with nonlocal (equal-time) operators

[Ji, PRL 110 (2013) 262002]

[A. Radyushkin, PRD 96, 034025 (2017)]

$$\tilde{q}_\mu^{\rm GPD}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-ixP_3 z} \, \langle N(P_f) \,|\, \bar{\Psi}(z)\, \gamma^\mu\, \mathcal{W}(z,0)\Psi(0) \,|\, N(P_i) \rangle_\mu$$

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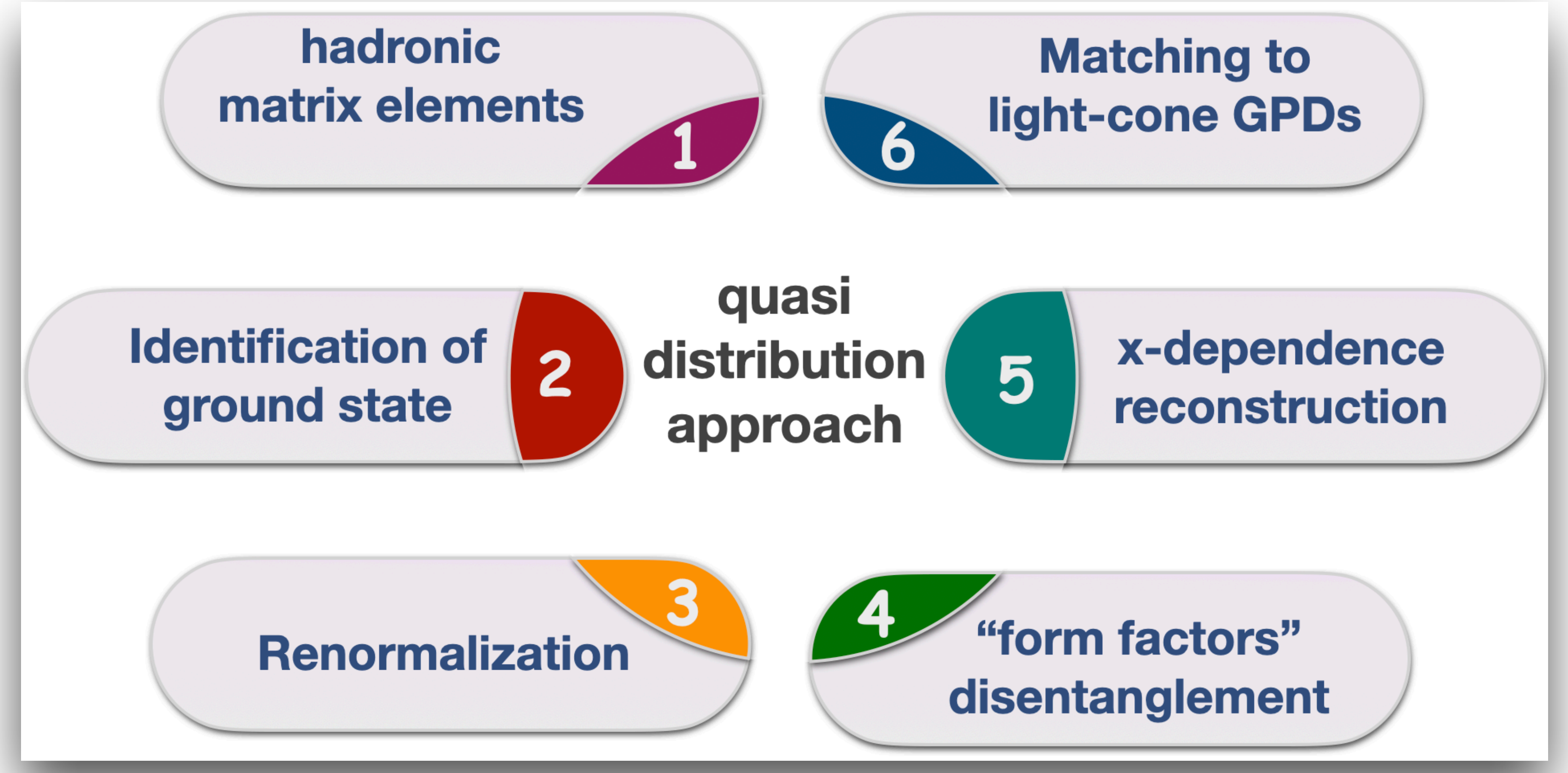

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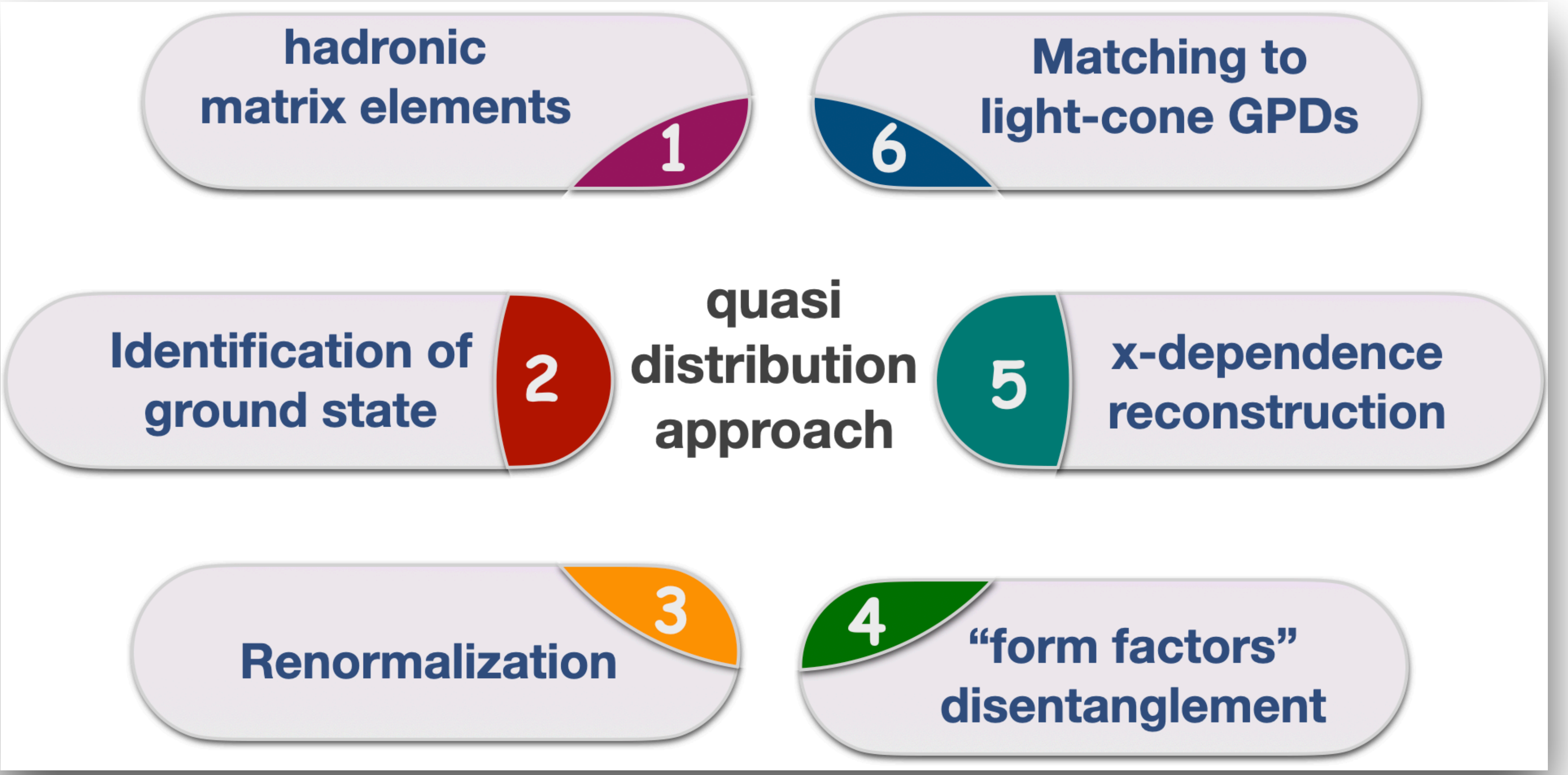

Calculation challenges

✦ Standard definition of GPDs in Breit (symmetric) frame
separate calculations at each t

✦ Statistical noise increases with P_3, t
Projection:
billions of core-hours at $P_3 = 3\,\mathrm{GeV}$

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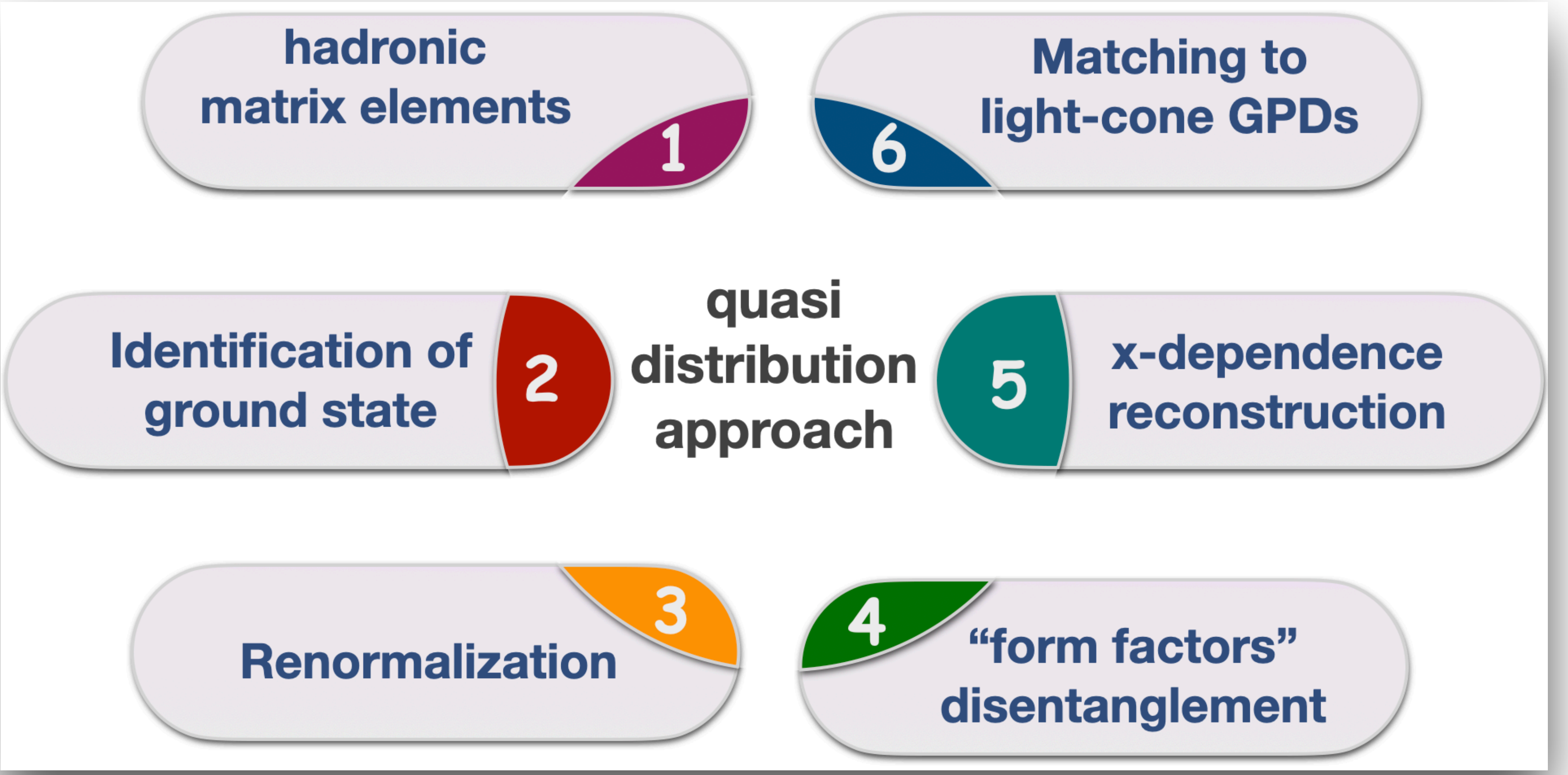

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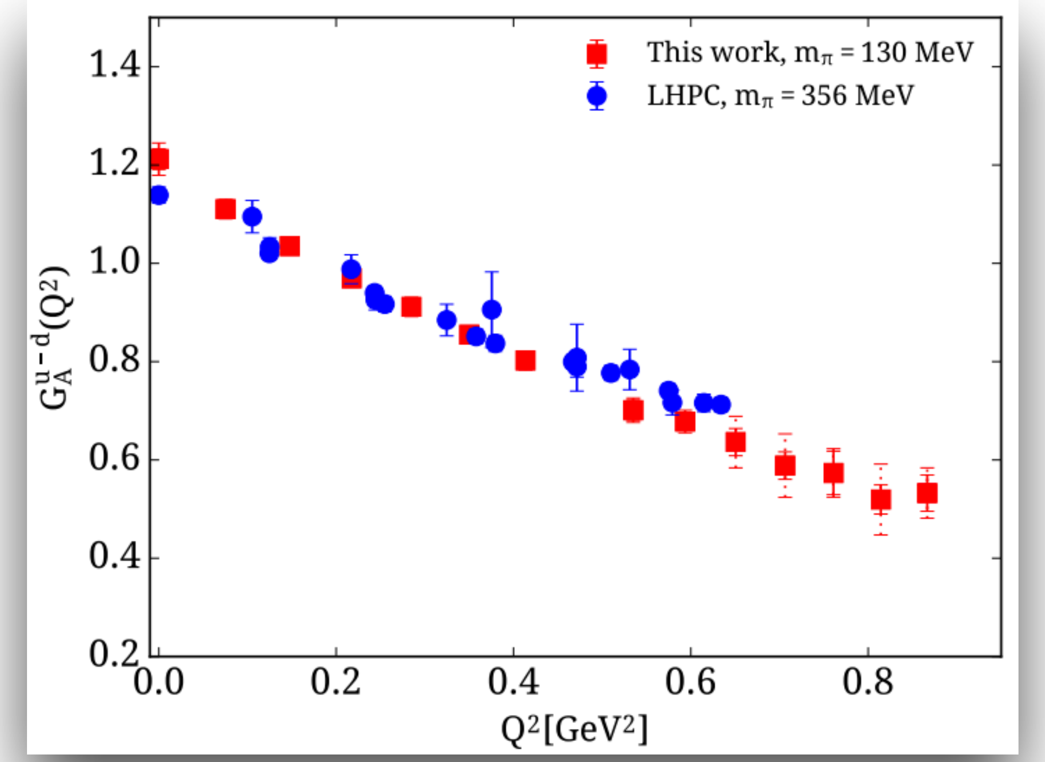

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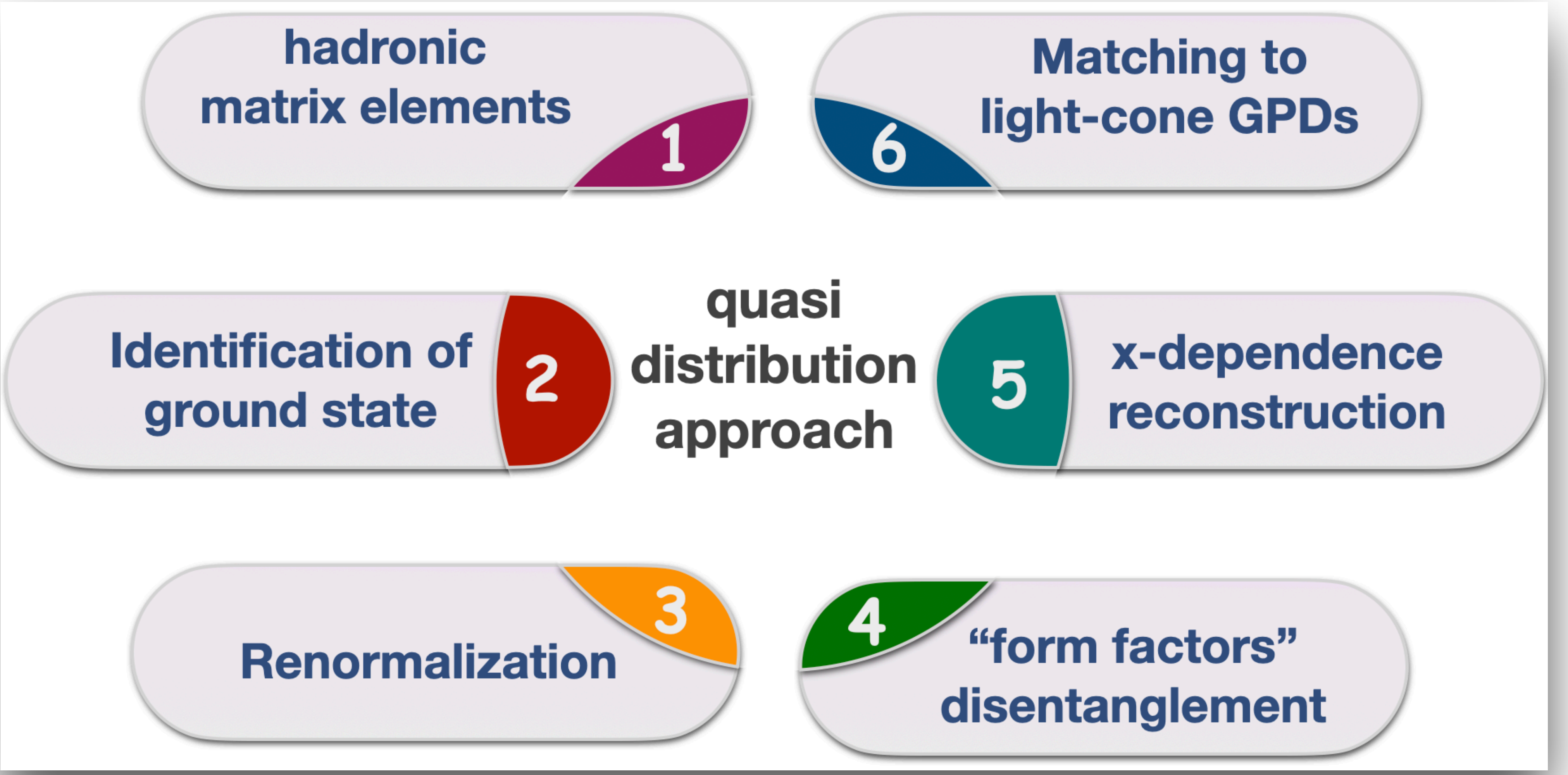

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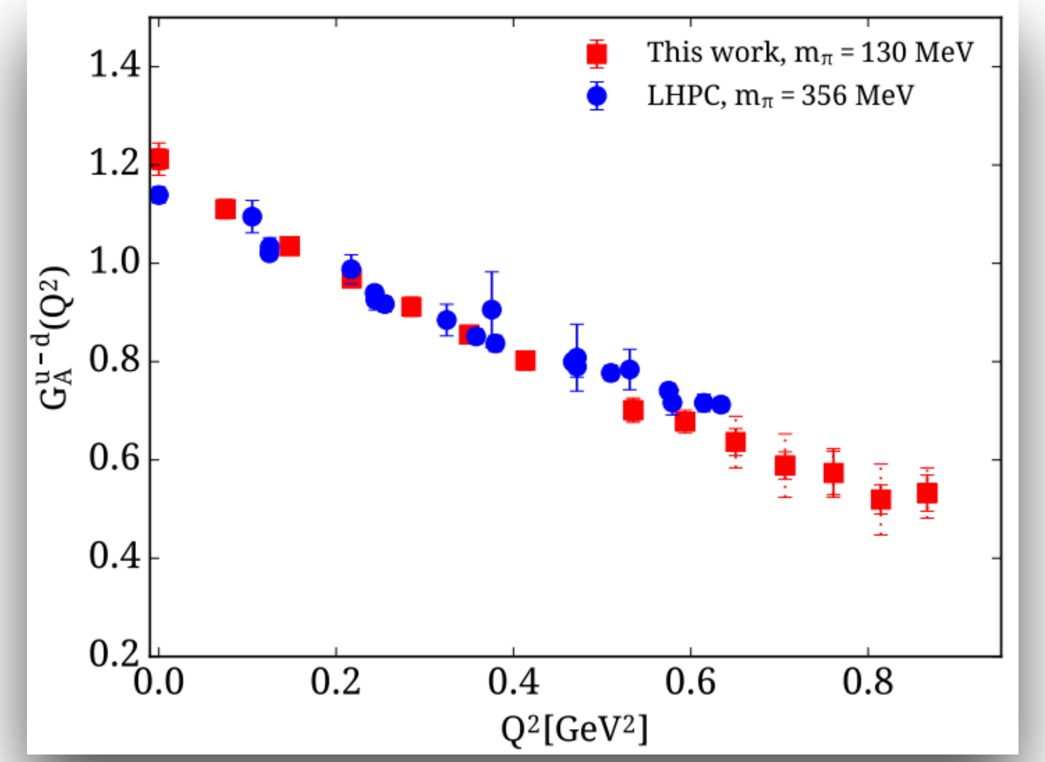

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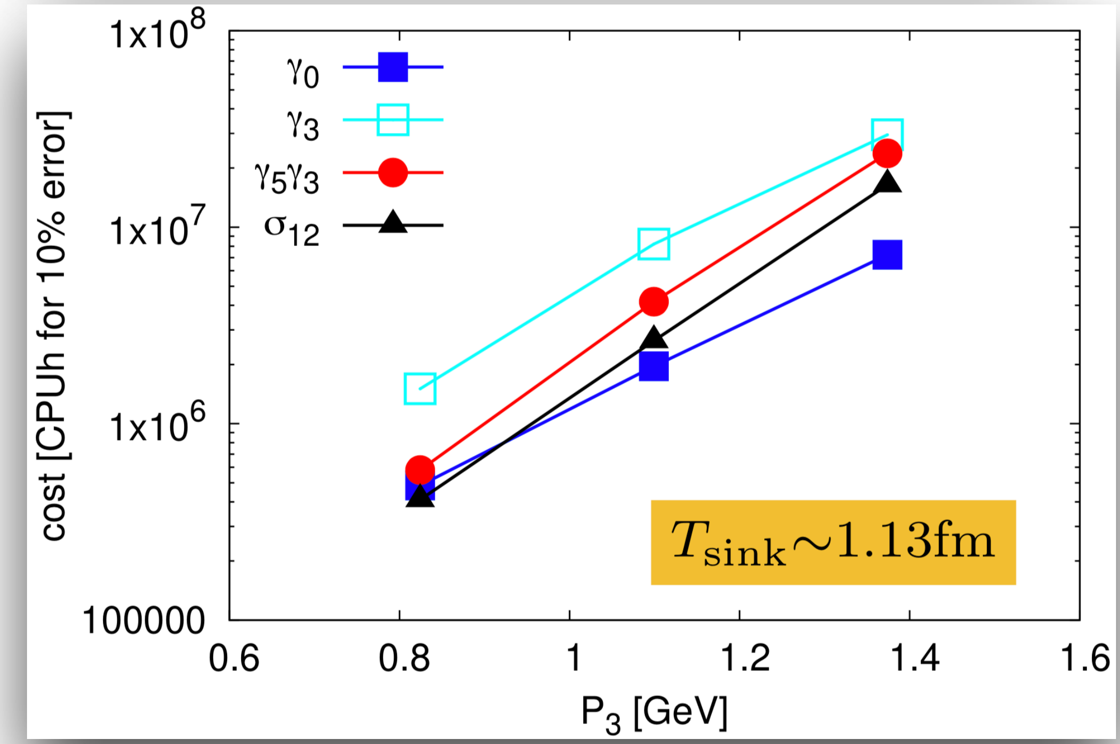
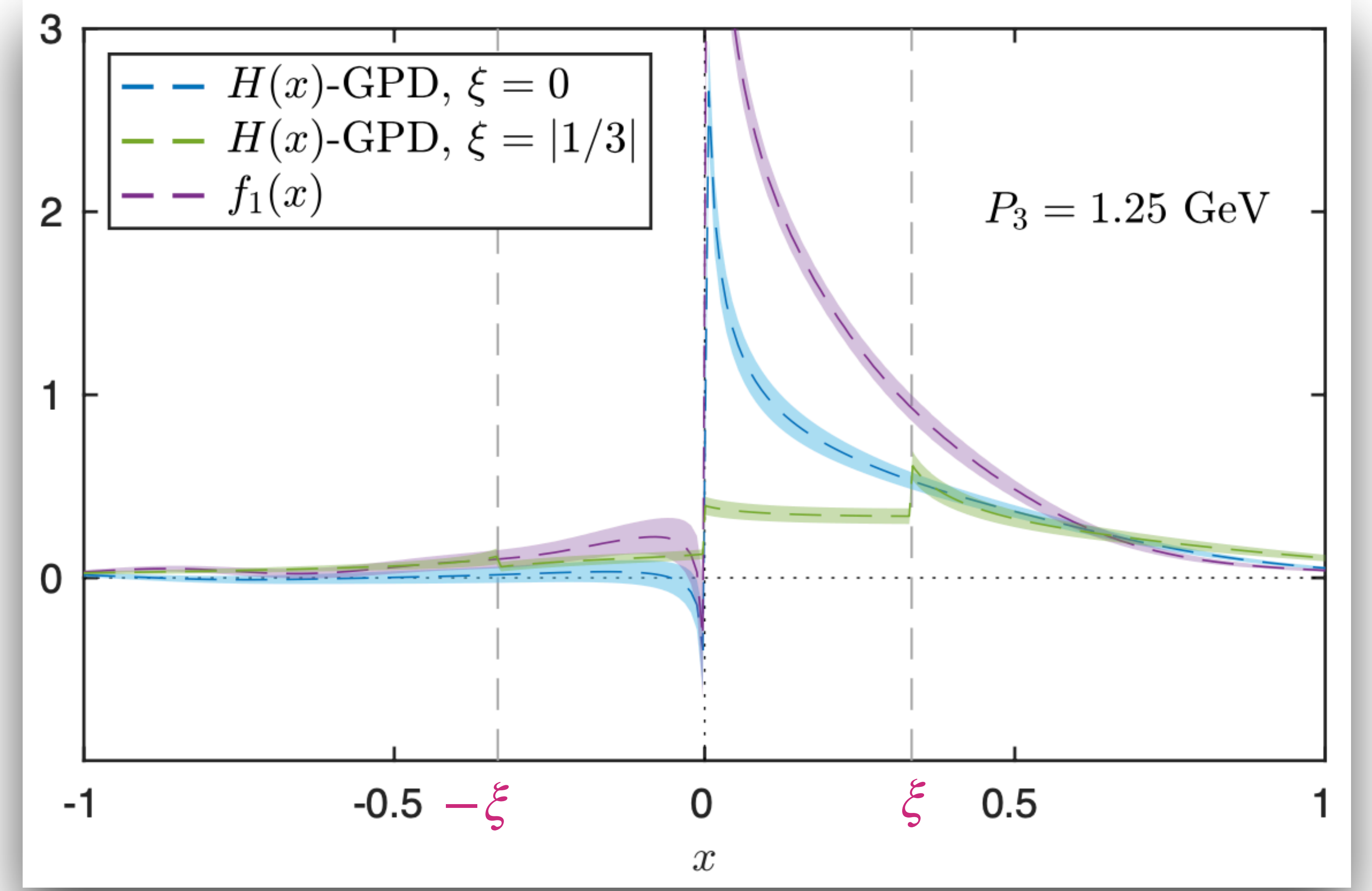


Progress in twist-2 GPDs

First ever calculation



[ETMC, PRL 125, 262001 (2020)]

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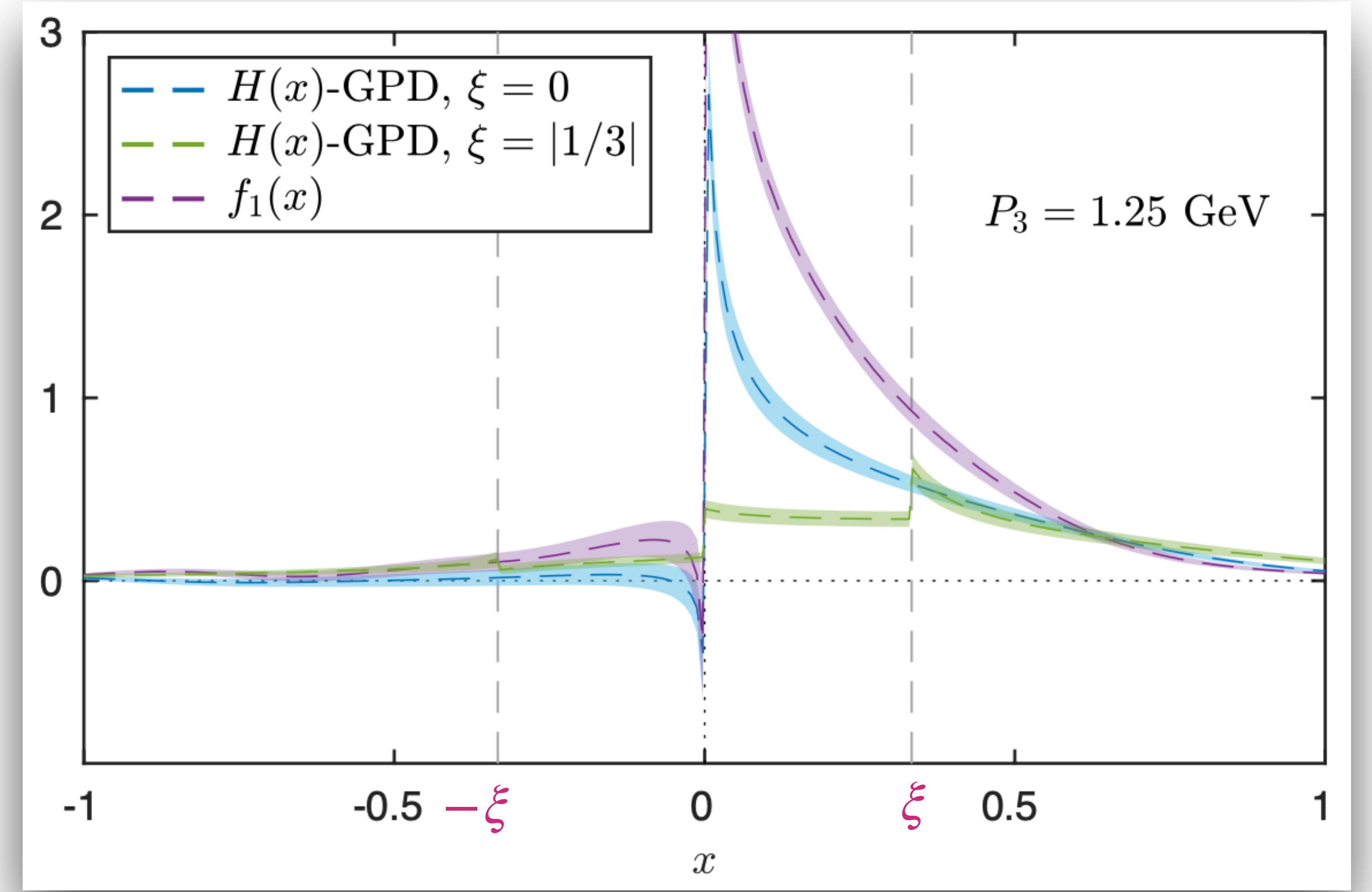

[ETMC, PRL 125, 262001 (2020)]

New approach [Bhattacharya et al., Phys.Rev.D 106 (2022) 11, 114512 ; Bhattacharya et al., arXiv:2310.13114]

★ Parametrization of matrix elements in Lorentz invariant amplitudes

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu M A_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

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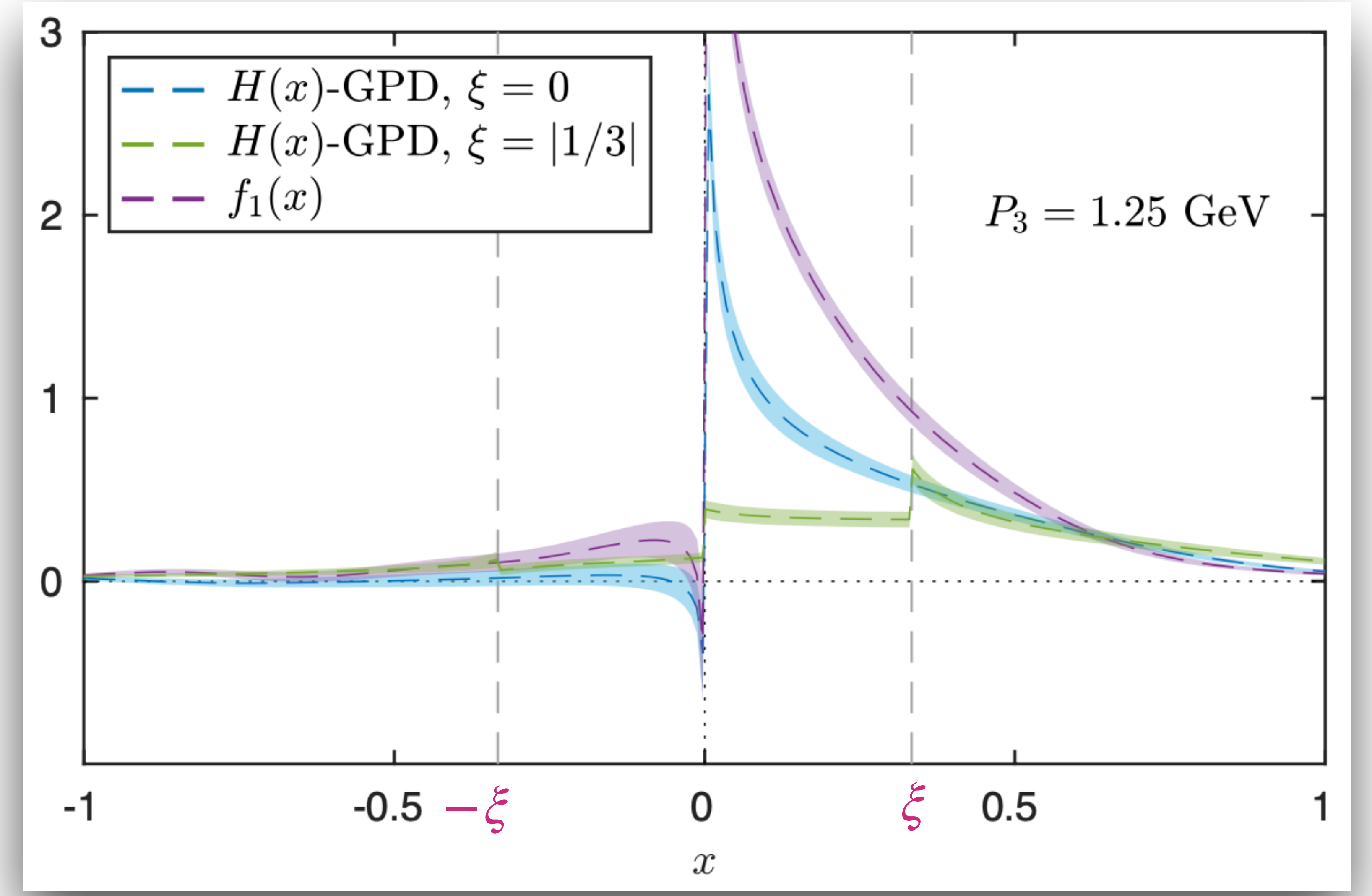

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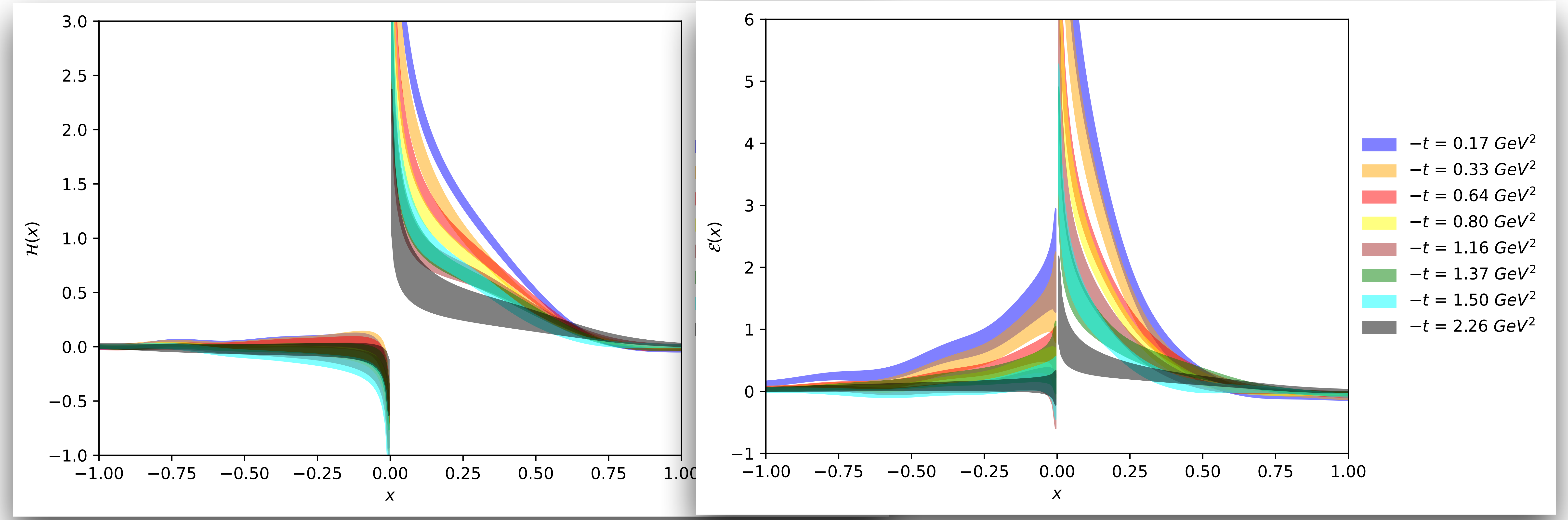

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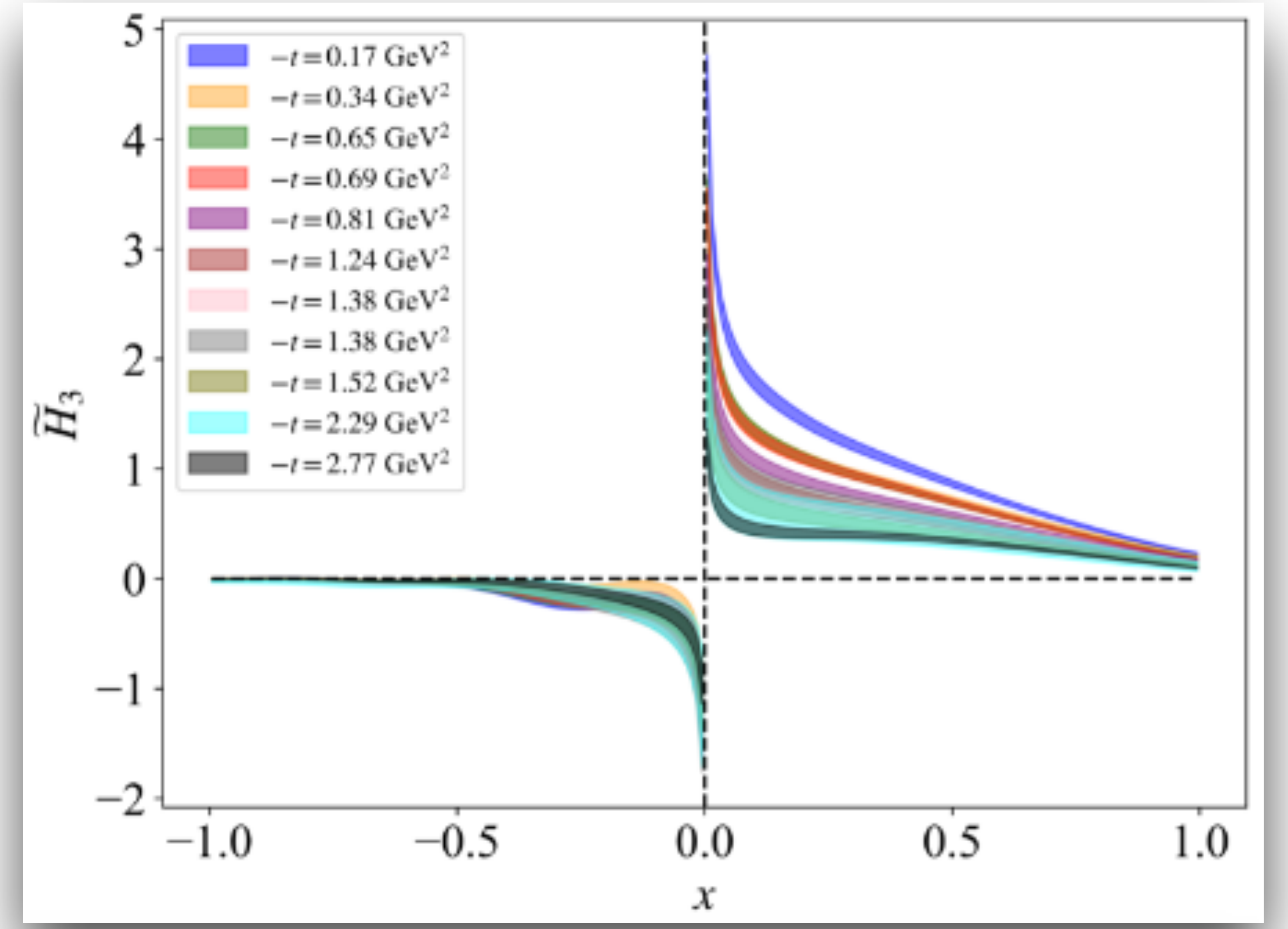
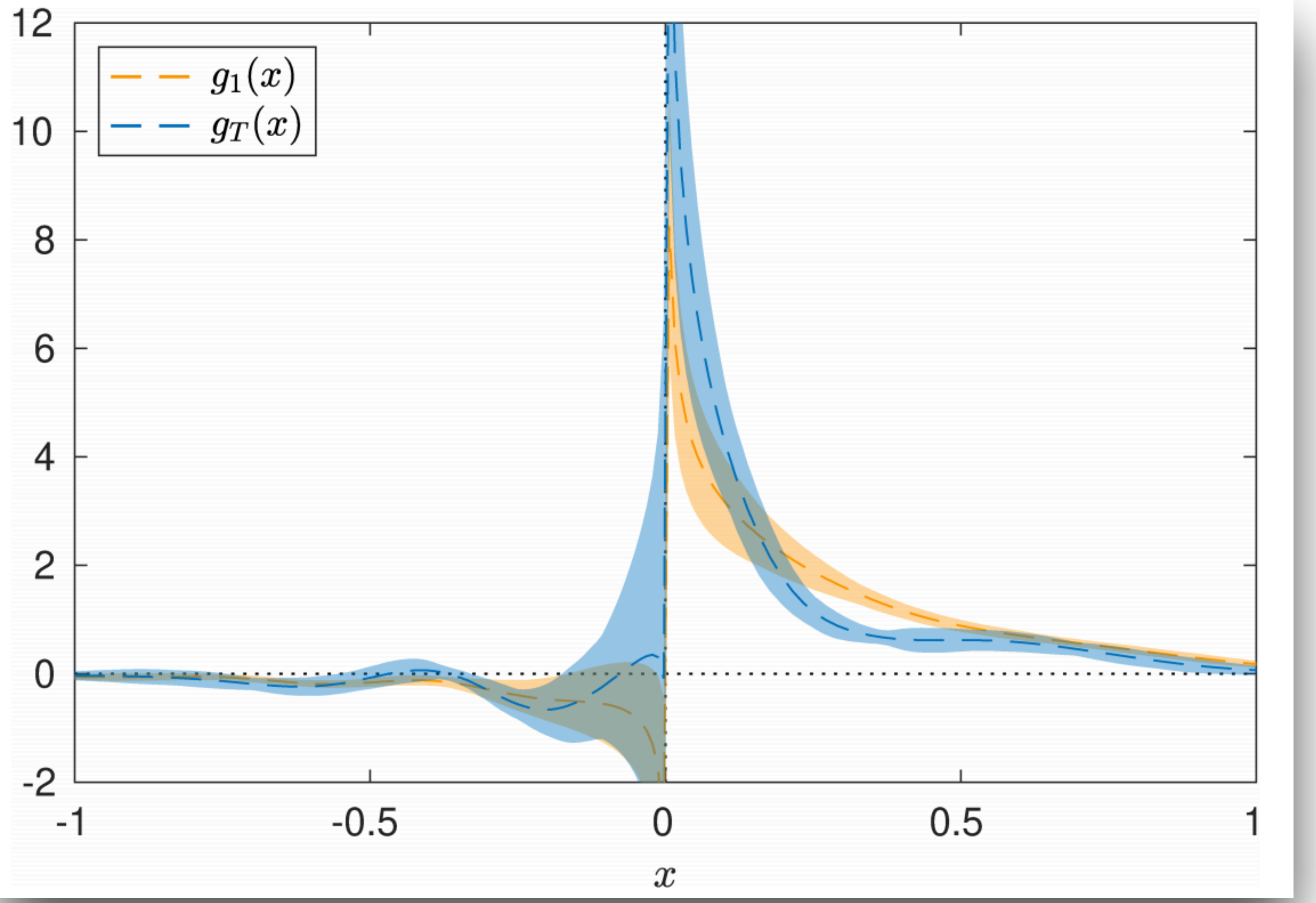


Investigations of Twist-3 PDFs/GPDs

Twist-3 exploration

Twist-3 exploration



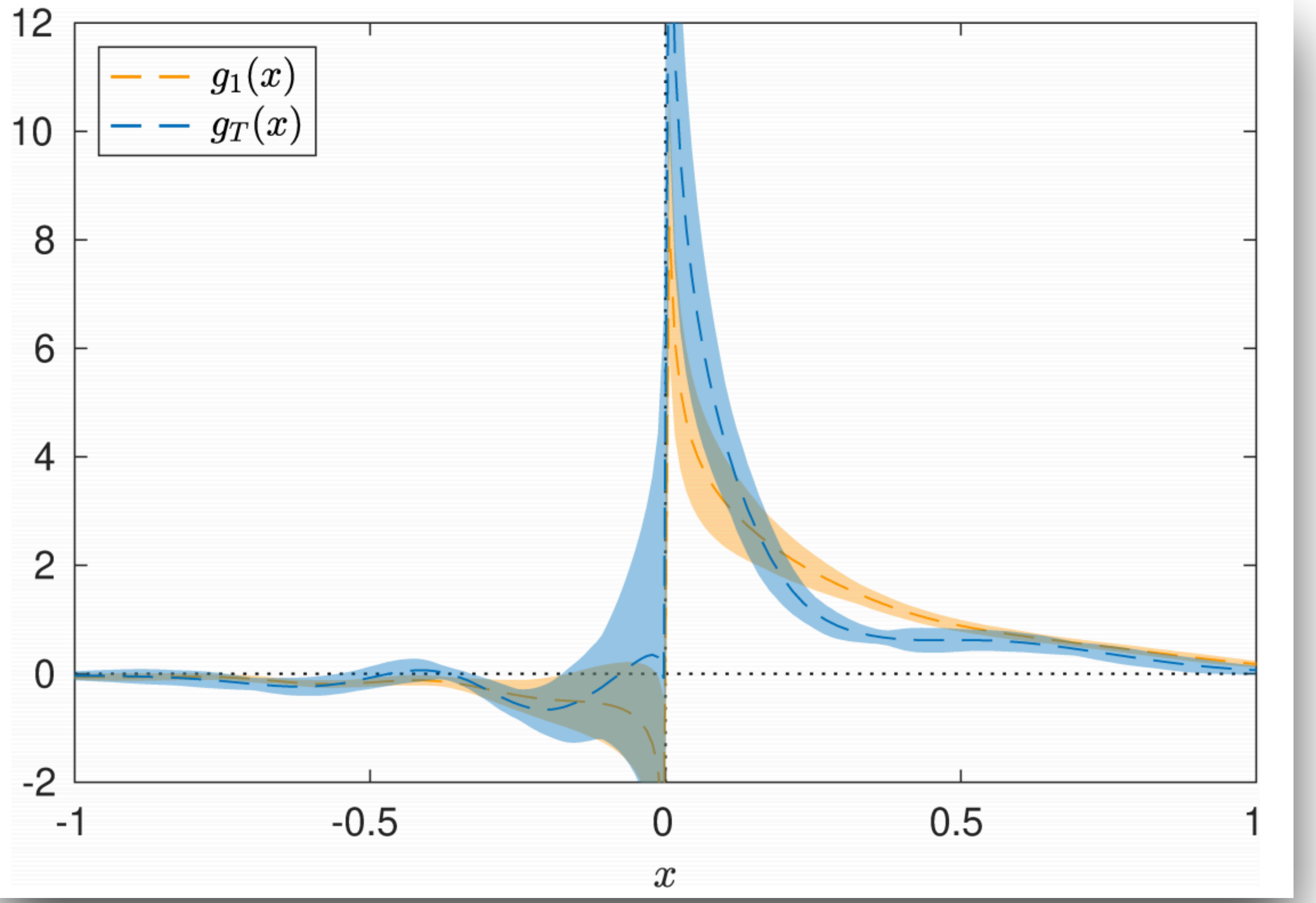
[S.Bhattacharya et al, PRD 102 (2020) 11, 111501]

Twist-3 counterpart as sizable as twist-2

Burkhardt-Cottingham sum rule important check

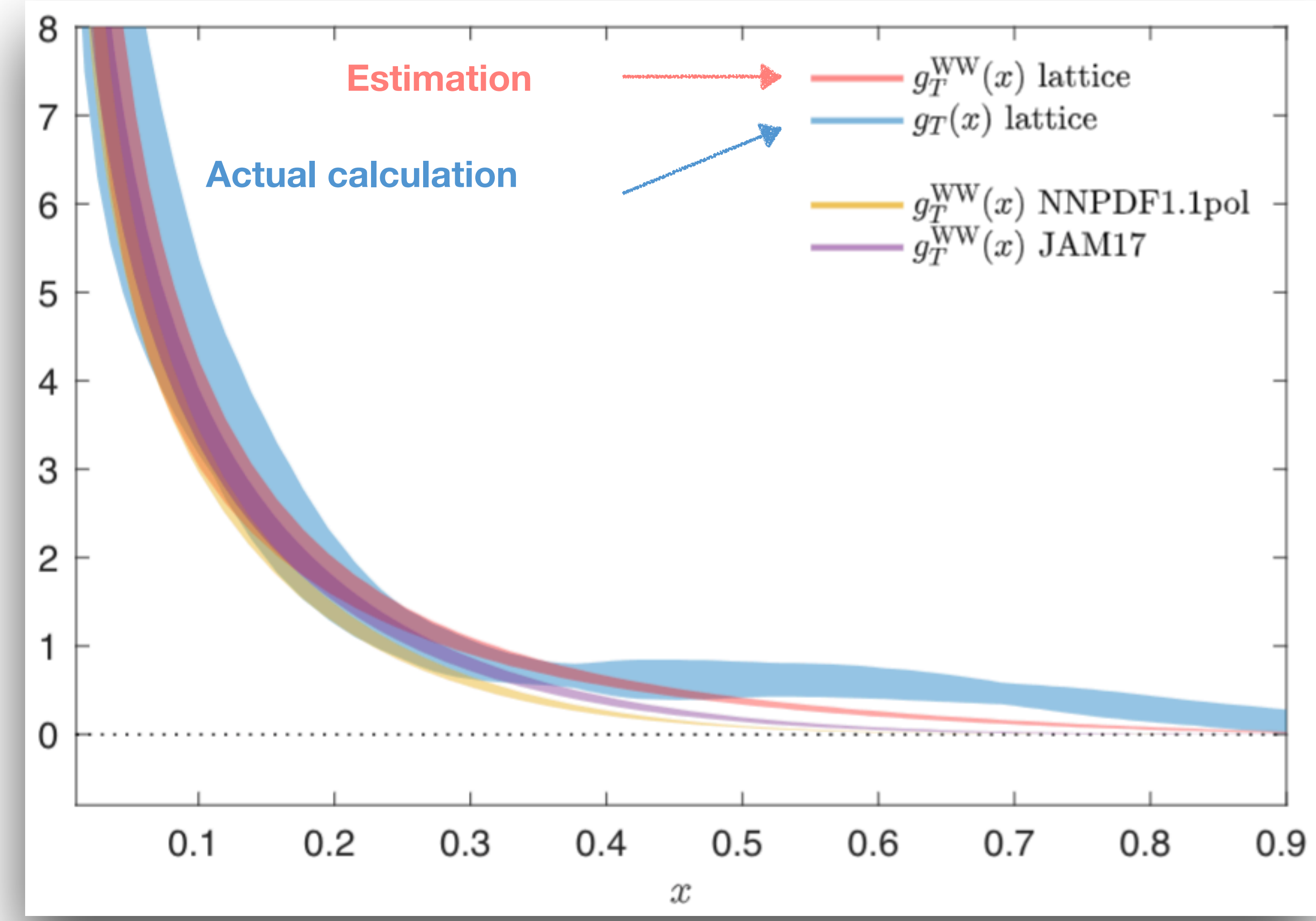
$$\int_{-1}^{1} dx\, g_1(x) - \int_{-1}^{1} dx\, g_T(x) = 0.01(20)$$

Twist-3 exploration



[S.Bhattacharya et al, PRD 102 (2020) 11, 111501]

★ WW approximation



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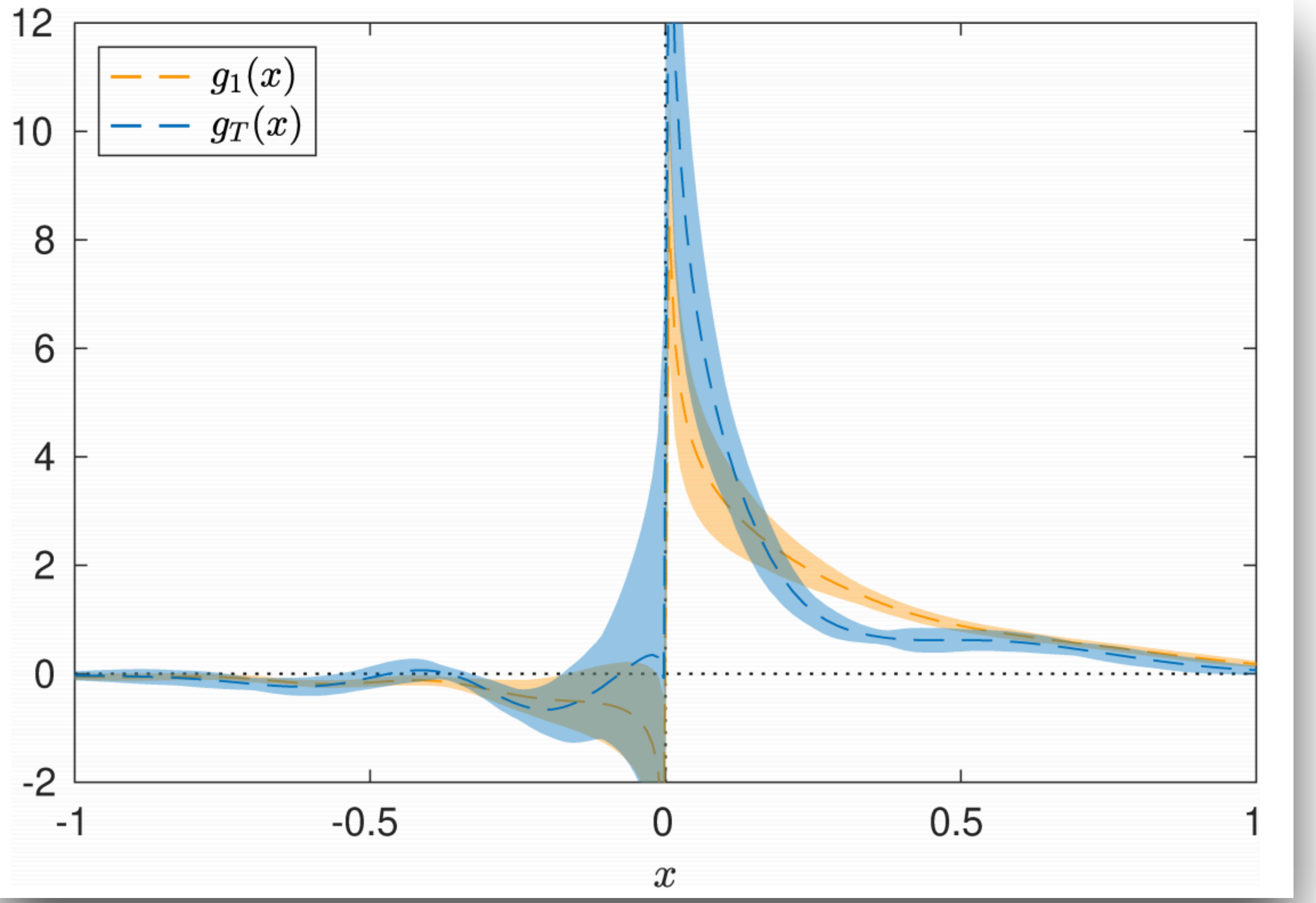

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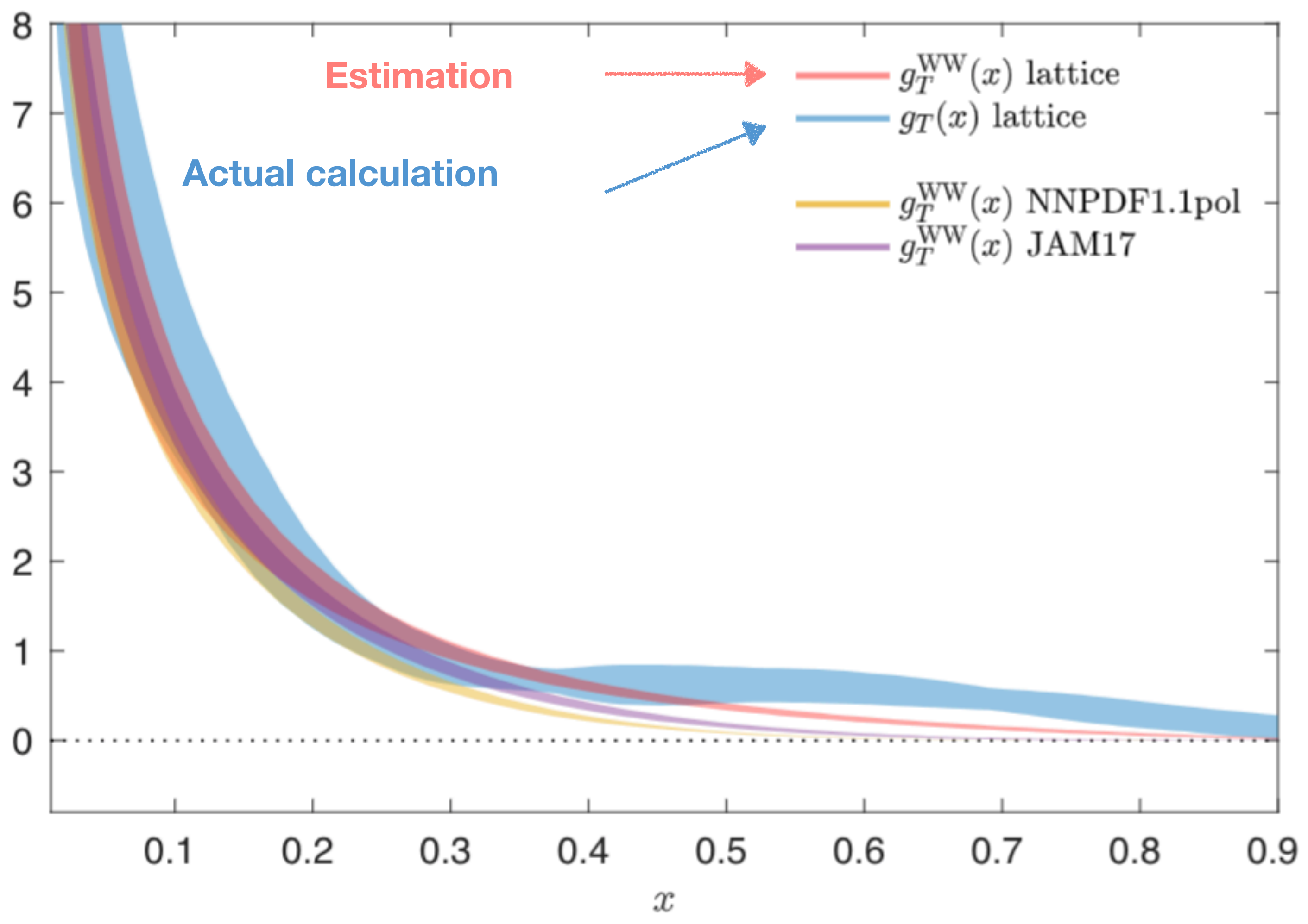
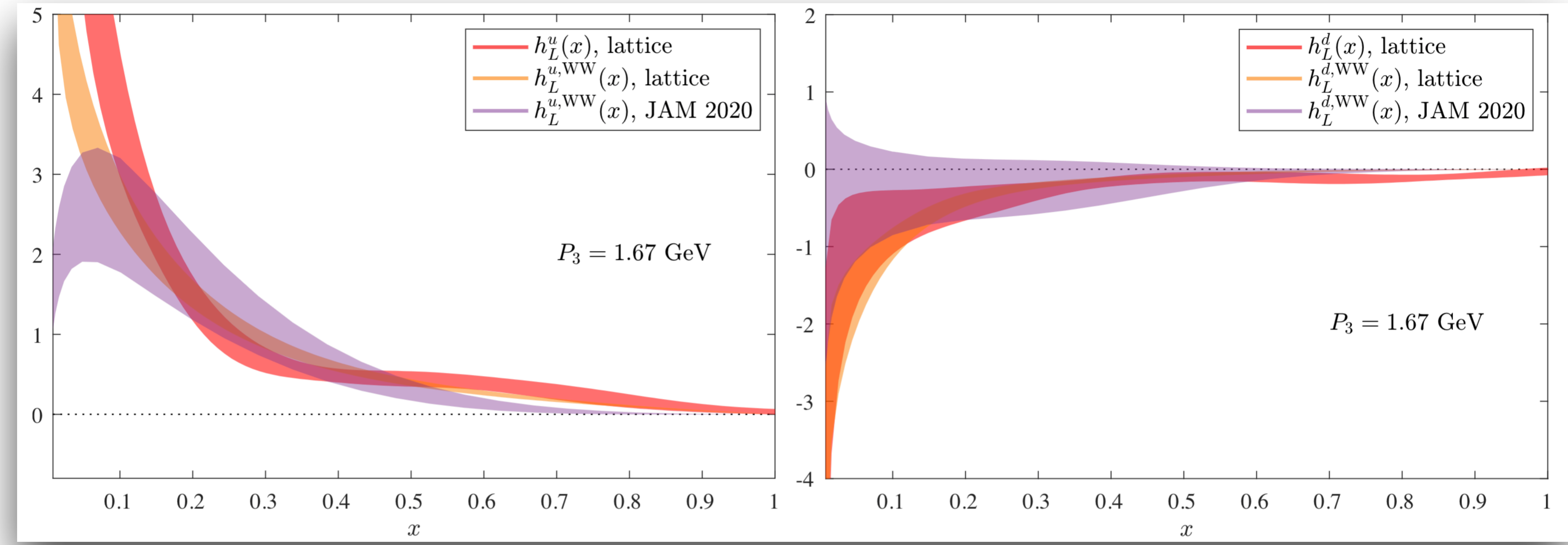


★ Flavor decomposition for $h_L(x)$

[S.Bhattacharya et al, *Phys.Rev.D* 104 (2021) 11, 114510]



Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

★ Calculation:
- isovector combination
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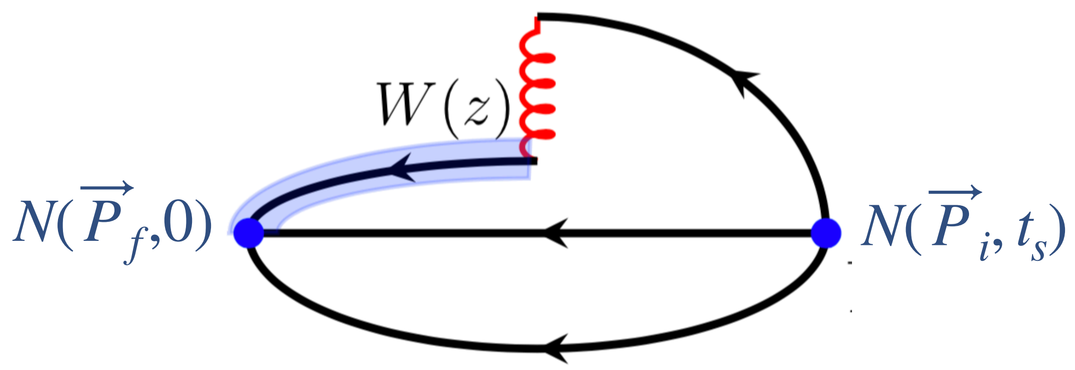

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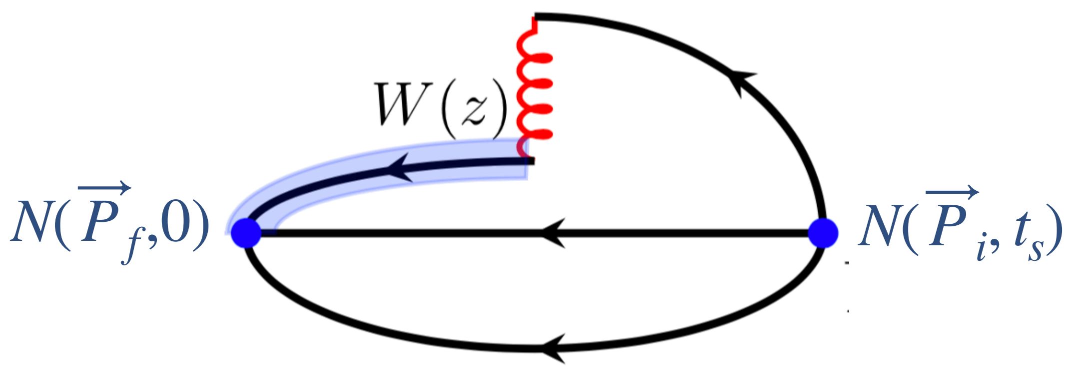

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±1.25	(0, 0, 0)	0	2	731	16	23392
±1.67	(0, 0, 0)	0	2	1644	64	210432
±0.83	(±2, 0, 0)	0.69	8	67	8	4288
±1.25	(±2, 0, 0)	0.69	8	249	8	15936
±1.67	(±2, 0, 0)	0.69	8	294	32	75264
±1.25	(±2, ±2, 0)	1.38	16	224	8	28672
±1.25	(±4, 0, 0)	2.76	8	329	32	84224

Symmetric frame computationally expensive

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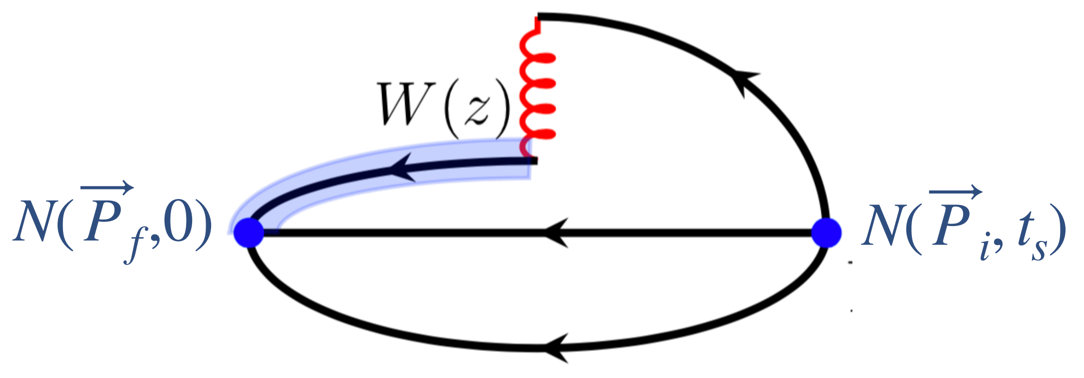

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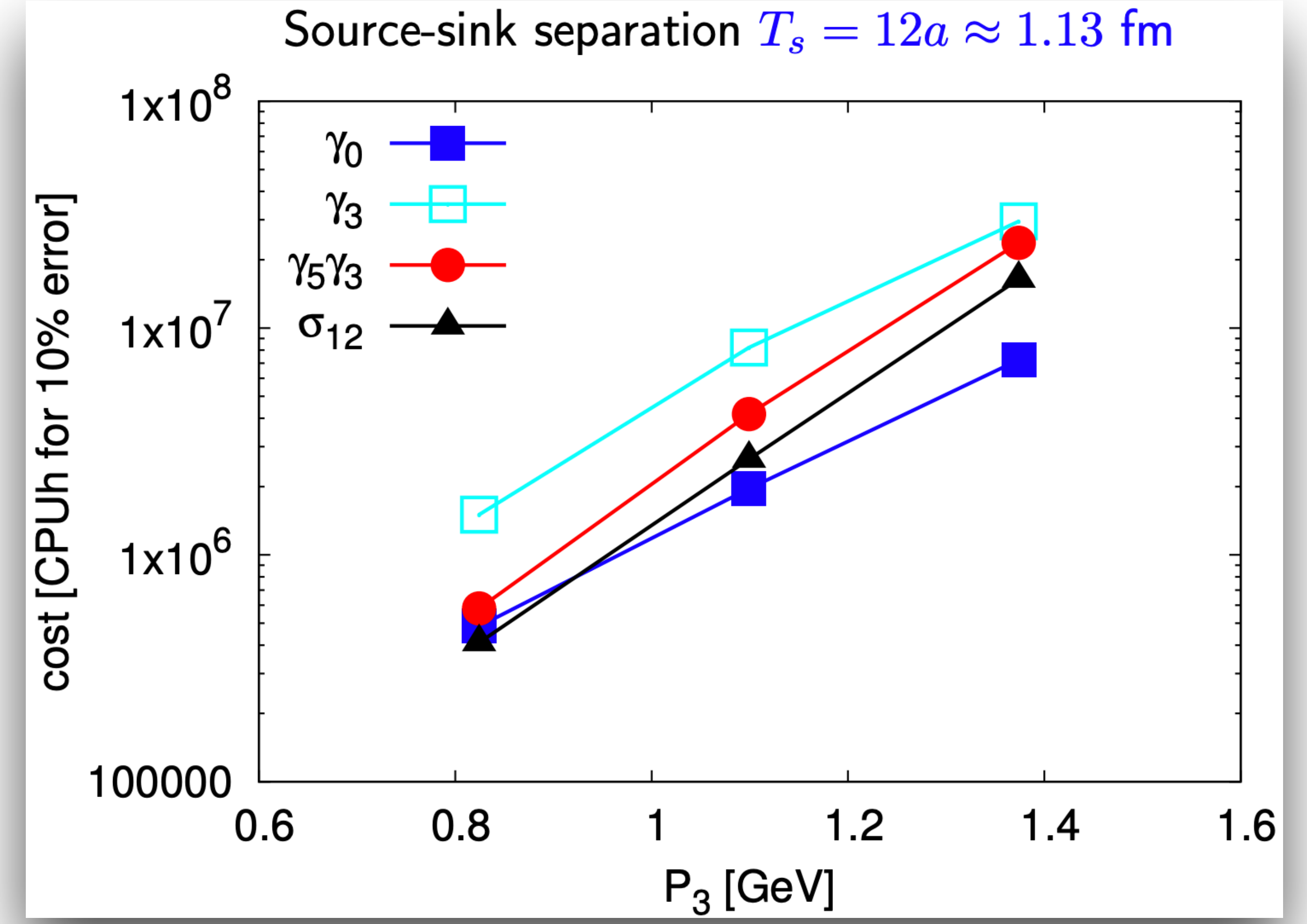

Symmetric frame computationally expensive

Suppressing gauge noise and reliably extracting the ground state comes at a significant computational cost

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3)=\frac{1}{2}\int\frac{dz^3}{2\pi}e^{ik\cdot z}\langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3)=\frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3)+P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
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★ Kinematic twist-three contributions to pseudo- and quasi-GPDs to restore translation invariance

[V. Braun et al., JHEP 10 (2023) 134]

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★ Twist-3 contributions to helicity GPDs: $\Gamma = \gamma^j\gamma_5,\ j=1,2$

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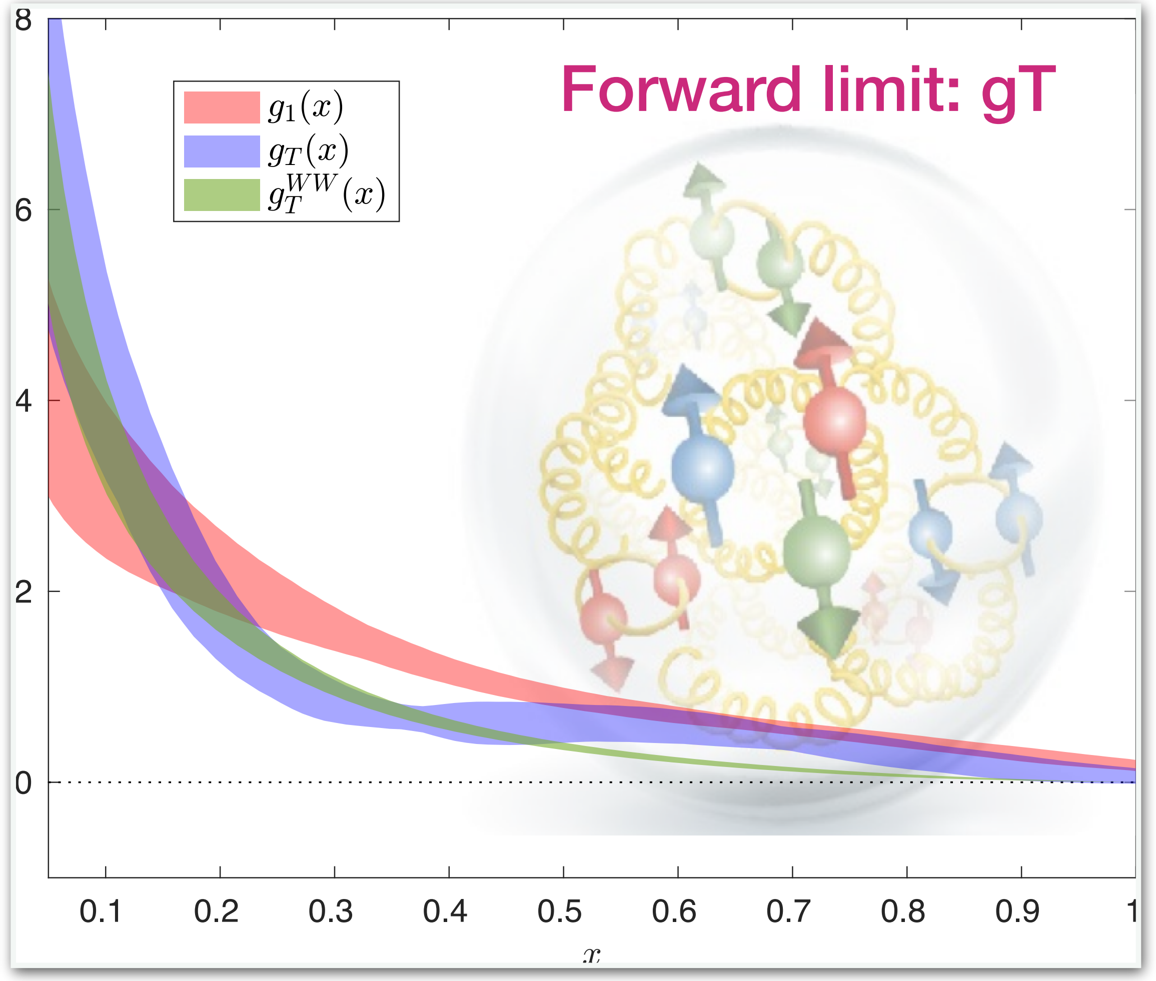

[S. Bhattacharya et al., PRD 102 (2020) 11] (Editors Highlight)

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Decomposition

★ Requirement: four independent matrix elements

$P_3\,[\mathrm{GeV}]$	$\vec{q}\,[\frac{2\pi}{L}]$	$-t\,[\mathrm{GeV}^2]$
±0.83	$(0,0,0)$	0
±1.25	$(0,0,0)$	0
±1.67	$(0,0,0)$	0
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±1.25	$(\pm2,\pm2,0)$	1.38
±1.25	$(\pm4,0,0)$	2.76

★ Average kinematically equivalent matrix elements

$$\Pi^1(\Gamma_0) = C\left(-F_{\widetilde{H}+\widetilde{G}_2}\frac{P_3\Delta_y}{4m^2} - F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_y(E+m)}{2m^2}\right),$$

$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{\left(4m(E+m)+\Delta_y^2\right)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2P_3}\right),$$

$$\Pi^1(\Gamma_2) = i\,C\left(-F_{\widetilde{H}+\widetilde{G}_2}\frac{\Delta_x\Delta_y}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x\Delta_y(E+m)}{8m^3} - F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_x\Delta_y(E+m)}{4m^2P_3}\right),$$

$$\Pi^1(\Gamma_3) = C\left(-F_{\widetilde{G}_3}\frac{E\Delta_x(E+m)}{2m^2P_3}\right),$$

$$\Pi^2(\Gamma_0) = C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{P_3\Delta_x}{4m^2} + F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_x(E+m)}{2m^2}\right),$$

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Consistency Checks

★ Sum Rules (generalization of Burkhardt-Cottingham)

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

$$\int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t) = G_A(t)\,, \quad \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

★ Sum Rules (generalization of Efremov-Leader-Teryaev)

[A. Efremov, O. Teryaev , E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_3(x,0,t) = \frac{\xi}{4} G_E \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,0,t) = \frac{1}{4} G_E(t)$$

G_E : electric FF

Lattice Results - Matrix Elements

★ **Bare matrix elements**

$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{(4m(E+m)+\Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\frac{\text{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2P_3}\right)$$

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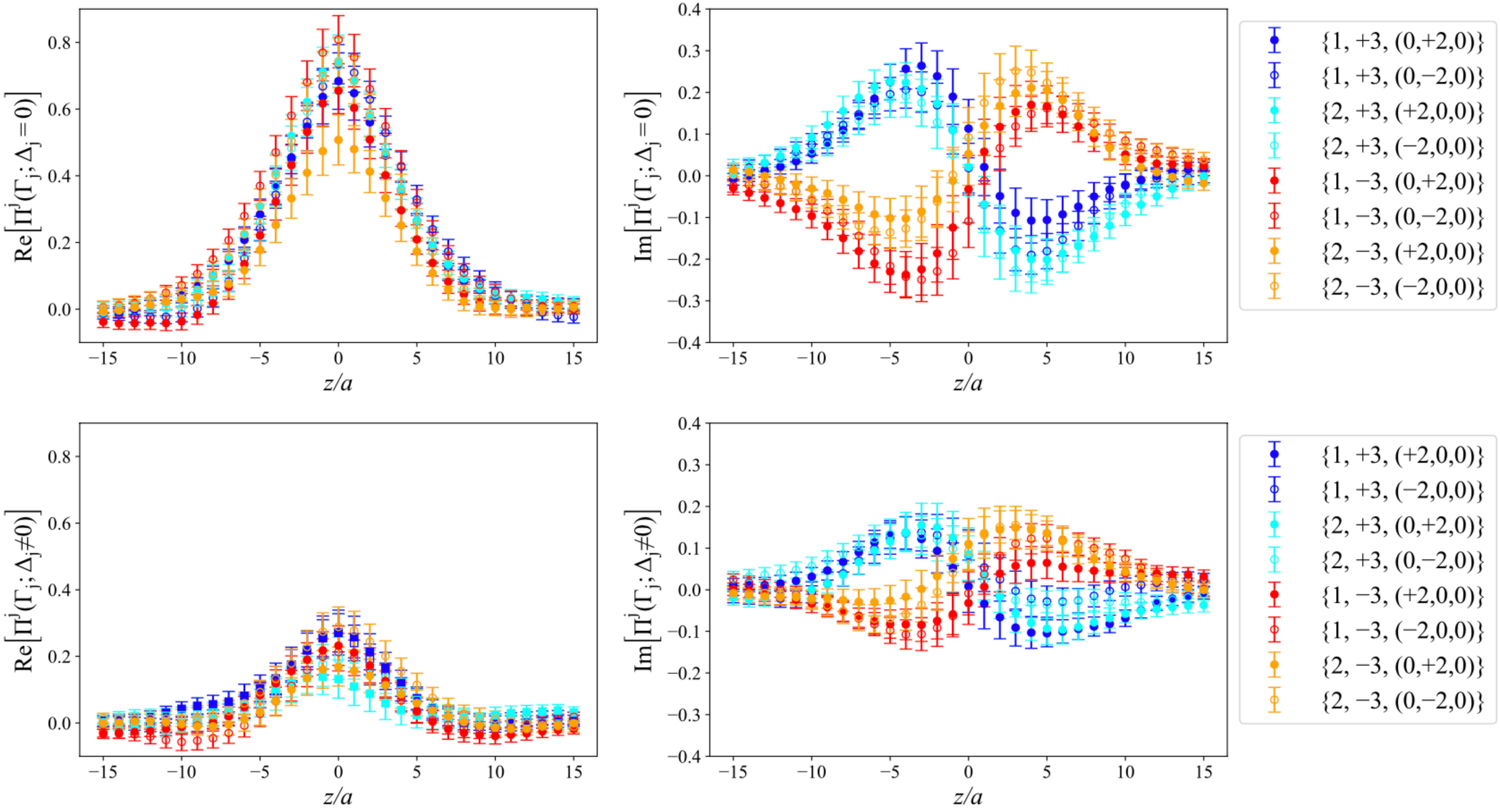

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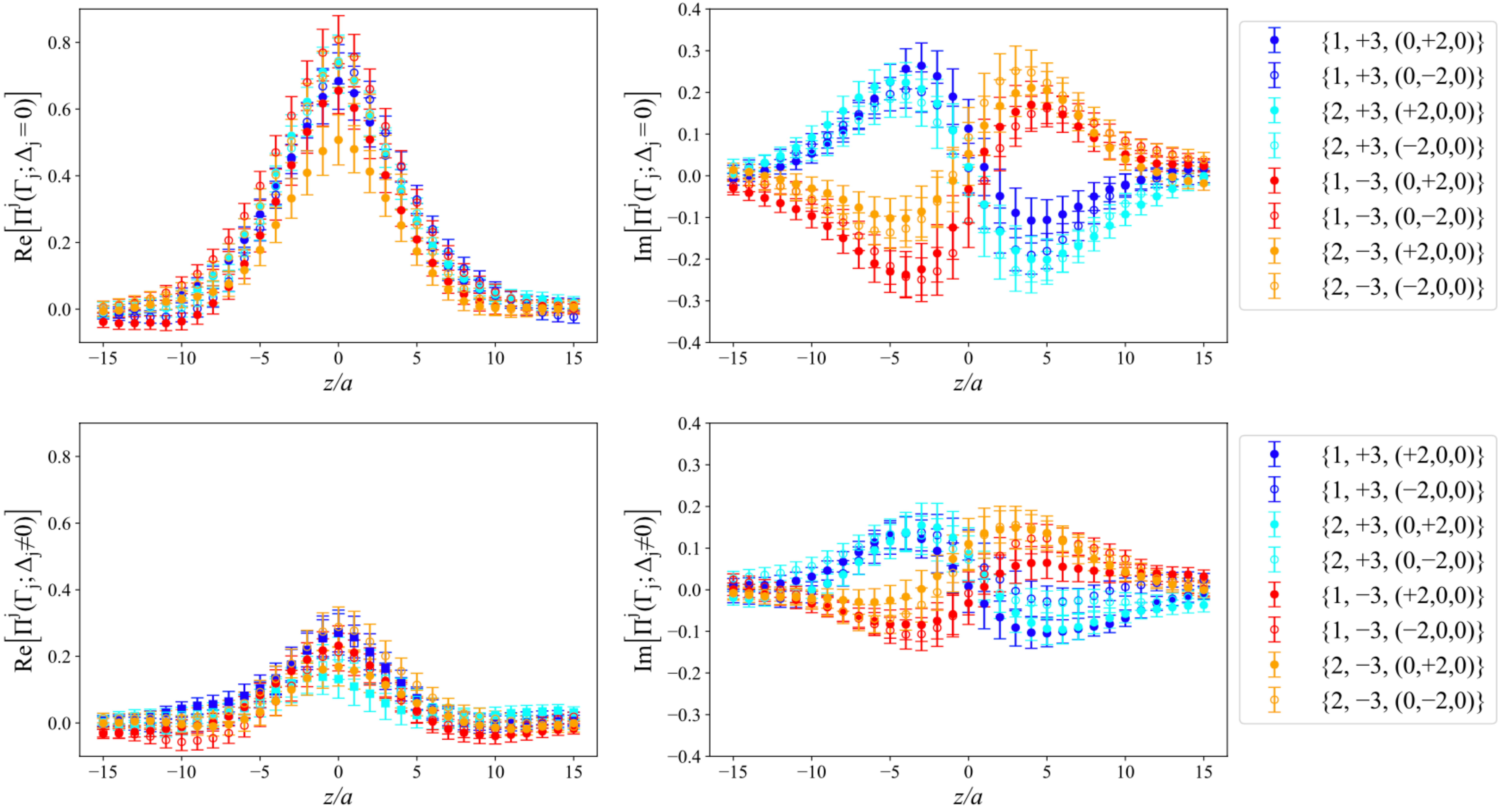


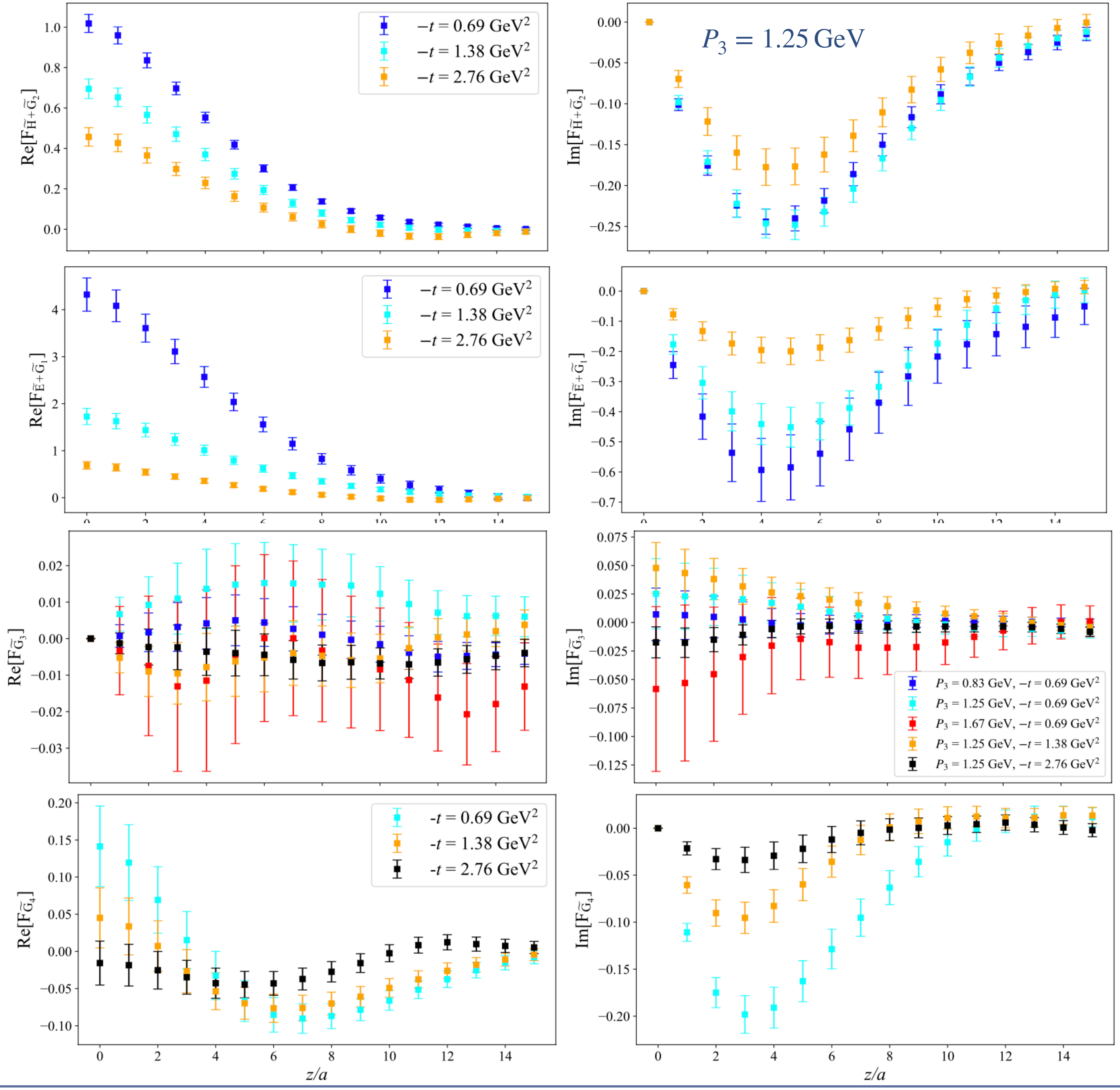
Lattice Results - quasi-GPDs

$F_{\widetilde{H}+\widetilde{G}_2}$

$F_{\widetilde{E}+\widetilde{G}_1}$

$F_{\widetilde{G}_3}$

$F_{\widetilde{G}_4}$



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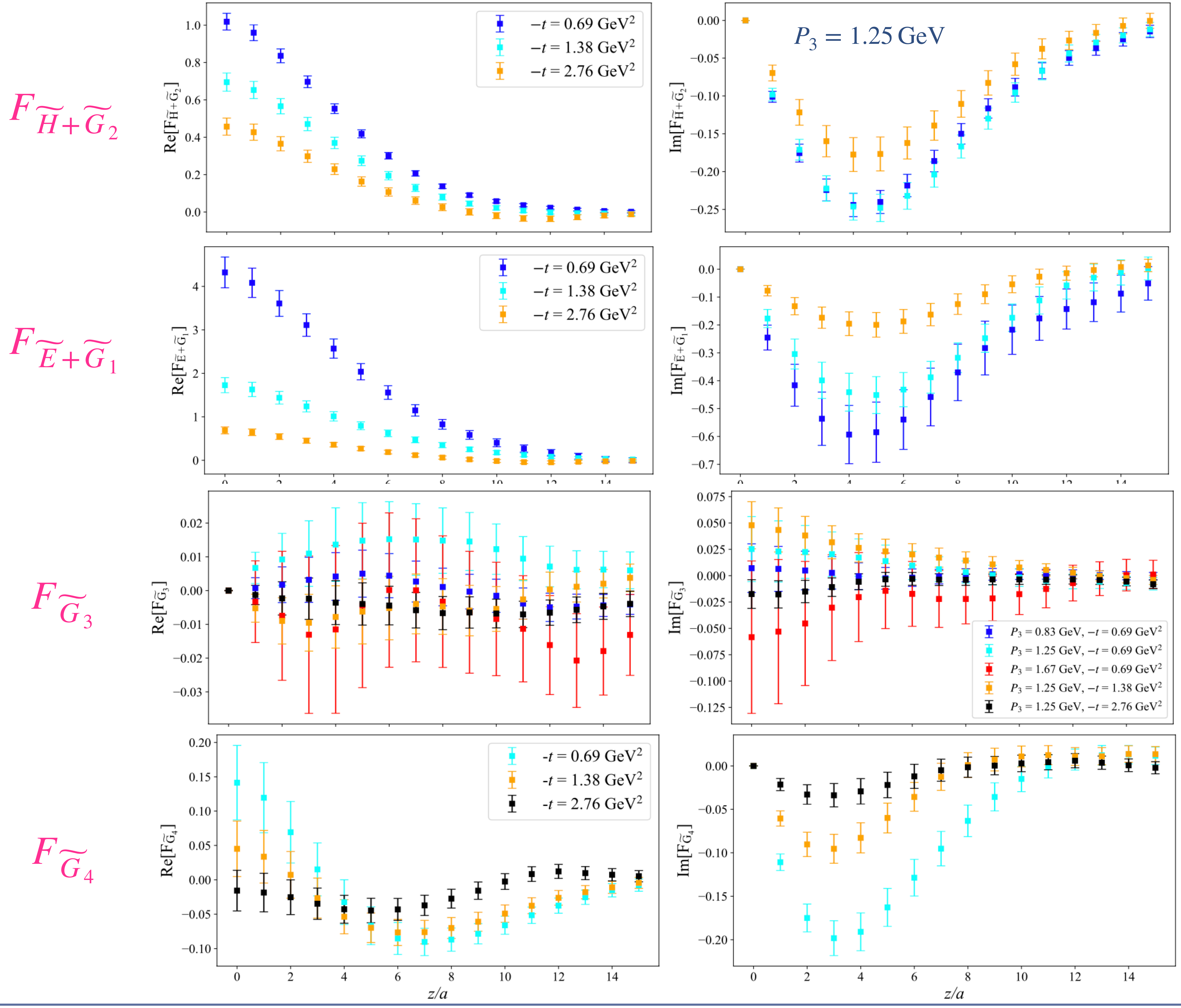


Indeed, numerically found to be zero within uncertainties at ξ=0

$$\int dx\, x\, \widetilde{G}_3 = \frac{\xi}{4} G_E(t)$$

Reconstruction of x-dependence & matching

★ quasi-GPDs transformed to momentum space using Backus Gilbert
[G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)]

★ Matching formalism to 1 loop accuracy level

$$F_X^{\mathrm{M}\overline{\mathrm{MS}}}(x,t,P_3,\mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C^{\mathrm{M}\overline{\mathrm{MS}},\overline{\mathrm{MS}}}_{\gamma_j\gamma_5}\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) G_X^{\overline{\mathrm{MS}}}(y,t,\mu) \; + \mathcal{O}\left(\frac{m^2}{P_3^2}, \frac{t}{P_3^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{x^2P_3^2}\right)$$

PHYSICAL REVIEW D **102,** 034005 (2020)

One-loop matching for the twist-3 parton distribution $g_T(x)$

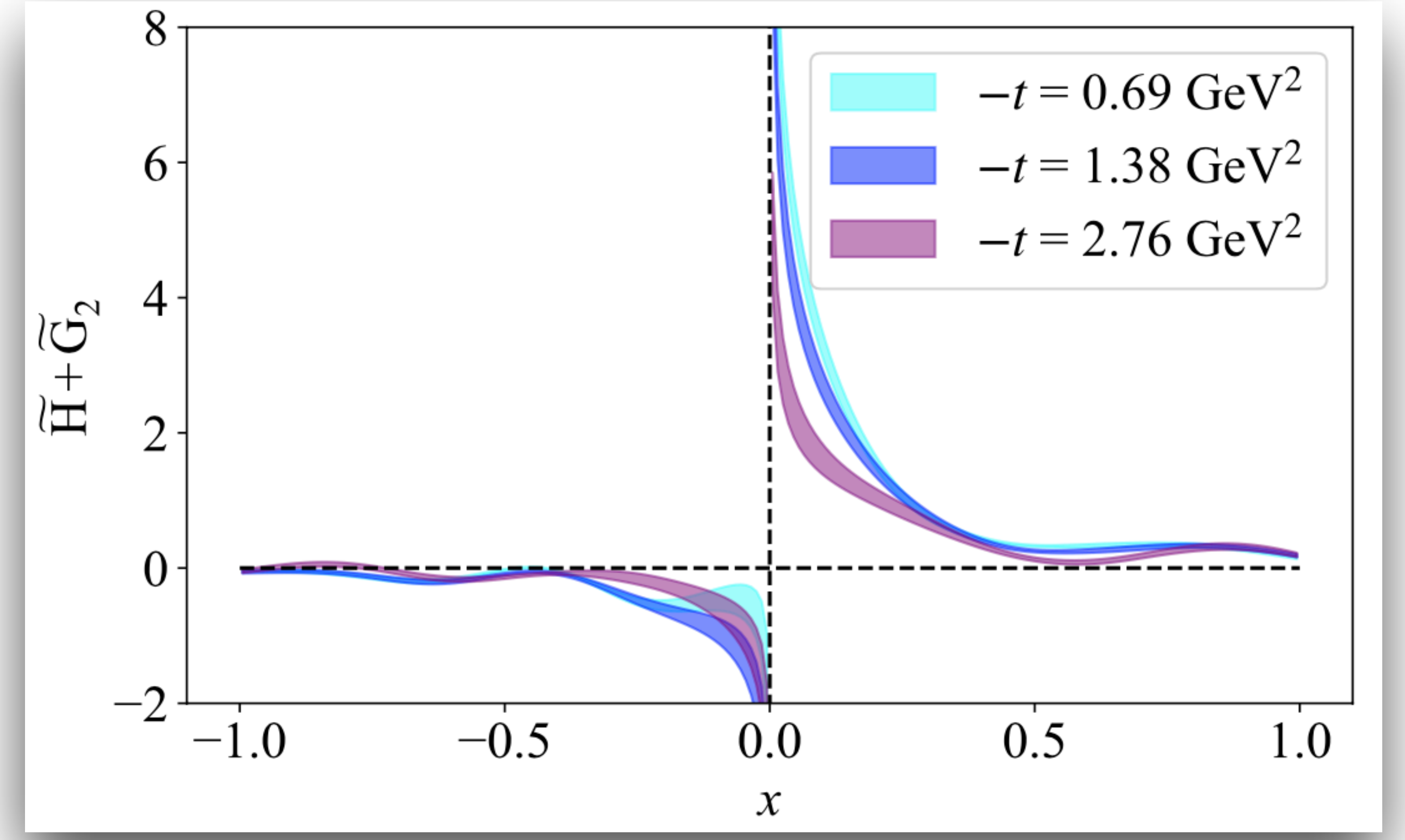
Shohini Bhattacharya[1], Krzysztof Cichy,[2] Martha Constantinou[1], Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

★ Operator dependent kernel

$$C^{(1)}_{\mathrm{M}\overline{\mathrm{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} 0 \\ \delta(\xi) \\ 0 \end{cases} + \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ & \xi > 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{4\xi(1-\xi)(xP_3)^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0<\xi<1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ & \xi<0\,, \end{cases}$$

★ Matching does not consider mixing with q-g-q correlators
[V. Braun et al., JHEP 05 (2021) 086]

Lattice Results - light-cone GPDs



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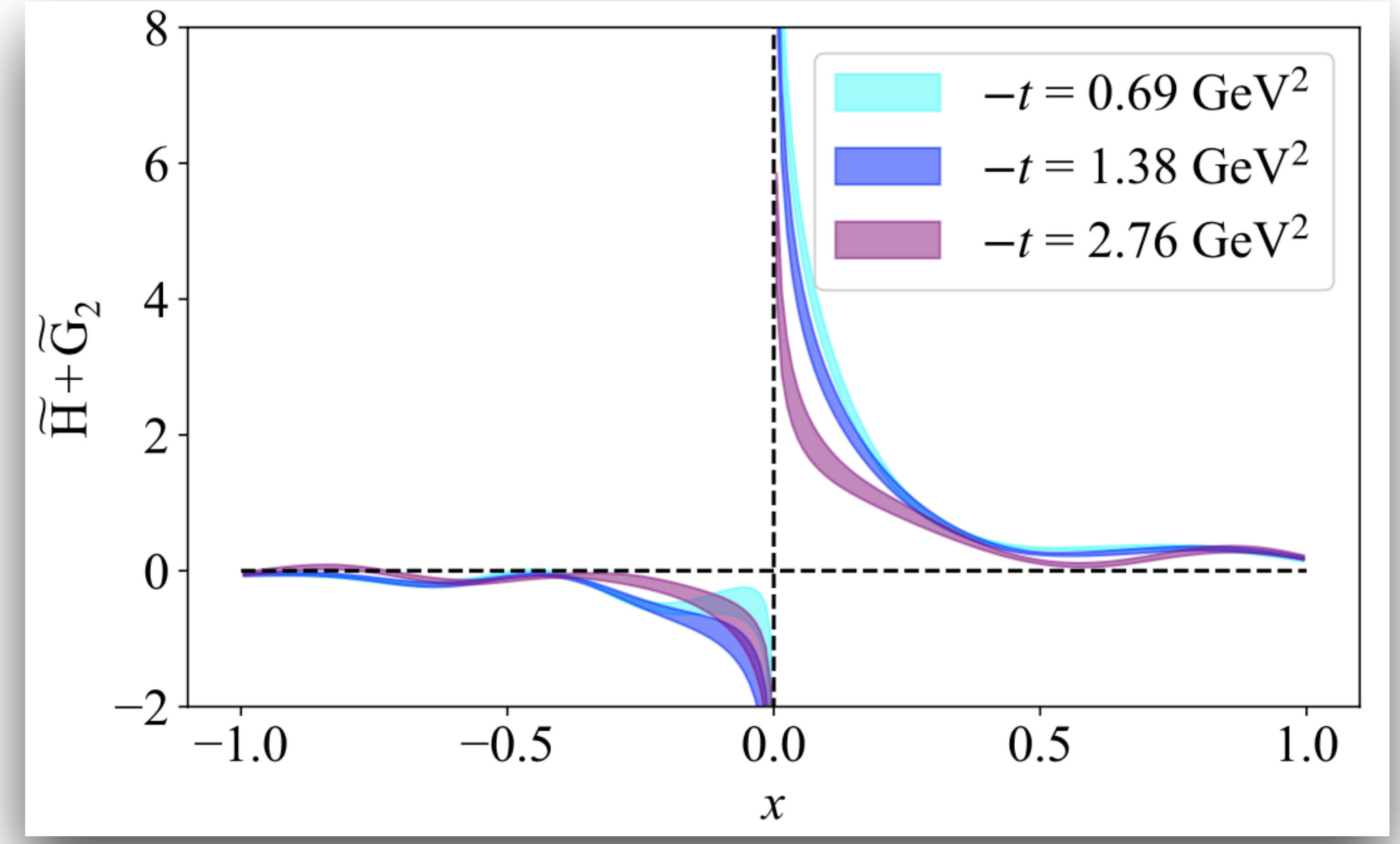
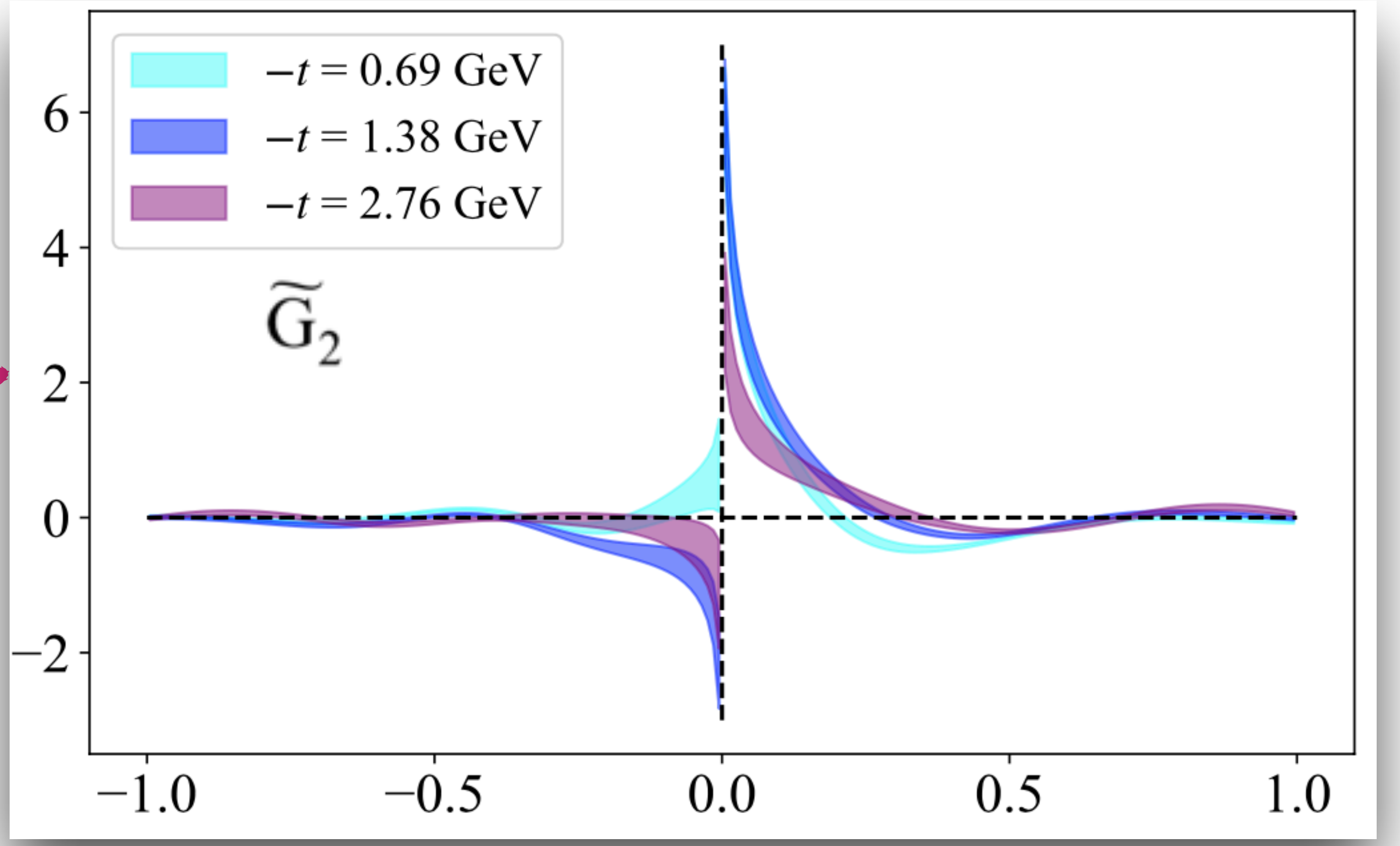


Isolating $\widetilde{G}_2$ using $\widetilde{H}$



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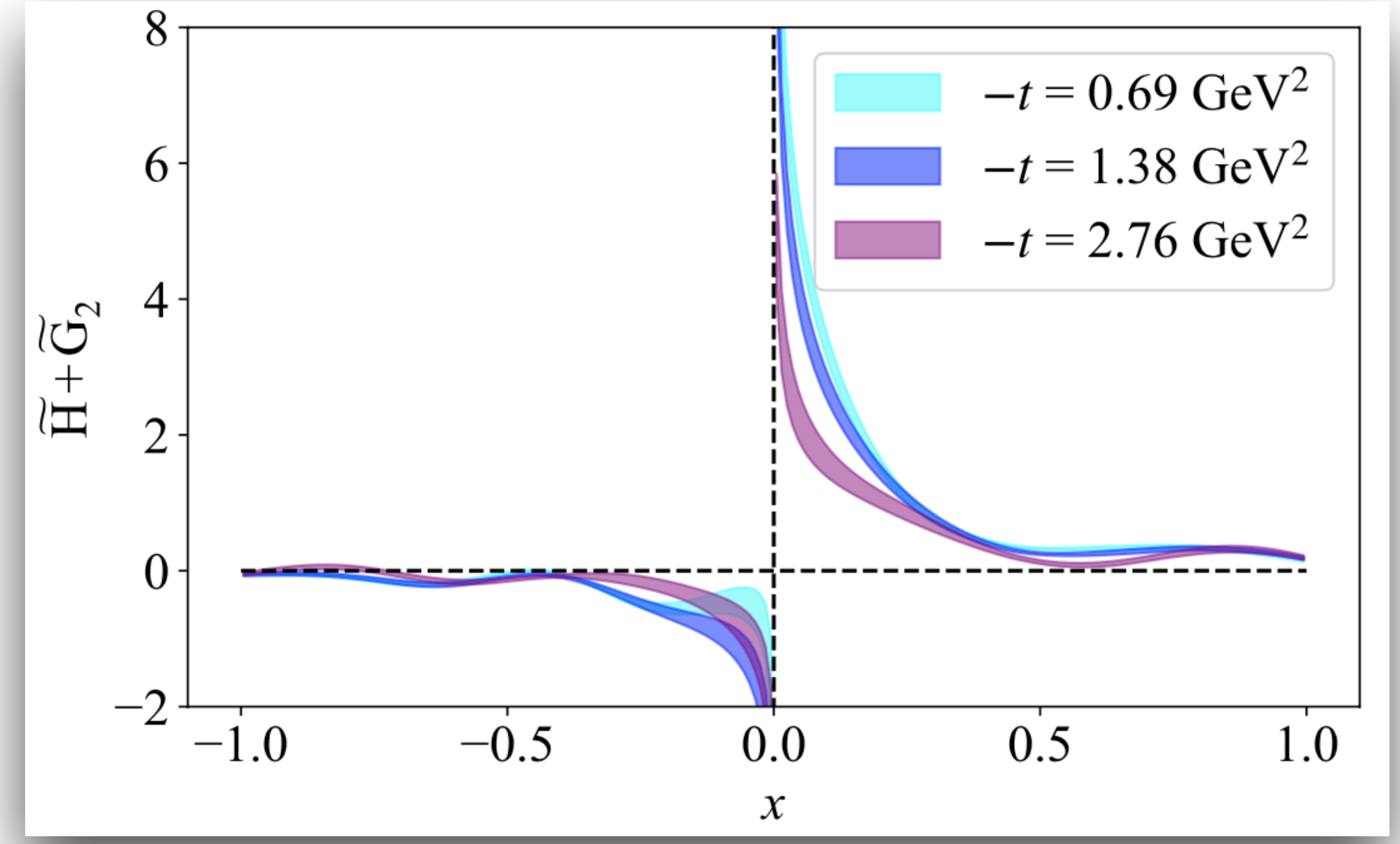

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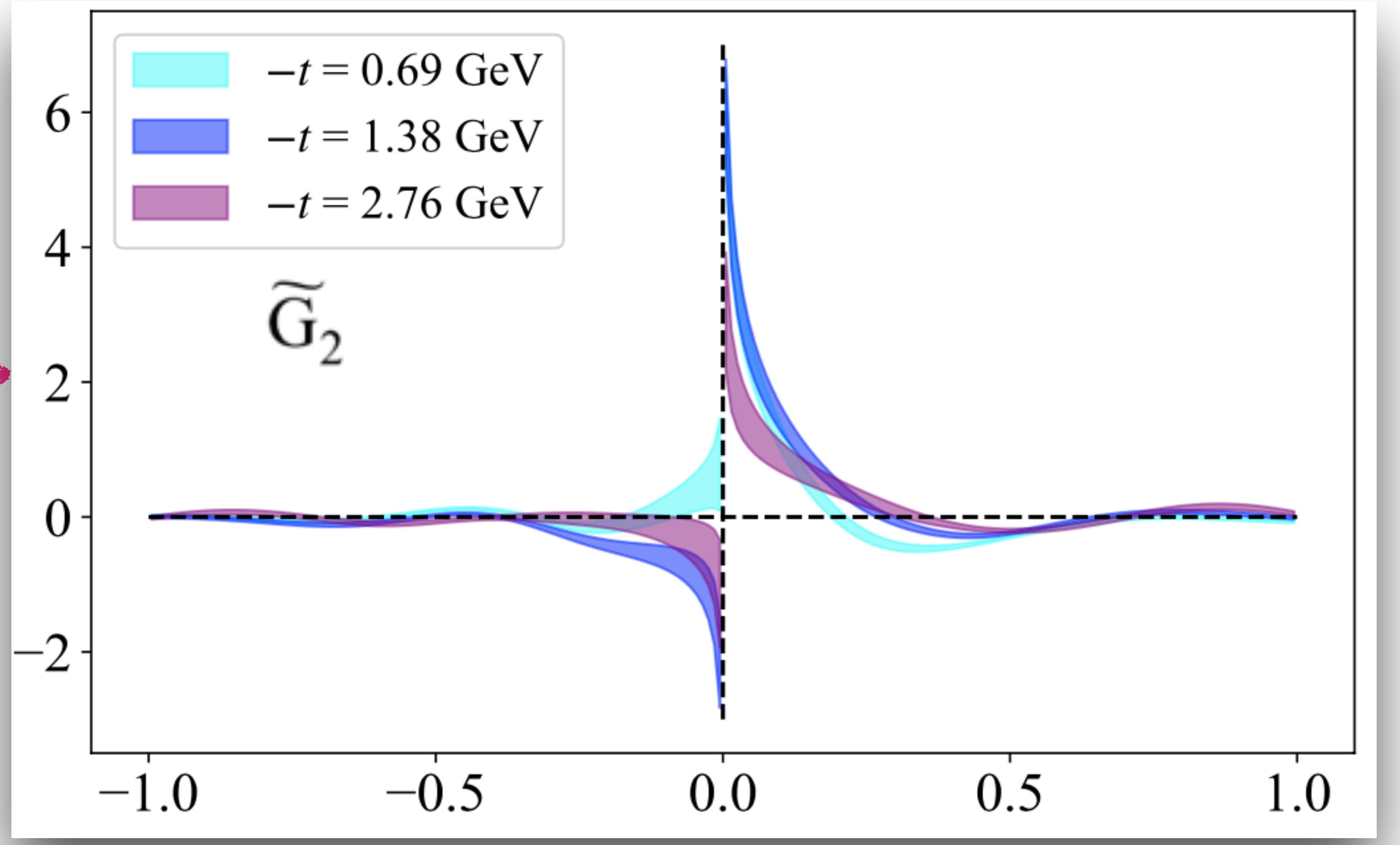

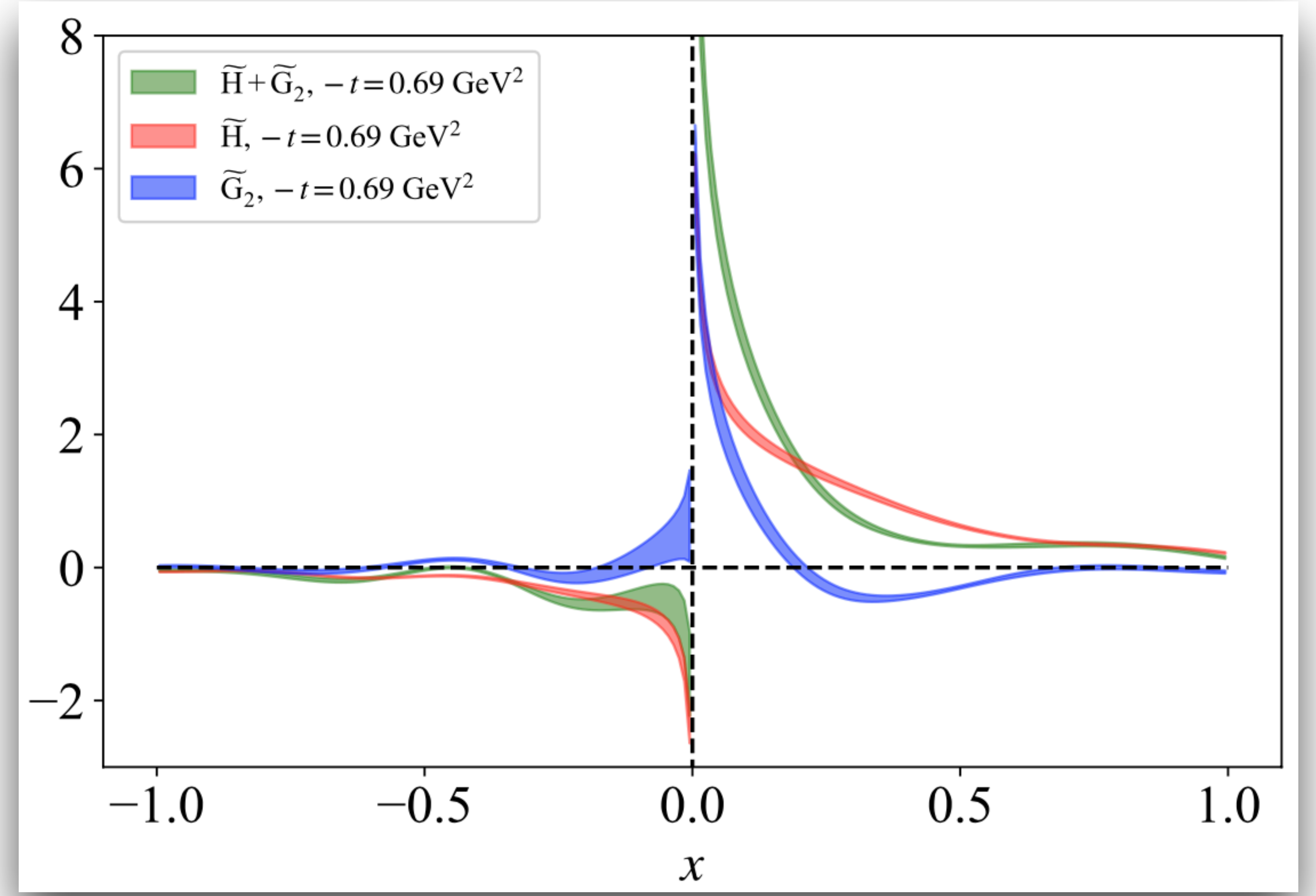

Negative areas in $\widetilde{G}_2$ theoretically anticipated:

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

Lattice Results - light-cone GPDs

★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

$$P^\mu \frac{\gamma^3\gamma_5}{P^0} F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu \frac{\Delta^3\gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

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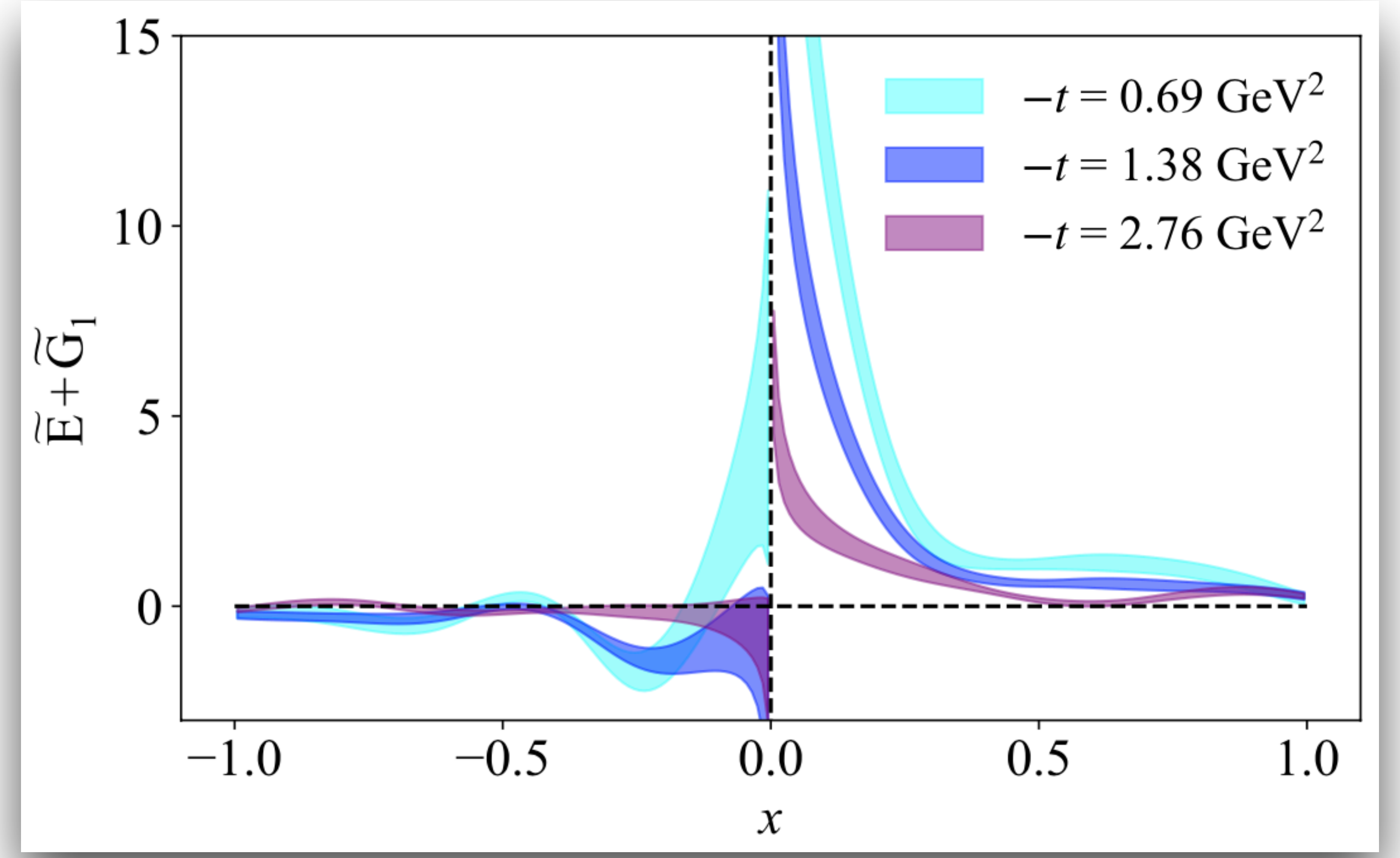

★ Sizable contributions as expected

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

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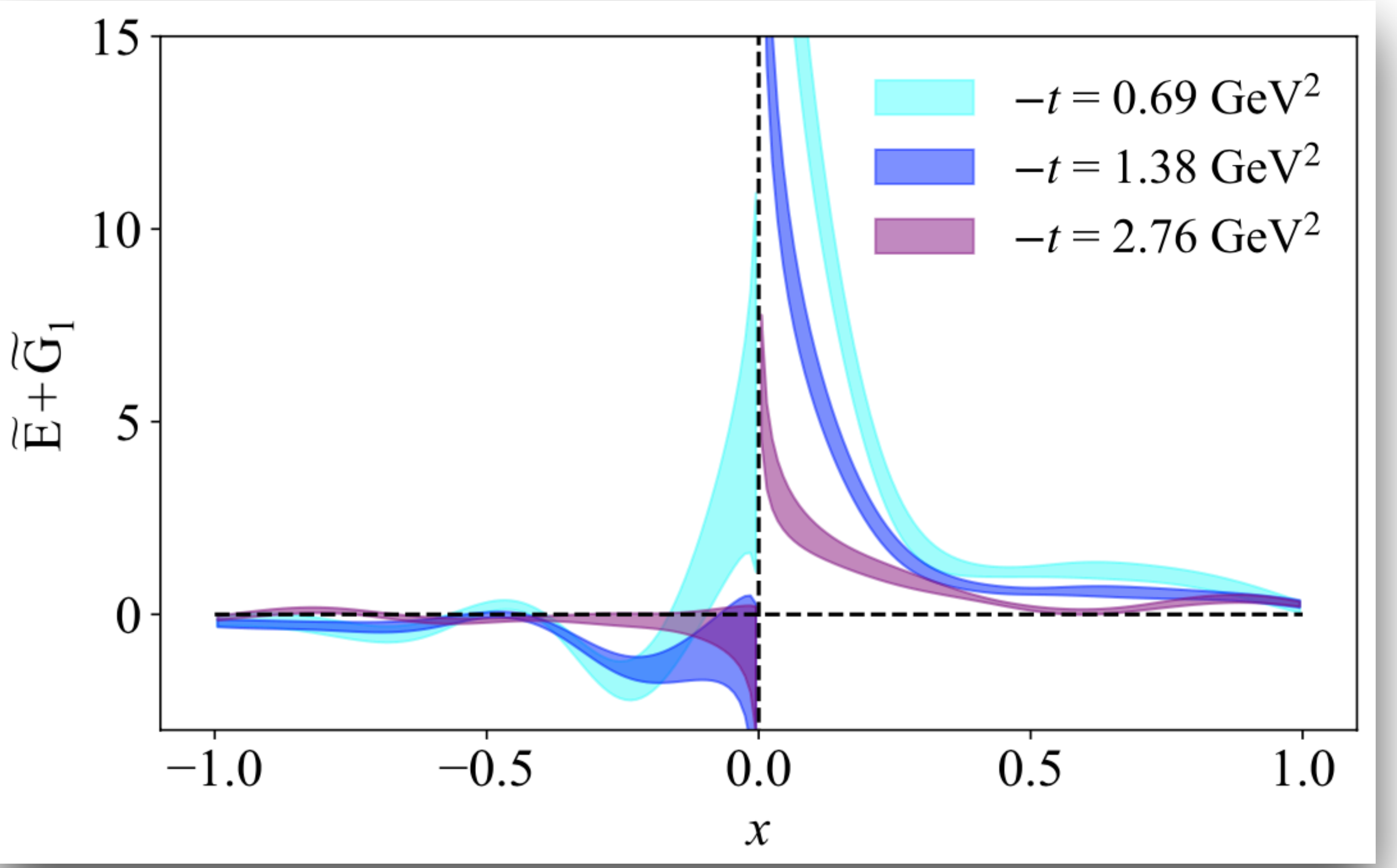
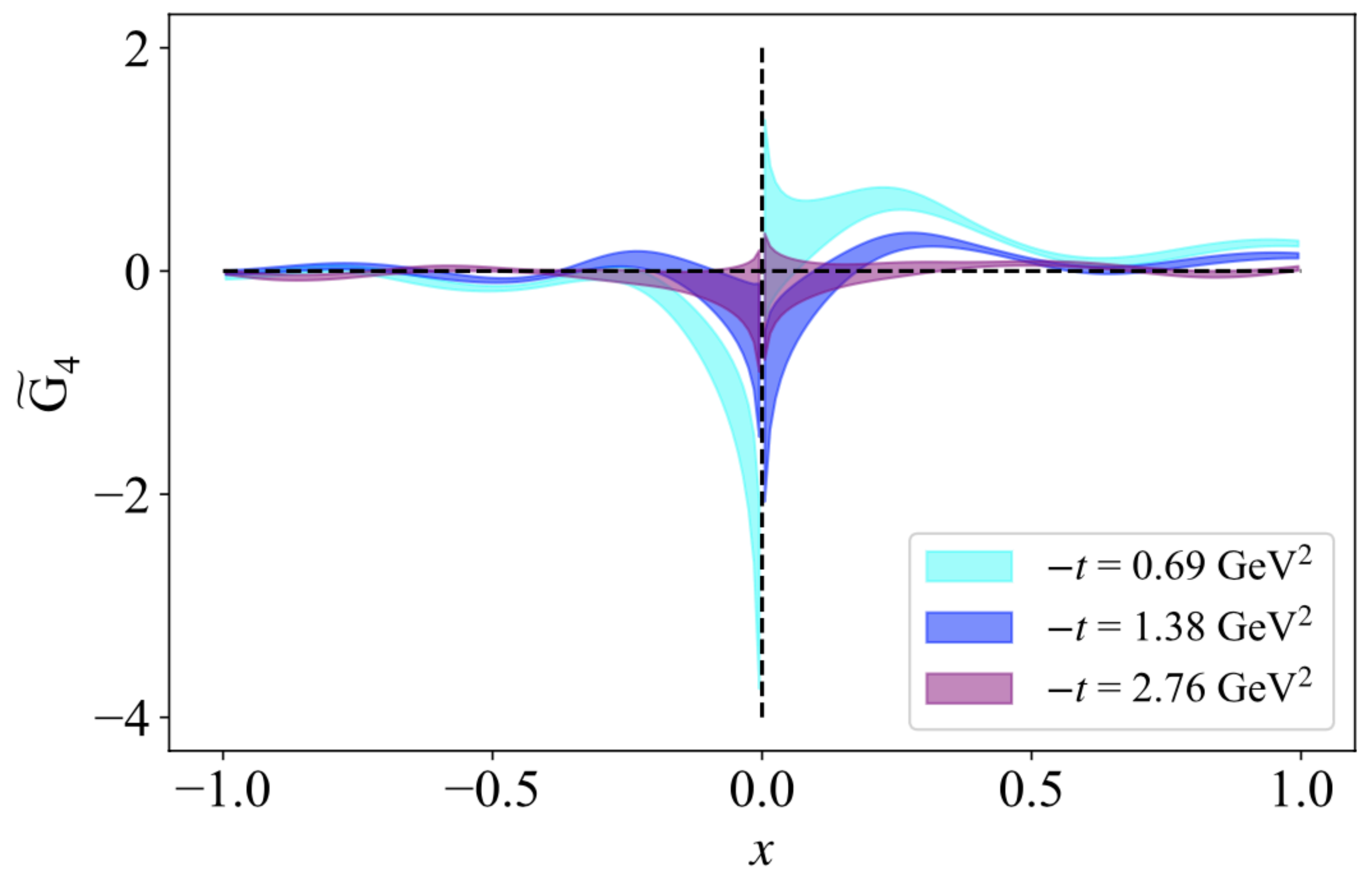


★ $\widetilde{G}_3(\xi = 0) = 0$, $\widetilde{G}_4$: small



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$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

★ $\widetilde{G}_4$ very small; no theoretical argument to be zero

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E$$

Consistency checks

★ Norms

$$\int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t) = G_A(t)\,, \quad \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

★ Consistency checks show encouraging results

★ Refining calculations is needed to address systematic effects and extract reliable numbers

Alternative setup

- Alternative kinematic setup can be utilized

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105]

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3) + \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3) + \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

[Bhattacharya et al., arXiv:2310.13114]

$$\widetilde{F}^\mu(z,P,\Delta) \equiv \langle p_f;\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\gamma^\mu\gamma_5\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i;\lambda\rangle = \bar{u}(p_f,\lambda')\Big[\frac{i\epsilon^{\mu Pz\Delta}}{m}\widetilde{A}_1 + \gamma^\mu\gamma_5\widetilde{A}_2 + \gamma_5\Big(\frac{P^\mu}{m}\widetilde{A}_3 + mz^\mu\widetilde{A}_4 + \frac{\Delta^\mu}{m}\widetilde{A}_5\Big) + m\not{z}\gamma_5\Big(\frac{P^\mu}{m}\widetilde{A}_6 + mz^\mu\widetilde{A}_7 + \frac{\Delta^\mu}{m}\widetilde{A}_8\Big)\Big]u(p_i,\lambda)\,,$$

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$$F^s_{\widetilde{G}_3} = zP_3\tilde{A}_8$$

$$F^s_{\widetilde{G}_4} = \frac{-EP_3}{m^2}\left(\frac{-E^2}{P_3} + P_3\right)z\tilde{A}_1$$

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★ Kinematic coefficients defined in symmetric frame

★ Amplitudes extracted from any frame.
Asymmetric frame calculations give $\tilde{A}_i$ at t^a, but F_i defined in t^s

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Lorentz transformation of kinematic factors

$$F^a_{\widetilde{E}+\widetilde{G}_1} = \frac{-E_f(E_f+E_i)}{P_3}z\tilde{A}_1 + 2\tilde{A}_5$$
$$F^a_{\widetilde{H}+\widetilde{G}_2} = \frac{-E_f^2(\Delta_x^2+\Delta_y^2)}{2m^2P_3}z\tilde{A}_1 + \tilde{A}_2$$
$$F^a_{\widetilde{G}_3} = zP_3\tilde{A}_8$$
$$F^a_{\widetilde{G}_4} = -\sqrt{\frac{E_f(E_f+E_i)}{2}}\frac{P_3}{m^2}\left(\frac{-E_f(E_f+E_i)}{2P_3}+P_3\right)z\tilde{A}_1$$

Amplitudes (proof-of-concept)

$$\vec{p}_f^{\,s} = \vec{P} + \frac{\vec{Q}}{2}, \qquad \vec{p}_i^{\,s} = \vec{P} - \frac{\vec{Q}}{2} \qquad t^s = -\vec{Q}^2$$

$$\vec{p}_f^{\,a} = \vec{P}, \qquad \vec{p}_i^{\,a} = \vec{P} - \vec{Q} \qquad t^a = -\vec{Q}^2 + (E_f - E_i)^2$$

[Bhattacharya et al., arXiv:2310.13114]

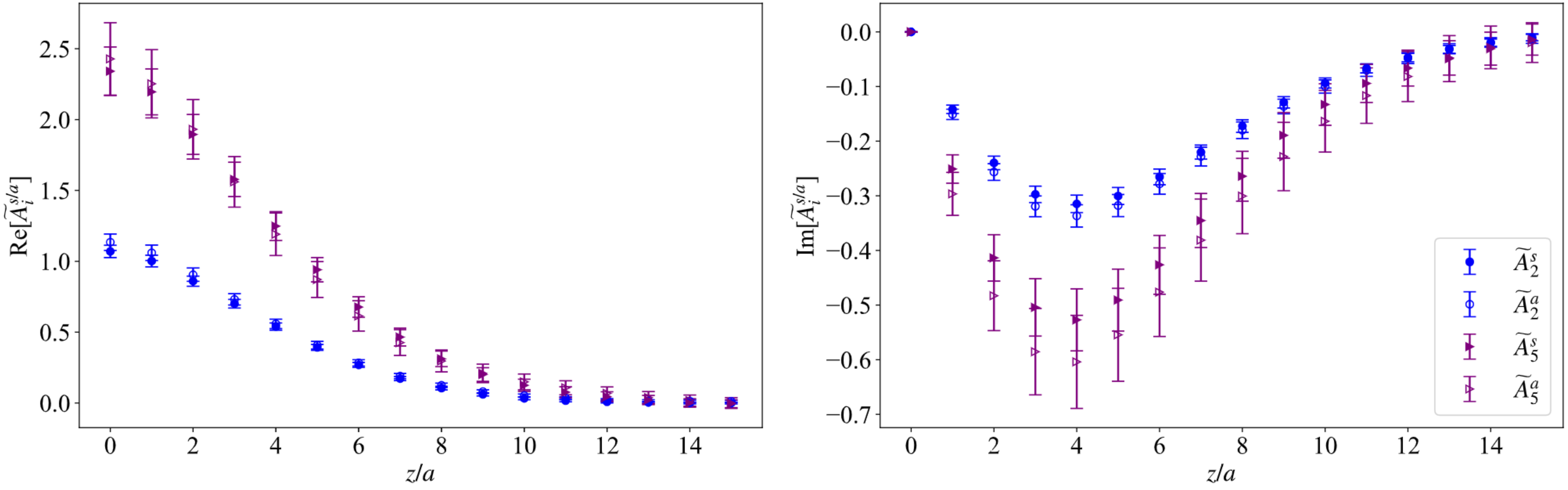

FIG. 5. Comparison of bare values of $\widetilde{A}_2$ and $\widetilde{A}_5$ in the symmetric (filled symbols) and asymmetric (open symbols) frame. The real (imaginary) part of each quantity is shown in the left (right) column. The data correspond to $|P_3| = 1.25$ GeV and $-t = 0.69$ GeV^2 ($-t = 0.65$ GeV^2) for the symmetric (asymmetric) frame.

Alternative setup

★ Separate calculation for each $-t$ value in symmetric frame

★ Asymmetric frame: 2 classes of $\vec{Q}$: $(Q_x,0,0), (Q_x, Q_y,0)$

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frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

★ Momentum transfer range is very optimistic

(some values have enhanced systematic uncertainties)

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- Asymmetric frame: 2 classes of $\vec{Q}$: $(Q_x,0,0), (Q_x, Q_y,0)$

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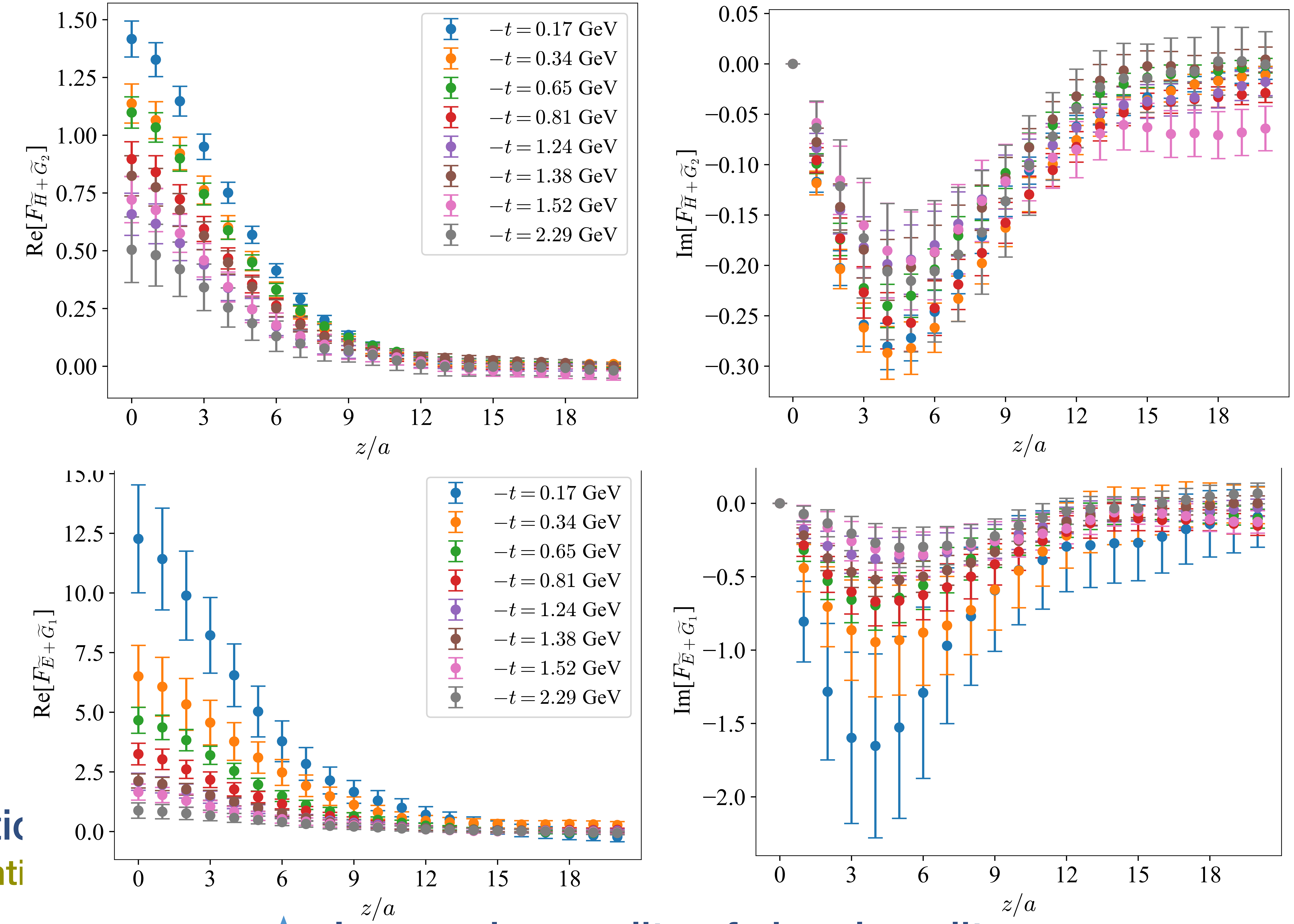
asymmetric frame



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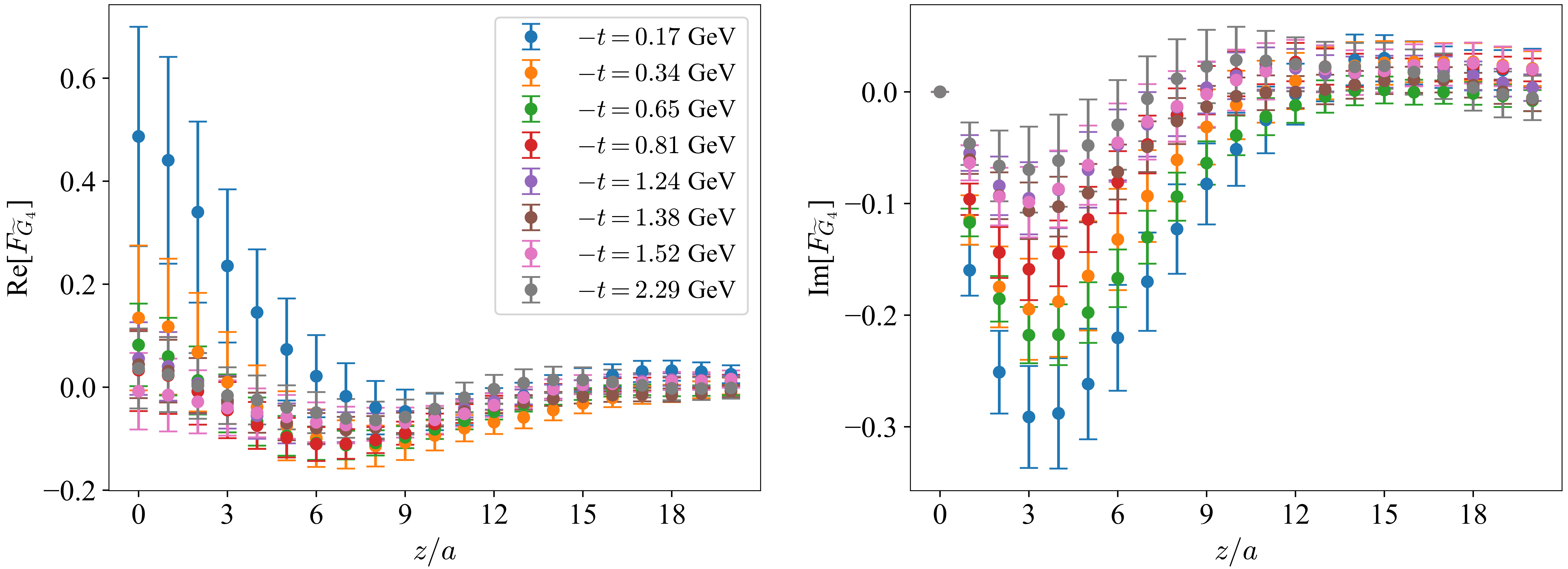
(some values have enhanced systematic uncertainti

★ Impressive quality of signal quality

$F_{\widetilde{G}_4}$



- Small, but not negligible
- Satisfies the sum rule: $\int_{-1}^{+1} dx\, x\widetilde{G}_3 = \frac{1}{4}G_E$

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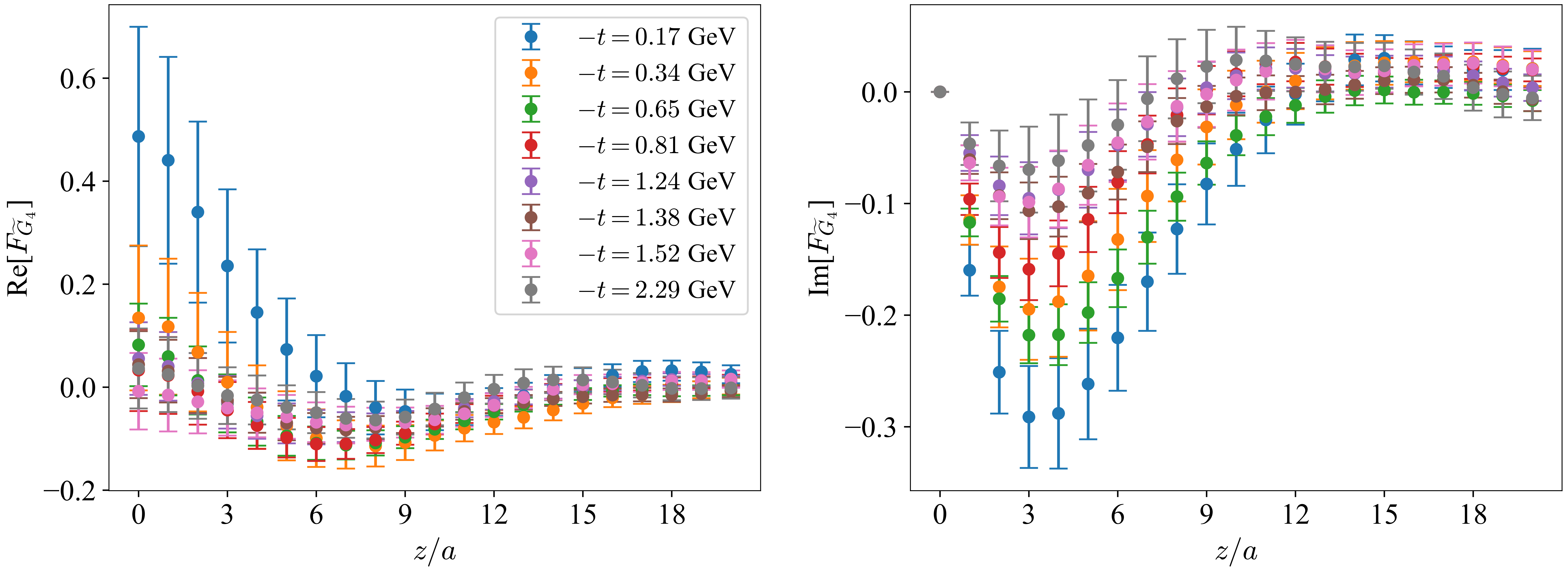
$F_{\widetilde{G}_4}$



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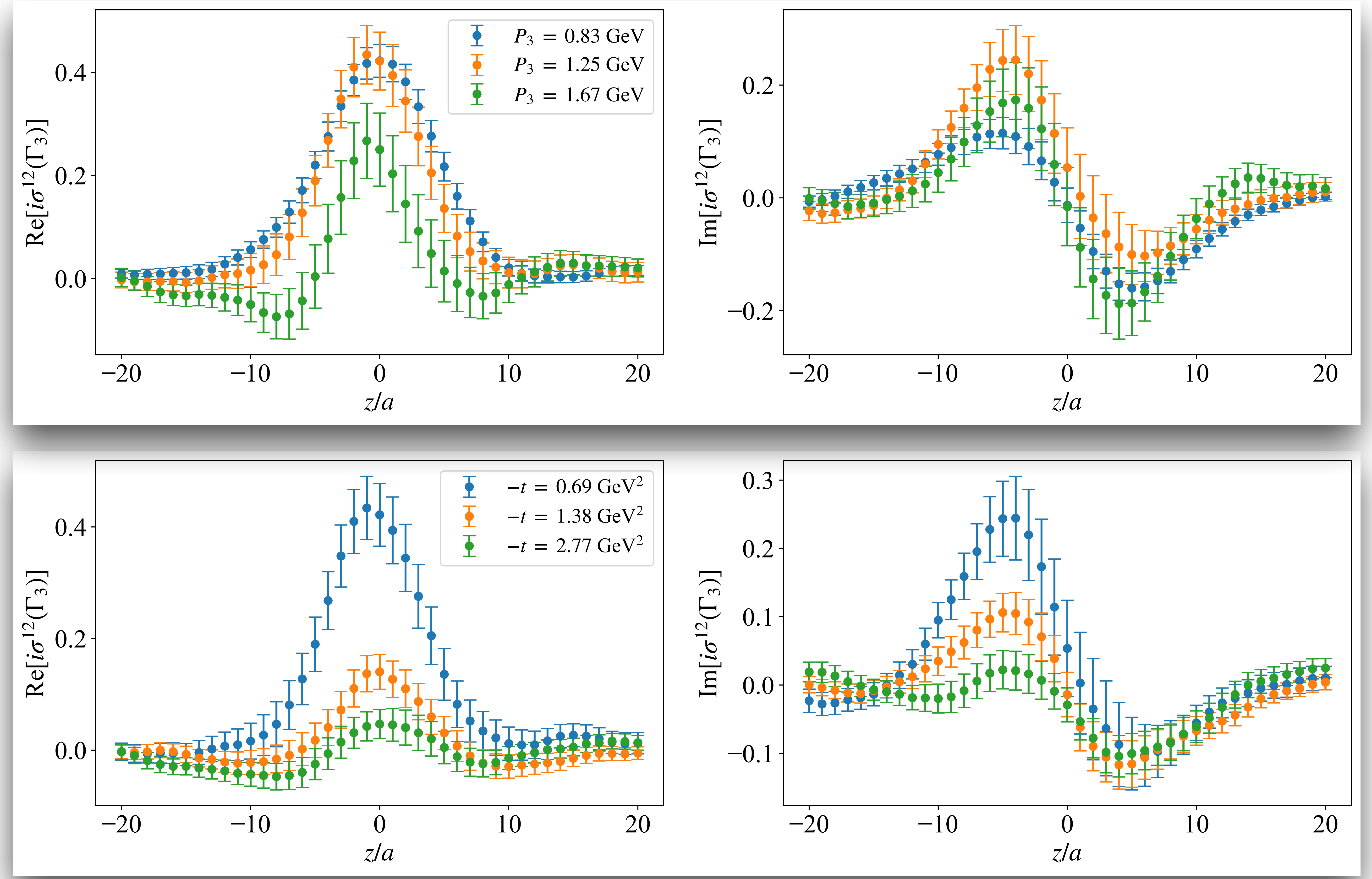
Small

Extension to twist-3 tensor GPDs

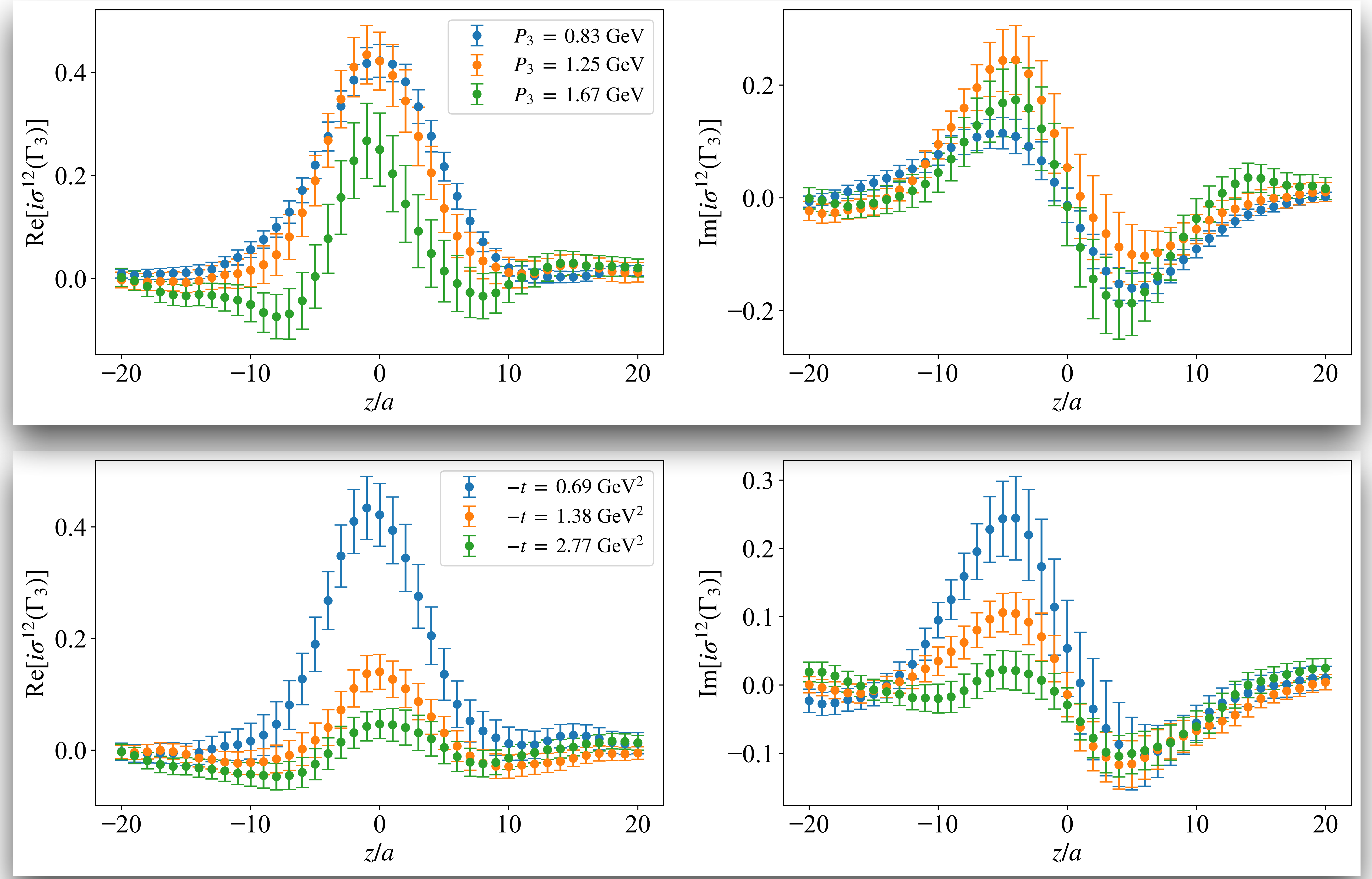
Extension to twist-3 tensor GPDs



Extension to twist-3 tensor GPDs

★ Parametrization [Meissner et al., *JHEP* 08 (2009) 056]

$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p') \left(\gamma^+ \gamma_5 \, \widetilde{H}_2' + \frac{P^+ \gamma_5}{M} \, \widetilde{E}_2' \right) u(p)$$



How to lattice QCD data fit into the overall effort for hadron tomography

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★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

How to lattice QCD data fit into the overall effort for hadron tomography

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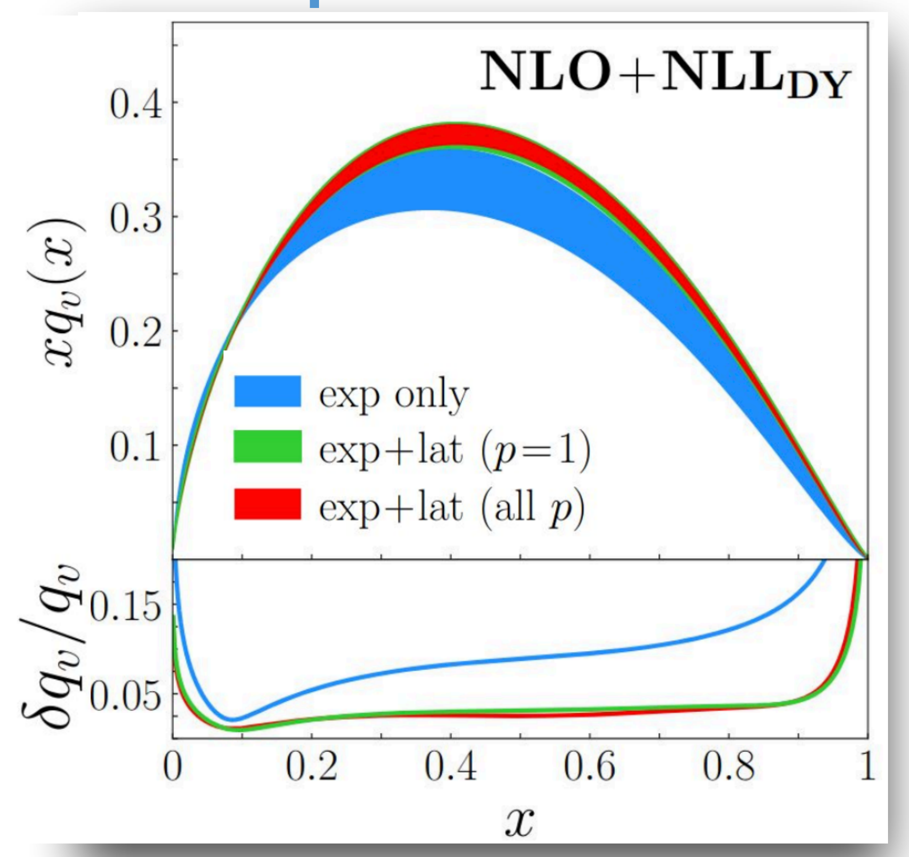
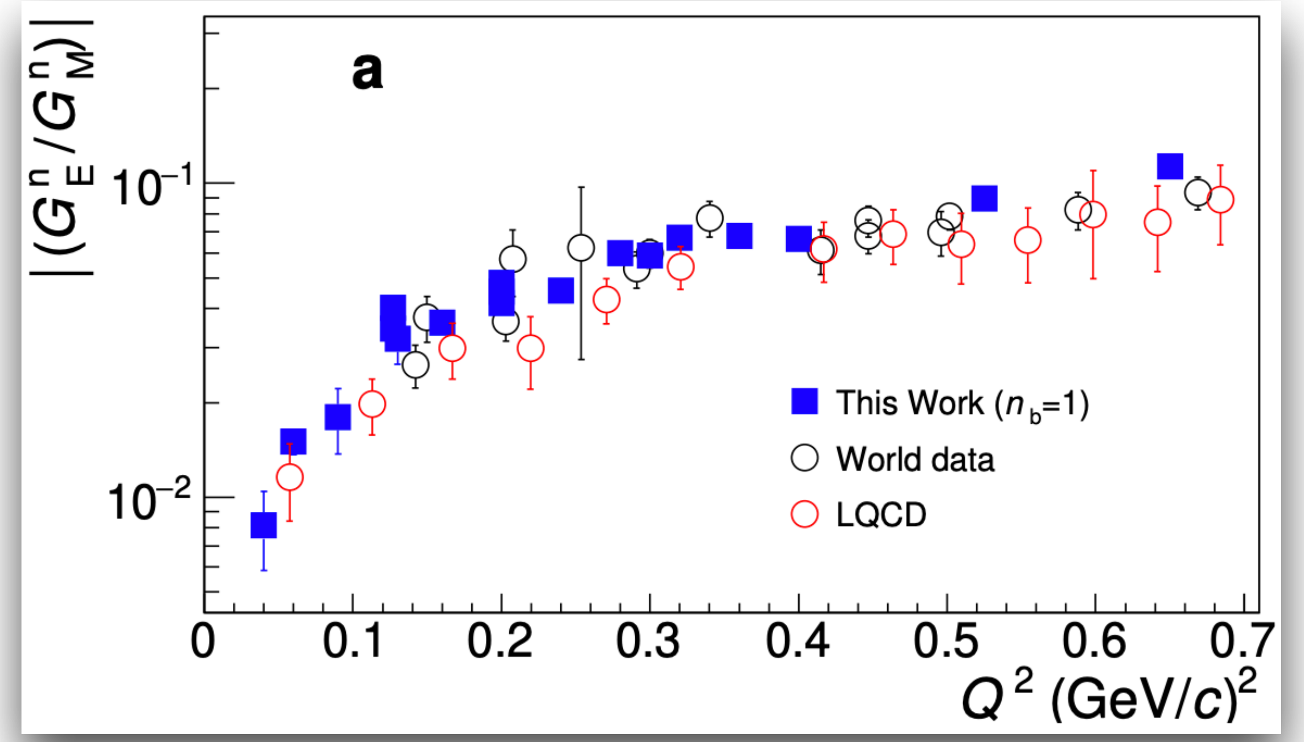
Constraints & predictive power of lattice QCD

pion PDF



[JAM/HadStruc, PRD105 (2022) 114051]

proton & neutron radius



[Atac et al., Nature Comm. 12, 1759 (2021)]

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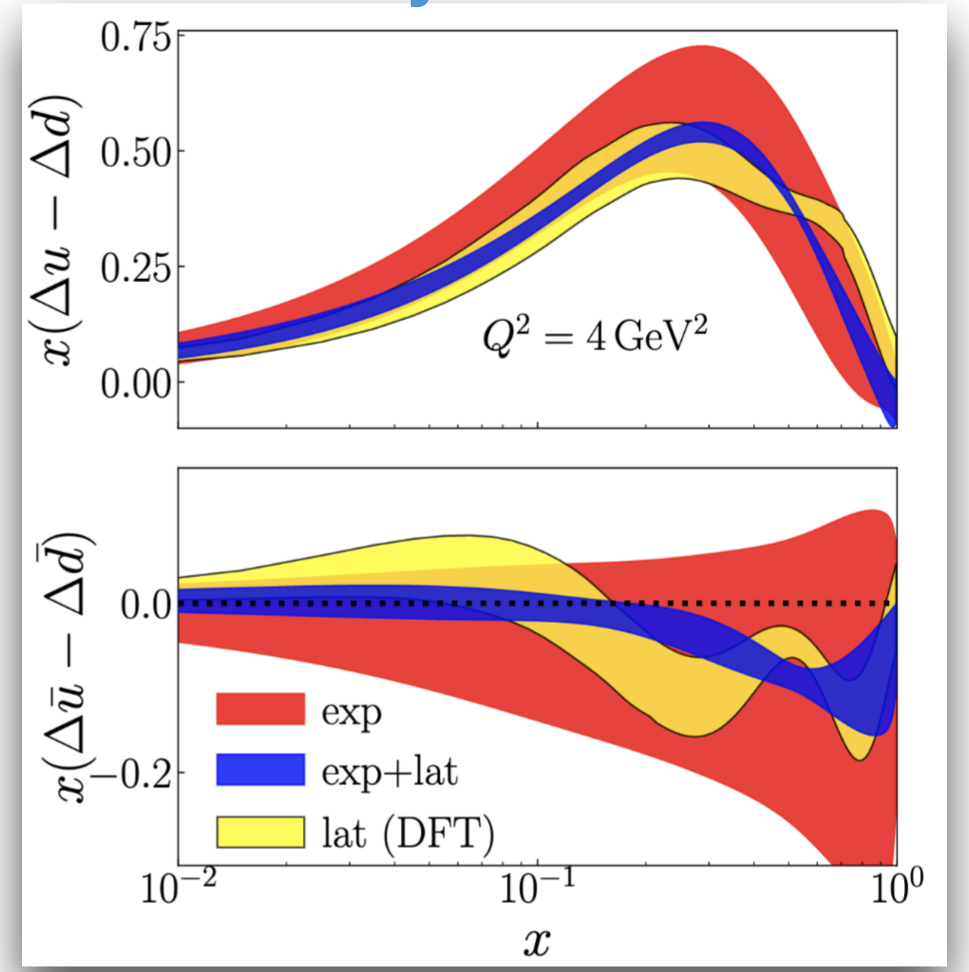

[JAM & ETMC, PRD 103 (2021) 016003]

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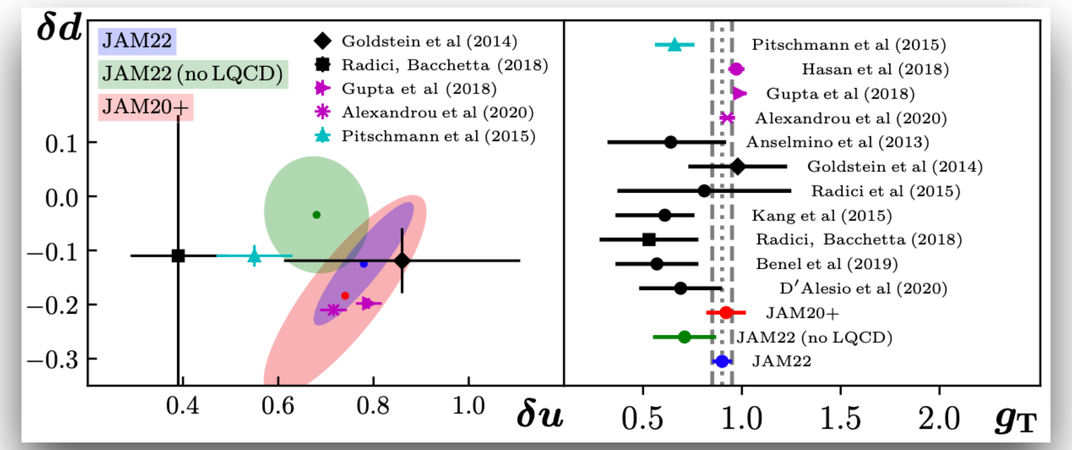

[JAM, PRD 106 (2022) 3, 034014]

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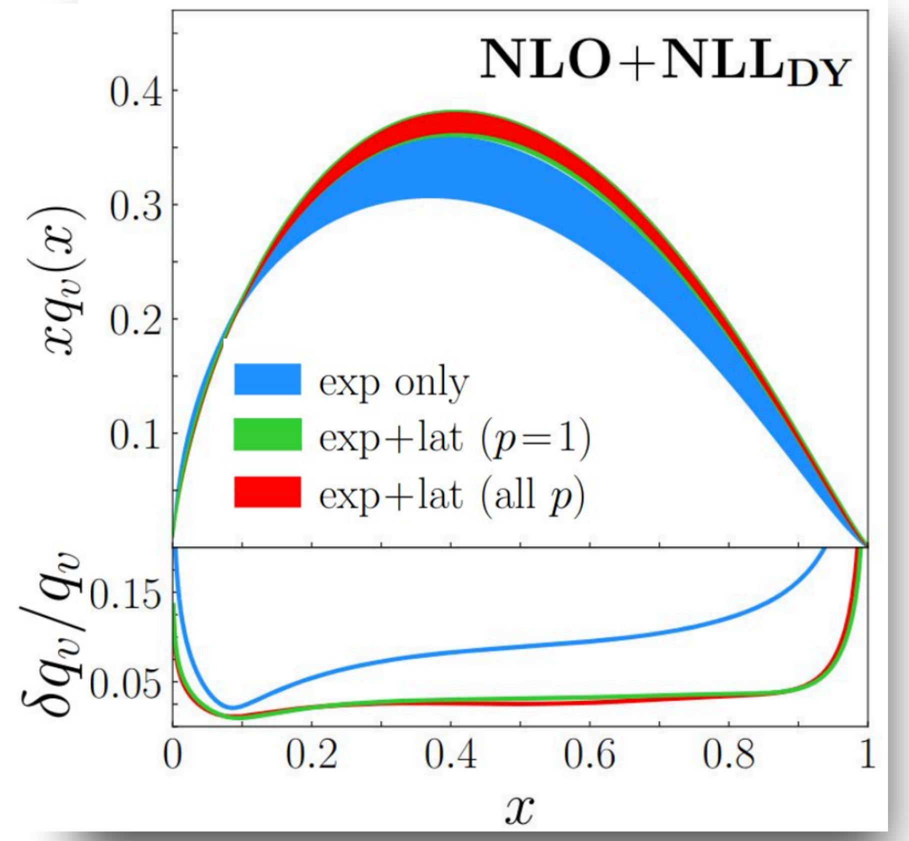

[JAM/HadStruc, PRD105 (2022) 114051]

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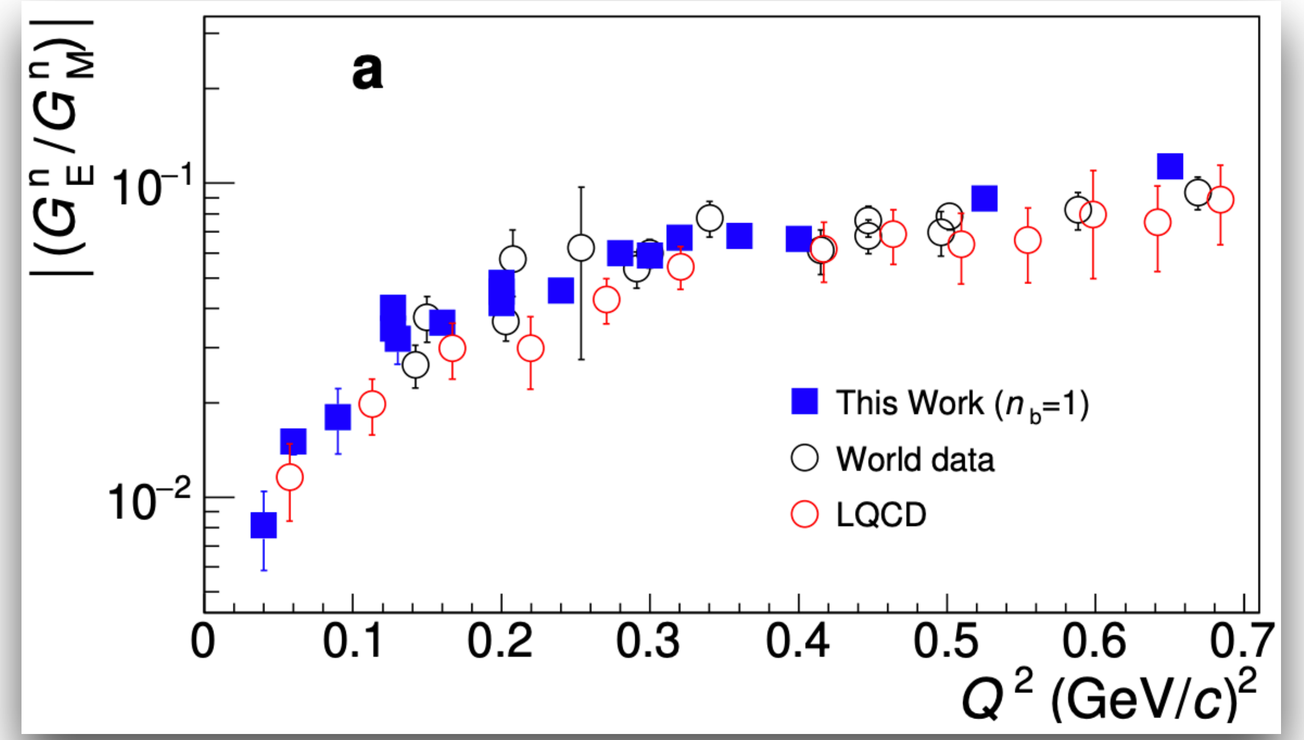

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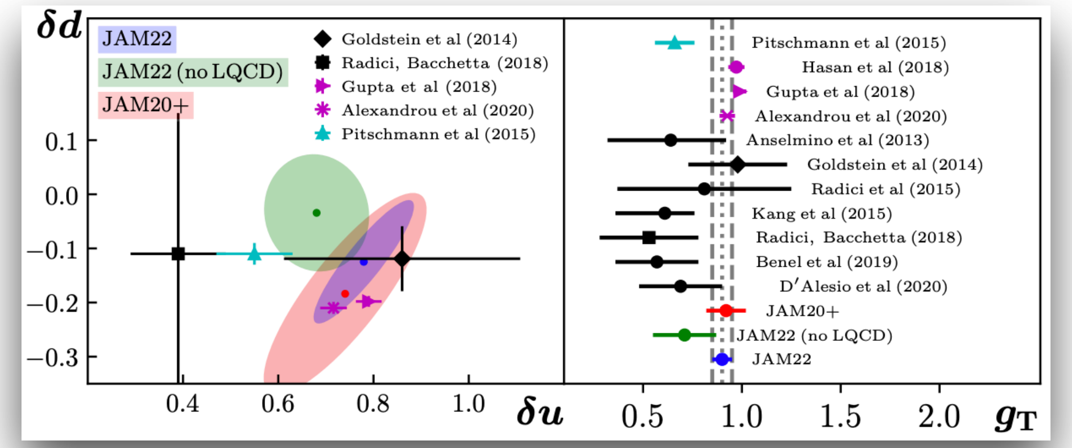

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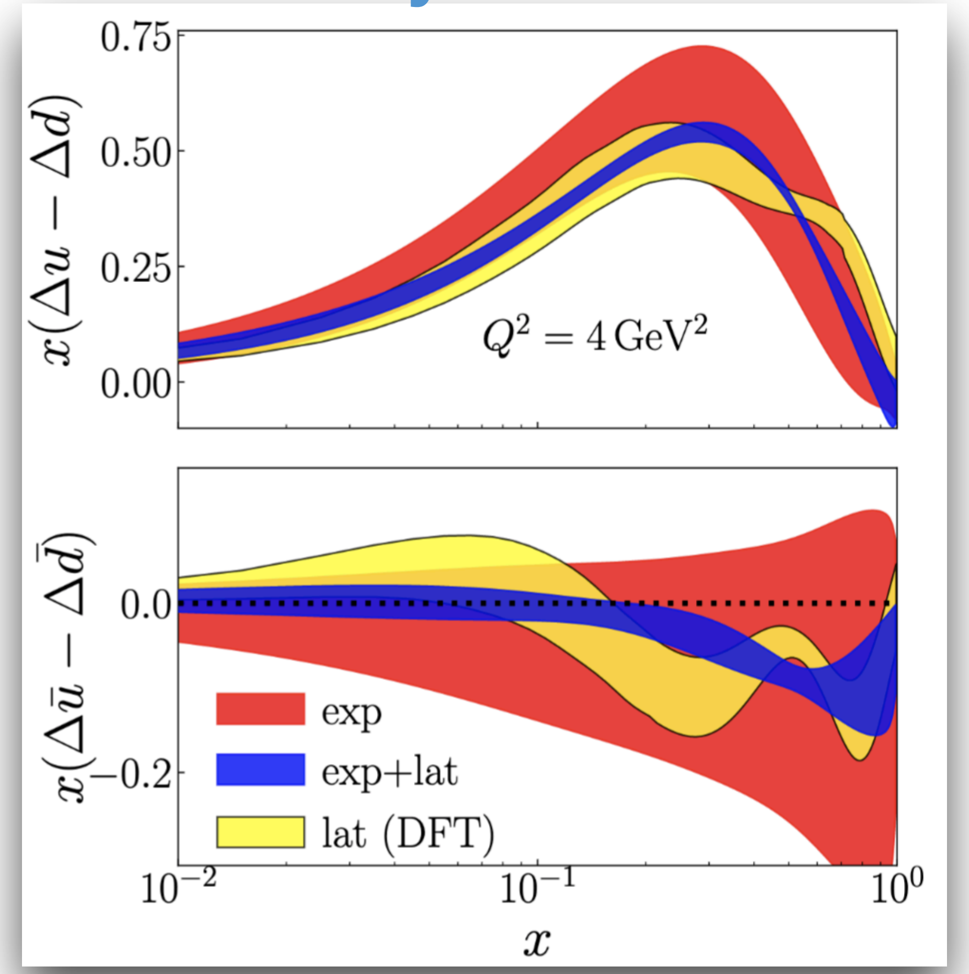

[JAM & ETMC, PRD 103 (2021) 016003]

And many more!

★ Three bridge faculty positions will be created in nuclear theory.

Stony Brook & Temple: Faculty positions in Fall 2024

QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number:
DE-SC0023646

The QGT Collaboration has a main goal of spearheading understanding and discovery in the quark and gluon tomography of hadrons, as well as the origin of their mass and spin.

1. **Theoretical studies** of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties
2. **Lattice QCD** calculations of GPDs and related structures
3. **Global analysis** of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

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Focus Areas - Composition & Expertise

QGT-related publications

1. “Gluon helicity in the nucleon from lattice QCD and machine learning”, Khan, Liu, Sabbir Sufian, *Physical Review D*, Accepted, 2023.

2. "Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO", Bhattacharya Cichy, Constantinou, Gao, Metz, Miller, Mukherjee, Petreczky, Steffens, Zhao, *Physical Review D*, DOI: 10.1103/PhysRevD.108.014507.

3. "Generalized parton distributions through universal moment parameterization: non-zero skewness case", Guo, Ji, Santiago, Shiells, Yang, *Journal of High Energy Physics*, DOI: 10.1007/JHEP05(2023)150.

4. "Hadronic structure on the light-front VI. Generalized parton distributions of unpolarized hadrons", Shuryak, Zahed, *Physical Review D*, DOI: 10.1103/PhysRevD.107.094005.

5. "Chiral-even axial twist-3 GPDs of the proton from lattice QCD", Bhattacharya, Cichy, Constantinou, Dodson, Metz, Scapellato, Fernanda Steffens, *Physical Review D*, DOI: 10.1103/PhysRevD.108.054501.

6. "Shedding light on shadow generalized parton distributions", Moffat, Freese, Cloët, Donohoe, Gamberg, Melnitchouk, Metz, Prokudin, Sato, *Physical Review D*, DOI: 10.1103/PhysRevD.108.036027.

7. "Colloquium: Gravitational Form Factors of the Proton", Burkert, Elouadrhiri, Girod, Lorcé, Schweitzer, Shanahan, *Physical Review D*, Under Review.

8. "Synchronization effects on rest frame energy and momentum densities in the proton", Freese, Miller, eprint: 2307.11165.

9. "Proton's gluon GPDs at large skewness and gravitational form factors from near threshold heavy quarkonium photo-production", Guo, Ji, Yuan, e-Print: 2308.13006.

10. "Parton Distributions from Boosted Fields in the Coulomb Gauge", Gao, Liu, Zhao, e-Print: 2306.14960.

11. "Exactly solvable models of nonlinear extensions of the Schrödinger equation", Dodge, Schweitzer, e-Print: 2304.01183.

12. "Role of strange quarks in the D-term and cosmological constant term of the proton", Won, Kim, Kim, e-Print: 2307.00740.

13. "Lattice QCD Calculation of Electroweak Box Contributions to Superallowed Nuclear and Neutron Beta Decays", Ma, Feng, Gorchtein, Jin, Liu, Seng, Wang, Zhang, e-Print: 2308.16755.

Synergy

★ The efforts from the three focus areas are interdependent and connected at multiple levels.

Courtesy: C. Weiss

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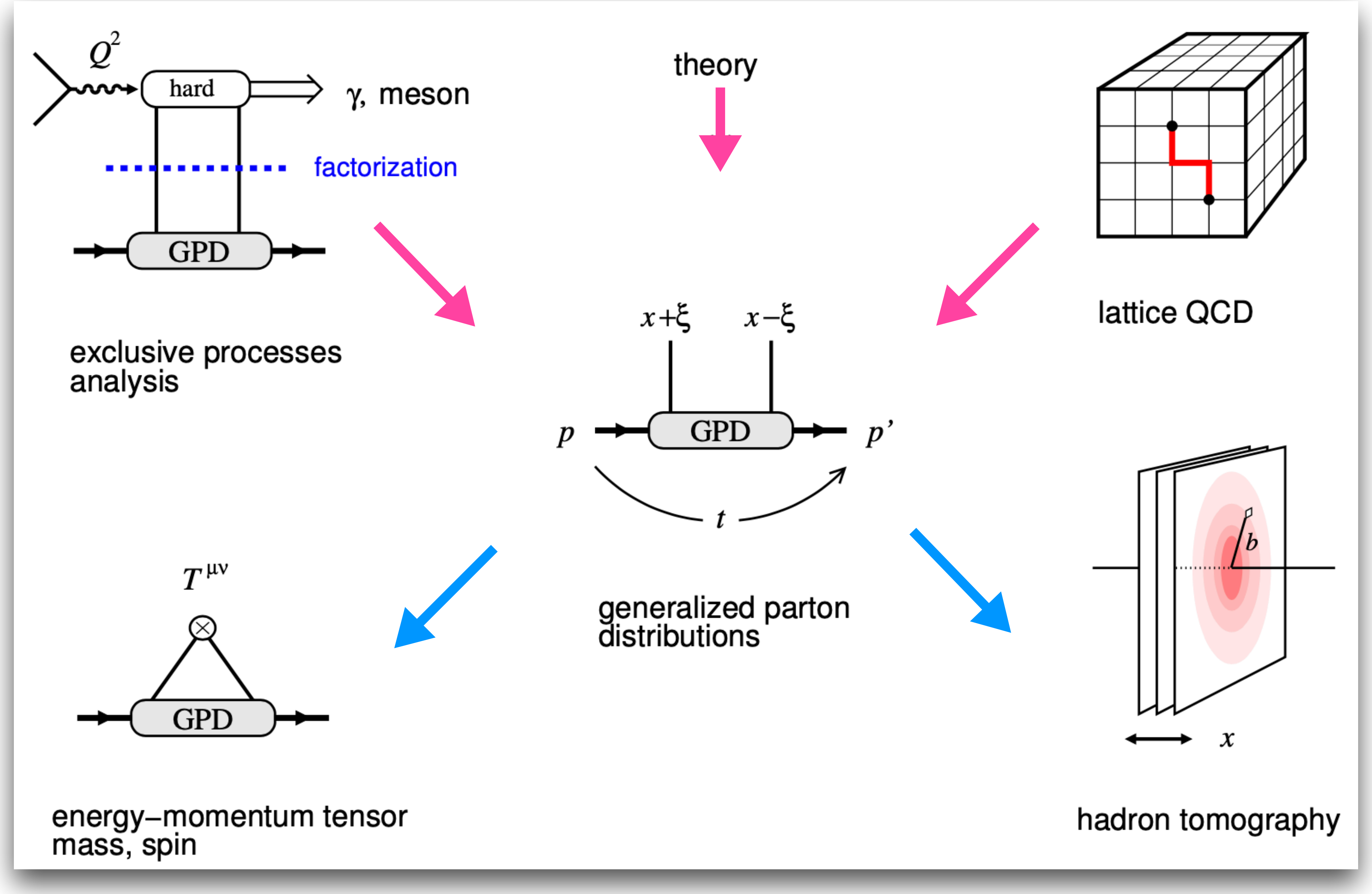

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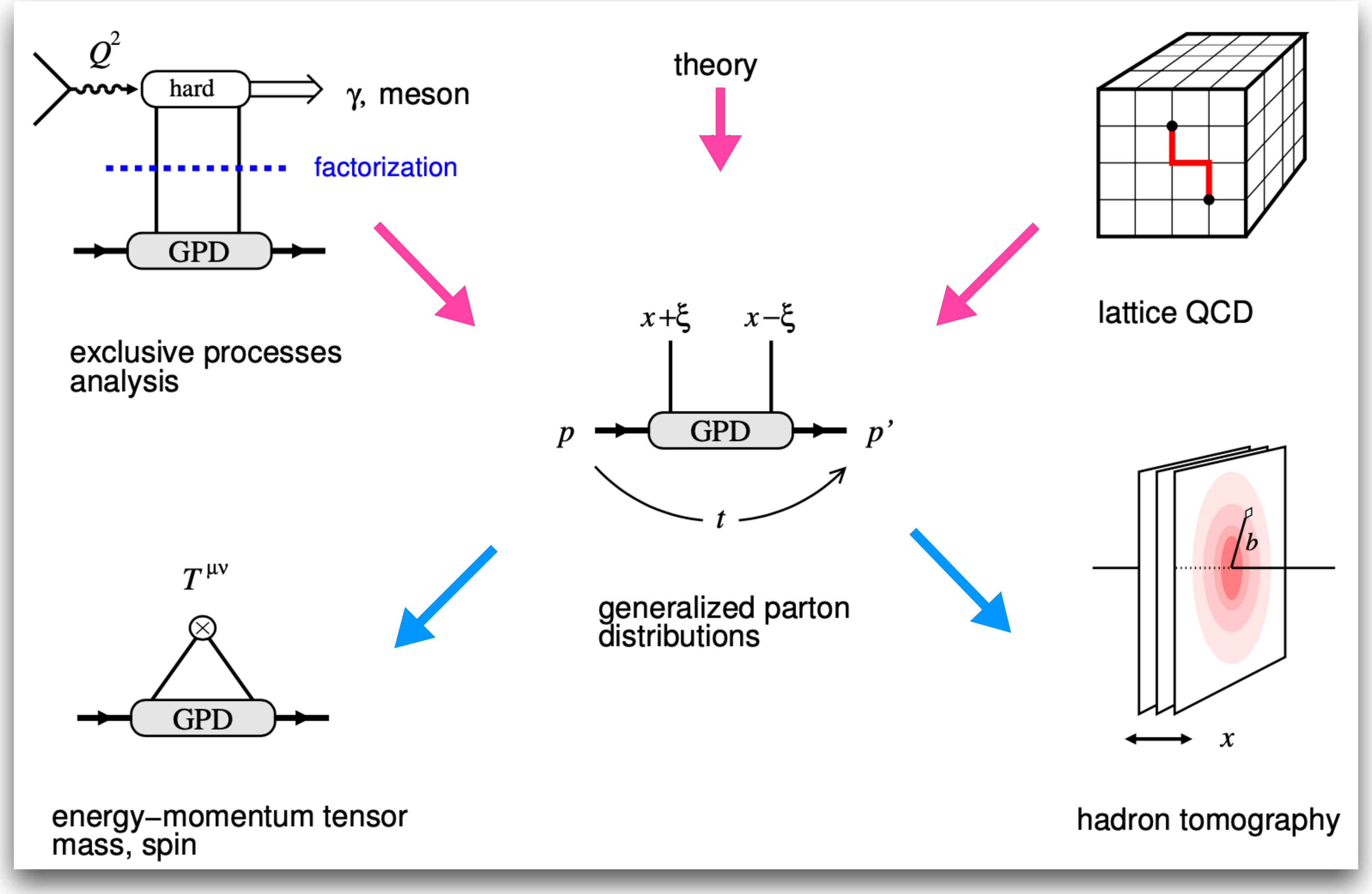

Courtesy: C. Weiss

★ Utilizing individual efforts from different focus areas and creating essential new synergies is a unique aspect of the topical collaboration

– impose constraints in global analysis guided by theory
– impose constraints by incorporating lattice data in global analysis
– address challenges by combining lattice & experimental data, as guided by theory

Summary

★ We address computationally expensive calculations
GPDs with signal comparable to PDFs

★ Several improvements needed (e.g., mixing with quark-gluon-quark correlators)

★ New proposal for Lorentz invariant decomposition has great advantages:
- significant reduction of computational cost
- access to a broad range of t and ξ

★ Future calculations have the potential to transform the field of GPDs
Lattice QCD data on GPDs will play an important role in the pre-EIC era
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★ Synergy with phenomenology is an exciting prospect

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Thank you

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