Froissart-Gribov projections in analysis of deeply virtual Compton scattering

K. Semenov-Tian-Shansky[1,2], P. Sznajder[3]

[1] Kyungpook National University, Daegu, Republic of Korea
[2] Petersburg Nuclear Physics Institute, Gatchina, Russia
[3] National Centre for Nuclear Research, Warsaw, Poland

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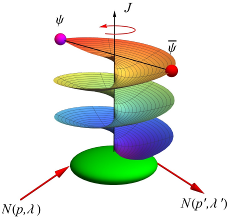
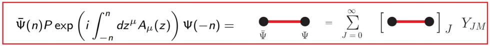
Introduction and motivation

- Generalized Parton Distributions (GPDs) encode hadron's response on excitations induced by QCD string quark and gluon operators on the light-cone $z^2 = 0$:

$$\langle N' | \bar{\psi}(0)[0;\ z]\psi(z) | N \rangle; \qquad \langle N' | G^a_{\alpha\beta}(0)\ [0, z]^{ab}\ G^b_{\mu\nu}(z) | N \rangle;$$

- Non-local QCD probe allows to mimic high spin elementary local probes: *e.g.* $J = 2$ G; see talk by H.-C. Kim ;
- Non-diagonal DVCS/DVMP: excitation of nucleon resonances with a QCD string, see talk by S. Son;
- Can we control the cross channel angular momentum of the non-local QCD probe?





- Froissart-Grbov projection in the context of DVCS: K. Kumericki, D. Müller, and K. Passek-Kumericki, Eur. Phys. J. C **58**, 193 (2008); M. Polyakov, Phys. Lett. B **659**, 542 (2008);
- Review the diagonal case before going to non-diagonal reactions;

Deconvolution problem

see talk by Paweł Sznajder today

- The elementary Leading Order (LO) amplitudes for $C = \pm 1$ GPDs:

$$\mathcal{H}_\pm(\xi,t) = \int_0^1 dx \left[\frac{1}{\xi - x - i\epsilon} \mp \frac{1}{\xi + x - i\epsilon} \right] H_\pm(x,\xi,t)$$

- The maximum information on GPDs one may expect to extract* is limited to GPDs on the $x = \xi$ trajectory

$$\mathrm{Im}\mathcal{H}_\pm(\xi,t) = \pi H_\pm(\xi,\xi,t);$$

and the D-term form factor, the subtraction constant of the fixed-t DR for the Compton amplitude. * From experiments performed at a fixed photon virtuality Q^2. More information from the QCD scale evolution. But lack of large enough lever arm in Q^2.

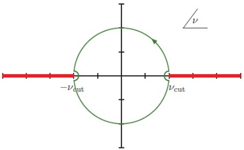
- Phenomenological parametrizations to extracting GPDs from the data: VGG, GK, KM, MMS, etc. (plenty of them).
- New observables selective with respect to GPD models?

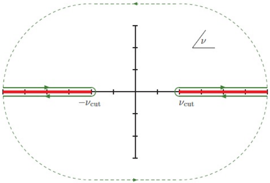
Fixed-t dispersion relations for Compton amplitude I

I. Anikin, M. Diehl, D. Ivanov, O. Teryaev'07

$$\gamma^*(q) + N(p) \to \gamma(q') + N(p');$$

- We discuss $q^2 = -Q^2 < 0$, $q'^2 = 0$; and consider $C = +1$ $P = +1$ CFF $\mathcal{H}_+(\nu, t|Q^2, 0)$.
- $\nu = \frac{P \cdot \bar{q}}{m_N} = \frac{s-u}{4m_N}$ with $P = \frac{1}{2}(p_1 + p_2)$, $\bar{q} = \frac{1}{2}(q_1 + q_2)$.
- Cuts: $\nu_{\text{cut}} = \frac{Q^2 + t + 2(m_N + 2m_\pi)^2 - 2m_N^2}{4m_N}$





Cauchy theorem

$$\mathcal{H}_+(\nu, t|Q^2, 0) = \frac{1}{2\pi i} \oint d\nu' \frac{1}{\nu' - \nu} \mathcal{H}_+(\nu', t|Q^2, 0),$$

Dispersion relations for Compton amplitude II

- Once subtracted DR:

$$\mathcal{H}_+(\nu,t|Q^2,0)=\mathcal{H}_0(t|Q^2,0)+\frac{1}{\pi}\int_{\nu_{\rm cut}}^{\infty}\frac{d\nu'}{\nu'}\frac{2\nu^2}{\nu'^2-\nu^2-i\epsilon}{\rm Im}\,\mathcal{H}_+(\nu',t|Q^2,0)\,.$$

- DVCS kinematics: large-Q^2, s, fixed $x_B=\frac{Q^2}{2p\cdot q}$,
- Scaling variable & integration limits:

$$\xi=\frac{Q^2}{4m_N\nu};\quad \xi\simeq\frac{x_B}{2-x_B}\quad \xi_{\rm cut}=\frac{Q^2}{4m_N\nu_{\rm cut}}\to 1;\quad \nu=\infty\to\xi=0;$$

DRs within scaling variables

$$\mathcal{H}_+(\xi,t)\;=\;\frac{1}{\pi}\int_0^1 d\xi'\frac{2\xi'}{\xi^2-\xi'^2-i\epsilon}{\rm Im}\mathcal{H}_+(\xi',t)+\underbrace{\mathcal{H}_0(t|Q^2,0)}_{4D(t)}\,.$$

Dispersive approach in the scaling regime versus pQCD approach

LO Compton FF within the pQCD approach

$$\mathcal{H}_+(\xi, t|\vartheta) \overset{\text{LO}}{=} \int_0^1 dx \, \frac{2x}{\xi^2 - x^2 - i\epsilon} H_+(x, \xi, t).$$

- The dispersion relation for the LO CFF:

$$\boxed{\operatorname{Re} \mathcal{H}_+(\xi, t) \overset{\text{LO}}{=} \mathcal{P} \int_0^1 dx \frac{2x H_+(x, x, t)}{\xi^2 - x^2} + 4D(t).}$$

GPD sum rule O. Teryaev'05

$$\boxed{4D(t) \overset{\text{LO}}{=} \int_0^1 dx \, \frac{2x}{x^2 - \xi^2} \left[H_+(x, x, t) - H_+(x, \xi, t) \right].}$$

- Integrable singularity at $x = \xi$ for $\xi \neq 0$

What is the Froissart- Gribov projection: case of spinless hadrons

Gribov'61, Froissart'61

- t-channel counterpart of the DVCS reaction:

$$\gamma^*(q) + \gamma(\bar{q}') \to h(p') + \bar{h}(\bar{p});$$

- θ_t: the angle between $\vec{q}$ and $\vec{\bar{p}}$ in the $\gamma^*\gamma$ CMS; in the DVCS kinematics:

$$\cos\theta_t \to -\frac{1}{\xi\sqrt{1-\frac{4m^2}{t}}} + \mathcal{O}\left(1/Q^2\right);$$

- At the moment we set $m = 0$. Effect of non-zero target mass: mixing of PWs (to be discussed later).

DR for the CFF analytically continued to the t-channel:

$$\mathcal{H}_+(\cos\theta_t, t) = \int_0^1 dx \frac{2x\cos^2\theta_t}{1 - x^2\cos^2\theta_t} H_+(x, x, t) + 4D(t).$$

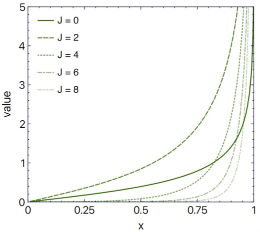
- Cross channel SO(3) PWAs

$$F_J(t) \equiv \frac{2J+1}{2} \int_{-1}^1 d(\cos\theta_t)\, P_J(\cos\theta_t)\mathcal{H}^{(+)}(\cos\theta_t, t).$$

Froissart- Gribov projection II

Neumann's integral representation for the Legendre functions $\mathcal{Q}_J$ ($J \geq 0$, integer):

$$\frac{1}{2}\int_{-1}^{1} dz'\, P_J(z')\frac{1}{z-z'} = \mathcal{Q}_J(z);$$



First weight functions $2(2J+1)\left(\frac{\mathcal{Q}_J(1/x)}{x^2} - \delta_{J0}\frac{1}{x}\right)$

- For even positive J

$$F_{J>0}(t) = 2(2J+1)\int_0^1 dx\,\frac{\mathcal{Q}_J(1/x)}{x^2} H_+(x,x,t)\,.$$

- Special case $J=0$:

$$F_{J=0}(t) = 2\int_0^1 dx\left[\frac{\mathcal{Q}_0(1/x)}{x^2} - \frac{1}{x}\right] H_+(x,x,t) + 4D(t)\,.$$

- N.B. $\frac{\mathcal{Q}_J(1/x)}{x^2} \sim x^{J-1}$ for small x.

On the physical content of the Froissart-Gribov FFs

An answer we want to have:

"FG FFS $F_J(t)$ quantifies hadron target response on the string-like QCD probe with fixed angular momentum J."

- What we have to support this point of view?
- Shortcomings we need to face:
  - Analytic continuation in t;
  - Target mass corrections;
  - Generalization for spin-$\frac{1}{2}$ target;
- Possible benefits of $F_J(t)$ as observable quantities?

Conformal PW expansion for GPDs

Conformal PW expansion for GPDs:

$$H(x,\xi,t)=\sum_{n=0}^{\infty} p_n(x,\xi) H_n(\xi,t); \quad p_n(x,\xi) \sim C_n^{\frac{3}{2}}\left(\frac{x}{\xi}\right) \theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right);$$

- Allows to factorize x, ξ and t dependence of GPDs.
- Scale dependence of the conformal moments is simply multiplicative:

$$H_n(\xi,t,\mu)=\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{\gamma_{0n}}{2\beta_0}} H_n(\xi,t,\mu_0)\,.$$

- Conformal moments are reproduced by this series.
- Restricted support property $\not\Rightarrow$ GPD vanishes in the outer region.
- The expansion is to be understand as an ill-defined sum of generalized functions.

Different ways to assign meaning to conformal PW expansion

1. Sommerfeld-Watson transform + Mellin-Barnes integral techniques D. Müller and A. Schäfer'05; A. Manashov, M. Kirch and A. Schafer'05;
2.
   - Shuvaev transform A. Shuvaev'99, J. Noritzsch'00;
   - Dual parametrization of GPDs M. Polyakov and A. Shuvaev'02;

Dual Parametrization: basic facts

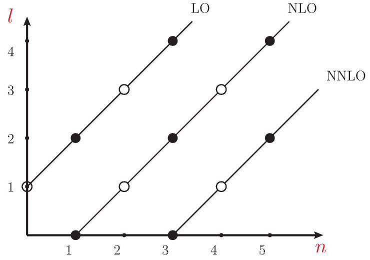
M. Polyakov, A. Shuvaev'02, D. Müller, M. Polyakov and K.S.'15:

- Mellin moments expanded in a set of suitable orthogonal polynomials: partial waves of the t-channel (t-channel refers to $\gamma^*\gamma \to \bar{h}h$):

$$N_n^{-1}\frac{(n+1)(n+2)}{2n+3}H_n(\xi,t) = \xi^{n+1}\sum_{l=0}^{n+1} B_{nl}(t)P_l\left(\frac{1}{\xi}\right)$$

Conformal PW expansion is then rewritten as:

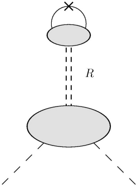
$$H_+(x,\xi,t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty}\sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}(t)\,\theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right)C_n^{\frac{3}{2}}\left(\frac{x}{\xi}\right)P_l\left(\frac{1}{\xi}\right)$$



- Polynomiality implemented via the Wigner-Ekkart theorem ($l \le n+1$).
- Discrete symmetries (C, T) through the selection rules for l^{PC} (X. Ji, R. Lebed'01).
- Generalized FFs $B_{nl}(t)$ are renormalized multiplicatively.

t-channel point of view and duality

- Conformal PW expansion converges for $\xi > 1$.
- Crossing relation between GPD and two particle GDA.
- Duality in the spirit of R. Dolen, D. Horn, C. Schmid'67. GPDs are presented as infinite series of t-channel Regge exchanges M. Polyakov'98:



$$\langle \pi(p')| \hat{O} |\pi(p)\rangle \sim \text{ Crossing of } \sum_{R_J} \sum_{\substack{\text{polarization} \\ \text{of } R_J}} \frac{1}{t - M_{R_J}^2}$$

$$\times \underbrace{\langle \pi(p')\pi(-p)|R_J\rangle}_{R_J\pi\pi \text{ effective vertex}} \underbrace{\langle R_J| \hat{O} |0\rangle}_{\text{F.T. of DA of } R_J}$$

- Spin sum of $R_J \sim P_J(\cos\theta_t)$
- Expansion in conformal PWs (DA of R_J) and in the t-channel PWs.

$$\cos\theta_t = -\frac{s-u}{\sqrt{1-\frac{4\,m^2}{t}}\,(Q^2+t)} = -\frac{1}{\xi\sqrt{1-\frac{4m^2}{t}}} + O(\frac{1}{Q^2}).$$

The basis for the Shuvaev transform & the dual parametrization

- How to restore $f(x)$ from its Mellin moments $M_n = \int dx x^n f(x)$?

- Formal solution:

$$f(x) = \sum_{n=0}^{\infty} M_n \delta^{(n)}(x) \frac{(-1)^n}{n!}\,.$$

✓ A trick: $\delta^{(n)}(x) = \frac{(-1)^n n!}{2\pi i}\left[\frac{1}{(x-i\epsilon)^{n+1}} - \frac{1}{(x+i\epsilon)^{n+1}}\right]$.

Define $F(z) = \sum_{n=0}^{\infty} \frac{M_n}{z^{n+1}}$; then $f(x) = \frac{1}{2\pi i}\left[F(x-i\epsilon) - F(x+i\epsilon)\right]$.

Idea of the Shuvaev transform (see A. Shuvaev'99, J. Noritzsch'00):

- Introduce $f_\xi(y)$ whose Mellin moments generate the Gegenbauer moments of GPD:

$$\int_0^1 dy y^n f_\xi(y) = H_n(\xi)$$

- One can explicitly construct the kernel $K(x,\xi;\, y)$ such that

$$H(x,\xi) = \int_0^1 dy\, K(x,\xi;\, y)\, f_\xi(y)\,.$$

Dual parametrization: summing up the formal series

- Mellin moments of $Q_{2\nu}(y,t)$ generate the generalized F.Fs. B_{nl}:

$$B_{n\ n+1-2\nu}(t) = \int_0^1 dy y^n Q_{2\nu}(y,t)\,.$$

- The difference between the conformal spin $n+2$ and usual spin l: $\boxed{n+2-l \equiv 2\nu+1}$
- $Q_0(x)$ is fixed in terms of (t-dependent) PDFs:

$$Q_0(x) = q(x) + \bar{q}(x) - \frac{x}{2}\int_x^1 \frac{dy}{y^2}\left(q(y)+\bar{q}(y)\right);$$

- GPD is given by the convolution with the set of kernels expressed through elliptic integrals:

$$H(x,\xi,t) = \sum_{\nu=0}^{\infty}\int_0^1 dy K^{(2\nu)}(x,\xi,y) Q_{2\nu}(y,t)\,.$$

Convolutions with hard kernels

- Consider the elementary amplitude:

$$\mathcal{H}_+(\xi,t) = \int_0^1 dx H_+(x,\xi,t)\left[\frac{1}{\xi - x - i0} - \frac{1}{\xi + x - i0}\right] = 4\sum_{\substack{n=1 \\ \text{odd}}}^{\infty}\sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{n\,l}(t) P_l\left(\frac{1}{\xi}\right);$$

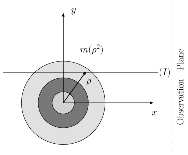
$$\mathrm{Im}\mathcal{H}_+(\xi,t) = 2\int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^{1} \frac{dy}{y}\, N(y,t)\, \frac{1}{\sqrt{\frac{2y}{\xi} - y^2 - 1}}\,.$$

- Explicit expression also exists for $\mathrm{Re}\mathcal{H}_+(\xi,t)$.
- $N(y,t) = \sum_{\nu=0}^{\infty} y^{2\nu} Q_{2\nu}(y,t) = Q_0(y,t) + x^2 Q_2(y,t) + x^4 Q_4(y,t) + \ldots$
- The amplitude satisfies the fixed-t DR with the subtraction constant given by the D-FF:

$$D(t) = \int_0^1 \frac{dy}{y}\left(\frac{1}{\sqrt{1+y^2}} - 1\right) Q_0(y,t) + \int_{(0)}^1 \frac{dy}{y}\left[N(y,t) - Q_0(y,t)\right]\frac{1}{\sqrt{1+y^2}}$$

- Note the possible regularization issue.
- $N(y,t)$ and D-FF is the maximal amount of info on a GPD to extract from the amplitude at fixed Q^2.

Abel transform tomography



The observer at ∞ looking along a line parallel to the x-axis a distance y above the origin sees the projection:

$$a(y^2) = \int_{-\infty}^{\infty} dx\, m(\rho^2) = \int_{y^2}^{\infty} d\rho^2 \frac{m(\rho^2)}{\sqrt{\rho^2 - y^2}}$$

- M. Polyakov'07: the Joukowski conformal map $\frac{1}{w} = \frac{1}{2}\left(y + \frac{1}{y}\right)$ allows to present the relation between $\mathrm{Im}\mathcal{H}(\xi, t)$ and $N(y, t)$ in form of the Abel integral equation.
- The inverse transform for $N(y)$:

$$N(y,t) = \frac{1}{\pi} \frac{y(1-y^2)}{(1+y)^{\frac{3}{2}}} \int_{\frac{2y}{1+y^2}}^{1} \frac{d\xi}{\xi^{\frac{3}{2}}} \frac{1}{\sqrt{\xi - \frac{2y}{1+y^2}}} \left\{ \frac{1}{2} \mathrm{Im}\mathcal{H}^{(+)}(\xi,t) - \xi \frac{d}{d\xi} \mathrm{Im}\mathcal{H}^{(+)}(\xi,t) \right\}.$$

Interpretation $N(x,t)$ and the Froissart- Gribov projection

$$N(y,t) = \underbrace{Q_0(y,t)}_{\text{PDFs}} + x^2 \underbrace{Q_2(y,t)}_{\text{FFs of EMT tensor}} + x^4 Q_4(y,t) + \ldots$$

- Only a principle possibility to separate $Q_{2\nu}$s via logarithmic scaling violation.
- Spin J expansion of the QCD string operator. For massless hadrons:

$$4\int_0^1 dy x^{J-1} N(y,t) = 4\left[B_{J-1\,J}(t) + B_{J+1\,J}(t) + B_{J+3\,J}(t) + \ldots\right] \equiv F_J(t).$$

- D. Müller, M. Polyakov, K.S.'14 Mellin moments of $N(y,t) \Leftrightarrow$ Froissart- Gribov projection (even $J > 0$):

$$4\int_0^1 dy\, y^{J-1} N(y,t) = \sum_{\substack{n=J-1 \\ \text{odd}}}^{\infty} B_{nJ}(t) = 4\int_0^1 dx \left[\frac{1}{\sqrt{x}} \frac{d}{dx} R_J(x)\right] H_+(x,x,t)\,,$$

where the auxiliary functions

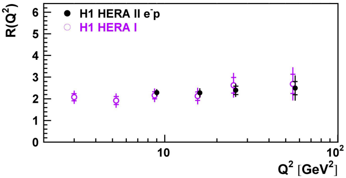
$$\frac{1}{\sqrt{x}} \frac{d}{dx} R_J(x) = \frac{1}{\sqrt{x}} \frac{d}{dx} \int_0^x \frac{dw}{\sqrt{2}\sqrt{w}} \left(\frac{1-\sqrt{1-w^2}}{w}\right)^{J+\frac{1}{2}} \frac{1}{\sqrt{x-w}} = \frac{2J+1}{2} \frac{\mathcal{Q}_J(1/x)}{x^2}\,.$$

$J = 0$ case

- For $J = 0$ it reads

$$F_{J=0}(t) = 4 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} B_{n0}(t) = 4 \,\text{Reg} \int_0^1 \frac{dy}{y} \left(N(y,t) - Q_0(y,t) \right)$$
$$= 4 \int_{(0)}^1 \frac{dy}{y} \left(N(y,t) - Q_0(y,t) \right) + 4 D^{\text{f.p.}}(t).$$

Skewness effect from H1'07 for fixed $W = 82$ GeV:



$$\mathbf{R} = 2^{\alpha_q} r^q = 2^{\alpha_q} \left. \frac{H^q(\xi,\xi)}{H^q(\xi,0)} \right|_{\xi \sim 0} \sim \frac{\sqrt{\sigma_{DVCS}}}{\sigma_{DIS}}$$

- Minimalist dual model accounting only $Q_0(y,t)$ contradicts data on skewness effect K. Kumericki et al. '08;
- $N(y,t) - Q_0(y,t)$ has to be singular as $y^{-\alpha_q}$ for $y \sim 0$.
- Minimally consistent model must include at least NLO conformal PW $\nu = 1$.
- Can we truncate the series in ν? How many conformal PWs we need?

Sum rules for the Mellin moments of GPDs

- Polynomiality property of GPDs ($h_{N,k}(t)$ are in one-to-one correspondence with $B_{n,l}(t)$)

$$\int_0^1 dx x^N H_+(x,\xi,t) = \sum_{\substack{k=0 \\ \text{even}}}^{N+1} h_{N,k}(t)\xi^k, \quad \text{for odd} \quad N;$$

- The sum rule for $J = 0,\, 2$ FG projection truncated at $\nu = 1$ ("next-to-minimalist" contribution):

$$F_{J=0}(t) = 4\left(B_{1,0}(t) + \ldots\right) = \frac{5}{3}h_{1,0}(t) + 5h_{1,2}(t) + \left\{\begin{matrix}\text{contribution of conformal PWs} \\ \text{with } \nu \geq 2\end{matrix}\right\};$$

$$\begin{aligned} F_{J=2}(t) &= 4\left(B_{1,2}(t) + B_{3,2}(t) + \ldots\right) \\ &= -\frac{7}{6}h_{1,0}(t) + 9h_{3,0}(t) + \frac{21}{2}h_{3,2}(t) + \left\{\begin{matrix}\text{contribution of conformal PWs} \\ \text{with } \nu \geq 2\end{matrix}\right\}, \end{aligned}$$

- Coefficients $h_{N,k}$ can be studied with methods of lattice QCD. A new way to connect lattice results to the data!

On mixing due to target mass $\neq 0$ I

- The modification of the double PW expansion (dual parametrization) explicit inclusion of threshold $\beta \equiv \sqrt{1 - \frac{4m^2}{t}}$ corrections in the summation of t-channel spin-l exchanges:

$$H_+(x,\xi,t) = 2 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} \beta^l \bar{B}_{n,l}(t)\, \theta\left(1 - \frac{x^2}{\xi^2}\right)\left(1 - \frac{x^2}{\xi^2}\right) C_n^{3/2}\left(\frac{x}{\xi}\right) P_l\left(\frac{1}{\xi\beta}\right).$$

- The appropriate system of orthogonal polynomials for the CFF is still $P_l(\beta \cos\theta_t)$, (see the integration limits in the dispersive integral for the LO CFF: $[0;\, 1]$ and not $[0;\, 1/\beta]$).
- Reexpansion of $P_l(\cos\theta_t)$ back to $P_l(\beta\cos\theta_t) \equiv P_l\left(\frac{1}{\xi}\right)$.
- Some guiding principles:
  - Two expansions give the same coefficients $h_{N,k}(t)$ at power ξ^k of N-th Mellin moments of a GPD.
  - The coefficients $h_{N,k}(t)$ must be regular in the $t \to 0$ limit $\Rightarrow$ assume specific singularities for the generalized FFs $\bar{B}_{n,l}(t)$ at $t = 0$.

On mixing due to target mass $\neq 0$ II

- Consider $h_{N,k=0}(t)$: no modification for $\nu = 0$ contribution ($Q_0(y,t)$):

$$B_{n,n+1}(t) = \bar{B}_{n,n+1}(t);$$

- Consider $h_{N,k=2}(t)$: for $\nu = 1$ ($Q_2(y,t)$): admixture of higher $J = l + 2$ spin:

$$\underbrace{B_{n,n-1}(t)}_{\text{Occurs in } J=n-1\text{-th FG projection}} = \underbrace{\bar{B}_{n,n-1}(t)}_{\text{spin } n-1} - (1-\beta^2)\left(\frac{1}{2} - n\right)\underbrace{\bar{B}_{n,n+1}(t)}_{\text{spin } n+1}.$$

- In general, the mixing for $B_{l+2\nu-1,l}(t)$ involves higher spin contributions up to $l + 2\nu$.
- FG projections $F_J(t)$ get admixture from higher spins:

$$F_J(t) = 4\int_0^1 dy x^{J-1} N(y,t) = 4\Big[\underbrace{B_{J-1\,J}(t)}_{\text{pure spin-}J} + \underbrace{B_{J+1\,J}(t)}_{\text{spin } J,\, J+2} + \underbrace{B_{J+3\,J}(t)}_{\text{spin } J,\, J+2,\, J+4} + \dots\Big]$$

- Mixing can be tamed once we may truncate summation in ν at some $\nu_{\max}$!
- This assumption can be tested through saturation of sum rules.

Can we treat the analytic properties in t better?

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Dispersive evaluation of the D-term form factor in deeply virtual Compton scattering

B. Pasquini[a,b,*], M.V. Polyakov[c], M. Vanderhaeghen[d,e]

- Unsubtracted DR in t for the D-term FF:

$$4D^q(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{+\infty} \mathrm{d}t' \frac{\mathrm{Im}_t\, F^q(0,t')}{t'-t} + \frac{1}{\pi}\int_{-\infty}^{-a} \mathrm{d}t' \frac{\mathrm{Im}_t\, F^q(0,t')}{t'-t};$$

- Phenomenological input for the absorptive part: $\pi\pi$ intermediate state

$$2\,\mathrm{Im}\, T^{\gamma^*\gamma\to N\bar{N}} = \frac{1}{(4\pi)^2}\frac{p_\pi}{\sqrt{t}}\int d\Omega_\pi \left[T^{\gamma^*\gamma\to\pi\pi}\right]\cdot\left[T^{\pi\pi\to N\bar{N}}\right]^*;$$

- Can one extend this kind of technique for the FG projection FFs?

What is new with the FG projection for spin-$\frac{1}{2}$ targets

- Combinations of GPDs suitable for the cross channel SO(3) PW expansion:

$$H^{(E)}(x,\xi,t) = H(x,\xi,t) + \frac{t}{4m_N^2} E(x,\xi,t) \quad \text{in} \quad P_J(\cos\theta_t)$$
$$H^{(M)}(x,\xi,t) = H(x,\xi,t) + E(x,\xi,t)\,, \quad \text{in} \quad P'_J(\cos\theta_t)$$

- Electric combination: same as spinless case;
- Cross channel SO(3)-PWs of magnetic combination (even $J \geq 2$):

$$F_J^{(M)}(t) = \frac{2J+1}{2J(J+1)} \int_{-1}^{1} d(\cos\theta_t) \mathcal{H}^{(M)}(\cos\theta_t, t) \frac{1}{\cos\theta_t} \left(1 - (\cos\theta_t)^2\right) C_{J-1}^{3/2}(\cos\theta_t).$$

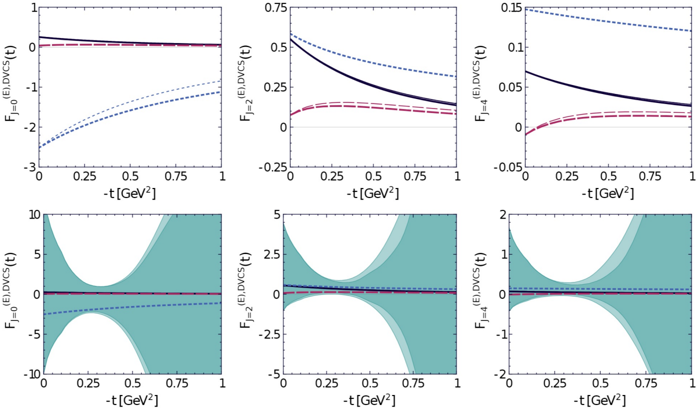
$$F_J^{(M)}(t) = 4 \int_0^1 dx H_+^{(M)}(x,x,t) \frac{J+\frac{1}{2}}{J(J+1)} \frac{(-1)}{x} \sqrt{\frac{1}{x^2} - 1}\, \mathcal{Q}_J^1(1/x).$$

- Associated Legendre functions of the second kind:

$$\mathcal{Q}_n^m(z) = \left(z^2 - 1\right)^{\frac{m}{2}} \frac{d^m \mathcal{Q}_n(z)}{dz^m};$$

defined in the complex plane with a cut along $[-1; 1]$ and $m \leq n$.

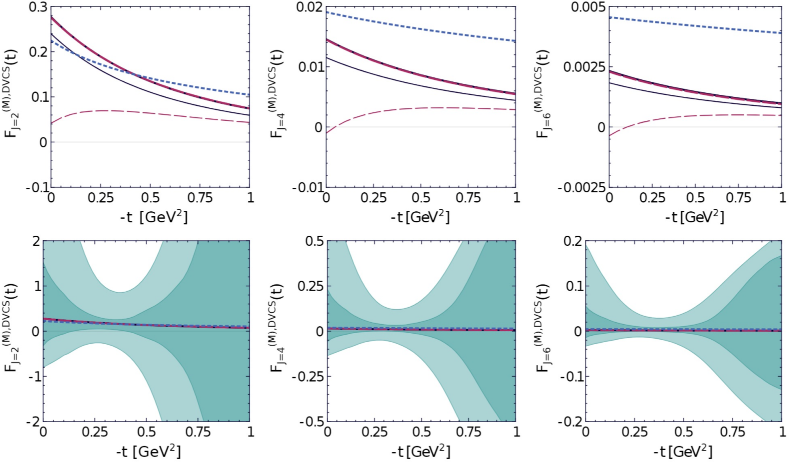
FG projection: phenomenological GPD models v.s. data I



- GK (solid black), MMS (dashed red) and KM (dotted blue) models. Thin lines denote estimates obtained with only GPD H.
- Comparison to the result from the global analysis of DVCS data H. Moutarde, P. Sznajder, and J. Wagner, Eur. Phys. J. C 79, 614 (2019) (light turquoise bands, corresponding to 68% confidence level). Dark bands are for results obtained with only CFF $\mathcal{H}$.

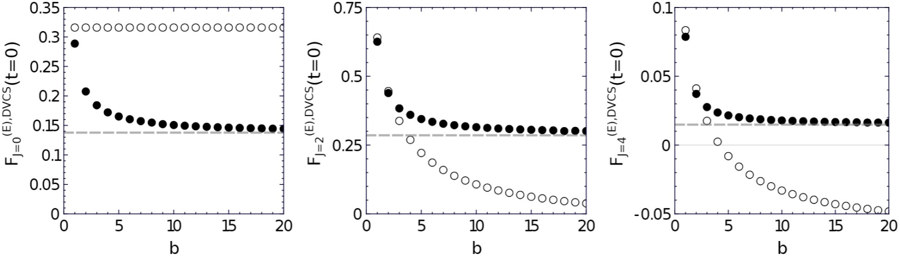
FG projection: phenomenological GPD models v.s. data II

- Magnetic FG projections $J = 2,4,6$.



Probing the effect of truncation in ν with sum rules

- GK model based on the RDDA with b-dependent profile used as a test ground;
- $q(x, t = 0) \sim 1/x^{\alpha}$ for $x \sim 0$;
- To recover the same skewness effect for $b = \alpha + M$, $M > 0$ one has to account $Q_{2\nu}$ with $\nu \leq M$;



- Result of the $\nu_{max} = 2$ SRs compared to the exact result of the FG projection.

Conclusions and Outlook

1. Froissar-Gribov projection provides a description of hadron's response to quark (and gluon, which is important for the EIC) string-like QCD probes characterised by different values of angular momentum J (up to the mixing issue).
   - Mixing can be put under control assuming the validity of truncation in ν.
2. A useful interpretation comes from the Abel transform tomography framework within the dual parametrization of GPDs.
3. A set of sum rules for the Mellin moments of GPDs. Possible contact to lattice QCD.
4. Generalization of the FG projections for the case of spin-$\frac{1}{2}$ target.
5. FG projections are very discriminative with respect to GPD models. First results for $J = 0, 2, 4$ electric and magnetic FG projections. Comparison to the result from the global analysis of DVCS data H. Moutarde, P. Sznajder, and J. Wagner'19.
6. "Electric" and "magnetic" spin-J radiuses $r_J^{(E,\,M)}$ of a nucleon: how nucleon is seen with a spin-J probe?

$$\frac{1}{F_J^{(E,M)}(0)} \frac{dF_J^{(E,M)}(t)}{dt}\Big|_{t=0} = -\frac{\left(r_J^{(E,M)}\right)^2}{6};$$