2nd APCTP Workshop on the Physics of Electron Ion Collider: ePIC Physics and Detectors

Generalized parton distributions and gravitational form factors of the kaon from the nonlocal chiral quark model

In collaboration with

Parada Hutauruk (PKNU)

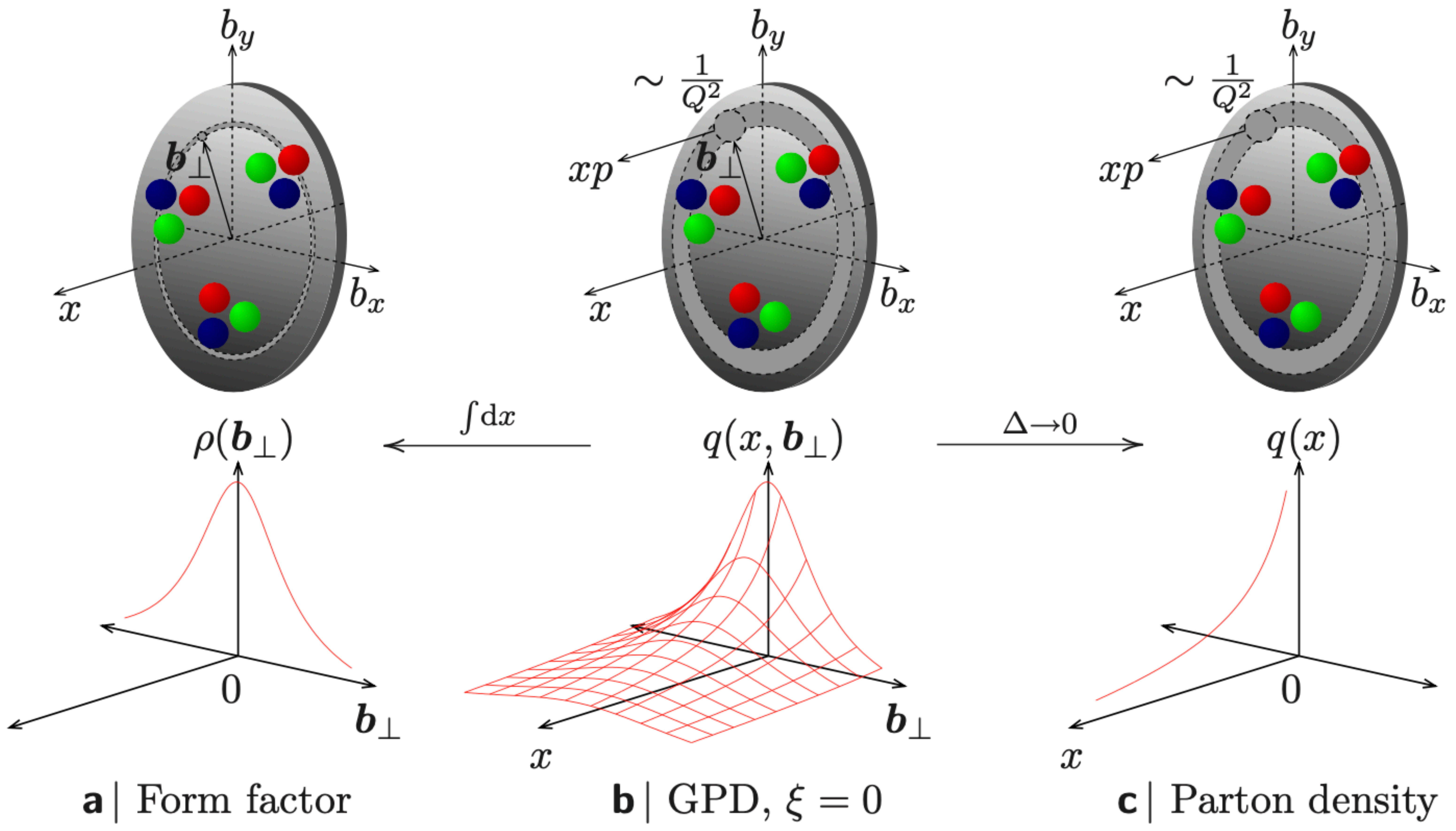
Hyeon-Dong Son

Hadron Theory Group, Inha University

Introduction

Generalized parton distributions: 3D-tomography of hadron structure

Generalized parton distributions (GPDs) [D. Mueller et al. 1994]



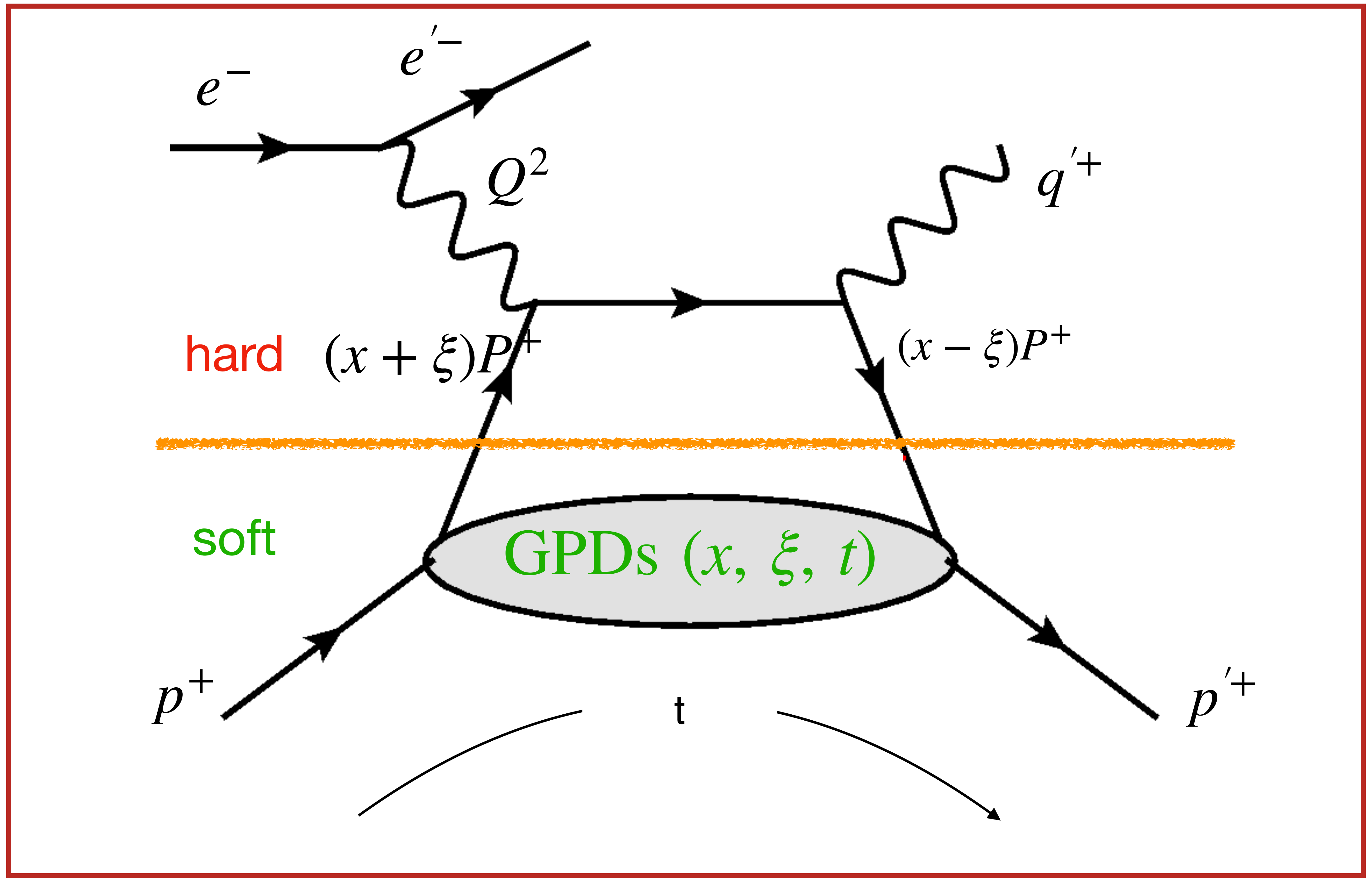
Spatial distribution in impact parameter space

Longitudinal momentum distribution

Hard exclusive reactions for GPDs

$$e + p \to e' + p' + \gamma/M$$

Deeply Virtual Compton Scattering / Meson Production (DVCS/DVMP)



Scattering cross-section factorizes as:

hard part (pQCD) $\otimes$ soft part

Q^2 : Virtuality → hard scattering limit $Q^2 \gg |t|, M_t^2, M_s^2, \cdots,$

p^+ : Light-front (LF) longitudinal momentum of incoming target,

p'^+ : Light-front (LF) longitudinal momentum of incoming target,

P^+ $= (p^+ + p^+)/2$, average hadron momentum,

ξ $= (p^+ - p'^+)/(p^+ + p'^+)$, skewness,

asymmetry of longitudinal momentum of target,

$x \pm \xi$: Longitudinal momentum fraction,

t : Squared momentum transfer, $\Delta^2 = (q' - q)^2 = (p - p')^2$,

"kick" transverse momentum depending on scattering angle.

Generalized parton distributions

[M. Diehl, Phys. Rept. 388 (2003) 41]

Interpretation of GPDs with respect to x and ξ

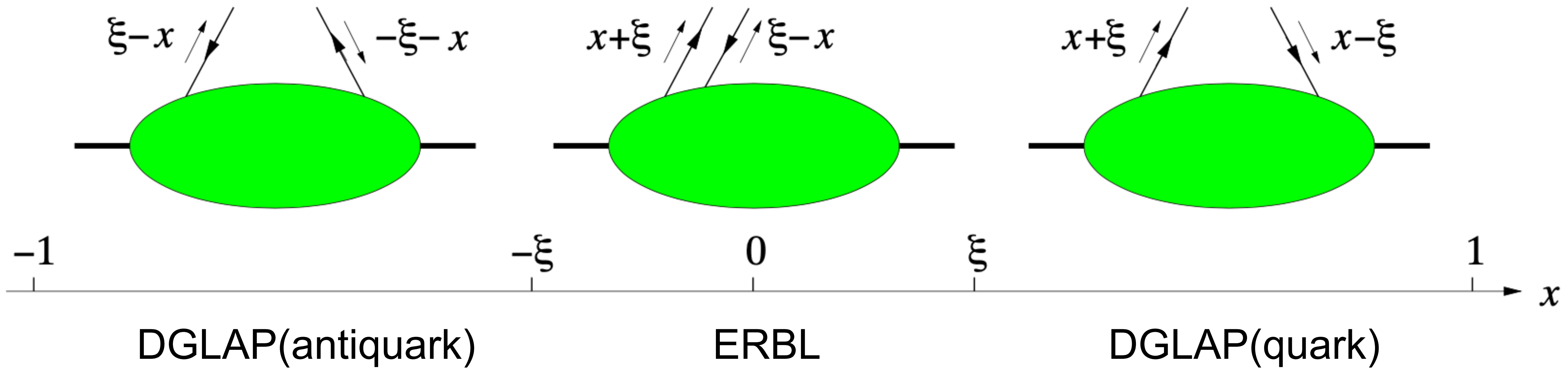


Figure 5: The parton interpretation of GPDs in the three x-intervals $[-1,-\xi]$, $[-\xi,\xi]$, and $[\xi,1]$.

1. $x \in [\xi, 1]$: Emission of a quark $x+\xi \geq 0$ and reabsorption $x-\xi \geq 0$.
2. $x \in [-\xi, \xi]$: Emission of a quark $x+\xi \geq 0$ and an antiquark $\xi - x \geq 0$ from the initial proton.
3. $x \in [-1, -\xi]$: Emission of an antiquark $\xi - x \geq 0$ and absorption of an antiquark $-\xi - x \geq 0$.

Scale evolution of GPDs

Limiting cases: present DGLAP ($\xi \to 0$) and ERBL ($\xi \to 1$) perturbative kernels

[A recent study:
V. Bertone et al. *Eur. Phys. J. C* 82 (2022) 10, 888
Also, references therein]

Chiral symmetry breaking and the Goldstone bosons

Hadron mass spectra: maximally broken chiral symmetry, eg. N(1/2+, 940) vs N(1/2-,1535).

Spontaneously broken chiral symmetry, $\langle \bar{\psi}\psi \rangle \neq 0 \rightarrow$ massless Goldstone boson (Pion)

Explicit chiral symmetry breaking by current quark masses $m \rightarrow$ Goldstone bosons acquire mass M

Gell-Mann - Oakes - Renner

$$M^2F^2 = -m\langle \bar{\psi}\psi \rangle + \mathcal{O}(m^2)$$

Including strangeness ($m_s \ll \Lambda$), SU(3)f: π, K, η

Breaking SU(3)f with $m_s \approx 100$ MeV may require significant correction in $\mathcal{O}(m^2)$

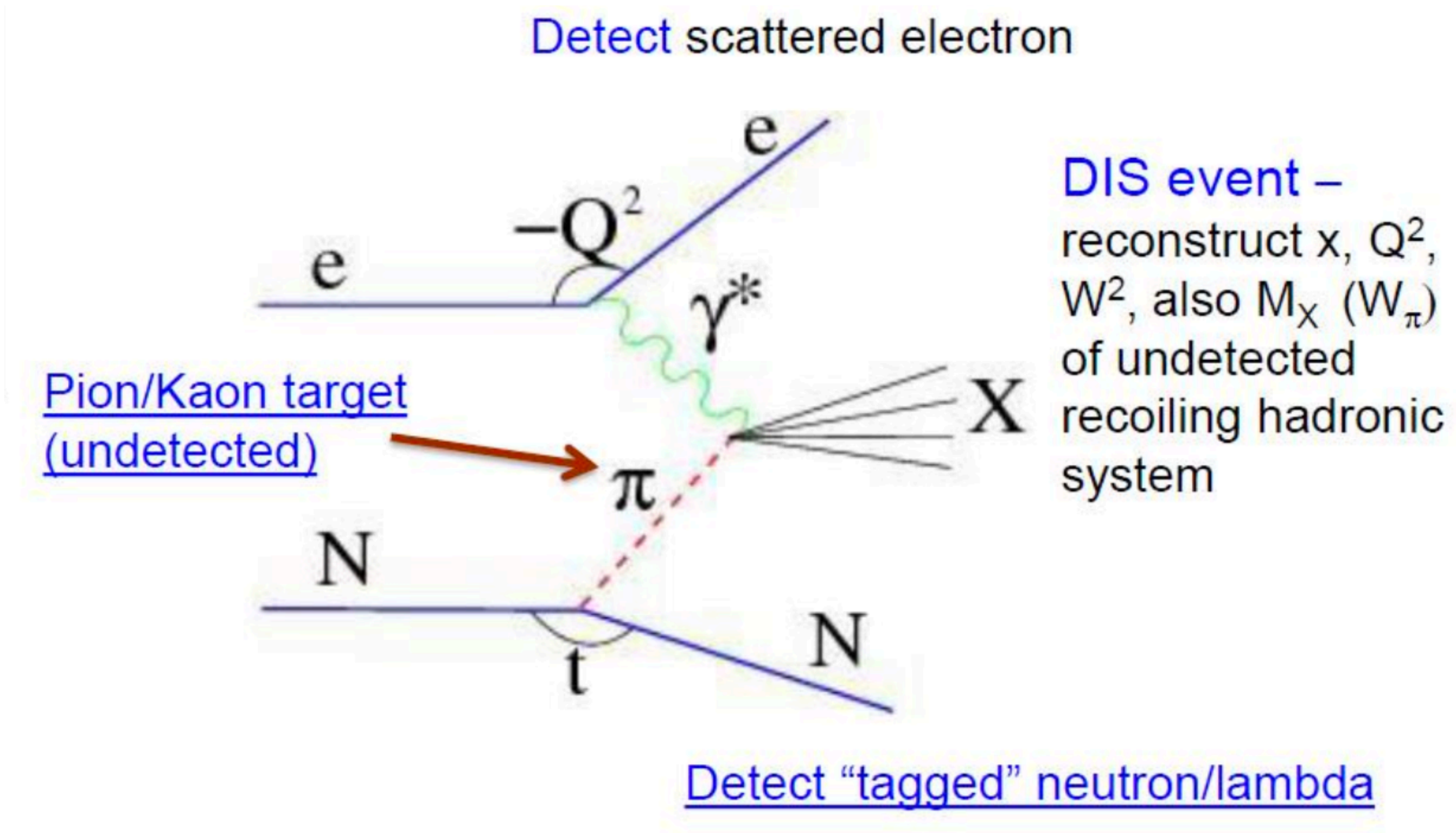
Quark structure of the kaon can be different from the pion

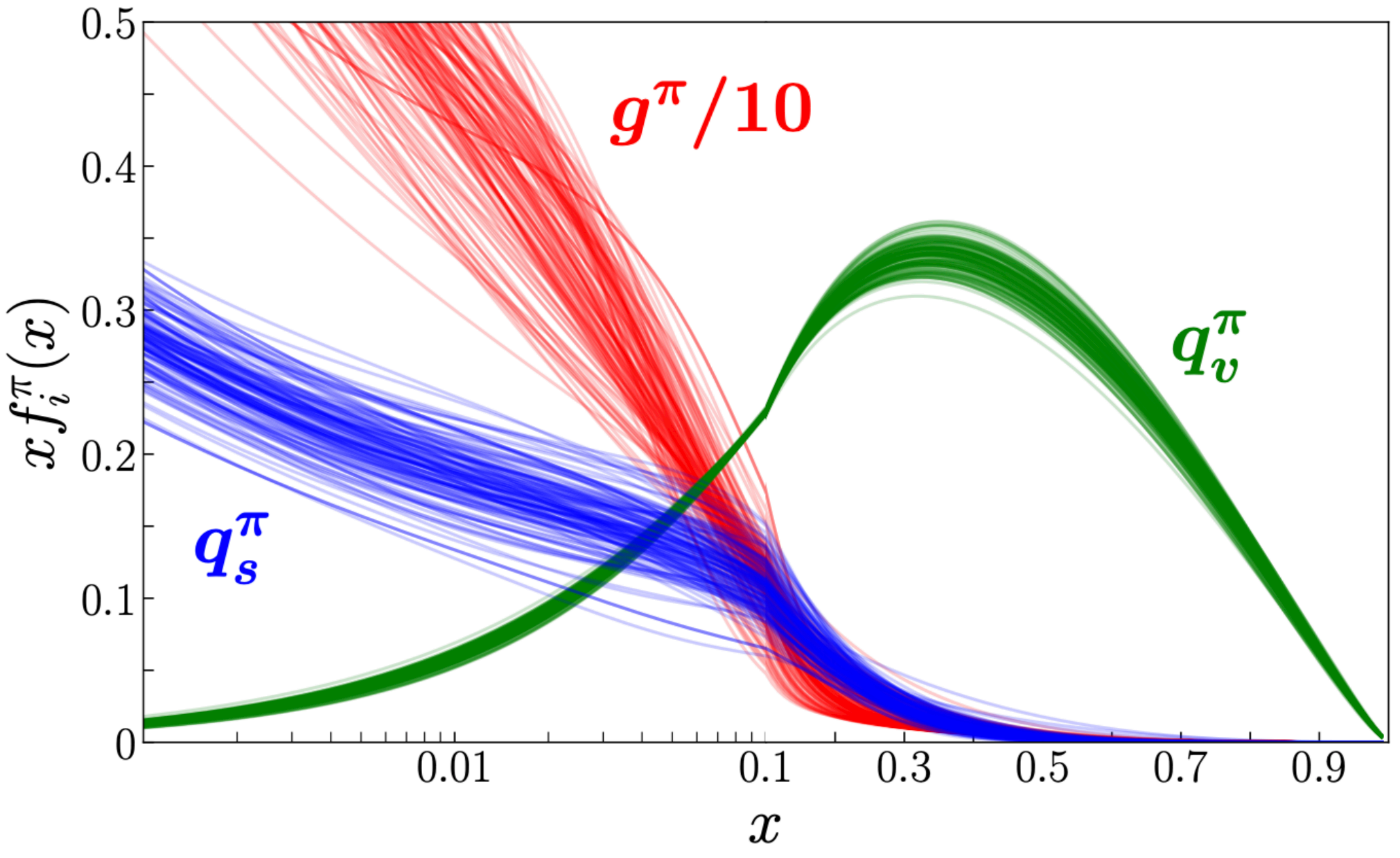
Pion and Kaon structures from Sullivan process

No meson target exists, let's do some collisions

Drell-Yan and Sullivan process to study the PDFs → global analysis

Eg.) JAM collaboration for pion PDFs





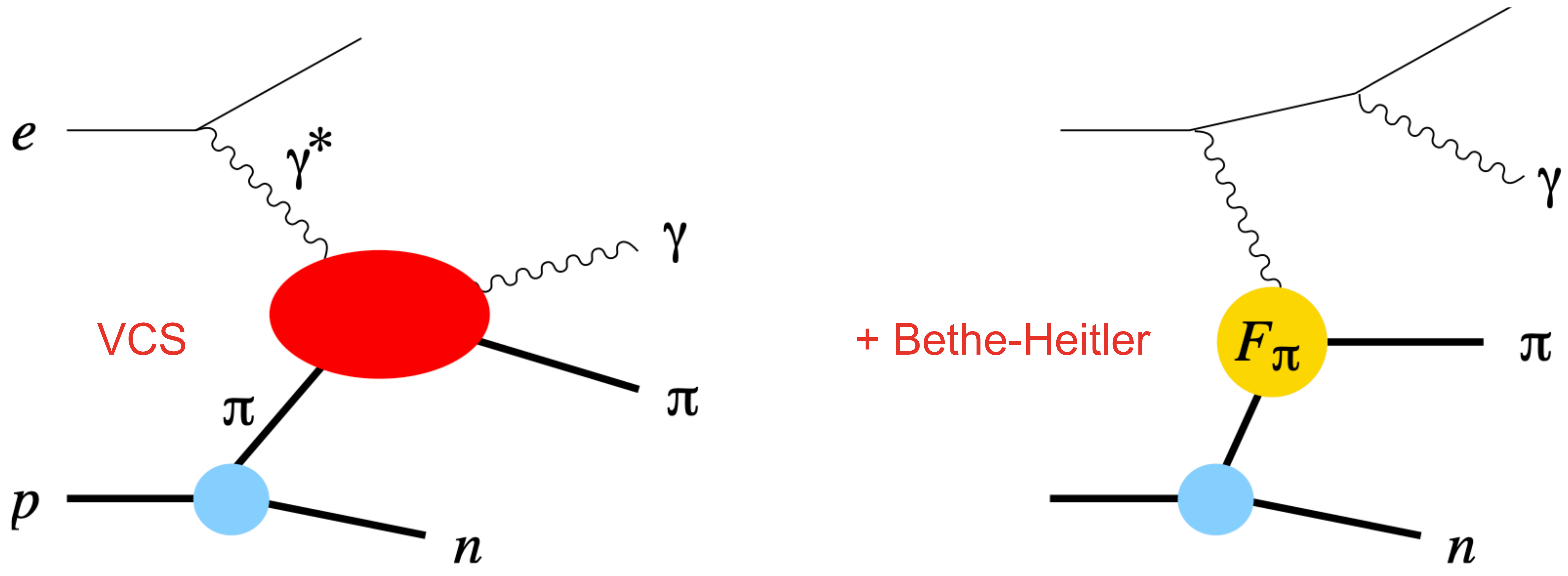
- **Pion-pole dominance**
- **Pion virtuality ~ nucleon momentum transfer**

Pion and Kaon GPDs from Sullivan process

[Amrath, Diehl, Lansberg, EPJ.C58,179-192]

DVCS in Sullivan process

$$ep \rightarrow e'\gamma\pi^+ n$$



Cross-section too small for JLAB 11GeV [Amrath, Diehl, Lansberg, EPJ.C58,179-192]

Feasibility study for EIC case [Chavez et al, PRL 128 (2022)]

Estimation study for the process involving the Kaon GPDs can be paralleled

Kaon generalized parton distributions

Quark GPDs for Kaon

Singlet generalized quark distributions in the kaon

$$\frac{1}{2}\int\frac{d\lambda}{2\pi}\exp\left(i\lambda x n\cdot\bar{P}\right)\langle K^+(p')|\left\{\begin{array}{c}\bar{u}(-\lambda n/2)\not{n}u(\lambda n/2)\\ \bar{s}(-\lambda n/2)\not{n}s(\lambda n/2)\end{array}\right\}|K^+(p)\rangle=\left\{\begin{array}{c}H^{u/K^+}(x,\xi,t)\\ -H^{\bar{s}/K^+}(-x,\xi,t)\end{array}\right\} \quad (3.1)$$

Symmetry properties

$$H^{u/K^+}(x,\xi,t)=H^{u/K^+}(x,-\xi,t)=-H^{u/K^+}(-x,\xi,t)$$

Mellin moments n=0

$$\int_{-1}^{+1}dx\; H^{u/K^+}(x,\xi,t)=A_{10}^{u/K^+}(t)$$
$$\int_{-1}^{+1}dx\; H^{\bar{s}/K^+}(x,\xi,t)=A_{10}^{\bar{s}/K^+}(t). \quad (2.3)$$

$$e_u A_{10}^{u/K^+}(t)+e_{\bar{s}}A_{10}^{\bar{s}/K^+}(t)=F_{K^+}(t)$$

Quark GPDs for Kaon

Singlet generalized quark distributions in the kaon

$$\frac{1}{2}\int\frac{d\lambda}{2\pi}\exp\left(i\lambda x n\cdot\bar{P}\right)\langle K^+(p')|\left\{\begin{array}{c}\bar{u}(-\lambda n/2)\not{n}u(\lambda n/2)\\ \bar{s}(-\lambda n/2)\not{n}s(\lambda n/2)\end{array}\right\}|K^+(p)\rangle=\left\{\begin{array}{c}H^{u/K^+}(x,\xi,t)\\ -H^{\bar{s}/K^+}(-x,\xi,t)\end{array}\right\} \tag{3.1}$$

Zero momentum transfer of the target hadron → Parton distribution functions

$H^q(x,\,0,\,0)=f_1(x)$ Unpolarized quark distribution

Mellin moments n=0

$$\int_{-1}^{+1}dx\,H^{u/K^+}(x,\xi,t)=A_{10}^{u/K^+}(t)$$
$$\int_{-1}^{+1}dx\,H^{\bar{s}/K^+}(x,\xi,t)=A_{10}^{\bar{s}/K^+}(t). \tag{2.3}$$

$$e_u A_{10}^{u/K^+}(t)+e_{\bar{s}}A_{10}^{\bar{s}/K^+}(t)=F_{K^+}(t)$$

Quark GPDs for Kaon

Singlet generalized quark distributions in the kaon

$$\frac{1}{2}\int\frac{d\lambda}{2\pi}\exp\left(i\lambda x n\cdot\bar{P}\right)\langle K^+(p')|\left\{\begin{array}{c}\bar{u}(-\lambda n/2)\not{n}u(\lambda n/2)\\ \bar{s}(-\lambda n/2)\not{n}s(\lambda n/2)\end{array}\right\}|K^+(p)\rangle=\left\{\begin{array}{c}H^{u/K^+}(x,\xi,t)\\ -H^{\bar{s}/K^+}(-x,\xi,t)\end{array}\right\} \quad (3.1)$$

Mellin moments n=1

$$\int_{-1}^{+1}dx\,H^{u/K^+}(x,\xi,t)=A_{20}^{u/K^+}(t)+\xi^2A_{22}^{u/K^+}(t),$$
$$\int_{-1}^{+1}dx\,H^{\bar{s}/K^+}(x,\xi,t)=A_{20}^{\bar{s}/K^+}(t)+\xi^2A_{22}^{\bar{s}/K^+}(t). \quad (2.5)$$

Momentum sum-rule

$$A_{20}^{u/K^+}(0)+A_{20}^{\bar{s}/K^+}(0)=M_2^{val}$$

A20 and A22 proportional to the gravitational form factors!

A20: mass distribution, A

A22: pressure and shear distribution, D

Theoretical Studies on the meson GPDs and Gravitational FFs

Pion structure is studied extensively,

(Methods:ChPT, Lattice QCD, Effective models as ChQM, LFWF, Dyson-Schwinger, …)

Pion gravitational form factors

Chiral perturbation theory [Donoghue and Leutwyler, ZPC52 (1991)]

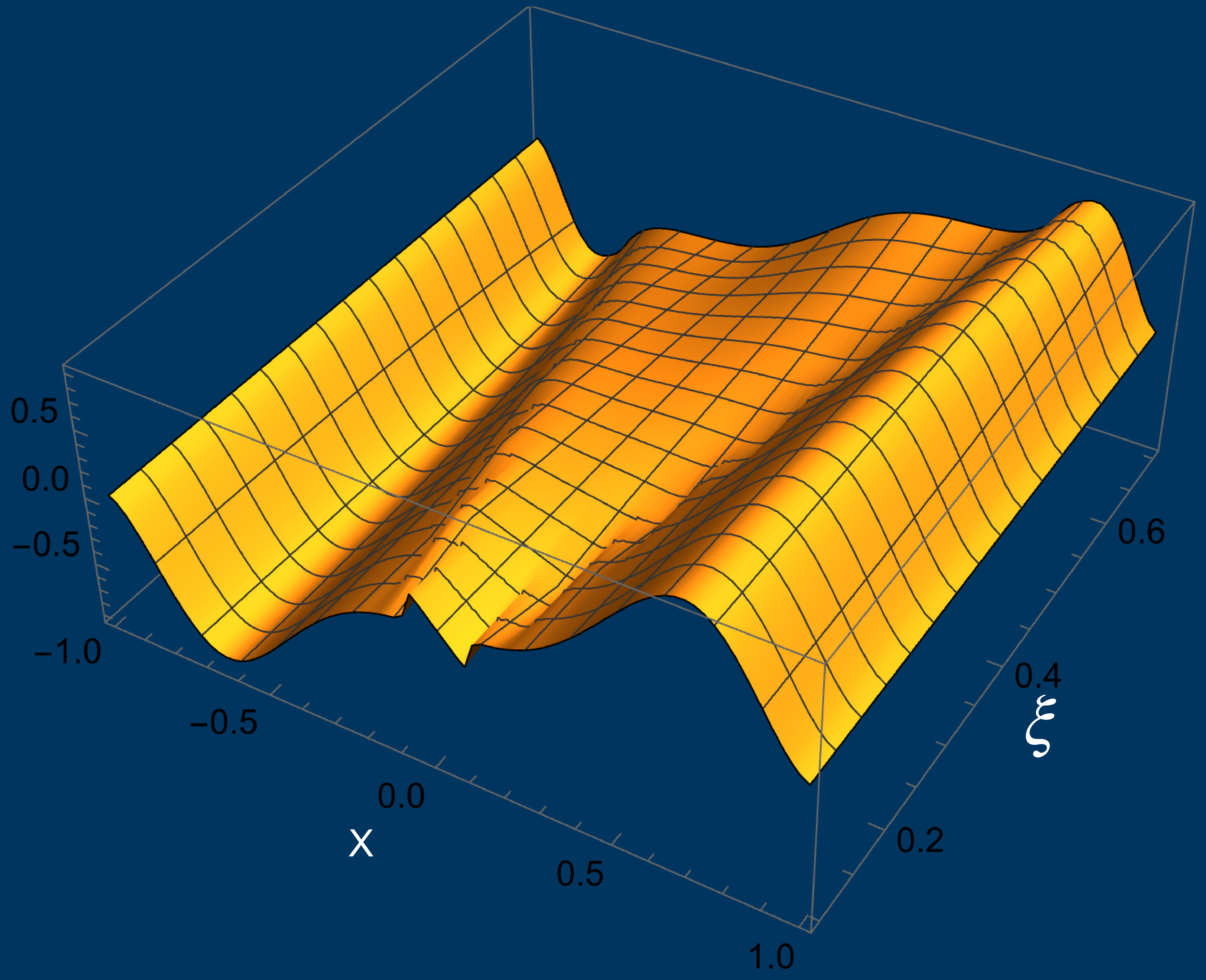
Crossing and GDAs (Belle data $\gamma\gamma^* \to \pi^0\pi^0$) [Kumano, Song, Teryaev ,PRD 97 (2018)] [Masuda et al, PRD 93 (2016)]

Chiral quark model (non-trivial cancellation of internal pressure) [HDS and H.-Ch. Kim, PRD 90 (2014)]
(… many other studies)

Studies on the Kaon GPDs appear only recently, mostly from LFWF, DSE (only DGLAP region)

We extend Praszalowicz and Rostworowski (pion GPDs in chiral limit) to study the Kaon GPDs [Acta Phys. Polon. B 34 (2003) 2699–2730]

Kaon GPDs and GFFs from the NLChQM



Why do we still rely on effective models? (My projection in this game)

Model independent approaches

Experiments, Lattice QCD, Effective theories (HQEFT, Large Nc QCD, ChPT)

What we can learn from a model

Complimentary study for experiment and lattice

Initial state of the partons inside a hadron at low energy scale,

insights via the effective degrees of freedom

A sound effective model should

be firmly planted to the first principle (symmetries),

clear and understandable limitation

eg. Instanton QCD vacuum

not have too much free parameters (self-consistency)

Quark one-loop effective action in the large Nc limit

$$S_{\rm eff} = \int \frac{d^4k}{(2\pi)^4}\bar{\psi}(k)(\not{k} - \hat{m})\psi(k) - \int \frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\bar{\psi}(p)\sqrt{M(p)}U^{\gamma_5}(p-k)\sqrt{M(k)}\psi(k), \tag{2.1}$$

$$M(k) = MF^2(k), \qquad U^{\gamma_5}(x) = \exp\left[\frac{i}{F_{\mathcal{M}}}\gamma^5\lambda^a\mathcal{M}^a\right], \qquad \hat{m} = \mathrm{diag}(m_u, m_d, m_s).$$

Inspired by the liquid instanton model at low-renormalization point $\mu \sim 1/\bar{\rho}$ (in Euclidean)

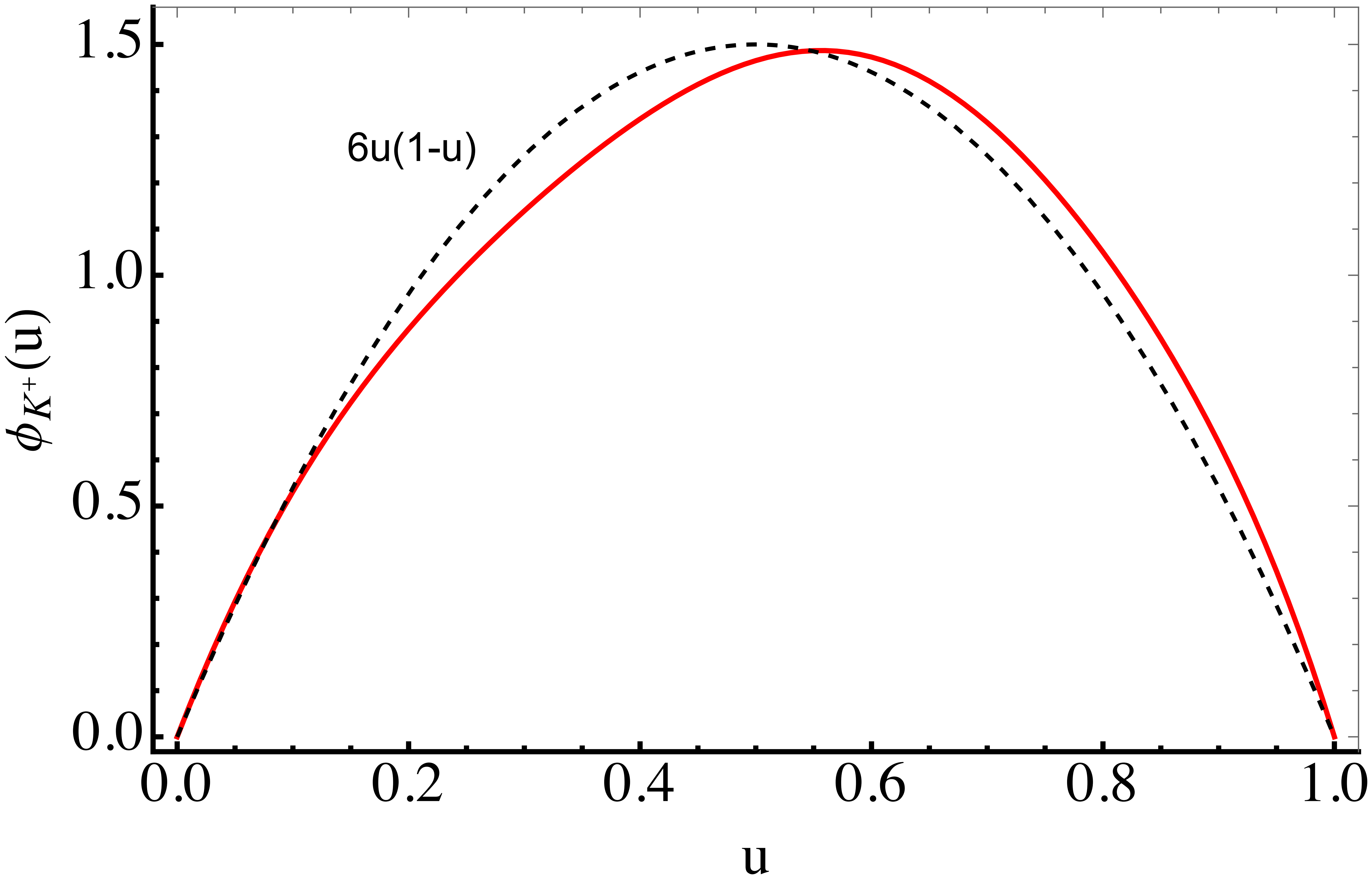
Assumed analytic continuation to the Minkowski space-time

n-pole type quark form factor: $F(k) = \left(\frac{1}{1 - k^2/\Lambda^2}\right)^n$ vs. large ~$1/k^3$ behavior of the instanton induced FFs

$M(0) = 350$ MeV is computed in the dilute instanton vacuum

n, Λ: model parameters fixed by the normalization of the pion light-cone DA ($F_\pi = 93$ MeV)

Kaon light-cone distribution amplitude



For the pion and kaon DA,

$$\int_0^1 du\ \phi(u) = 1$$

For n=1, we fix $\Lambda = 1.2$ GeV and $m_s = 100$ MeV

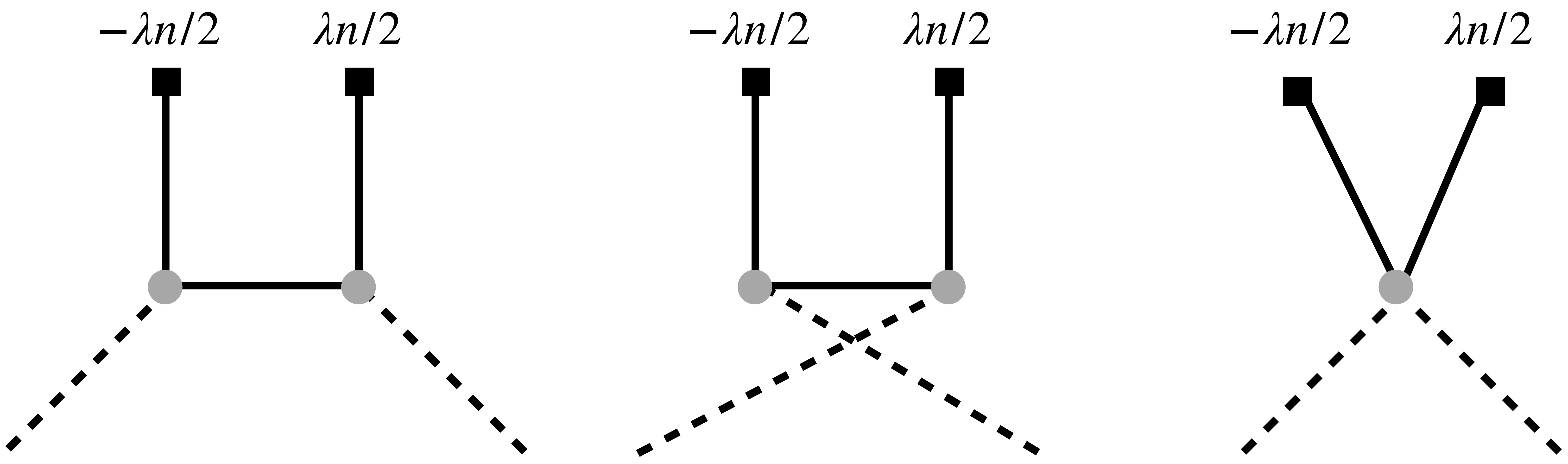
using the normalization conditions

(Kaon DA is skewed towards u=1 slightly,

due to explicit chiral symmetry breaking:

$(m_u, m_s, m_{K^+}) = (5{,}100{,}494)$ MeV.)

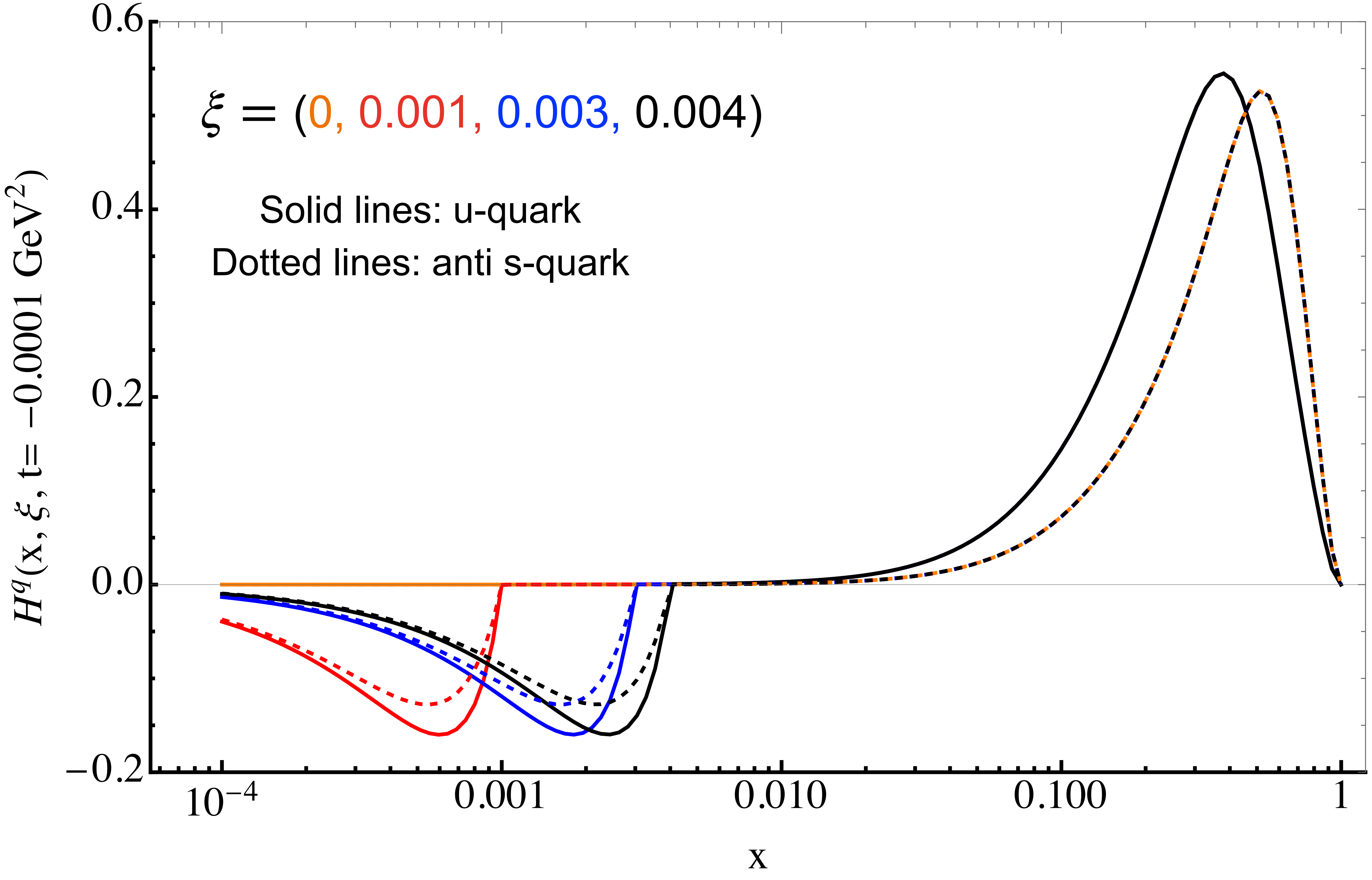
Leading Nc quark-loop diagrams for the kaon valence-quark GPDs



$$\frac{1}{2}\int\frac{d\lambda}{2\pi}\exp\left(i\lambda x n\cdot\bar{P}\right)\langle K^+(p')|\left\{\begin{array}{c}\bar{u}(-\lambda n/2)\not{n}u(\lambda n/2)\\ \bar{s}(-\lambda n/2)\not{n}s(\lambda n/2)\end{array}\right\}|K^+(p)\rangle=\left\{\begin{array}{c}H^{u/K^+}(x,\xi,t)\\ -H^{\bar{s}/K^+}(-x,\xi,t)\end{array}\right\} \quad (3.1)$$

- **The hadronic matrix elements with the quark bilinear operator are computed covariantly in the model**
- **DGLAP (PDF) region governed by the first and second diagrams**
- **Third diagram contributes only to** $-\xi < x < \xi$

Kaon GPD (-t = 0.0001 GeV²)

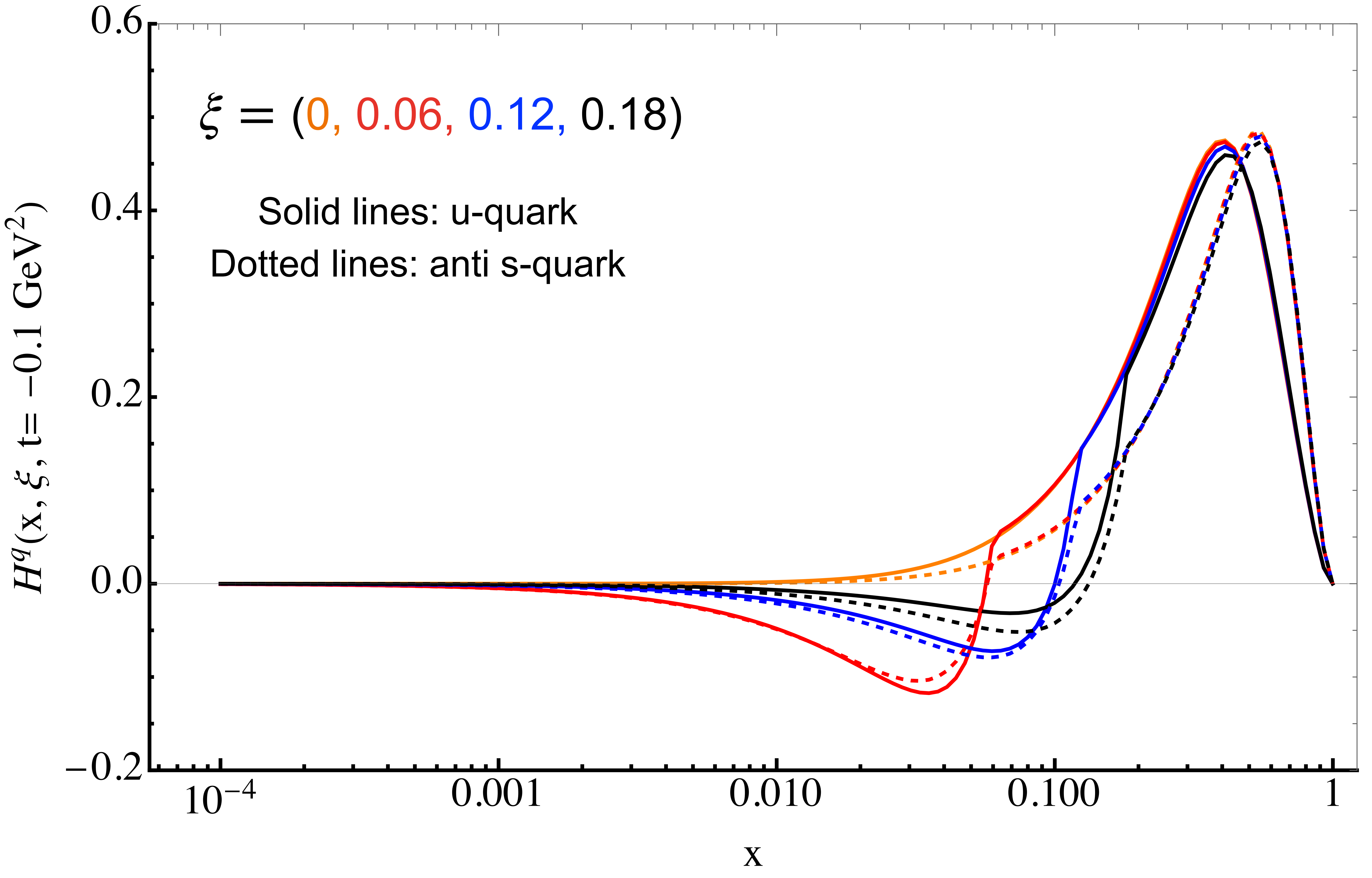


- Continuous GPDs at cross-over points $x = \xi$ but derivatives are not (common in many other studies)
- Physically allowed skewness

$$|\xi| \leq \sqrt{\frac{-t}{-t + 4m_K^2}}$$

- **At very small |t|, GPDs ~ valence PDF**
- **Strange quarks have larger momentum (→momentum sum)**

Kaon GPD (-t = 0.1 GeV²)

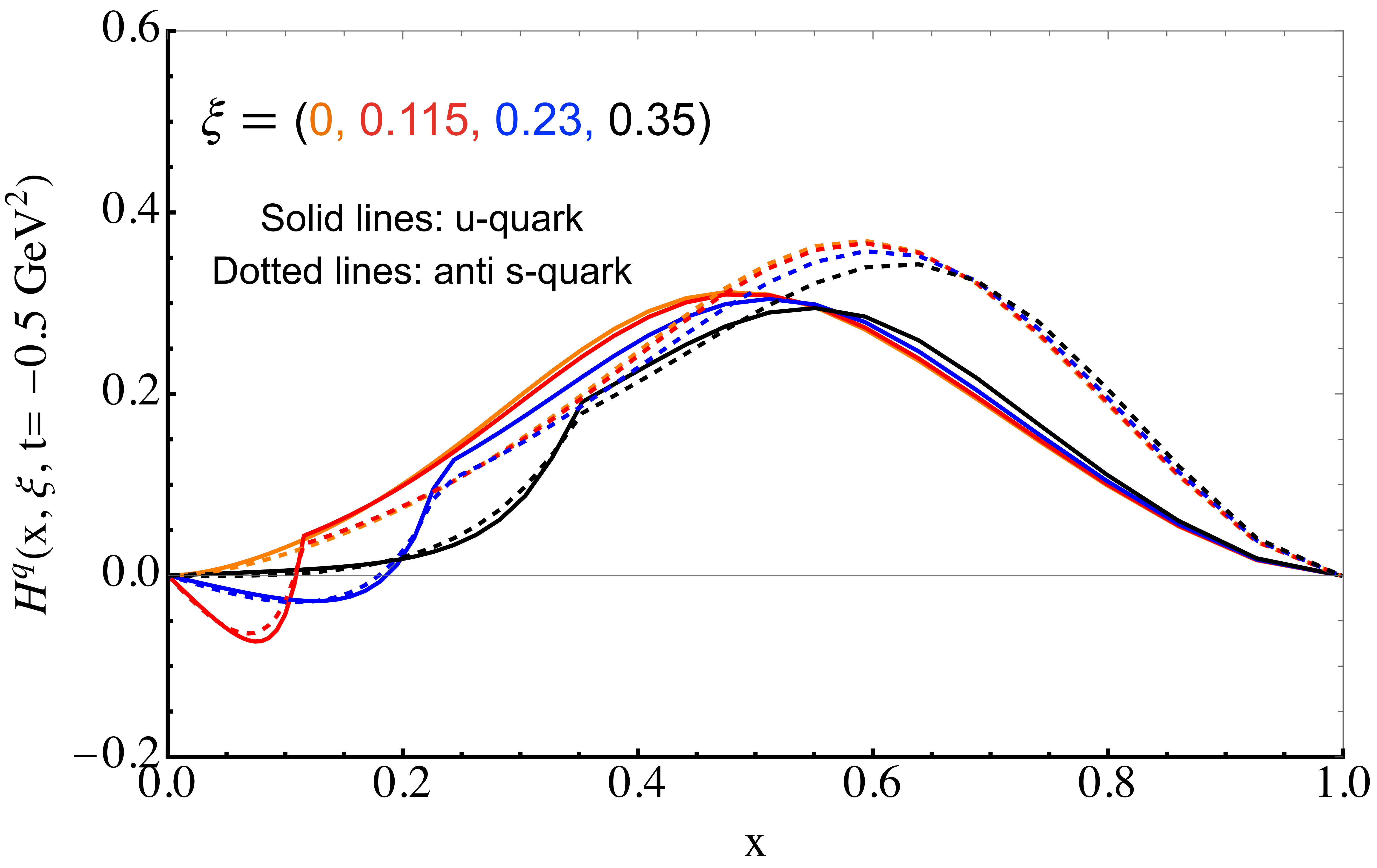


- Continuous GPDs at cross-over points $x = \xi$ but derivatives are not (common in many other studies)
- Physically allowed skewness

$$|\xi| \leq \sqrt{\frac{-t}{-t + 4m_K^2}}$$

- **As |t| gets larger, u-quark has stronger t dependence**
- **Cross-over point $x = \xi$ splits, growing in larger x**

Kaon GPD (-t = 0.5 GeV^2)



- Continuous GPDs at cross-over points $x = \xi$ but derivatives are not (common in many other studies)
- Physically allowed skewness

$$|\xi| \le \sqrt{\frac{-t}{-t + 4m_K^2}}$$

- **As |t| gets larger, u-quark has stronger t dependence**
- **Cross-over point $x = \xi$ splits, growing in larger x**

QCD energy-momentum tensor operator

$\hat{T}^a_{\mu\nu}$: **QCD energy-momentum tensor operator (a: quarks and gluon), symmetric, gauge-invariant**

Quark

$$\hat{T}^{\mu\nu}_q = \frac{1}{4}\bar{\psi}_q\left(-i\overleftarrow{\mathcal{D}}^\mu\gamma^\nu - i\overleftarrow{\mathcal{D}}^\nu\gamma^\mu + i\overrightarrow{\mathcal{D}}^\mu\gamma^\nu + i\overrightarrow{\mathcal{D}}^\nu\gamma^\mu\right)\psi_q - \eta^{\mu\nu}\bar{\psi}_q(i\overleftrightarrow{\not{D}}/2 - m_q)\psi_q$$

Gluon

$$\hat{T}^{\mu\nu}_g = -F^{\mu\alpha}F^\nu_\alpha + \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$

Symmetric ($\mu \leftrightarrow \nu$), gauge invariant (not in the canonical derivation)

Not conserved separately (renormalization scale dependent), but total operator $\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_q + \hat{T}^{\mu\nu}_g$ is conserved

Trace anomaly: the renormalized operator $\hat{T}^\mu{}_\mu = \frac{\beta(g)}{2g}F^2 + (1+\gamma_m)m\bar{\psi}\psi$ non-vanishing in the chiral limit

Mass decomposition $2M^2 = \langle P|\frac{\beta(g)}{2g}F^2|P\rangle + \langle P|(1+\gamma_m)\bar{\psi}m\psi|P\rangle$

Gravitational form factors of the Kaon

$$\langle K^+(p') | \hat{T}^a_{\mu\nu}(0) | K^+(p) \rangle = \left[4P_\mu P_\nu \, A^a(t) + (q^\mu q^\nu - g^{\mu\nu} q^2) \, D^a(t) + g^{\mu\nu} 4\Lambda^2 \bar{c}^a(t) \right]$$

$A^a(t)$

Mass distribution of the quarks and gluons inside the kaon

At t=0, second Mellin moment of the unpolarized PDF

Normalization $A^q(0) + A^g(0) = 1$

$D^a(t)$ (D-term)

Dispersion relation of the DVCS (and DVMP) amplitudes

Fundamental, but not related to an obvious symmetry

Internal pressure and shear distributions

Negative for hadrons to satisfy the stability conditions

[Polyakov, Shuvaev hep-ph/0207153]
[Polyakov PLB555 (2003)]
[Polyakov, Schweitzer IJMPA33 (2018)]

$\bar{c}^a(t)$

Non-conservation of quark and gluon parts of EMT ~ $g_{\mu\nu}$

Contributes to the mass(00) and the pressure(ii) (quark and gluon portions)

[M. Polyakov, HDS, JHEP 156 (2018)]

$\sum_q \bar{c}^q + \bar{c}^g = 0$, Smallness of $\sum_q \bar{c}^q(0)$ at low scale, suppressed by instanton packing fraction

Gravitational form factors of the Kaon

$$\langle K^+(p')\,|\,\hat{T}^a_{\mu\nu}(0)\,|\,K^+(p)\rangle = \left[4P_\mu P_\nu\, A^a(t) + (q^\mu q^\nu - g^{\mu\nu}q^2)\, D^a(t) + g^{\mu\nu}4\Lambda^2\bar{c}^a(t)\right]$$

Low energy theorem: χPT to O(p²) [Donoghue and Leutwyler, ZPC52 (1991)]

$$A(t) = 1 - 2\, L^r_{12}\frac{t}{F^2}$$

$$-D(t) = 1 + 2\frac{t}{F^2}(4L^4_{11} + L^r_{12})$$

$$-16\frac{m_K^2}{F^2}(L^4_{11} - L^r_{13}) + \frac{3t}{4F^2}I_\pi(t) + \frac{3t}{2F^2}I_K(t) + \frac{9t - 8m_K^2}{12F^2}I_\eta(t)$$

$$I(q^2) = \frac{1}{48\pi^2}\left[\ln\frac{\mu^2}{m^2} - 1 + \frac{q^2}{5m^2}\right] + \mathcal{O}(q^4)$$

GFF LECs: L_{11}, L_{12}, L_{13}

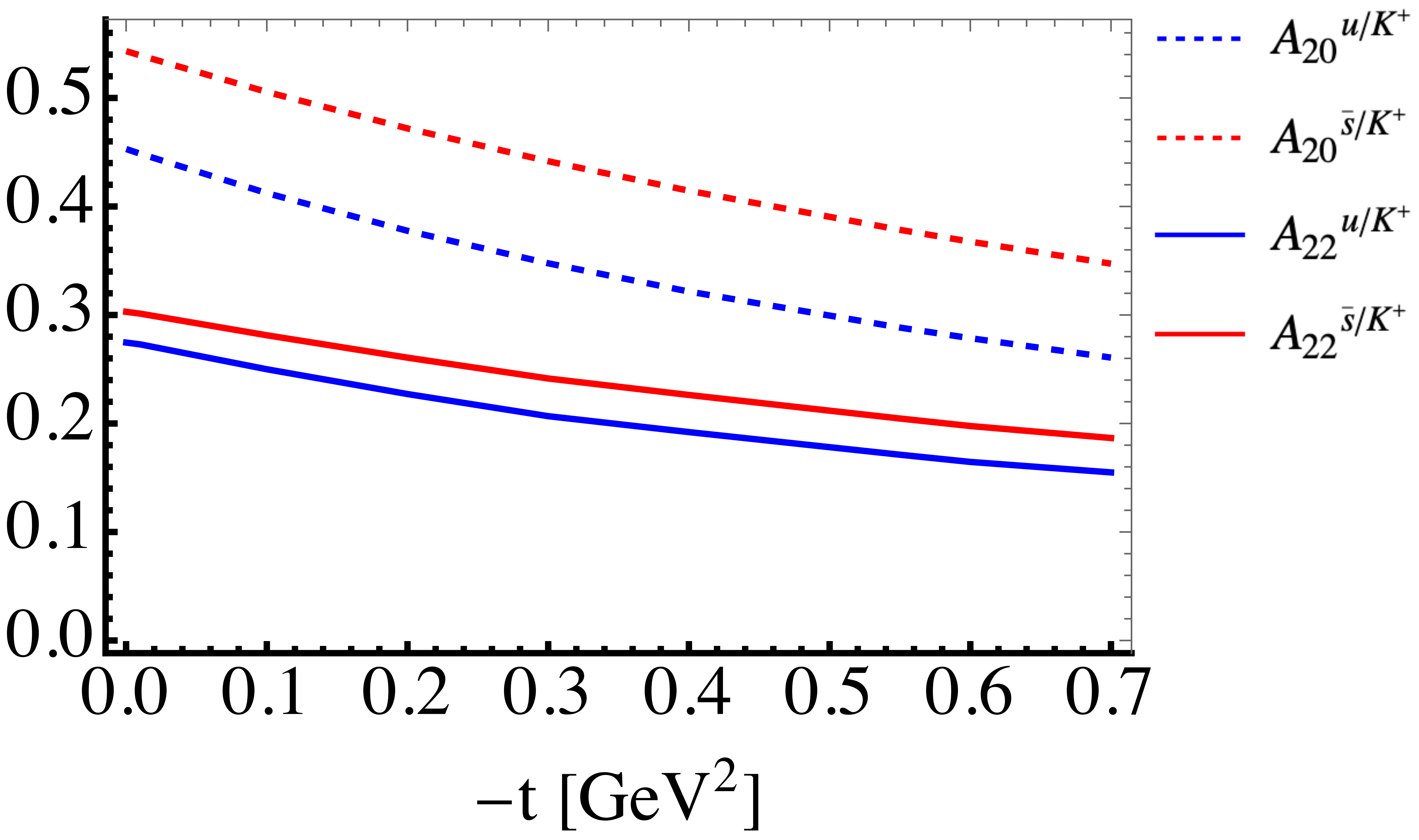
A and D have different sign but same normalization with meson mass correction

$$A(0) + D(0) = \boxed{\frac{16m_K^2}{F^2}(L^r_{11} - L^r_{13})} + \frac{m_K^2}{72\pi^2F^2}\left[\ln\frac{\mu^2}{m_\eta^2} - 1\right] + \ldots \approx 0.77 \pm 0.15 \quad (\mu = m_\eta)$$

[Hudson and Schweitzer, Phys. Rev. D 96, 114013 (2017)]

Leading Nc result in the quark model, magnitude is amplified by larger kaon mass (vs. A+D=0.03 for the pion)

Kaon gravitational form factors



Values at -t=0	s	u	Total
A_{20}(=A)	0.54	0.45	0.99
A_{22}(=-D)	0.30	0.27	0.57

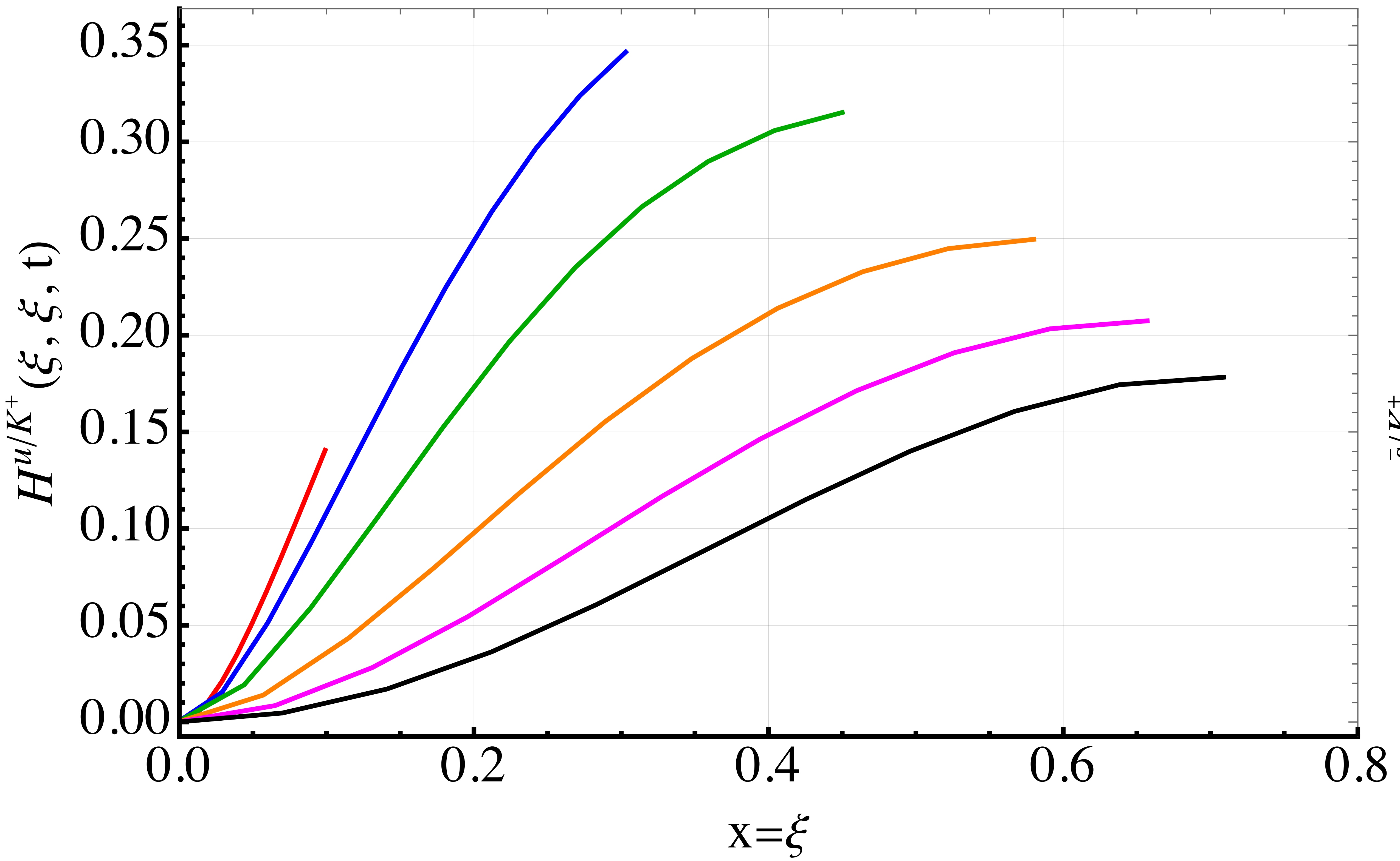
Comparison with other works

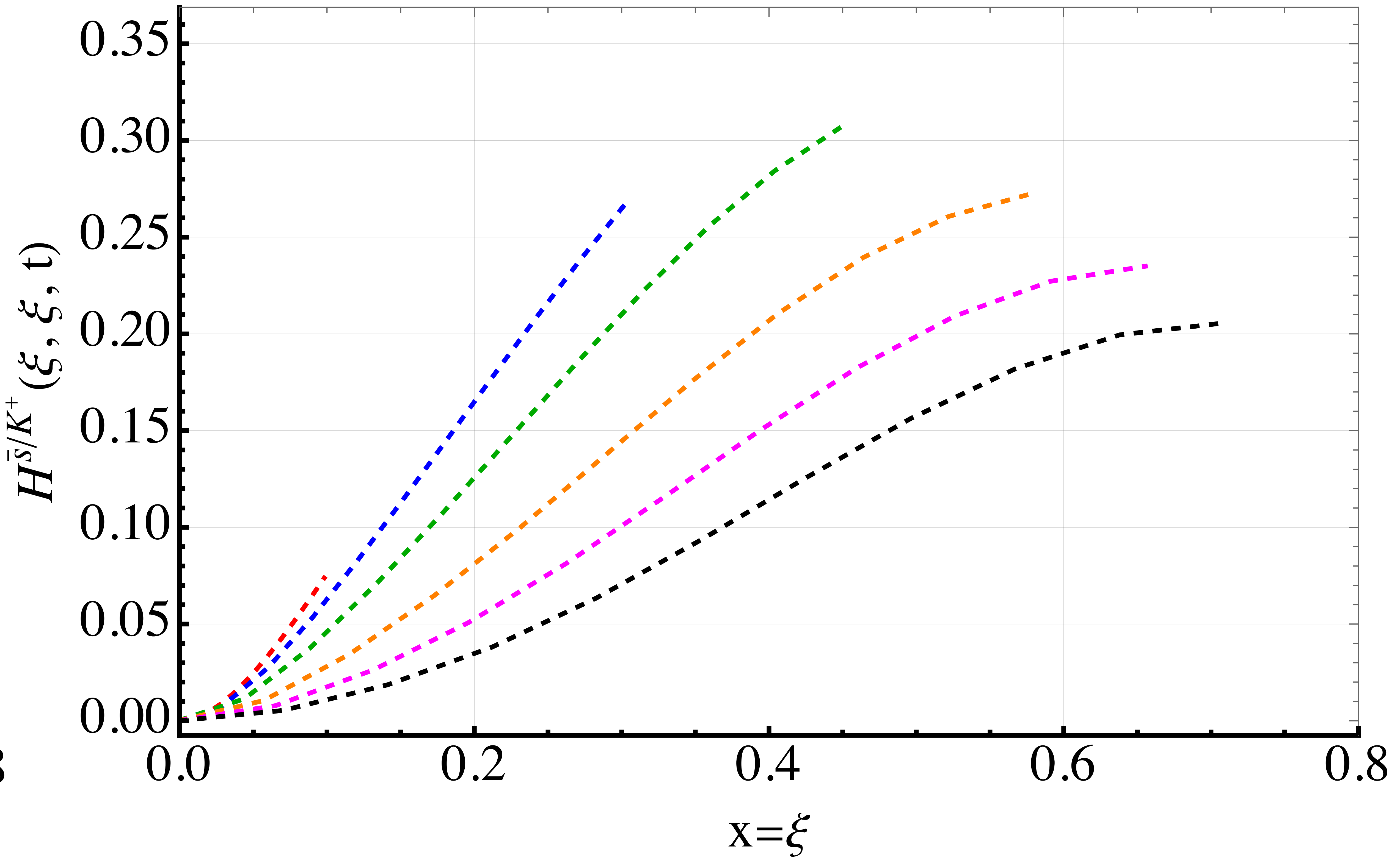
- ChPT: Donoghue and Leutwyler D(0) = - 0.77 +- 0.15
- Raya et al, LFWFs (2021), CPC 46 (2022)
  $|D_u(0)|$=0.8 $|D_s(0)|$, but D(0)=-1?
- Y.-Z. Xu et al, DS-BS, D(0) = - 0.77 & Du/Ds=0.8
- A_s/A_u is consistent with other works,
  Eg.) P. Hutauruk et al, NJL model, PRC 94 (2016)

Kaon GPD ($x = \xi$)

$-t = (0.01, 0.1, 0.25, 0.5, 0.75, 1)\ \mathrm{GeV}^2$

Solid lines: u-quark
Dotted lines: anti s-quark





Quark GPDs along the cross-over line ($x = \xi$)

~ Imaginary part of the Compton form factor

Example) Pion GPD evolution (isoscalar I=0, see red-dashed curves)

[Shastry et al, hep-ph/2308.09236]

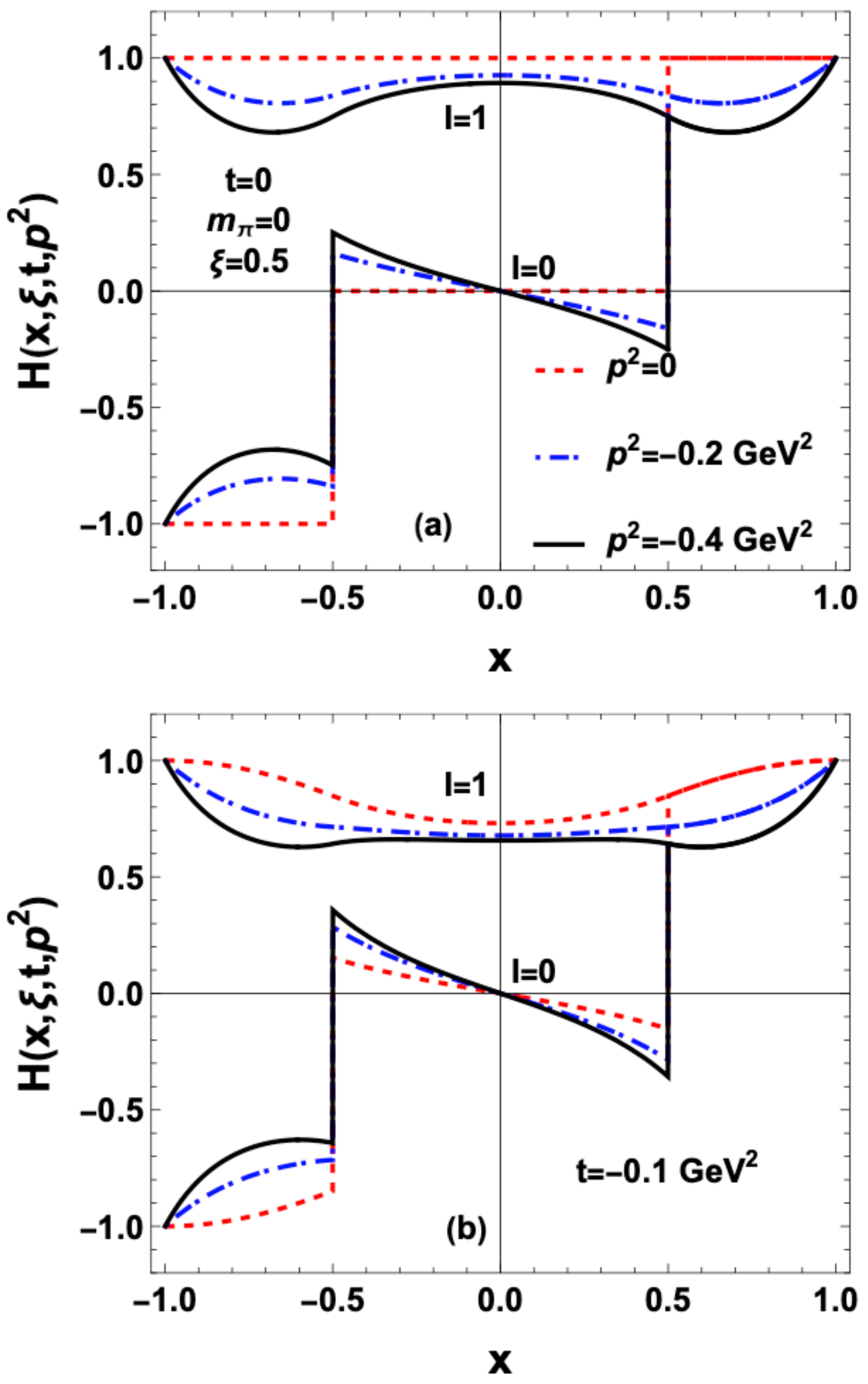


FIG. 4. Half-off-shell pion GPDs at $\xi = 0.5$ for (a) $t = 0$ and (b) $t = -0.1$ GeV^2, evaluated in the chiral limit in SQM at the quark model scale for several values of the off-shell parameter p^2.

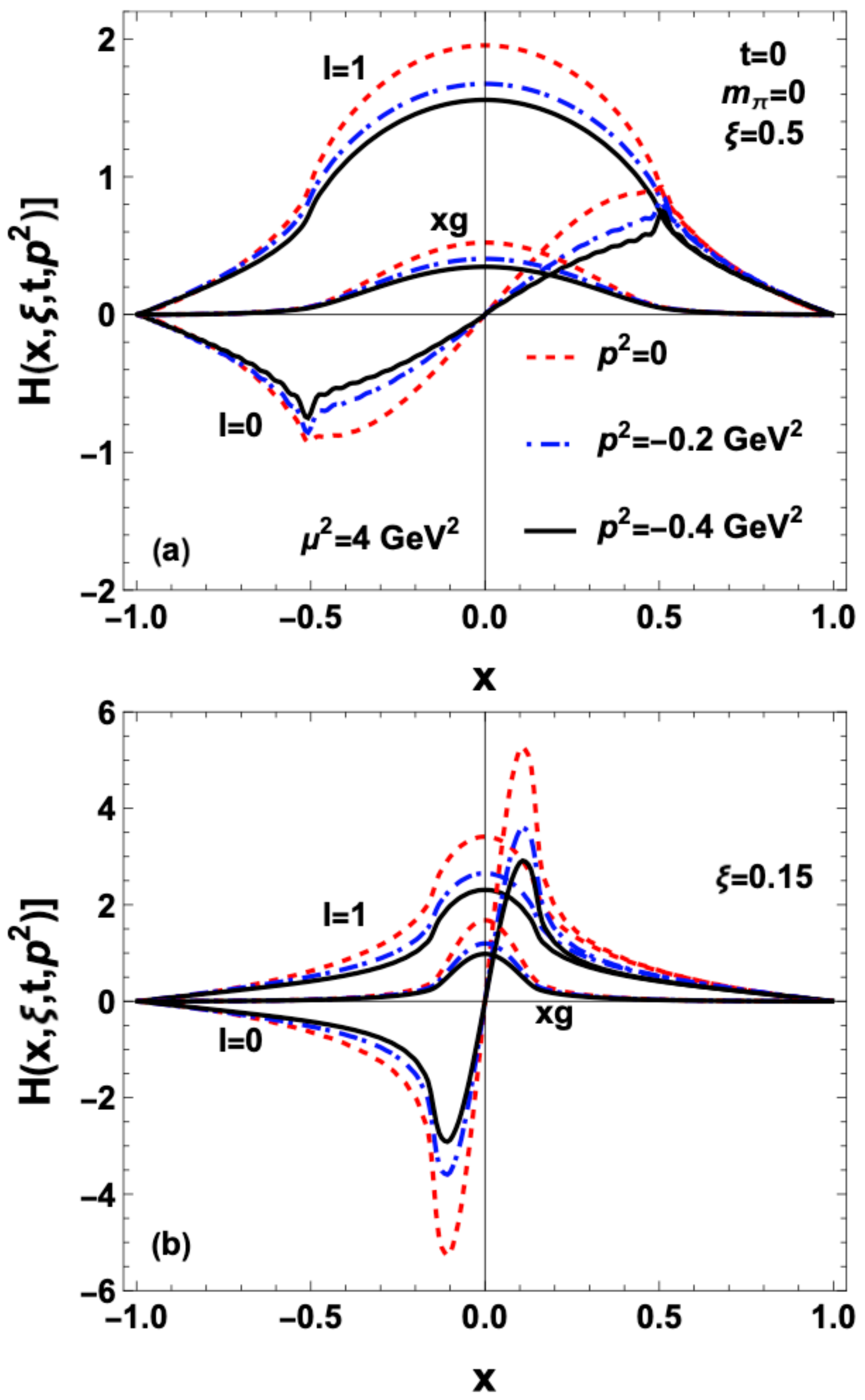


FIG. 5. Half-offshell pion GPD for $t = 0$ at $\xi = 0.5$ and $\xi = 0.15$, evolved to $Q^2 = 4$ GeV^2 with LO DGLAP-ERBL equations.

- Decreasing GPD at $x = \xi$, as -t larger
- Evolution leading an enhancement at $x = \xi$, especially at small x, possibly due to the gluon and sea quarks
- Nb. LO calculation, could be not enough
- Similar tendency in more realistic picture (current study & NLO)?

Summary and outlook

Observations

We computed the Kaon GPDs within the nonlocal chiral quark model

Cross-over $x = \xi$ point is continuous but not smooth

Light quark distribution presents stronger t-dependence than strange quark

Gravitational form factors $D^u/D^s \sim 0.9$, $D^{u+s} \sim 0.6$ can be compared with the ChPT prediction ~0.77

Tasks

Perturbative evolution of the GPDs

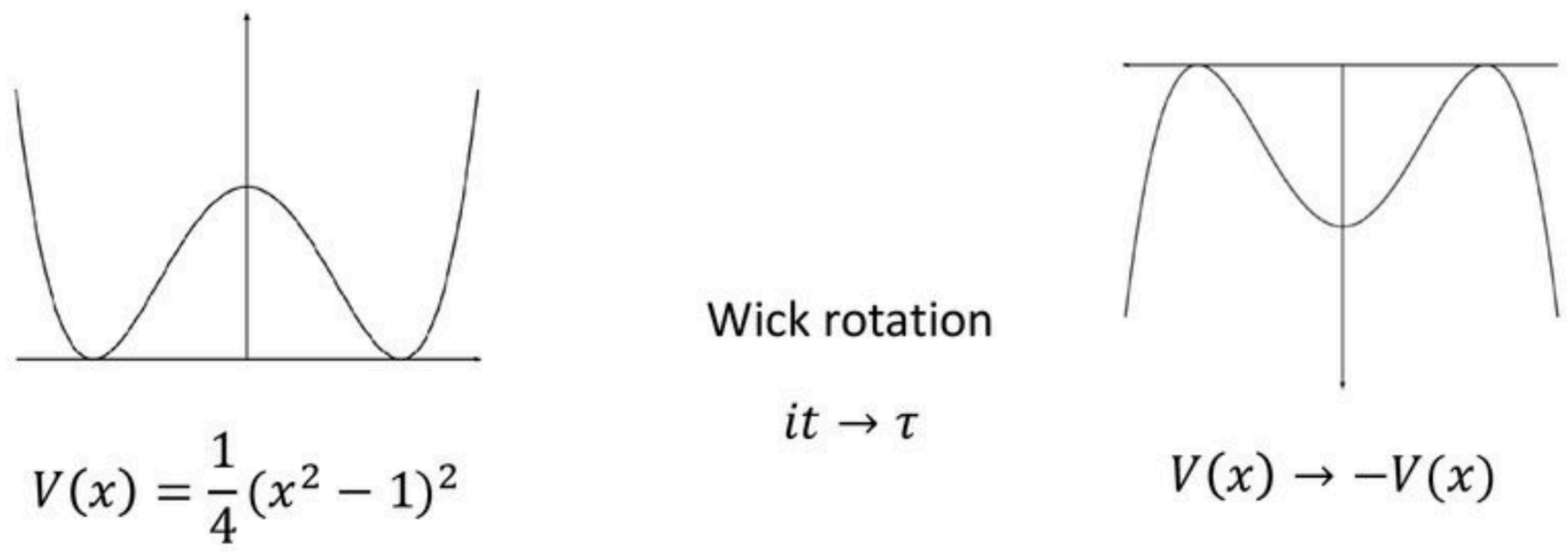
Restoration of gauge invariance, broken due to the nonlocal quark interaction

Detailed study on the kaon Sullivan-DVCS process in EIC (can we use our model result for the σ estimation?)

Model can be used for the transition GPDs ($\pi \to \pi\pi$, in progress with K. Semenov-Tyan-Shanskiy)

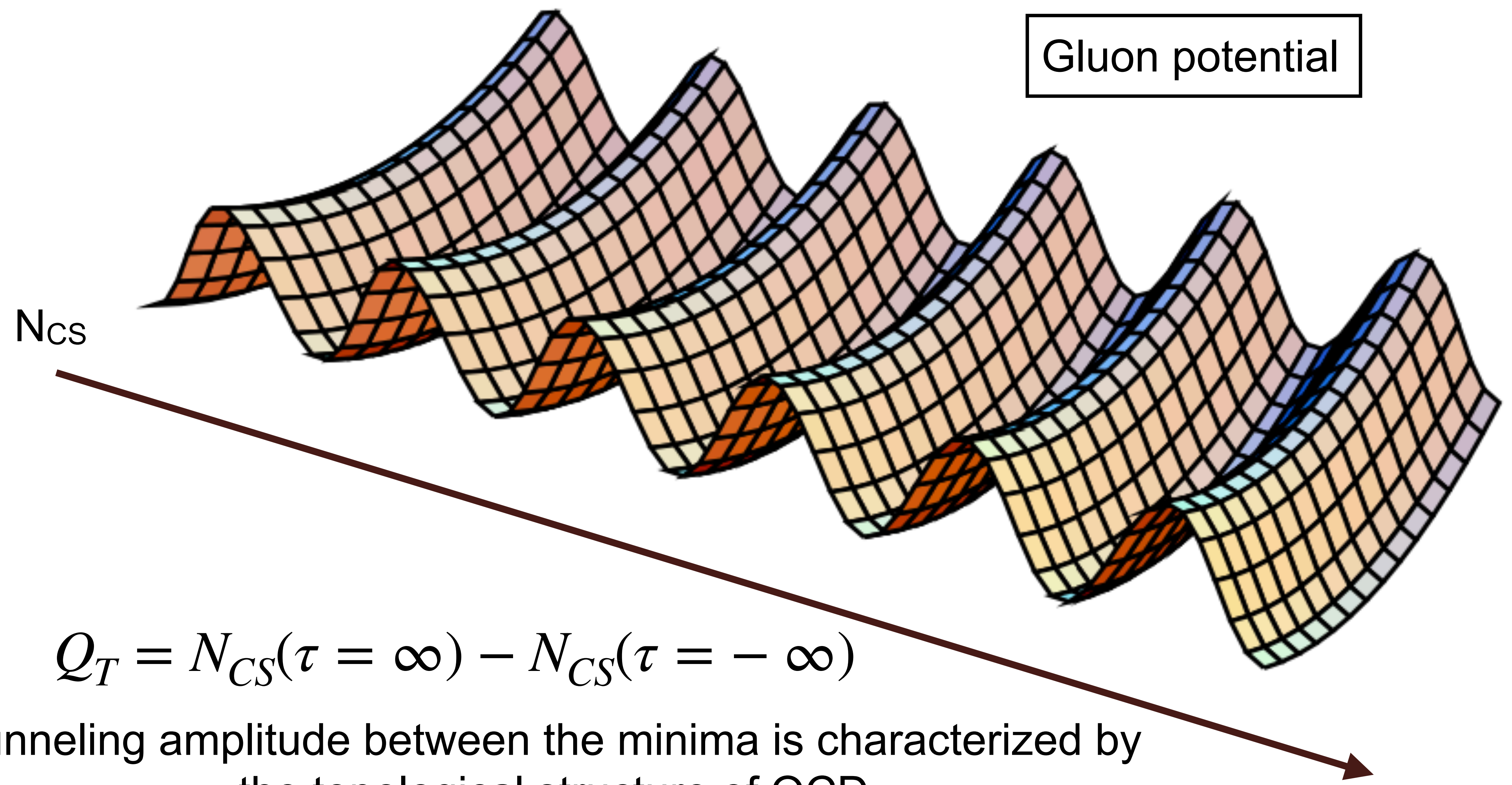
Thank you very much!

Nonlocal chiral quark model from the Instanton QCD-vacuum



Tunneling amplitude between the minima

Classical path between the apexes



$$Q_T = N_{CS}(\tau = \infty) - N_{CS}(\tau = -\infty)$$

Tunneling amplitude between the minima is characterized by the topological structure of QCD

Classical solution minimizes the Euclidean YM's action

$$F = \tilde{F}$$

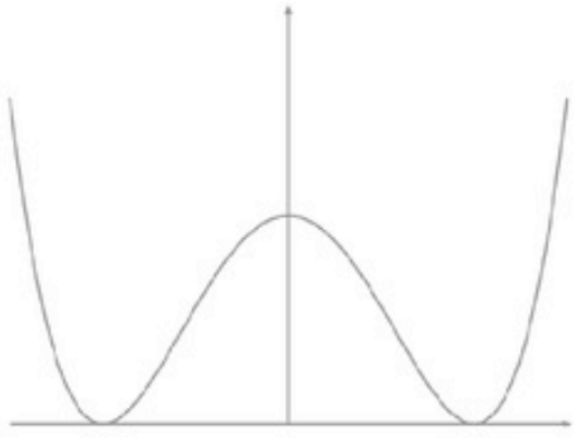
Spatial distribution of the instanton is characterized by

$$\bar{\rho} \approx 0.5/\Lambda_{\overline{MS}}$$

$$\bar{R} \approx 1.35/\Lambda_{\overline{MS}}$$

Diluteness is assumed

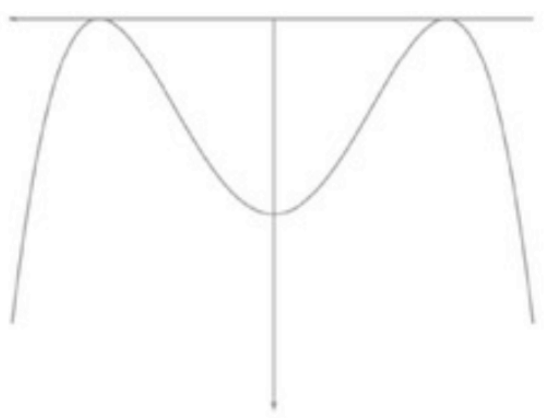
Nonlocal chiral quark model from the Instanton QCD-vacuum



$V(x) = \frac{1}{4}(x^2 - 1)^2$

Wick rotation

$it \to \tau$



$V(x) \to -V(x)$

Tunneling amplitude between the minima

Classical path between the apexes

Classical solution minimizes the Euclidean YM's action

$$F = \tilde{F}$$

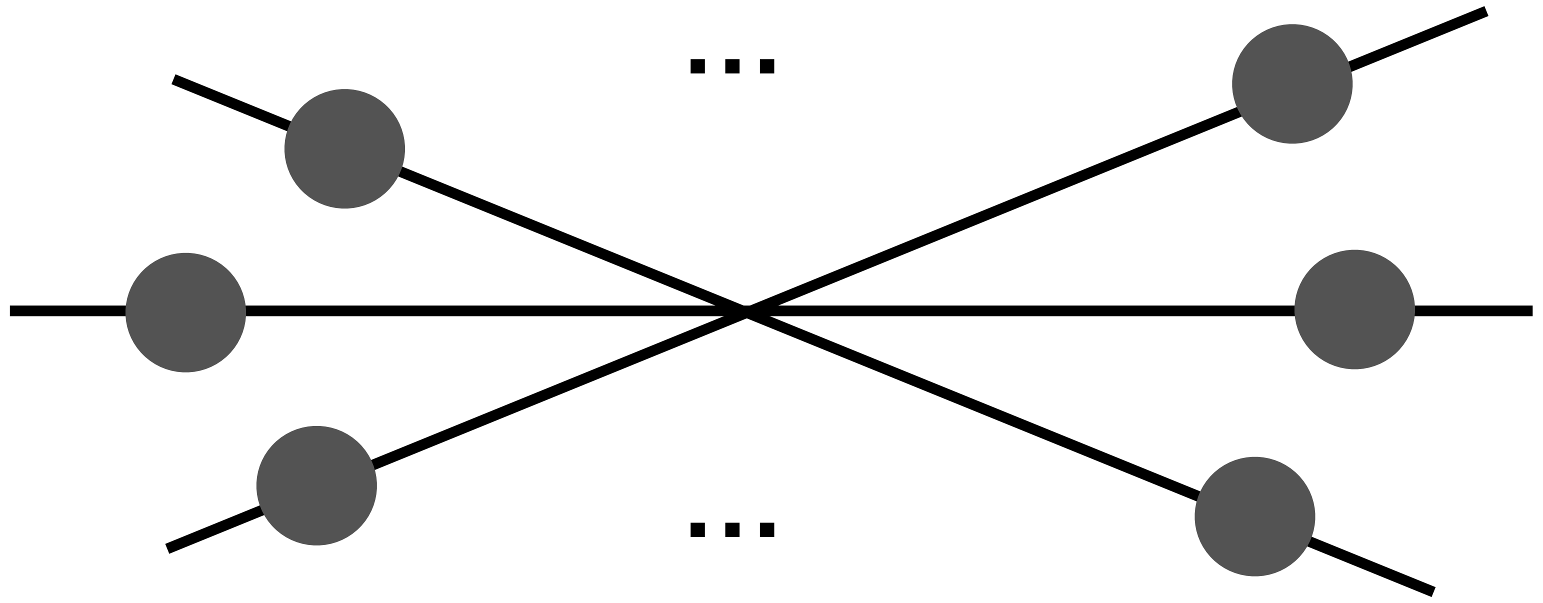
Spatial distribution of the instanton is characterized by

$$\bar{\rho} \approx 0.5/\Lambda_{\overline{MS}}$$

$$\bar{R} \approx 1.35/\Lambda_{\overline{MS}}$$

Diluteness is assumed

't Hooft like 2-N_f quark effective interactions



Quark form-factor

$$F(k) = 2t \left[I_0(t)K_1(t) - I_1(t)K_0(t) - \frac{1}{t} I_1(t)K_1(t) \right] \Bigg|_{t=\frac{k\rho}{2}}$$

Dynamical quark mass

$$M \approx 350 \text{ MeV}$$

Definition: Pion

M. Polyakov & Ch. Weiss, Phys.Rev. D60 (1999) 114017, arXiv:hep-ph/9902451

$$\langle \pi^a(p-r/2)|\, \bar{\psi}(-z/2) \left\{ \begin{array}{c} 1 \\ \\ \tau^c \end{array} \right\} \hat{z}\, [-z/2, z/2]\, \psi(z/2)\, |\pi^b(p+r/2)\rangle$$

$$= \left\{ \begin{array}{l} 2\delta^{ab}\, \mathcal{M}^{I=0}(p\cdot z,\ r\cdot z;\ t) \\ \\ 2i\varepsilon^{abc}\mathcal{M}^{I=1}(p\cdot z,\ r\cdot z;\ t). \end{array} \right. \qquad (1)$$

$$\mathcal{M}^{I=0,1}(p\cdot z,\ r\cdot z = \xi p\cdot z;\ t) \quad = \quad 2p\cdot z \int_{-1}^{1} dX\ e^{-iXpz}\ H^{I=0,1}(X,\xi;t), \qquad (8)$$

$$H(X,\xi;t) \quad = \quad \frac{1}{2}\int \frac{d\tau}{2\pi} e^{i\tau X\, p\cdot n}$$
$$\times \langle \pi(p-r/2)|\, \bar{\psi}(-\tau n/2)\ \hat{n}\ [-\tau n/2, \tau n/2]\, \psi(\tau n/2)\ |\pi(p+r/2)\rangle \qquad (9)$$

Properties: Pion

M. Polyakov & Ch. Weiss, Phys.Rev. D60 (1999) 114017, ,arXiv:hep-ph/9902451

$$H^{I=0}(X,\,\xi=0;\,t=0) \;=\; \frac{1}{2}\left[\theta(X)\,q_{\rm s}(X) \;-\; \theta(-X)\,q_{\rm s}(-X)\right],$$

$$H^{I=1}(X,\,\xi=0;\,t=0) \;=\; \theta(X)q_{\rm v}(X) \;+\; \theta(-X)q_{\rm v}(-X), \qquad (10)$$

$$q_{\rm s}(X) \;=\; [u+\bar{u}]_{\pi^\pm}(X) \;=\; [d+\bar{d}]_{\pi^\pm}(X) \;=\; [u+\bar{u}]_{\pi^0}(X) \;=\; [d+\bar{d}]_{\pi^0}(X),$$

$$q_{\rm v}(X) \;=\; \pm[u-\bar{u}]_{\pi^\pm}(X) \;=\; \mp[d-\bar{d}]_{\pi^\pm}(X). \qquad (11)$$

$$\int_{-1}^{1} dX\, X\, \left[H^{I=0} + H^{G}\right](X,\xi;t=0) \;=\; \frac{1}{2}\left(1 \;-\; \frac{\xi^2}{4}\right). \qquad (12)$$

$$\int_{-1}^{1} dX\, H^{I=1}(X,\xi;t) \;=\; F_{\pi}^{\rm e.m.}(t).$$