Non-diagonal DVCS off pion with $\pi \to \pi\pi$ transition GPDs

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2nd APCTP Workshop on the physics of Electron Ion Collider 2023.12.01

Non-diagonal DVCS and DVMP

Study of $\gamma^* N \to \gamma(M)\pi N$ with transition GPD

I. Excitation of hadrons with non-local QCD probe (resonance region).

II. Natural test ground of the chiral dynamics (near the threshold region).

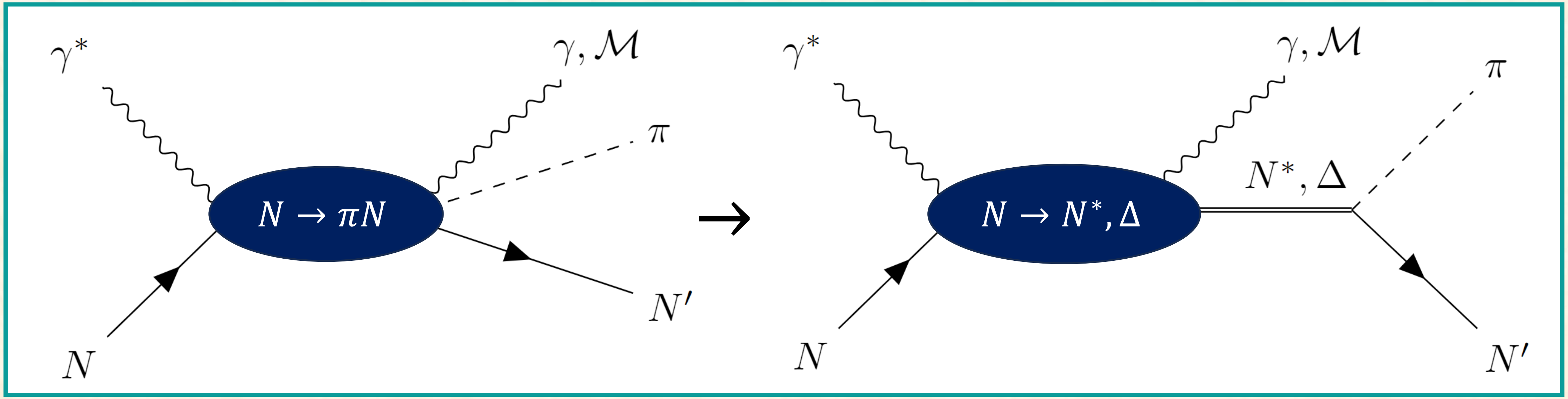
III. Non-diagonal matrix elements of the QCD energy-momentum tensor can be probed.

K. Goeke, M. Polyakov, and M. Vanderhaeghen,
Prog. Part. Nucl. Phys. 47, 401 (2001)

Recent developments in the non-diagonal hard exclusive reactions

- DVCS of $\gamma^* N \to \gamma N' \pi$ with $N \to \Delta, P_{11}, D_{13}, S_{11}$ transition GPDs — K. Semenov-Tyan-Shanskiy and M. Vanderhaeghen, Phys. Rev. D 108, 034021 (2023)
- DVMP of $\gamma^* p \to \pi^- \Delta^{++}$ with $p \to \Delta^{++}$ transition GPDs — P. Kroll and K. Passek-Kumericki, Phys. Rev. D 107, 054009 (2023)
- Measurement of $\gamma^* p \to \pi^- \Delta^{++} \to \pi^- p \pi^+$ BSA by the CLAS collaboration — S. Diehl et. al. (CLAS collaboration), Phys. Rev. Lett 131, 021901 (2023)

Non-diagonal DVCS and DVMP



Transition GPD description with *resonance state* in non-diagonal reactions

- *Angular structure* of $N \to \pi N$ GPDs is investigated by the $\pi N R$ interaction.
- Dependence on the *invariant mass* is induced by the Breit-Wigner resonance formula.

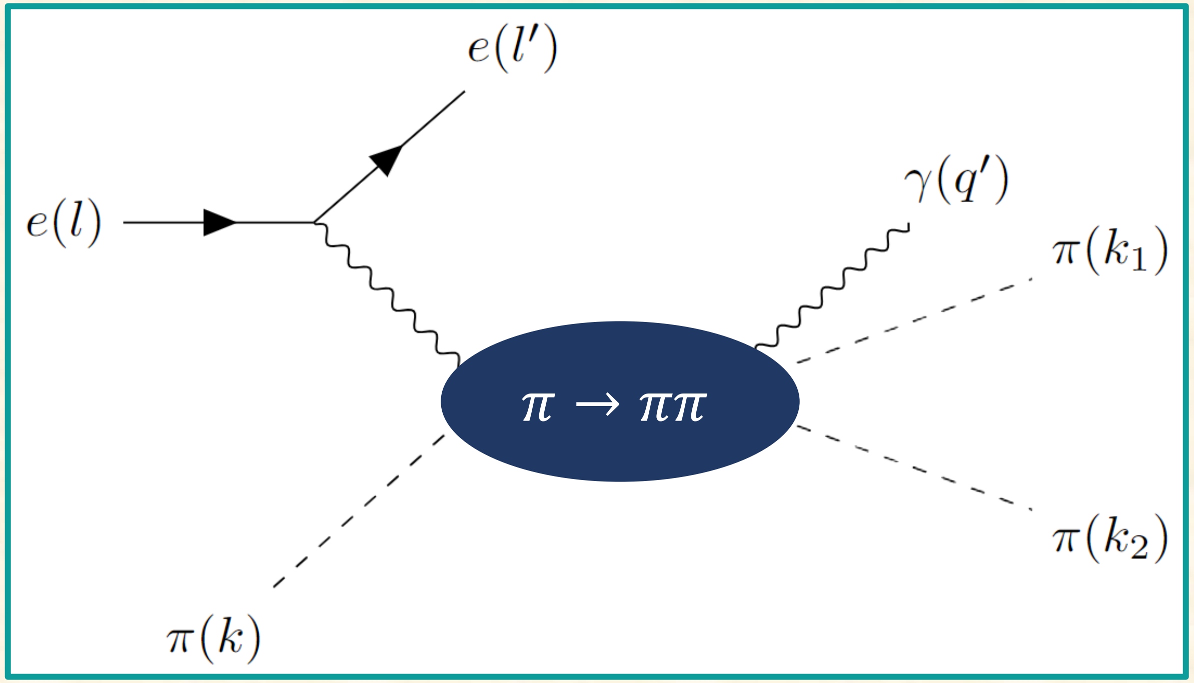
✓ Transition GPDs such as $N \to \pi N$ GPDs depend on more arguments;
the invariant mass and *the solid angles* of produced hadronic states

Non-diagonal DVCS and DVMP

✓ In this work, we study the non-diagonal DVCS of $\gamma^*\pi \to \gamma\pi\pi$ to avoid complications due to hadron spin.

- Study the $\pi \to \rho$ contribution to the $\pi \to \pi\pi$ GPDs. Express the $e\pi \to e\gamma\pi\pi$ cross section and work out its angular distribution near $W_{\pi\pi} \simeq m_\rho$.

- Apply the soft-pion theorem to determine the normalization condition for the $\pi \to \pi\pi$ GPDs near $W_{\pi\pi} \simeq 2m_\pi$.



An exercise to develop the framework of the partial wave analysis of transition GPDs for further generalization for $N \to \pi N$ GPDs

$\pi \to \pi\pi$ transition GPDs

- The GPDs are defined through hadronic matrix elements of the longitudinal projections of the light cone operator of the leading twist-2.

- The variables α and t' are related to the decay angles $\cos\phi_\pi^*$ and $\cos\theta_\pi^*$ of the $\pi\pi$ system.

Unpolarized and polarized isoscalar $\pi \to \pi\pi$ GPDs

$$\begin{aligned}
&\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix\lambda n\cdot\bar{P}}\langle\pi^b(k_1)\pi^c(k_2)|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\psi\left(\frac{\lambda n}{2}\right)|\pi^a(k)\rangle \\
= \quad &\frac{1}{2\bar{P}\cdot n}i\epsilon(n,\bar{P},\Delta,k_1)\frac{1}{f_\pi^3}i\epsilon^{abc}H^{(S)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2), \\
&\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix\lambda n\cdot\bar{P}}\langle\pi^b(k_1)\pi^c(k_2)|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\gamma_5\psi\left(\frac{\lambda n}{2}\right)|\pi^a(k)\rangle \\
= \quad &\frac{1}{2\bar{P}\cdot n}(\bar{P}\cdot n)\frac{1}{f_\pi}i\epsilon^{abc}\tilde{H}^{(S)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2),
\end{aligned}$$

$$t' = (k - k_1)^2$$

$$\alpha = \frac{k_2\cdot n}{(k_1+k_2)\cdot n} = \frac{k_2\cdot n}{1-\xi}$$

$\epsilon(a,b,c,d) = \epsilon^{\mu\nu\alpha\beta}a_\mu b_\nu c_\alpha d_\beta$

$f_\pi = 93$ MeV: the pion decay const.

$\pi \to \pi\pi$ transition GPDs

- The GPDs are defined through hadronic matrix elements of the longitudinal projections of the light cone operator of the leading twist-2.

- The variables α and t' are related to the decay angles $\cos\phi_\pi^*$ and $\cos\theta_\pi^*$ of the $\pi\pi$ system.

Unpolarized and polarized isovector $\pi \to \pi\pi$ GPDs

$$\begin{aligned}
&\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix\lambda n\cdot\bar{P}}\langle\pi^b(k_1)\pi^c(k_2)|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\tau^d\psi\left(\frac{\lambda n}{2}\right)|\pi^a(k)\rangle\\
=\ &\frac{1}{2\bar{P}\cdot n}i\epsilon(n,\bar{P},\Delta,k_1)\frac{1}{f_\pi^3}\Big[\delta_{ab}\delta_{cd}H_1^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2)+\delta_{ac}\delta_{bd}H_2^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2)\\
&+\delta_{ad}\delta_{bc}H_3^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2)\Big],\\
&\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix\lambda n\cdot\bar{P}}\langle\pi^b(k_1)\pi^c(k_2)|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\gamma_5\tau^d\psi\left(\frac{\lambda n}{2}\right)|\pi^a(k)\rangle\\
=\ &\frac{1}{2\bar{P}\cdot n}(\bar{P}\cdot n)\frac{i}{f_\pi}\Big[\delta_{ab}\delta_{cd}\tilde{H}_1^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2)+\delta_{ac}\delta_{bd}\tilde{H}_2^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2)\\
&+\delta_{ad}\delta_{bc}\tilde{H}_3^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2)\Big],
\end{aligned}$$

Soft-pion theorem

- Near the two-pion threshold, $W_{\pi\pi} = 2m_\pi$, the emitted pion is *soft*.
- The soft-pion theorem provides the normalization conditions of $\pi \to \pi\pi$ transition GPDs at threshold in terms of the pion GPD.
- PCAC relation allows us to write the pion field in terms of the axial current and by the LSZ reduction *soft pion reduces to the chiral rotation of the operator.*

➡ ***Soft-pion theorem***

P. Pobylitsa, M. Polyakov, and M. Strikman, Phys. Rev. Lett. 87, 022001 (2001)

$$\boxed{\langle \pi^b(k_1)\pi^c(k_2)|\mathcal{O}(z)|\pi^a(k)\rangle\Big|_{k_2\to 0} = -\frac{i}{f_\pi}\langle \pi^b(k_1)|[Q_5^c,\mathcal{O}(z)]|\pi^a(k)\rangle + k_2^\mu R_\mu^c(k_2)\Big|_{k_2\to 0}}$$

✓ The chiral rotation of the isoscalar (isovector) lightcone operator

$$\boxed{[Q_5^a,\bar\psi(0)\gamma^\mu(1,\gamma_5)\tau^b\psi(z)] \;=\; i\epsilon^{abc}\bar\psi(0)\gamma^\mu(\gamma_5,1)\tau^c\psi(z)}$$

$$\boxed{[Q_5^a,\bar\psi(0)\gamma^\mu(1,\gamma_5)\psi(z)] = 0}$$

Q_5^a: axial charge

$R^a(k_2)$: pole contribution

Soft-pion theorem

Pion GPD of the leading twist

$$\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{iy\lambda n\cdot\bar{P}_\pi}\langle\pi^b(p'_\pi)|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\tau^c\psi\left(\frac{\lambda n}{2}\right)|\pi^a(p_\pi)\rangle \quad = \quad 2(\bar{P}_\pi\cdot n)i\epsilon^{abc}H_\pi^{(V)}(y,\zeta,t_\pi)$$

- ✓ $\pi \to \pi\pi$ transition GPDs is normalized by the diagonal pion GPD at the threshold.

$$\zeta_1 = \frac{2\xi-(1-\xi)\alpha}{2-(1-\xi)\alpha} \quad \text{and} \quad \zeta_2 = \frac{2\xi-(1-\xi)(1-\alpha)}{2-(1-\xi)(1-\alpha)}$$

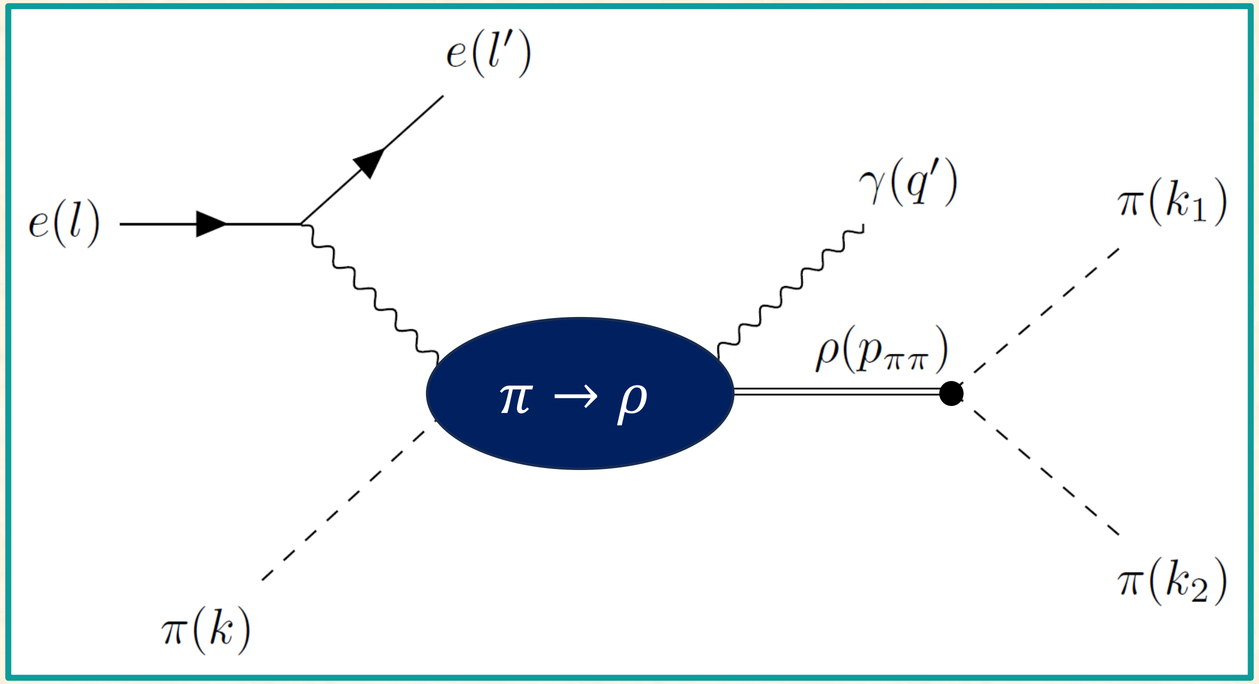
Ex) In the case that $\pi(k_2)$ is taken to be soft

$$\begin{aligned}
\tilde{H}_1^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) &= 0\\
\tilde{H}_2^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) &= 2n\cdot(k+k_1)H_\pi^{(V)}\left(\frac{2x}{n\cdot(k+k_1)},\zeta_1,t'\right)\theta\left(1-\left|\frac{2x}{n\cdot(k+k_1)}\right|\right)\\
\tilde{H}_3^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) &= -2n\cdot(k+k_1)H_\pi^{(V)}\left(\frac{2x}{n\cdot(k+k_1)},\zeta_1,t'\right)\theta\left(1-\left|\frac{2x}{n\cdot(k+k_1)}\right|\right)
\end{aligned}$$

$\pi \to \rho$ transition

- As ρ-meson is likely to decay into two pions the $\pi \to \rho$ transition in the intermediate resonance state can be included.

- $\pi \to \rho$ transition GPDs (FFs) are accessed through the VCS (BH) amplitude.



✓ The $\rho \to \pi\pi$ decay is described by the effective $\rho\pi\pi$ Lagrangian

$\pi \to \rho$ transition

Effective Lagrangian for $\rho\pi\pi$ interaction

O. Dumbrajs et al., Nucl. Phys. B 216, 277 (1983)

$$\boxed{\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi}\epsilon_{abc}\rho^a_\mu \pi^b \partial^\mu \pi^c}$$

$g_{\rho\pi\pi}$: $\rho\pi\pi$ coupling

Γ_ρ : decay width

$\mathcal{E}(p_\rho, \lambda)$: polarization vector with helicity λ

ρ-meson contribution to the $e\pi \to e\gamma\pi\pi$ amplitude

$$\boxed{\mathcal{M}(e\pi \to e\gamma\pi\pi) = g_{\rho\pi\pi} C_{\rm iso} \frac{i}{W_{\pi\pi}^2 - m_\rho^2 + i m_\rho \Gamma_\rho} \sum_\lambda \mathcal{M}_\lambda(e\pi \to e\gamma\rho(p_{\pi\pi}, \lambda))\mathcal{E}^*(p_{\pi\pi}, \lambda) \cdot (k_1 - k_2)}$$

- Near $W_{\pi\pi} \simeq m_\rho$, the $\pi \to \rho$ transition will dominantly contribute. We describe it with the Breit-Wigner form of the propagator and $\rho\pi\pi$ coupling.

$\pi \to \rho$ transition

$e\pi \to e\gamma\pi\pi$ amplitude squared

- Integrating over the azimuthal angle ϕ_π^* of $\pi\pi$ system relates the $e\pi \to e\gamma\pi\pi$ cross section to that of $e\pi \to e\gamma\rho$.

$$\boxed{\begin{aligned}\int d\phi_\pi^* |\mathcal{M}(e\pi \to e\gamma\rho \to e\gamma\pi\pi)|^2 \quad = \quad & C_{\rm iso}^2 \frac{g_{\rho\pi\pi}^2}{(W_{\pi\pi}^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2} \frac{16\pi}{3} |\vec{k}_1^*|^2 \\ & \times \sum_\lambda |\mathcal{M}_\lambda(e\pi \to e\gamma\rho)|^2 \left[\frac{3}{2}\cos^2\theta_\pi^* \delta_{\lambda,0} + \frac{3}{4}\sin^2\theta_\pi^* (\delta_{\lambda,1} + \delta_{\lambda,-1})\right]\end{aligned}}$$

- Yields the angular distribution of the $e\pi \to e\gamma\pi\pi$ cross section in the vicinity of the ρ-meson mass

- Explicit form of $e\pi \to e\gamma\rho$ amplitude squared for each helicity state λ is required

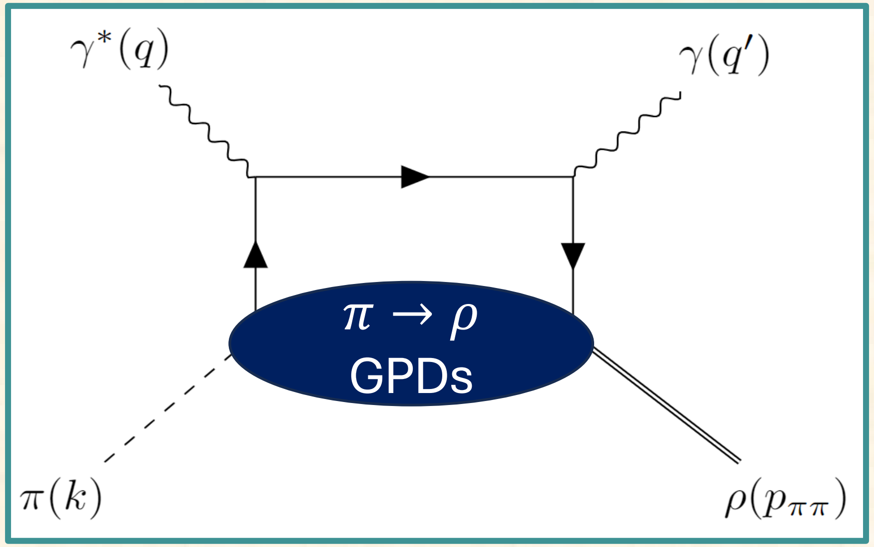
$$|\vec{k}_1^*| = \frac{\Lambda(W_{\pi\pi}^2, m_\pi^2, m_\pi^2)}{2W_{\pi\pi}}$$

Λ : Mandelstam ftn.

$\pi \to \rho$ transition

$e\pi \to e\gamma\rho$ amplitude (BH + DVCS)

$$
\begin{aligned}
\mathcal{M}_{VCS} &= \frac{ie^3}{q^2}\mathcal{E}^*_\mu(q')\bar{u}(l')\gamma_\nu u(l) T^{\mu\nu}(\gamma^*\pi \to \gamma\rho) \\
\mathcal{M}_{BH} &= \frac{e^3 C_V}{\Delta^2} F_{\pi\to\rho}(\Delta^2)\epsilon_{\mu\alpha\beta\gamma} p^\alpha_{\pi\pi} k^\beta \mathcal{E}^{*\gamma}(p_{\pi\pi})\mathcal{E}^*_\nu(q') \\
&\quad \times \bar{u}(l')\left[\gamma^\nu \frac{1}{\not{l}' + \not{q}'}\gamma^\mu + \gamma^\mu \frac{1}{\not{l} - \not{q}'}\gamma^\nu\right] u(l)
\end{aligned}
$$



$T^{\mu\nu}$: the hadronic tensor

C_V : the $\gamma\pi\rho$ coupling

Unpolarized $\pi \to \rho$ GPD

$$
\begin{aligned}
&\frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \sum_q e_q^2 \langle \rho(p_{\pi\pi})|\bar{q}\left(-\frac{\lambda n}{2}\right)\not{n} q\left(\frac{\lambda n}{2}\right)|\pi(k)\rangle \\
= &\frac{1}{2\bar{P}\cdot n} C_V \epsilon(n, \mathcal{E}^*, \bar{P}, \Delta) H^{\pi\to\rho}(x,\xi,t)
\end{aligned}
$$

Polarized $\pi \to \rho$ GPDs

$$
\begin{aligned}
&\frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \sum_q e_q^2 \langle \rho(p_{\pi\pi})|\bar{q}\left(-\frac{\lambda n}{2}\right)\not{n}\gamma_5 q\left(\frac{\lambda n}{2}\right)|\pi(k)\rangle \\
= &\frac{1}{2\bar{P}\cdot n}\frac{i}{f_\pi}\left[(\mathcal{E}^*\cdot\Delta)(\bar{P}\cdot n)\tilde{H}_1^{\pi\to\rho}(x,\xi,t) + m_\rho^2(\mathcal{E}^*\cdot n)\tilde{H}_2^{\pi\to\rho}(x,\xi,t)\right]
\end{aligned}
$$

Phenomenological models for $\pi \to \rho$ GPDs

- Simple Ansatz for GPD: factorized t-dependence
- Forward limit and Mellin moment conditions are satisfied

$$F(t) = \int_{-1}^{1} dx\, H(x,\xi,t) \qquad q(x) = H(x,\xi,t=0)$$

1. Factorized model for the unpolarized GPD

$$\boxed{H^{\pi\to\rho}(x,\xi,t) = q(x,\xi)F_{\pi\to\rho}(t)}$$

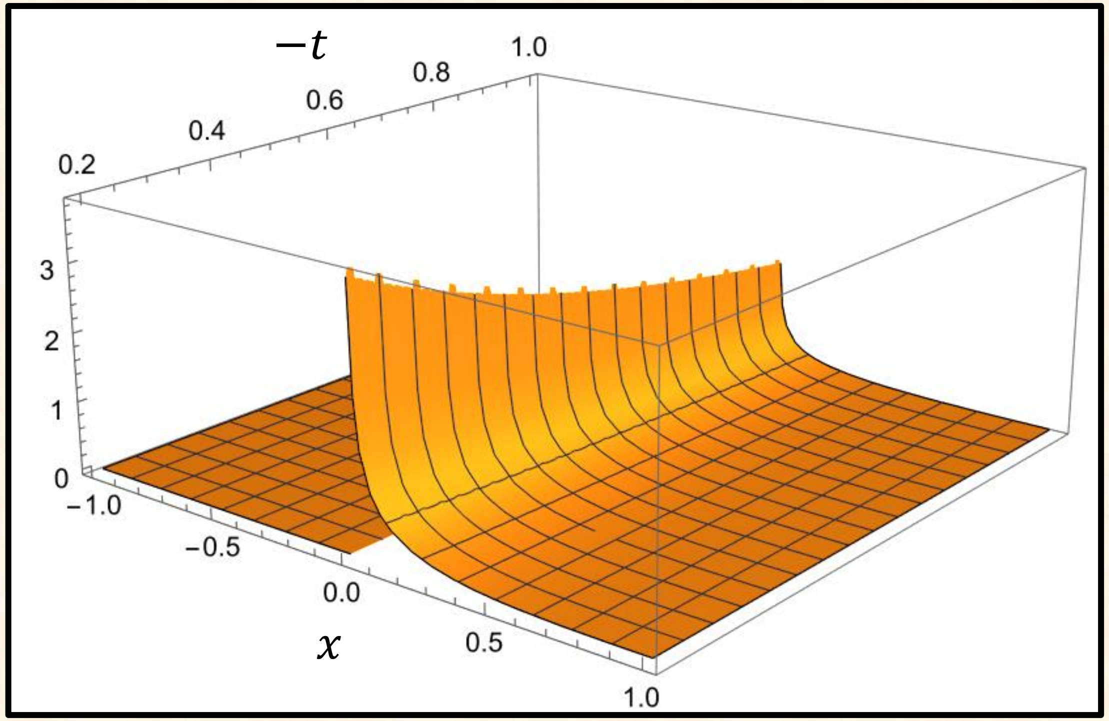
ξ-independent forward parton distribution

$$\boxed{q(x) = Nx^{-1/2}(1-x)^3\theta(x)}$$

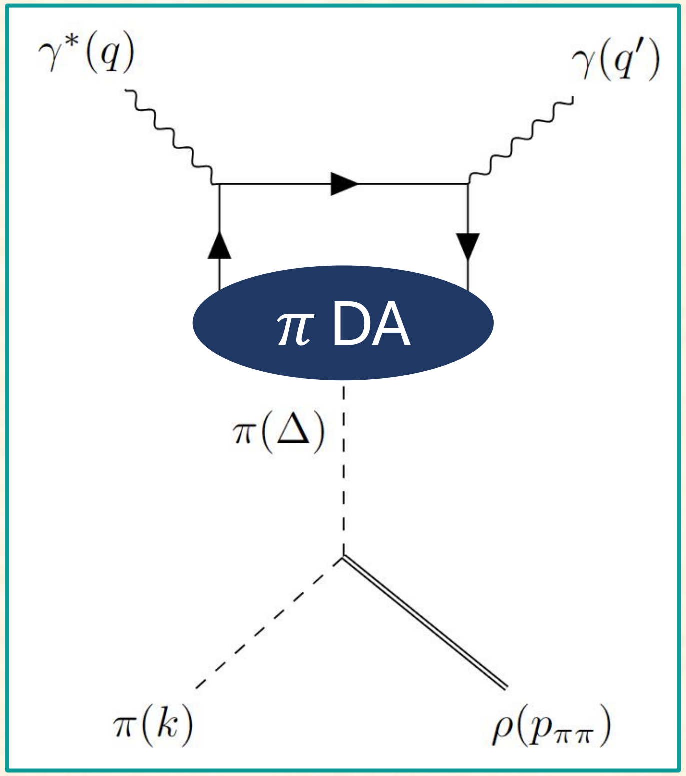
✓ Valence-type quark distribution

$$N = 1.09375 \quad \Leftarrow \quad 1 = \int_{-1}^{1} dx\, q(x)$$

Unpolarized GPD, $H^{\pi\to\rho}(x,\xi,t)$



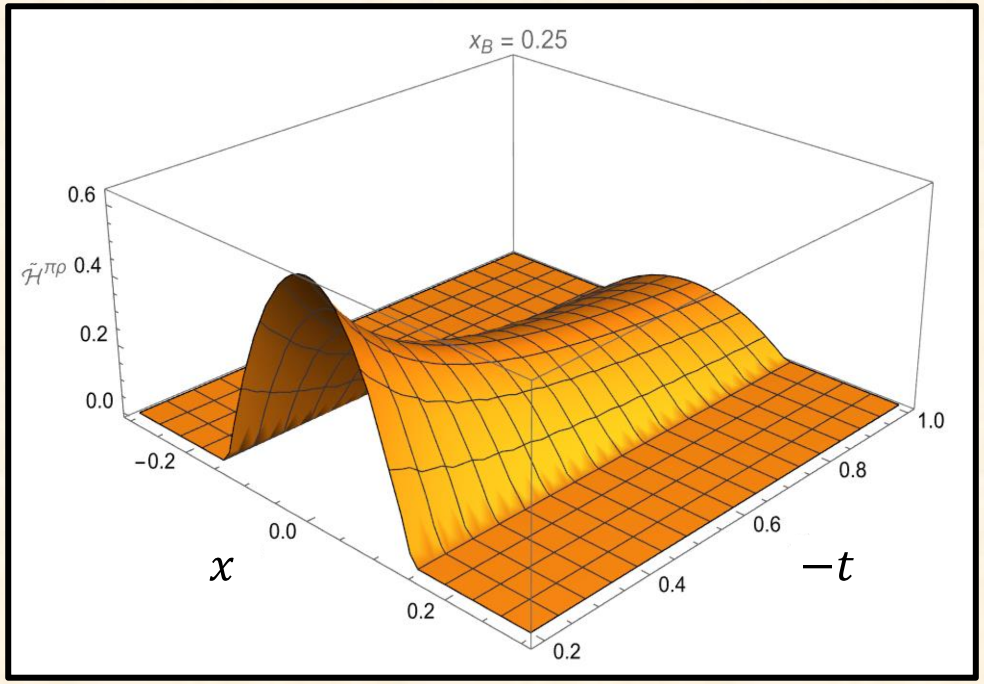
Phenomenological models for $\pi \to \rho$ GPDs



2. Pion pole model for the polarized GPD

- Pion pole contribution to the matrix elements of the non-local operator, $\langle \rho | \bar{\psi}(x) \gamma^\mu \gamma_5 \psi(0) | \pi \rangle$
- expressed through the pion distribution amplitude (DA) and the $\rho\pi\pi$ coupling

Polarized GPD, $\tilde{H}_1^{\pi\to\rho}(x,\xi,t)$

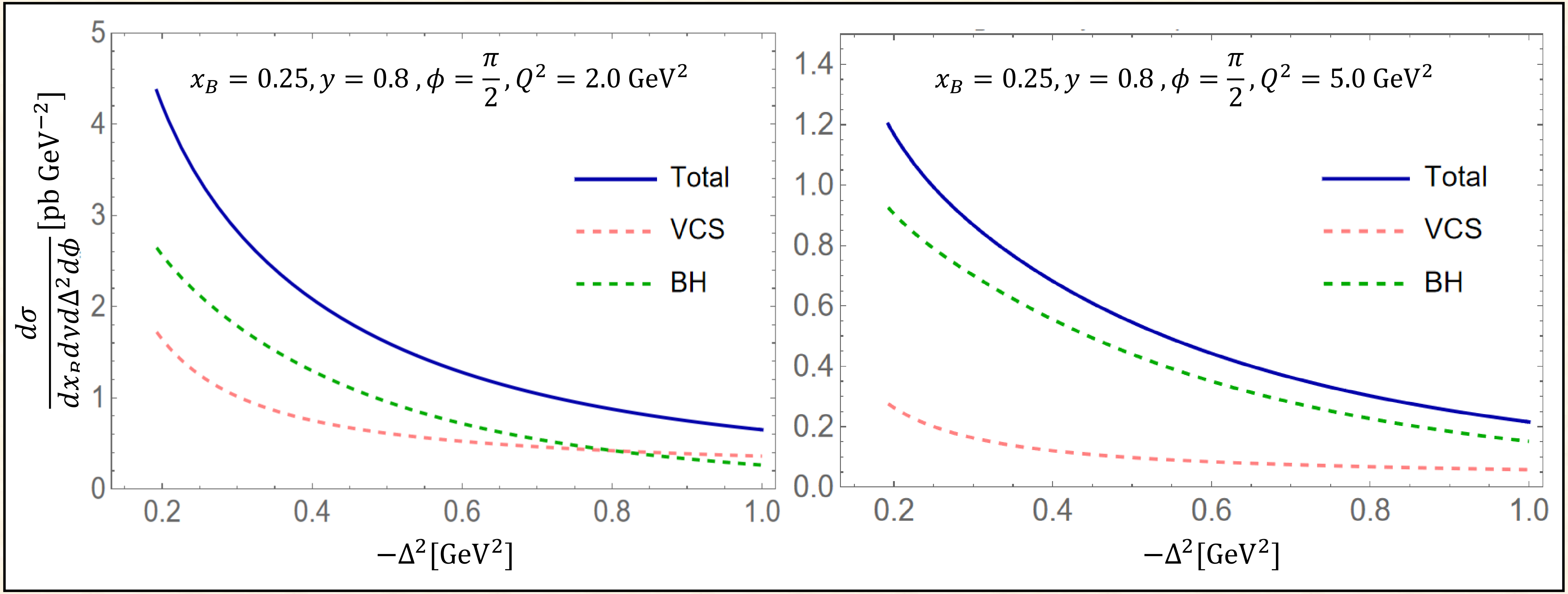


$$
\begin{aligned}
\tilde{H}_1^{\pi\to\rho}(x,\xi,t) &= \phi_\pi\left(\frac{x}{\xi}\right)\theta(\xi-|x|)\frac{1}{6}\frac{4f_\pi^2 g_{\rho\pi\pi}}{m_\pi^2-t} \\
\tilde{H}_2^{\pi\to\rho}(x,\xi,t) &= 0
\end{aligned}
$$

Asymptotic form of the pion DA

$$
\phi_\pi(u) = \frac{3}{4}(1-u^2)
$$

$e\pi \to e\gamma\rho$ cross section



Differential cross section in the unit of $\mathrm{pb\ GeV}^{-2}$

$$\frac{d\sigma(e\pi \to e\gamma\rho)}{dx_B dy d\Delta^2 d\phi}$$

y : lepton energy loss
x_B: Bjorken x
ϕ : azimuthal angle between leptonic and hadronic plane

Summary

- Transition GPDs arise in a description of non-diagonal hard exclusive reactions provide information on the dynamics of the hadron excitations in terms of partonic degrees of freedom.

- We study the $\pi \to \pi\pi$ GPDs describing the $e\pi \to e\gamma\pi\pi$ reaction near the regions where $W_{\pi\pi} \simeq 2m_\pi$ and $W_{\pi\pi} \simeq m_\rho$.

- The $\pi \to \rho$ contribution is included and the dependence of the $e\pi \to e\gamma\pi\pi$ cross section on the angles of the $\pi\pi$ c.m. system is studied in the vicinity of ρ mass.

- Phenomenological model for GPDs based on the pion pole dominance is adopted to estimate the $e\pi \to e\gamma\rho$ cross section.

Back up

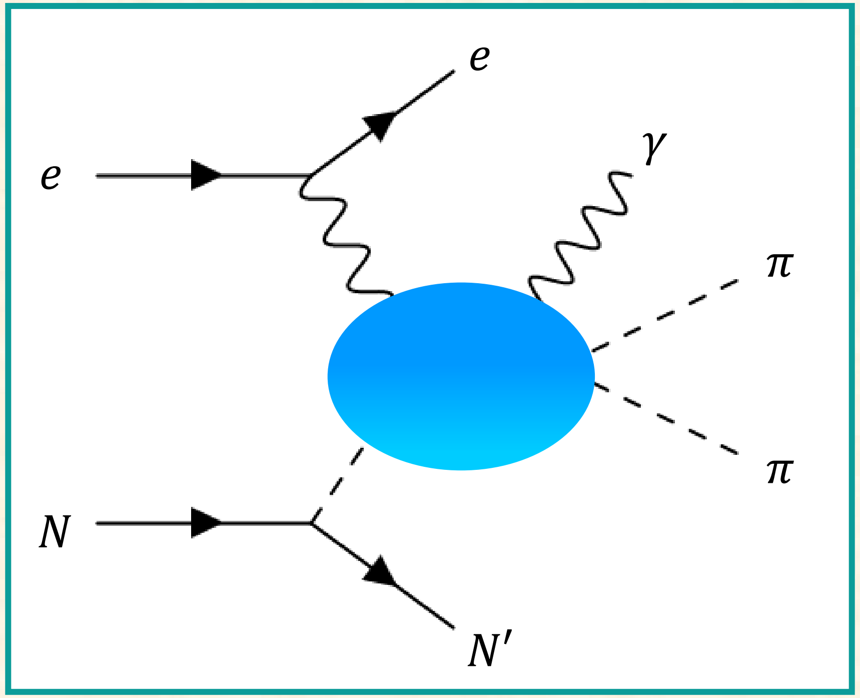
The Sullivan-type of process, $eN \to e\gamma N'\pi\pi$

- DVCS reaction of $e\pi \to e\gamma\pi\pi$ can be accessed through the pion emission from the Sullivan process.

- Near the threshold of pion production, the momentum transfer between nucleons is small.

Factorized into two subprocesses

$$\mathcal{M}_{eN\to eN\gamma\pi\pi} = \mathcal{M}_{N\to N\pi}\mathcal{M}_{e\pi\to e\gamma\pi\pi}$$



✓ *The meson cloud can be approximated by the one-pion-exchange.*

✓ *The intermediate pion is slightly off-shell.*

D. Amrath, M. Diehl, and J. P. Lansberg, Eur. Phys. J. C 58, 179 (2008)
J. Morgado et al., arXiv.2203.169472