Pion form factor and transverse momentum dependent distributions beyond the leading twist in the light-front quark model

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Work in progress (with Prof. Chueng-Ryong Ji)
Outline

1. Motivation

2. Why Light-Front?

3. Form Factors on the LF:
   - New Development of self-consistent LFQM
   - Pion Form Factor

4. Unpolarized Transverse Momentum Distributions (TMDs) of pion

5. QCD evolution of Pion Parton Distribution Functions (PDFs)

6. Conclusions
The Electron-Ion Collider (EIC) aims to provide detailed insights into the behavior of quarks and gluons...

Physics Topics at the EIC

1. Precision 3D imaging of protons and nuclei.
2. Search for gluon saturation.
3. Solving the proton spin puzzle.
The Electron-Ion Collider (EIC) aims to provide detailed insights into the behavior of quarks and gluons...

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Need to measure positions and momenta of the partons transverse to its direction of motion.
3D nucleon structure from 5D tomography

\[ W(x, k_\perp, b_\perp) : \text{Wigner distribution} \]

\[ \int dB_\perp \quad \int dk_\perp \]

\[ f(x, k_\perp) \quad f(x, b_\perp) \]

TMDs: semi-inclusive processes

Impact parameter distributions

\[ \int dk_\perp \quad \int db_\perp \]

\[ f(x) \]

PDFs: inclusive and semi-inclusive processes

\[ W(x, k_\perp, b_\perp) : \text{Wigner distribution} \]
### 3D nucleon structure from 5D tomography

**Wigner distribution**

\[ W(x, k_\perp, b_\perp) \]:

\[
\int d b_\perp \\
\int d k_\perp \\
\int d k_\perp
\]

**TMDs**: semi-inclusive processes

\[ f(x, k_\perp) \]

**Impact parameter distributions**

\[ f(x, b_\perp) \]

**FT**

\[ b_\perp \leftrightarrow \Delta \]

\[ H(x, \zeta, t = -\Delta^2) \]

**GPDs**: exclusive processes

\[ \zeta = \frac{(P - P')^+}{P^+} \]

\[ \int dx \]

**Form factors**: elastic scattering

\[ F(t) \]

**PDFs**: inclusive and semi-inclusive processes

\[ f(x) \]
3D nucleon structure from 5D tomography

\[ W(x, k_\perp, b_\perp): \text{Wigner distribution} \]

\[ \int db_\perp \quad \int dk_\perp \quad \int db_\perp \]

\( f(x, k_\perp) \quad f(x, b_\perp) \quad f(x) \)

TMDs: semi-inclusive processes

Impact parameter distributions

PDFs: inclusive and semi-inclusive processes

\[ b_\perp \leftrightarrow \Delta \]

\[ \text{FT} \]

\[ H(x, \zeta, t = -\Delta^2) \]

GPDs: exclusive processes

\[ \zeta = \frac{(P - P')^+}{P^+} \]

\[ \int dx \]

Form factors: elastic scattering

\[ (x, k_\perp): \text{Light front (LF) variables} ... \]
1. Motivation

- Light-Front Dynamics (LFD) has been quite successful in describing various hadron properties such as Decay constants, DAs, PDFs, Form Factors, GPDs and TMDs etc.

\[ F_{\pi\pi^-}(Q^2) = \sum_n \frac{x_n k_{\perp}}{4\pi^2} \frac{1}{k_{\perp}^2} \]

- We have developed a new self-consistent covariant LFQM for decay constants, DAs, and weak form factors (e.g. $B \rightarrow Dll$) of mesons:

PRD 89, 033011(14); PRD91, 014018(15); PRD95, 056002(17) by HMC and C.-R. Ji
PRD 103, 073004(21); Adv. High Energy Phys., 4277321(21) by HMC
PRD 107, 053003(23); PRD108, 013006(23) by A. J. Arifi, HMC and C.-R. Ji

In this work, we apply our new developed LFQM to compute Form Factor, TMDs, and PDFs of Pion!
2. Why Light-Front?

**Hamiltonian**  
\[ P^0 = P^0 - P^3 \]

**Momentum**  
\[ P_{\perp} = (P^1, P^2) \]
\[ P_{\perp} = P^0 + P^3 \]

**Energy-Momentum Dispersion Relation**  
\[ P^0 = \sqrt{M^2 + \vec{p}^2} \]
\[ P^- = \frac{M^2 + P_{\perp}^2}{P^+} \]

Irrational vs. Rational

Light-Front Dynamics (LFD) (by Dirac in 1949)
• Advantage of LFD in the calculation of Form Factors:

Equal-\( t \) vs Equal Light-front \( \tau \) formulations

Equal-\( t \) (Instant form)

\[ = \]
• Advantage of LFD in the calculation of Form Factors:
  Equal-$t$ vs Equal Light-front $\tau$ formulations

Equal $t$ (Instant form)

\[ k^0 = \sqrt{m^2 + \vec{k}^2} \]

Allowed!

\[ \vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0 \]
• Advantage of LFD in the calculation of Form Factors:
  Equal-\(t\) vs Equal Light-front \(\tau\) formulations

Equal \(\tau\) (Front form)

\[
\begin{align*}
&= \\
\end{align*}
\]
• Advantage of LFD in the calculation of Form Factors: 
  Equal-\( t \) vs Equal Light-front \( \tau \) formulations

Equal \( \tau \) (Front form)

\[
\text{Equal } \tau = t \frac{z}{c}
\]

\[
k^- = \frac{m^2 + k_{1\perp}^2}{k^+}
\]

\[
k_1^+ + k_2^+ + k_3^+ = 0
\]
• Advantage of LFD in the calculation of Form Factors:

**Equal-\( t \)** vs **Equal Light-front \( \tau \)** formulations

### Equal \( \tau \) (Front form)

\[ \text{Equal } \tau \text{ (Front form)} = \]

### LF nonvalence (higher Fock state)

\[ \text{LF nonvalence (higher Fock state)} \]

### LF valence
3. Form Factors on the Light-Front

\[ J^\mu (p,k) \rightarrow q \]

\[ p \leftrightarrow k \rightarrow p + q \]

\[ q^+ \rightarrow 0 \]

- LF valence
- LF nonvalence
3. Form Factors on the Light-Front

\[ J^\mu \Rightarrow \text{facilitates the partonic interpretation of the amplitude!} \]
3. Form Factors on the Light-Front

\[ J^\mu \]

facilitates the partonic interpretation of the amplitude!

(e.g.) E&M form factors of pseudoscalar and vector mesons

\[ \frac{k^+}{p^+} = x \]

\[ F(Q^2) = \int [dx][d^2 k_{\perp}] \psi_i^*(x, k'_{\perp}) \psi_i(x, k_{\perp}) \quad \text{for } J^+ \& J^\perp \]
3. Form Factors on the Light-Front

\[ J(\mathbf{q}) = q \cdot \mathcal{L} \]

\( \mathcal{L} \) valence

\( \mathcal{L} \) nonvalence

\( \mathcal{L} \rightarrow 0 \)

\( P \)

\( P + q \)

\( q^+ \rightarrow 0 \)

Nonvanishing : LF Zero-Mode!

☞ One has to take into account of the zero mode in using \( J^- \) current!
Light-Front Quark Model (LFQM)

Meson state: Noninteracting "on-mass" shell $Q$ & $\bar{Q}$ representation consistent with Bakamjian-Thomas (BT) construction!

The interaction is added to the mass operator

$$M_{Q\bar{Q}} = \langle \psi | H_{Q\bar{Q}} | \psi \rangle$$

$$H_{Q\bar{Q}} = \sqrt{m_Q^2 + \vec{k}^2} + \sqrt{m_{\bar{Q}}^2 + \vec{k'}^2} + V_{Q\bar{Q}}$$

$$V_{Q\bar{Q}} = a + br - \frac{4\kappa}{3r} + \frac{2\vec{S}_Q \cdot \vec{S}_{\bar{Q}}}{3m_Q m_{\bar{Q}}} \nabla^2 V_{\text{Coul}}$$

Refs.) PRD59, 074015(99); PLB460, 461(99) by HMC and CRJ; PRC92, 055203(2015) by HMC, CRJ, Z. Li, and H. Ryu

PRD106, 014009(2022) by A. J. Arifi, HMC, and CRJ
New Development of including the LF Zero-Mode in the LFQM

General structure for $P(P) \rightarrow P(P')$ transition:

$$\langle P'|\bar{q} \gamma^\mu q|P \rangle = \mathcal{O}^\mu F(q^2) + q^\mu \left( \frac{M^2 - M'^2}{q^2} \right) H(q^2), \quad \mathcal{O}^\mu = (P + P')^\mu - q^\mu \left( \frac{M^2 - M'^2}{q^2} \right)$$

$$q^\mu = (P - P')^\mu$$
New Development of including the LF Zero-Mode in the LFQM

General structure for $P(P) \rightarrow P(P')$ transition:

$$\langle P'|q\gamma^\mu q|P \rangle = \mathcal{B}^\mu F(q^2) + q^\mu \frac{(M^2 - M'^2)}{q^2} H(q^2), \quad \mathcal{B}^\mu = (P + P')^\mu - q^\mu \frac{(M^2 - M'^2)}{q^2}$$

$$q^\mu = (P - P')^\mu$$

For elastic process ($M = M'$),

only gauge invariant form factor $F(q^2)$ survives!

$$\langle P'|\bar{q}\gamma^\mu q|P \rangle = \mathcal{B}^\mu F_P(q^2) \quad \mathcal{B} \cdot q = 0$$
New Development of including the LF Zero-Mode in the LFQM

\[ \langle P' | \bar{q} \gamma^\mu q | P \rangle = \not{\partial}^\mu F_P(q^2), \quad \not{\partial}^\mu = (P + P')^\mu - q^\mu \frac{(M^2 - M'^2)}{q^2} \]

In $q^+ = 0$ frame,

\[ \langle P' | \bar{q} \gamma^\mu q | P \rangle = \int_0^1 dp_1^+ \int \frac{d^2k_\perp}{16\pi^3} \phi'(x, k_\perp)\phi(x, k_\perp) \sum_{\lambda', s} R_{\lambda_2\lambda}^+ \overline{u}(p_2) \gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}} R_{\lambda_1\bar{\lambda}}. \]

\[ F_P^{(\mu)}(Q^2) = \int_0^1 dp_1^+ \int \frac{d^2k_\perp}{16\pi^3} \phi'(x, k_\perp)\phi(x, k_\perp) \frac{1}{\not{\partial}^\mu} \sum_{\lambda', s} R_{\lambda_2\lambda}^+ \overline{u}(p_2) \gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}} R_{\lambda_1\bar{\lambda}}. \]
New Development of including the LF Zero-Mode in the LFQM

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In \( q^+ = 0 \) frame,

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Apply \( P^- = p^-_q + p^\perp_q \) (i.e. \( M^2 \to M_0^2 \))

New Effective Method

\[ F_P^{(\mu)}(Q^2) = \int_0^1 dp_1^+ \int \frac{d^2k^\perp}{16\pi^3} \phi'(x, k^\perp)\phi(x, k^\perp) \frac{1}{\not{\partial}^\mu} \sum_{\lambda', \lambda} R^\dagger_{\lambda, \lambda'} \left[ \frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}} \gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}} \right] R_{\lambda_1, \lambda'} \]

Then we get \( F^{(+)}_\pi(Q^2) = F^{(\perp)}_\pi(Q^2) = F^{(-)}_\pi(Q^2) \)
The first proof of the pion form factor's independence from current components in the LFQM!
The final result of the axial vector framework, standing, we present the essential aspect required to attain the LFQM in a process-independent and current component-in the small proportional to the very low constituent quark mass.

$$f_\pi^{\text{LFQM}} = 130 \text{ MeV}$$

(Exp. = 131 MeV)

$$r_\pi^{\text{LFQM}} = 0.654 \text{ fm}$$

(Exp. = 0.659(4)fm)

$$F_\pi^{(+)}(Q^2) = F_\pi^{(\perp)}(Q^2) = F_\pi^{(-)}(Q^2)$$
4. Unpolarized TMDs of pion

\[ \int \frac{[dz]}{2(2\pi)^3} e^{ip \cdot z} \langle P|\bar{\psi}(0)\gamma^+\psi(z)|P\rangle|_{z^+=0} = f_1^q(x, p_T), \]

\[ \int \frac{[dz]}{2(2\pi)^3} e^{ip \cdot z} \langle P|\bar{\psi}(0)\gamma_T^j\psi(z)|P\rangle|_{z^+=0} = \frac{p_T^j}{P^+} f_3^q(x, p_T), \]

\[ \int \frac{[dz]}{2(2\pi)^3} e^{ip \cdot z} \langle P|\bar{\psi}(0)\gamma^-\psi(z)|P\rangle|_{z^+=0} = \left(\frac{m_\pi}{P^+}\right)^2 f_4^q(x, p_T), \]

which are related with the forward matrix elements \( \langle P|\bar{q} \gamma^\mu q|P\rangle \) as

\[ 2P^+ \int dx f_1^q(x) = \langle P|\bar{\psi}(0)\gamma^+\psi(0)|P\rangle, \]

\[ 2p_T \int dx f_3^q(x) = \langle P|\bar{\psi}(0)\gamma_T^\perp\psi(0)|P\rangle, \]

\[ 4P^- \int dx f_4^q(x) = \langle P|\bar{\psi}(0)\gamma^-\psi(0)|P\rangle, \]
4. Unpolarized TMDs of pion

\[ \int \frac{[dz]}{2(2\pi)^3} e^{i p \cdot z} \langle P|\bar{\psi}(0)\gamma^+ \psi(z)|P\rangle_{z^+ = 0} = f_1^q(x, p_T), \]

\[ \int \frac{[dz]}{2(2\pi)^3} e^{i p \cdot z} \langle P|\bar{\psi}(0)\gamma_T^j \psi(z)|P\rangle_{z^+ = 0} = \frac{P_T^j}{P^+} f_3^q(x, p_T), \]

\[ \int \frac{[dz]}{2(2\pi)^3} e^{i p \cdot z} \langle P|\bar{\psi}(0)\gamma^- \psi(z)|P\rangle_{z^+ = 0} = \left(\frac{m_\pi}{P^+}\right)^2 f_4^q(x, p_T), \]

which are related with the forward matrix elements \( \langle P|\bar{q} \gamma^\mu q|P\rangle \) as

\[
\begin{align*}
2P^+ \int dx f_1^q(x) &= \langle P|\bar{\psi}(0)\gamma^+ \psi(0)|P\rangle, \\
2P_T \int dx f_3^q(x) &= \langle P|\bar{\psi}(0)\gamma^T \psi(0)|P\rangle, \\
4P^- \int dx f_4^q(x) &= \langle P|\bar{\psi}(0)\gamma^- \psi(0)|P\rangle,
\end{align*}
\]

PDF

TMD

\[
f(x) = \int d^2p_T f(x, p_T).
\]
4. Unpolarized TMDs of pion

C. Lorcé, B. Pasquini, and P. Schweitzer, EPJC76,415(2016)

Sum rules : 
\[ \int dx \ f_1^q(x) = N_q \]

Positivity inequalities:
\[ f_1^q(x, p_T) \geq 0 \]

\[ \sum_q \int dx \ x f_1^q(x) = 1 \]

\[ f_4^q(x, p_T) \geq 0 \]

\[ 2 \int dx \ f_4^q(x) = N_q \]

\[ (e.g. N_u = N_{\bar{u}} = 1 \text{ in } \pi^+) \]
4. Unpolarized TMDs of pion

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\[ f_4^q(x, p_T) \geq 0 \]
\[ 2 \int dx \ f_4^q(x) = N_q \]

In free quark model:
\[ xf_3^q(x, p_T) = f_1^q(x, p_T) \]

(e.g. \( N_u = N_{\bar{u}} = 1 \) in \( \pi^+ \))
LF Zero-Mode for twist-4 PDF and its Resolution

\[ 2 \int dx \, f_4^q(x) \neq 1 \]

Fails to meet the sum rule due to the absence of a zero mode!

\[ 2 \int dx \, f_4^q(x) = 1 \]

Zero mode is well taken!

C. Lorcé, B. Pasquini, and P. Schweitzer,
EPJC76, 415 (2016)

This work!
FIG. 6: The unpolarized TMDs for pion, $f_q(x, k_\perp)$ ($i = 1, 3, 4$) (top panel) and $x f_q(x, k_\perp)$ ($i = 1, 3, 4$) (middle panel), as a function of $x$ and $k_\perp^2$, and the corresponding PDFs, $f_q(x)$ and $x f_q(x)$ ($i = 1, 3, 4$) as a function of $x$ (bottom panel) at the scale $\mu^2 = 1$ GeV$^2$.

- For definiteness, we consider $\pi^+$ ($\pi^- = u \bar{d}$), and denote $f_q(x)$ and $f_{\bar{q}}(x)$ as the single-flavor distributions of quarks and antiquarks. In our LFQM, we define $f_q(x)$ and $f_{\bar{q}}(x)$ as the valence (or nonsinglet) quark distribution of $\pi^+$ for each twist ($i = 1, 4$) is $f_{\pm}^{val}(x) \equiv f_{\pm}^{u}(x) + f_{\pm}^{d}(x)$.

\begin{align*}
x f_1^q(x, k_\perp^2) &= f_1^q(x, k_\perp^2) \\
x f_3^q(x, k_\perp^2) &= f_3^q(x, k_\perp^2) \\
x f_4^q(x, k_\perp^2) &= f_4^q(x, k_\perp^2)
\end{align*}
Unpolarized PDFs for Pion

\[\int d^2 k_\perp \] at the initial scale \(\mu_0 = 1 \text{ GeV}\)
4. QCD Evolution of Pion PDFs

Evolved from $\mu_0^2 = 1$ GeV$^2$ to $\mu^2 = 4$ and 27 GeV$^2$

We use the Higher Order Perturbative Parton Evolution toolkit (HOPPET) to solve the NNLO DGLAP equation.
Twist-2 PDF

\[
\begin{array}{c|ccccc}
\mu^2 = 4 \text{ GeV}^2 & \langle x \rangle_{t_2}^u & \langle x^2 \rangle_{t_2}^u & \langle x^3 \rangle_{t_2}^u & \langle x^4 \rangle_{t_2}^u \\
\hline
\text{This work} & 0.236 & 0.101 & 0.055 & 0.033 \\
[64] & 0.2541(26) & 0.094(12) & 0.057(4) & 0.015(12) \\
[65] & 0.2075(106) & 0.163(33) & - & - \\
[39] & 0.24(2) & 0.098(10) & 0.049(7) & - \\
[40] & 0.24(2) & 0.094(13) & 0.047(8) & - \\
\end{array}
\]

\[
\begin{array}{c|cccc}
\mu^2 = 27 \text{ GeV}^2 & \langle x \rangle_{t_2}^u & \langle x^2 \rangle_{t_2}^u & \langle x^3 \rangle_{t_2}^u & \langle x^4 \rangle_{t_2}^u \\
\hline
\text{This work} & 0.182 & 0.069 & 0.034 & 0.019 \\
[66] & 0.18(3) & 0.064(10) & 0.030(5) & - \\
[40] & 0.20(2) & 0.074(10) & 0.035(6) & - \\
[48] & 0.184 & 0.068 & 0.033 & 0.018 \\
[53] & 0.217(11) & 0.087(5) & 0.045(3) & - \\
\end{array}
\]

Mellin moments: \( \langle x^n \rangle = \int_0^1 dx \ x^n f(x) \)
5. Conclusions

We developed a new method for the covariant analysis of LFQM.

Our LFQM: Noninteracting $Q$ & $\bar{Q}$ representation consistent with the Bakamjian-Thomas (BT) construction!

\[ P^- = p_q^- + p_{\bar{q}}^- \], i.e. $M^2 \rightarrow M_0^2$

\[ \langle 0 | \bar{q} \Gamma^\mu q | P \rangle = \mathcal{F} \mathcal{P}^\mu \]

$\mathcal{F}$: physical observables ($\mathcal{F} = f_p, F \ldots$)

$\mathcal{P}^\mu$: Lorentz factors ($\mathcal{P} = P^\mu \ldots$)

\[ \mathcal{F} = \left\langle 0 \left| \bar{q} \frac{\Gamma^\mu}{\mathcal{P}^\mu} q \right| P \right\rangle = \int dx \; d^2 k_\perp \; \left( \frac{\Gamma^\mu}{\mathcal{P}^\mu} \right) \ldots \]

Constrained by BT construction!

We obtain physical observables independent of $\mu$, paving the way for the development of the self-consistency in the LFQM.
GPDs of Pion at zero skewness

\[ F^q(t) = \int_0^1 dx \, H^q(x, t) \]

\[ H^q(x, t) = H^q(x, \zeta = 0, -t = Q^2) \]

GPD at \( \zeta = 0 \)

\[ H_1^q(x, t) = H^{(+)}(x, t) \]

\[ H_4^q(x, t) = H^{(-)}(x, t) \]

\[ H_1^q(x, 0) = f_1^q(x) \]

\[ H_4^q(x, 0) = 2 f_4^q(x) \]