Quasi-distribution amplitudes for pseudo-scalar mesons

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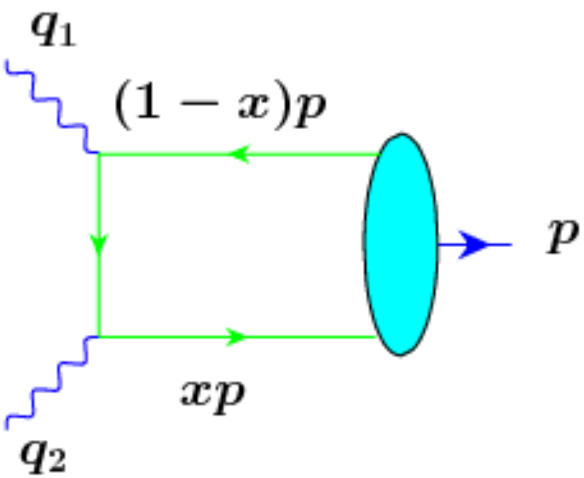
Contents based on **SiN, Modern Physics Letters A32, 1750218 (2017)**

Theory

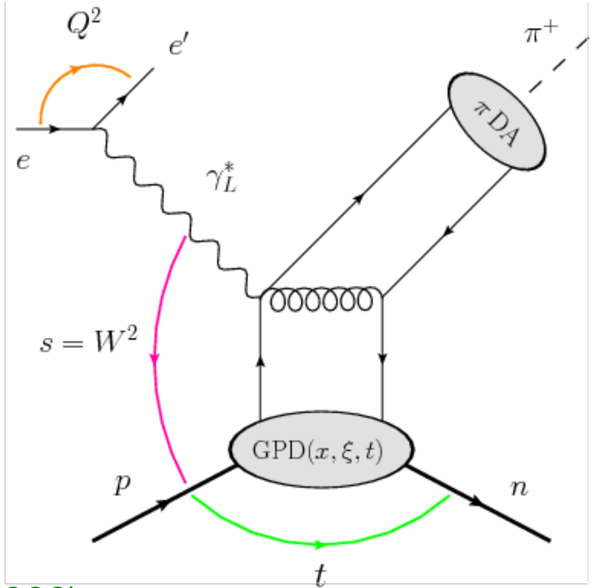
Quark distribution amplitude (DA) for PS mesons

Pion-photon transition form factor

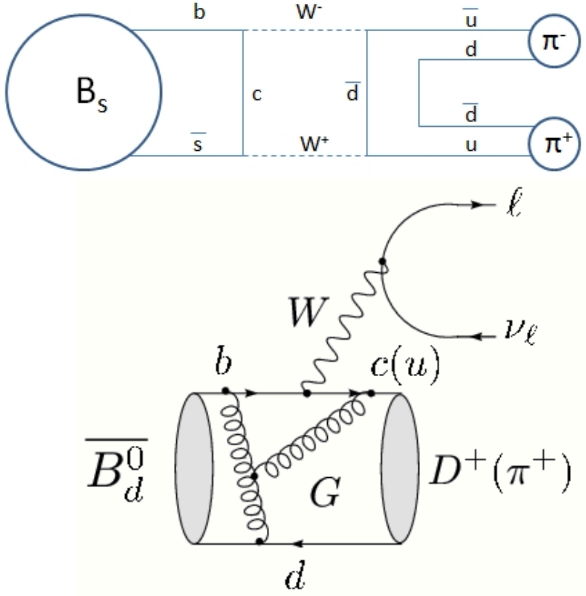


A.V.Radyushkin, PRD80, 094009 (2009)

Hard exclusive pion electro-production



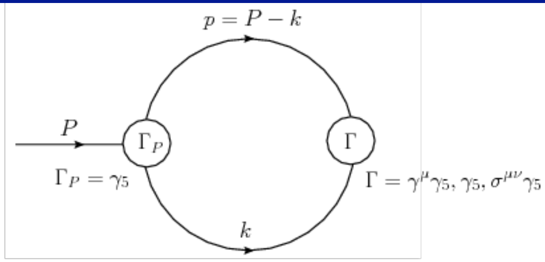
Heavy-meson weak decay



K.Park et al. [CLAS Collaboration], PLB780, 340 (2018)

Structural (nonpert.) information of DA as func. of x

Theory



PS-meson DA defined in light-front (LF) formalism

$$\langle 0|\bar{q}_f(\tau\hat{n})\gamma_\mu\gamma_5 q_g(-\tau\hat{n})|\mathcal{M}(P)\rangle = i\sqrt{2}F_{\mathcal{M}}P_\mu \int_0^1 du\, e^{i(2u-1)P\cdot\tau\hat{n}}\phi_{\mathcal{M}}(u),$$

Well defined Fock-states for meson but not covariant

LF formalism can not applied for lattice QCD (LQCD)

Instead, LQCD computes *moments* of DA

X.Ji, A.V.Radyushkin have developed quasi-DA (QDA) in terms of Large-momentum effective theo. (LaMET)

X. Ji, PRL110, 262002 (2013), A. V. Radyushkin, PRD93, 056002 (2016).

Theory (LaMET)

A. V. Radyushkin, PRD93, 056002 (2016).

Defining virtuality distribution amplitude (VDA)

$$\langle 0|\bar{q}(0)\gamma_5\gamma_+ q(z_-)|\mathcal{M}(p)\rangle = \frac{p_+}{\sqrt{2}F_{\mathcal{M}}}\int_0^\infty d\sigma \int_0^1 dx\, \Phi_{\mathcal{M}}(x,\sigma)e^{-ixp_+z_-} .$$

VDA

Relation between DA and VDA

$$\phi_{\mathcal{M}}(x) = \int_0^\infty d\sigma\, \Phi_{\mathcal{M}}(x,\sigma)\,, \qquad \int_0^1 dx\, \phi_{\mathcal{M}}(x) = 1$$

Fourier transform (FT) of matrix element: TMDA

$$\langle 0|\bar{q}(0)\gamma_5\gamma_+ q(z_-)|\mathcal{M}(p)\rangle = \frac{p_+}{\sqrt{2}F_{\mathcal{M}}}\int_0^\infty d^2k_\perp \int_0^1 dx\, \Psi_{\mathcal{M}}(x,k_\perp^2)\, e^{-ixp_+z_-} .$$

TMDA

Theory (LaMET)

TMDA in terms of VDA

$$\Psi_{\mathcal{M}}(x, k_\perp^2) = \frac{i}{\pi} \int_0^\infty \frac{d\sigma}{\sigma} \Phi_{\mathcal{M}}(x, \sigma) e^{-ik_\perp^2/\sigma}$$

TMDA integrated over k_T gives DA

$$\phi_{\mathcal{M}}(x) = \int_0^\infty dk_\perp^2 \, \Psi_{\mathcal{M}}(x, k_\perp^2) = 2\pi \int_0^\infty k_\perp \, dk_\perp \, \Psi_{\mathcal{M}}(x, k_\perp^2)$$

DA

VDA ⇔ TMDA ⇔ DA

Theory (LaMET)

Now, matrix element at equal time $z = (0, 0, 0, z_3)$

VDA

$$\langle 0|\bar{q}(0)\gamma_5\gamma_0 q(z_3)|\mathcal{M}(p)\rangle = \frac{p_0}{\sqrt{2}F_{\mathcal{M}}}\int_0^\infty d\sigma \int_0^1 dx\, \Phi_{\mathcal{M}}(x,\sigma)e^{-ixp_3z_3+i\sigma z_3^2/4}\,.$$

Similarly, FT of equal-time matrix element: QDA [-∞,∞]

QDA

$$\langle 0|\bar{q}(0)\gamma_5\gamma_0 q(z_3)|\mathcal{M}(p)\rangle = \frac{p_0}{\sqrt{2}F_{\mathcal{M}}}\int_{-\infty}^\infty dy\, Q_{\mathcal{M}}(y,p_3)e^{-iyp_3z_3}\,.$$

QDA in terms of VDA: Constrained x, while not for y

$$Q_{\mathcal{M}}(y,p_3) = \int_0^1 dx \int_0^\infty d\sigma \sqrt{\frac{ip_3^2}{\pi\sigma}}e^{-i(x-y)^2p_3^2/\sigma}\Phi_{\mathcal{M}}(x,\sigma)\,,$$

Theory (LaMET)

By equating them, TMDA and VDA related as

$$p_3 \int_{-\infty}^{\infty} dk_1 \, \Psi_{\mathcal{M}}(x, k_1^2 + (x-y)^2 p_3^2) = \int_0^{\infty} d\sigma \sqrt{\frac{ip_3^2}{\pi\sigma}} e^{-i(x-y)^2 p_3^2/\sigma} \Phi_{\mathcal{M}}(x, \sigma) \, .$$

A. V. Radyushkin, PRD93, 056002 (2016).

Thus, QDA given in terms of TMDA

TMDA

$$Q_{\mathcal{M}}(y, p_3) = p_3 \int_{-\infty}^{\infty} dk_1 \int_0^1 dx \, \Psi_{\mathcal{M}}(x, k_1^2 + (x-y)^2 p_3^2) \, .$$

QDA

A useful limit $p_3 \to \infty$

$$\lim_{p_3 \to \infty} \left[\sqrt{\frac{ip_3^2}{\pi\sigma}} e^{-i(x-y)^2 p_3^2/\sigma} \right] = \delta(x-y)$$

Theory (LaMET)

Due to the limit, DA-like func. (covariant) relates to VDA

~DA

$$\lim_{p_3\to\infty} Q_{\mathcal{M}}(y,p_3) \equiv \varphi_{\mathcal{M}}(y) = \int_0^1 dx \int_0^\infty d\sigma\, \delta(x-y)\Phi_{\mathcal{M}}(x,\sigma)$$

$$= \int_0^\infty d\sigma\, \Phi_{\mathcal{M}}(y,\sigma)\,.$$

DA-like func. and DA satisfy similar normalizations

$$\int_{-\infty}^{\infty} dy\, \varphi_{\mathcal{M}}(y) = \int_0^1 dx\, \phi_{\mathcal{M}}(x) = 1$$

Theory (LaMET with a model for LFWF)

Introducing LFWF for DA, previous equation becomes

$$\lim_{p_3\to\infty} p_3 \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2\, \Psi_{\mathcal{M}}(x, k_\perp^2)\delta(k_2 - (x-y)p_3)$$

TMDA

$$= \int_0^{\infty} d^2k_\perp\, \psi_{\mathcal{M}}(x, k_\perp^2)$$

LFWF

After performing integration, arriving at

$$\lim_{p_3\to\infty} \Psi_{\mathcal{M}}(x, k_\perp^2)|_{x=y+\frac{k_2}{p_3}} = \psi_{\mathcal{M}}(x, k_\perp^2)\,.$$

If k_2 is small (nonpert.), $\lim_{p_3\to\infty} k_2/p_3 = 0$

Theory (LaMET with a model for LFWF)

As far as we are interested in NP region, we have

$$\Psi^{\rm NP}_{\mathcal{M}}(y, k_\perp^2) = \psi^{\rm NP}_{\mathcal{M}}(y, k_\perp^2) \quad \text{for} \;\; y = x = [0, 1]$$

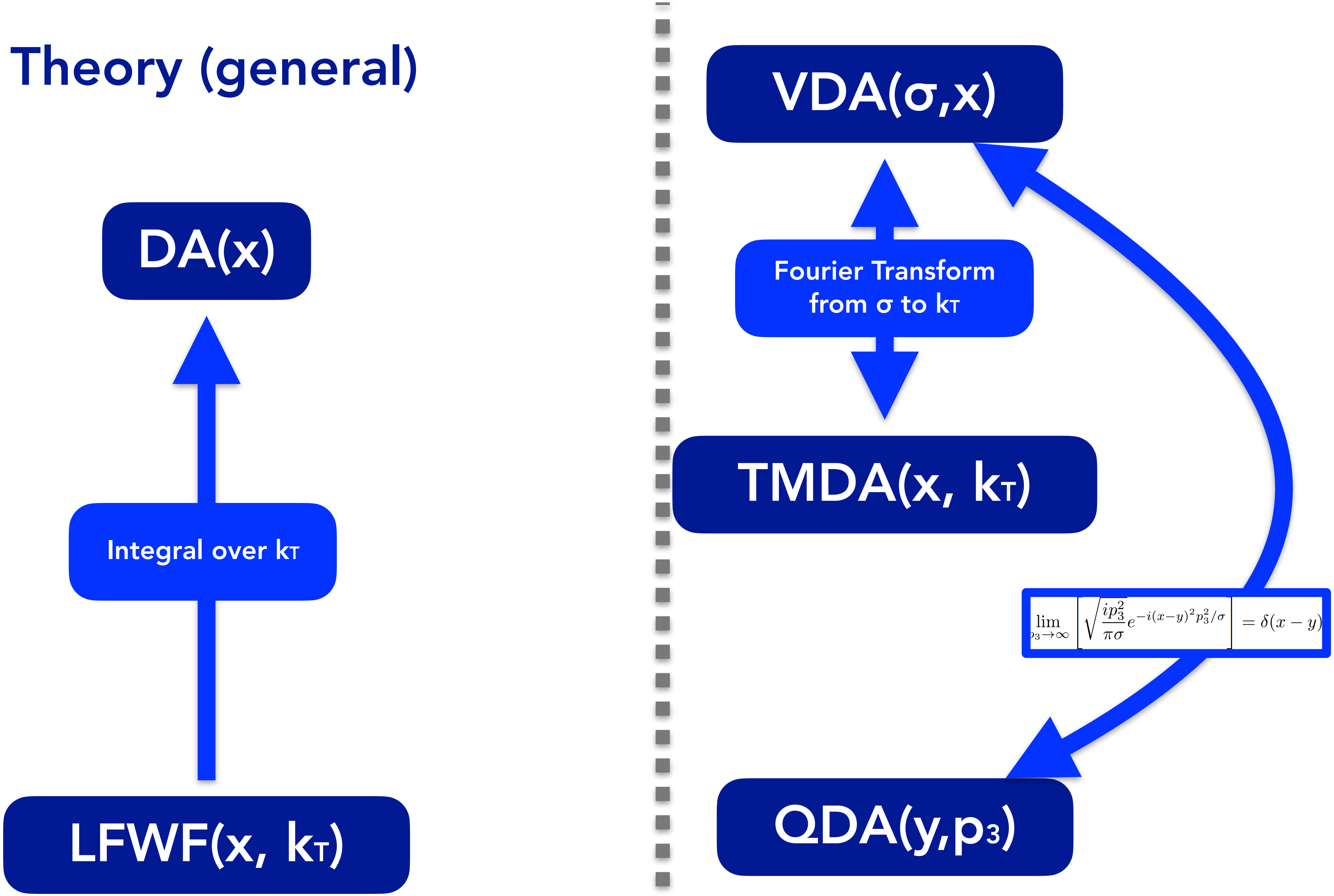
Saying, LFWF ~ TMDA in NP region

How can we test this relation?

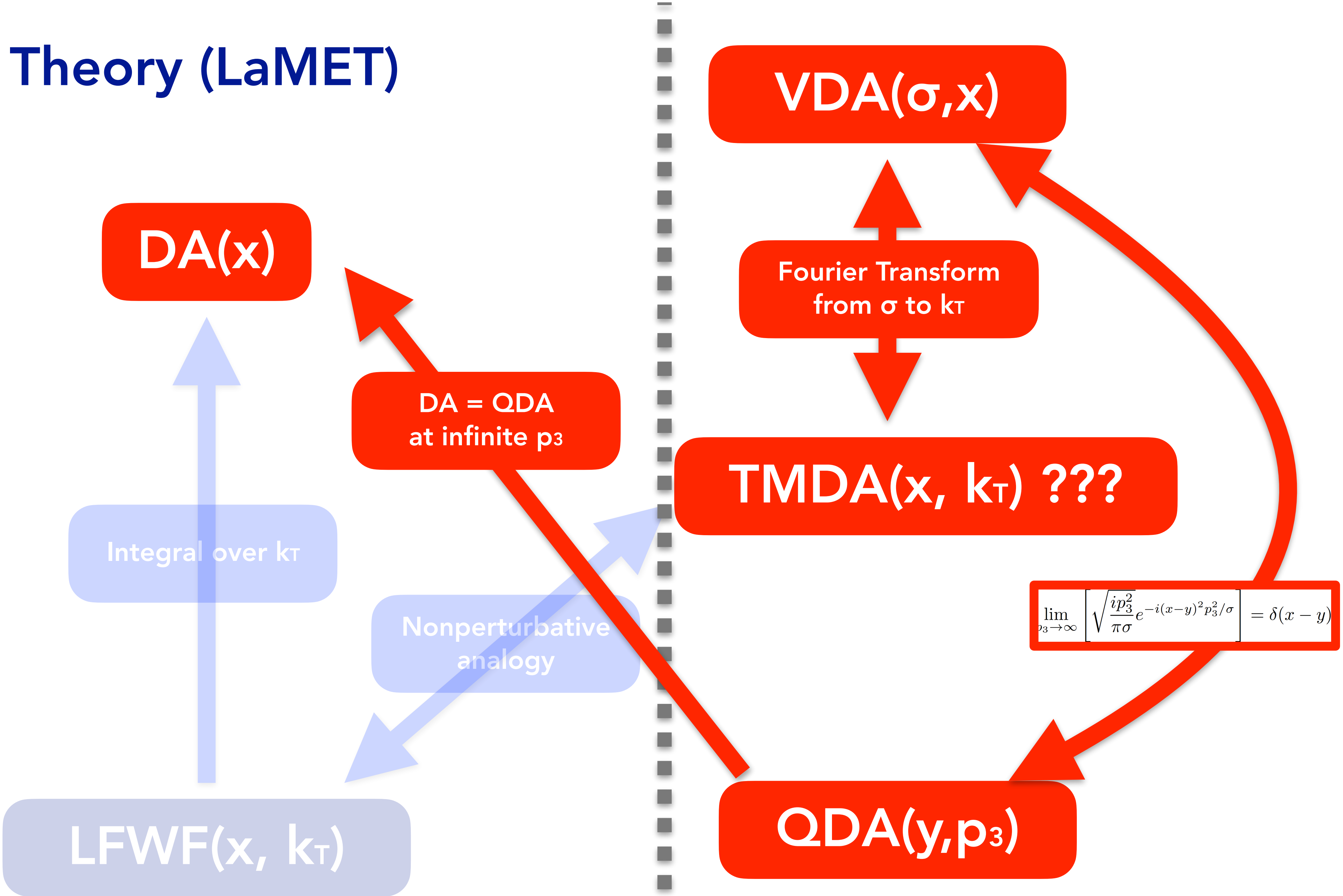
$$F_{\gamma\gamma^*\pi^0}(Q^2) = \frac{2F_\pi}{3}\int_0^1 \frac{dx}{xQ^2}\int_0^{xQ^2}\frac{dk_\perp^2}{xQ^2}\int_0^{k_\perp} d^2p_\perp\, \underbrace{\Psi(x, p_\perp^2)}_{\text{TMDA}}$$

Replacing TMDA with LFWF, data reproduced?

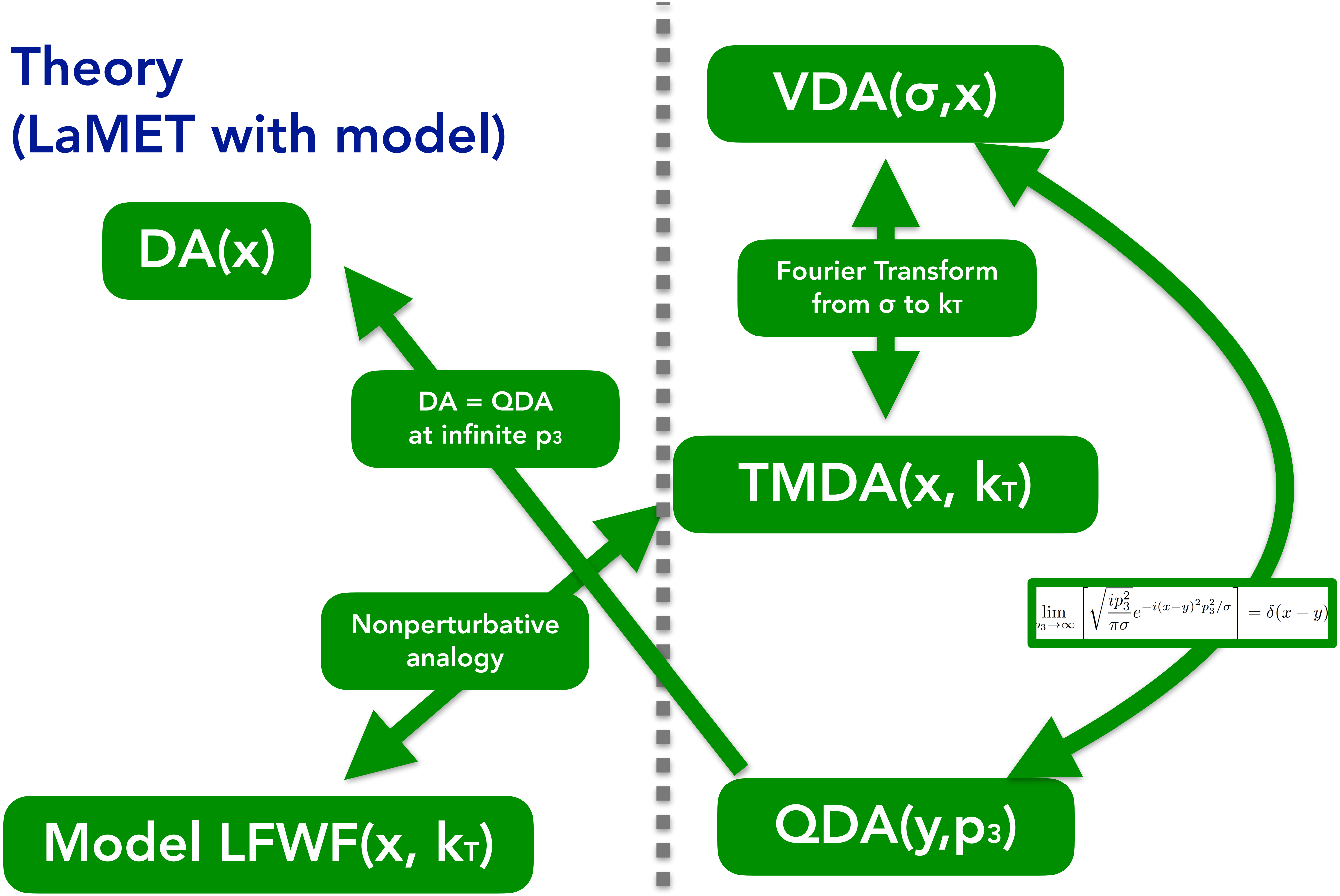
Theory (general)



Theory (LaMET)



Theory
(LaMET with model)



Model: Non-local chiral-quark model (NLChQM)

Based on liquid-instanton model (LIM)

D. Diakonov, Prog. Part. Nucl. Phys. 51, 173 (2003).

Nonlocal inter-quark interactions via instanton

$$\mathcal{S}_{\rm eff}[m_q,\mathcal{M}] = -\mathrm{Sp}\,\ln\left[i\partial\!\!\!/ + im_q + i\sqrt{M_q(\partial^2)}U^{\gamma_5}(\mathcal{M})\sqrt{M_q(\partial^2)}\right]$$

Effective model describing SCSB at $\Lambda_{\rm NLChQM}$ ~ 1.0 GeV

Performing Wick rotation from Euclidean to Minkowski

Then, Minkowski to LF frame with light-like vector n

Model: Non-local chiral-quark model (NLChQM)

PS-meson DA within NLChQM reads

$$\phi_{\mathcal{M}}^{\mathrm{NP}}(x) = -\frac{2iN_c}{F_{q\bar{q}'}^2}\int \frac{d^4k}{(2\pi)^4}\sqrt{M_q(k)M_{q'}(k-p)}\delta[\bar{x}p\cdot n - k\cdot n]$$

SiN et al., Phys. Rev. D 74, 014019 (2006)

$$\times \frac{[M_{q'}k - M_q(k-p)]\cdot n}{[k^2 - M_q^2][(k-p)^2 - M_{q'}^2]}.$$

And, LFWF from NLChQM reads

SiN, MPLA32, 1750218 (2017)

$\bar{x} = 1 - x$

$$\psi_{\mathcal{M}}^{\mathrm{NP}}(x, k_\perp^2)$$

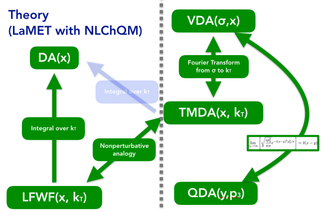
$$= \frac{\bar{x}N_c\Lambda^4\sqrt{M_qM_{q'}}[xM_q + \bar{x}M_{q'}]}{4\pi^3F_{\mathcal{M}}^2[M_q^2 - \Lambda^2][k_\perp^2 + \Lambda^2 - x\bar{x}M_{\mathcal{M}}^2][k_\perp^2 + \bar{x}M_{q'}^2 + x\Lambda^2 - x\bar{x}M_{\mathcal{M}}^2]}$$

$$+ \frac{\bar{x}N_c\Lambda^4\sqrt{M_qM_{q'}}[xM_q + \bar{x}M_{q'}]}{4\pi^3F_{\mathcal{M}}^2[\Lambda^2 - M_q^2][k_\perp^2 + xM_q^2 + \bar{x}\Lambda^2 - x\bar{x}M_{\mathcal{M}}^2][k_\perp^2 + xM_q^2 + \bar{x}M_{q'}^2 - x\bar{x}M_{\mathcal{M}}^2]},$$

Model: Non-local chiral-quark model (NLChQM)

An analytic expression for QDA beyond chiral limit within NLChQM

SiN, MPLA32, 1750218 (2017)



$$Q^{\rm NP}_{\mathcal{M}}(y,p_3) = \frac{N_c M_0^2 \Lambda^4}{8\pi^2 F_\pi^2 \eta^4}$$

$$\times \left\{ \ln\left[\frac{[\eta^2+2p_3 f_-(\bar{y},\Lambda)]^2[\eta^2-2p_3 f_-(\bar{y},M_0)]^2 f_+(\bar{y},\Lambda) f_+(\bar{y},M_0) f_+(y,\Lambda) f_+(y,M_0)}{[\eta^2+2p_3 f_+(y,\Lambda)]^2[\eta^2-2p_3 f_+(y,M_0)]^2 f_-(\bar{y},\Lambda) f_-(\bar{y},M_0) f_-(y,\Lambda) f_-(y,M_0)}\right]\right.$$

$$- \frac{\Delta M}{M_0}\left[(2y-3)\ln\left[\frac{f_-(\bar{y},\Lambda) f_-(\bar{y},M_0)[\eta^2+2p_3 f_+(y,\Lambda)][\eta^2-2p_3 f_+(y,M_0)]}{f_+(y,\Lambda) f_+(y,M_0)[\eta^2+2p_3 f_-(\bar{y},\Lambda)][\eta^2-2p_3 f_-(\bar{y},M_0)]}\right]\right.$$

$$\left.\left. + \frac{\eta^2}{p_3^2}\ln\left[\frac{[\eta^2+2p_3 f_+(y,\Lambda)][\eta^2+2p_3 f_-(\bar{y},\Lambda)]}{[\eta^2-2p_3 f_+(y,M_0)][\eta^2-2p_3 f_-(\bar{y},M_0)]}\right]\right]\right\} + \mathcal{O}(\Delta M^2),$$

$$\Delta M = |M_q - M_{q'}| = |m_q - m_{q'}|$$

$$f_\pm(y,M_0) = xp_3 \pm \sqrt{M_0^2 + y^2 p_3^2},$$

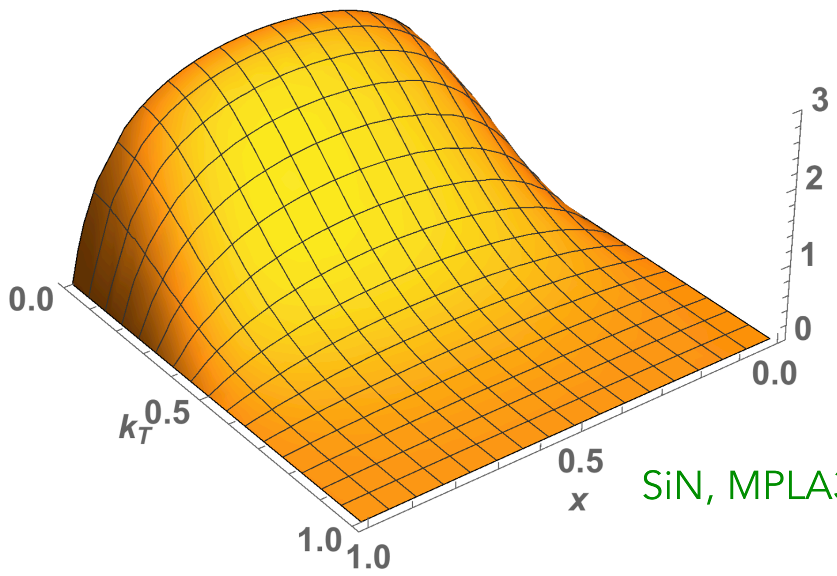
$$\eta^2 = \Lambda^2 - M_0^2$$

Numerical results

Table 1. Model parameters for the present calculations. With these values, the pion and kaon DAs satisfy the normalization condition, i.e. $\int dx\, \phi_{\pi,K}(x) = 1$.

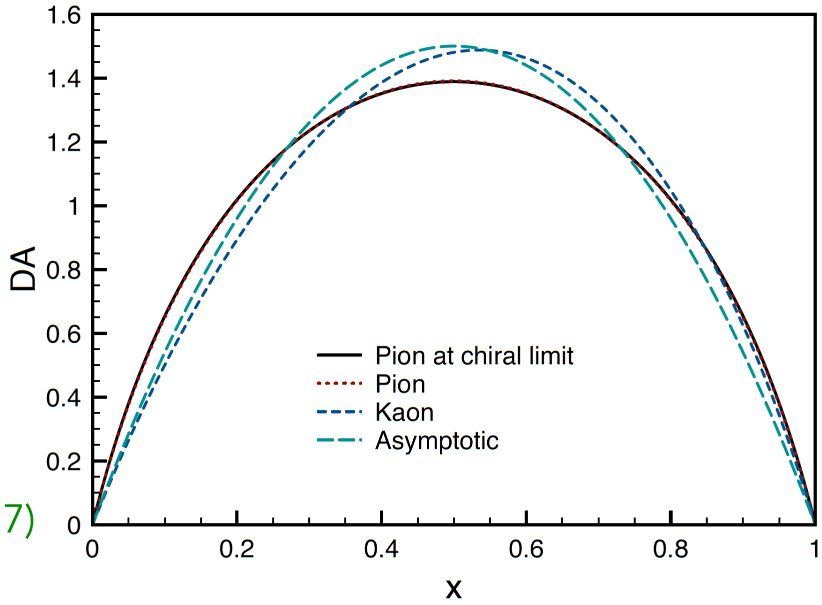
	$M_0 = 350$ MeV	$m_{u,d} = 5$ MeV	$m_s = 135$ MeV		
Pion at CL	$F_\pi = 93$ MeV	$M_q = M_0$	$M_{q'} = M_0$	$M_\pi = 0$ MeV	$\Lambda = 1.02$ GeV
Pion	$F_\pi = 93$ MeV	$M_q = (m_{u,d} + M_0) = 355$ MeV	$M_{q'} = (m_{u,d} + M_0) = 355$ MeV	$M_\pi = 140$ MeV	$\Lambda = 1.01$ GeV
Kaon	$F_K = 113$ MeV	$M_q = (m_{u,d} + M_0) = 355$ MeV	$M_{q'} = (m_s + M_0) = 485$ MeV	$M_K = 495$ MeV	$\Lambda = 1.05$ GeV

TMDA ~ LFWF as a func. of (x, k_T)



SiN, MPLA32, 1750218 (2017)

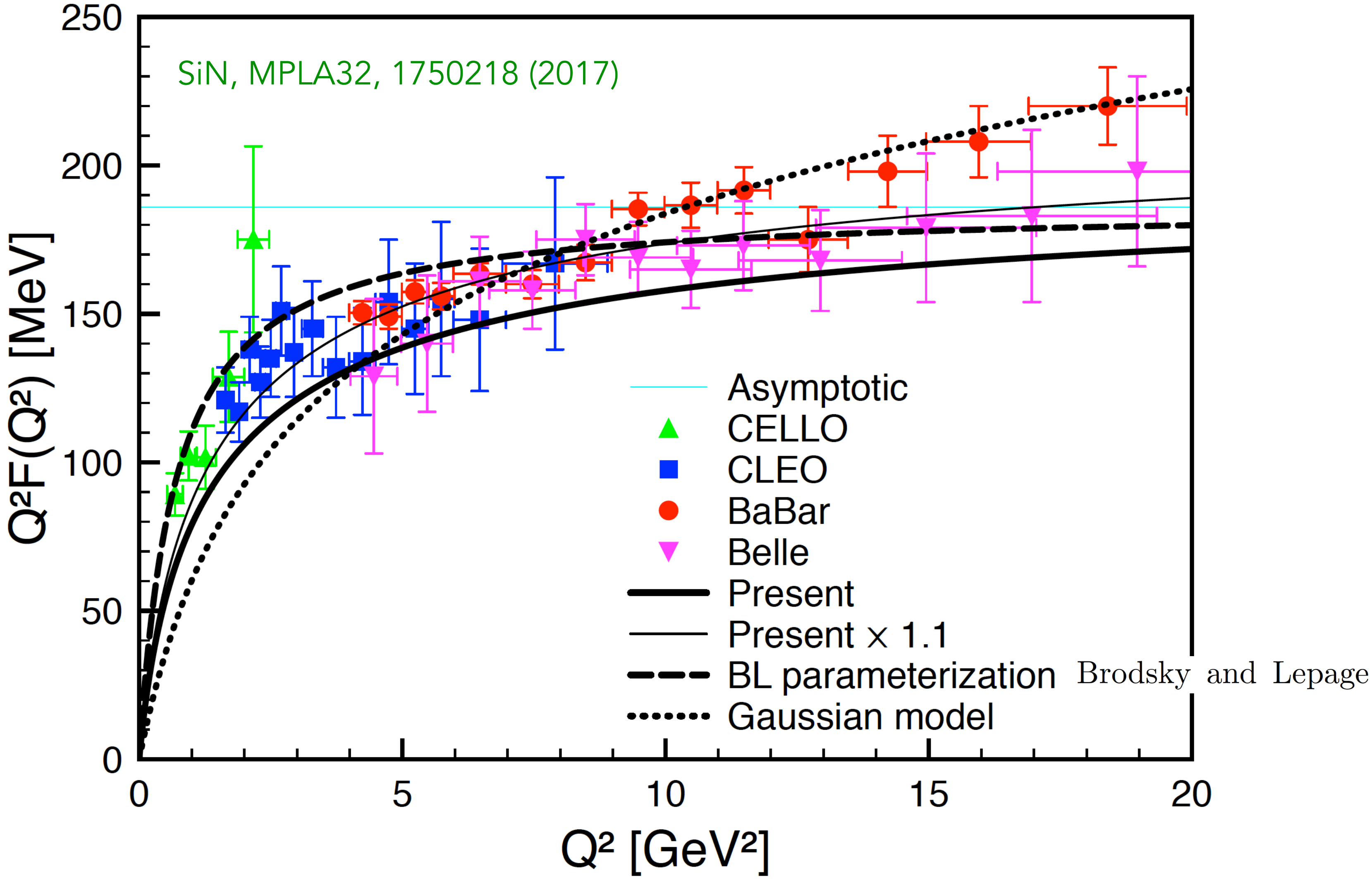
DA as a func. of x



Numerical results

$$F_{\gamma\gamma^*\pi^0}(Q^2) = \frac{2F_\pi}{3}\int_0^1 \frac{dx}{xQ^2}\int_0^{xQ^2}\frac{dk_\perp^2}{xQ^2}\int_0^{k_\perp} d^2p_\perp\,\Psi(x,p_\perp^2)$$

Photon-pion transition FF with TMDA ~ LFWF



Numerical results

Non-zero at $Q^2=0$, via Adler-Bell-Jackiw axial anomaly for real photons: $F_{\gamma\gamma\pi^0}(0) = (4\pi^2 F_\pi)^{-1} \approx 0.272\ \mathrm{GeV}^{-1}$

From NLChQM gives $\lim_{Q^2\to 0} F^{\mathrm{NLChQM}}_{\gamma\gamma^*\pi^0}(Q^2) = 0.191\ \mathrm{GeV}^{-1}$

SiN, MPLA32, 1750218 (2017)

The ratio becomes $F^{\mathrm{NLChQM}}_{\gamma\gamma^*\pi^0}(0)/F_{\gamma\gamma\pi^0}(0) = 0.702$

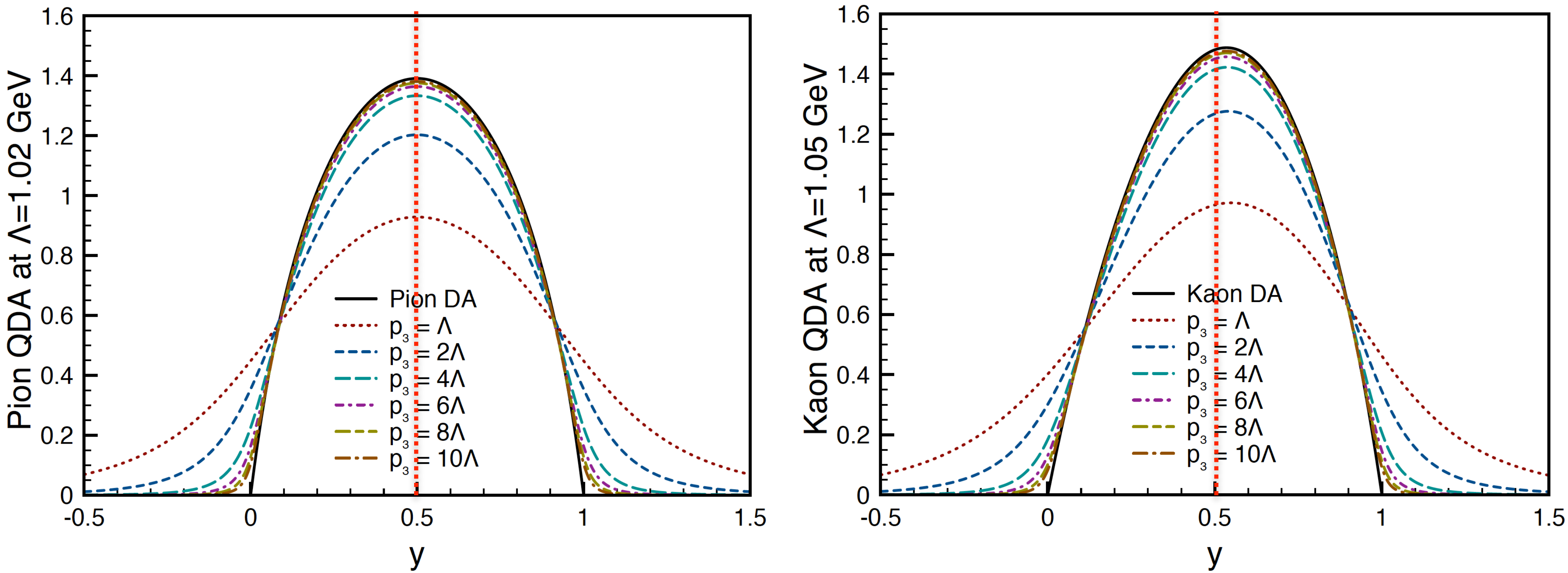
Slightly larger than 0.5

G. P. Lepage and S. J. Brodsky, PRD22, 2157 (1980)
A. V. Radyushkin, PRD93, 056002 (2016).

E. Ruiz Arriola and W. Broniowski, PRD74, 034008 (2006)
A. G. Oganesian et al., PRD93, 054040 (2016).

Numerical results

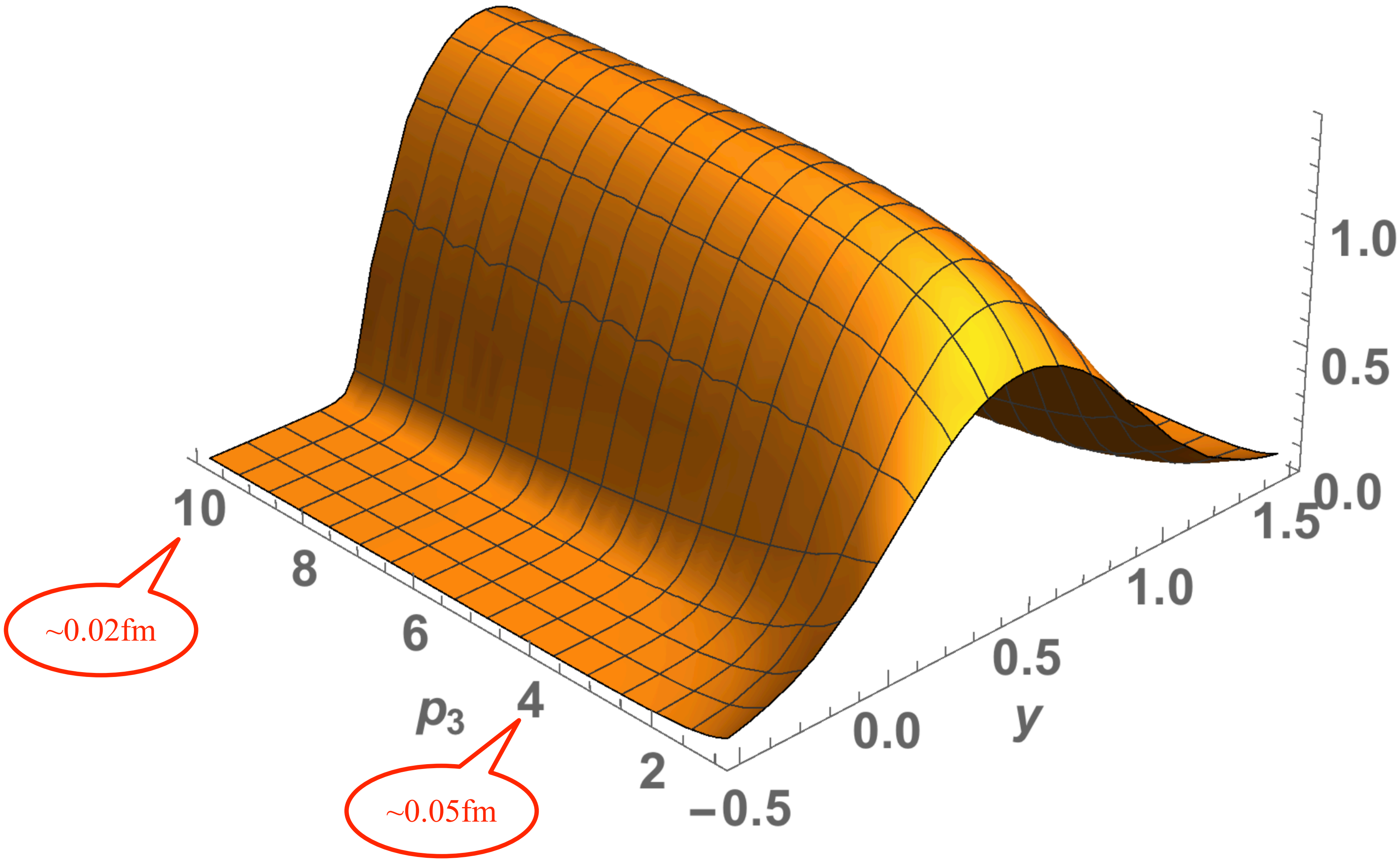
Pion and kaon QDA for p_3 [GeV]



Slightly tilted curves for kaon, due to $m_s > m_u$, m_d

Numerical results

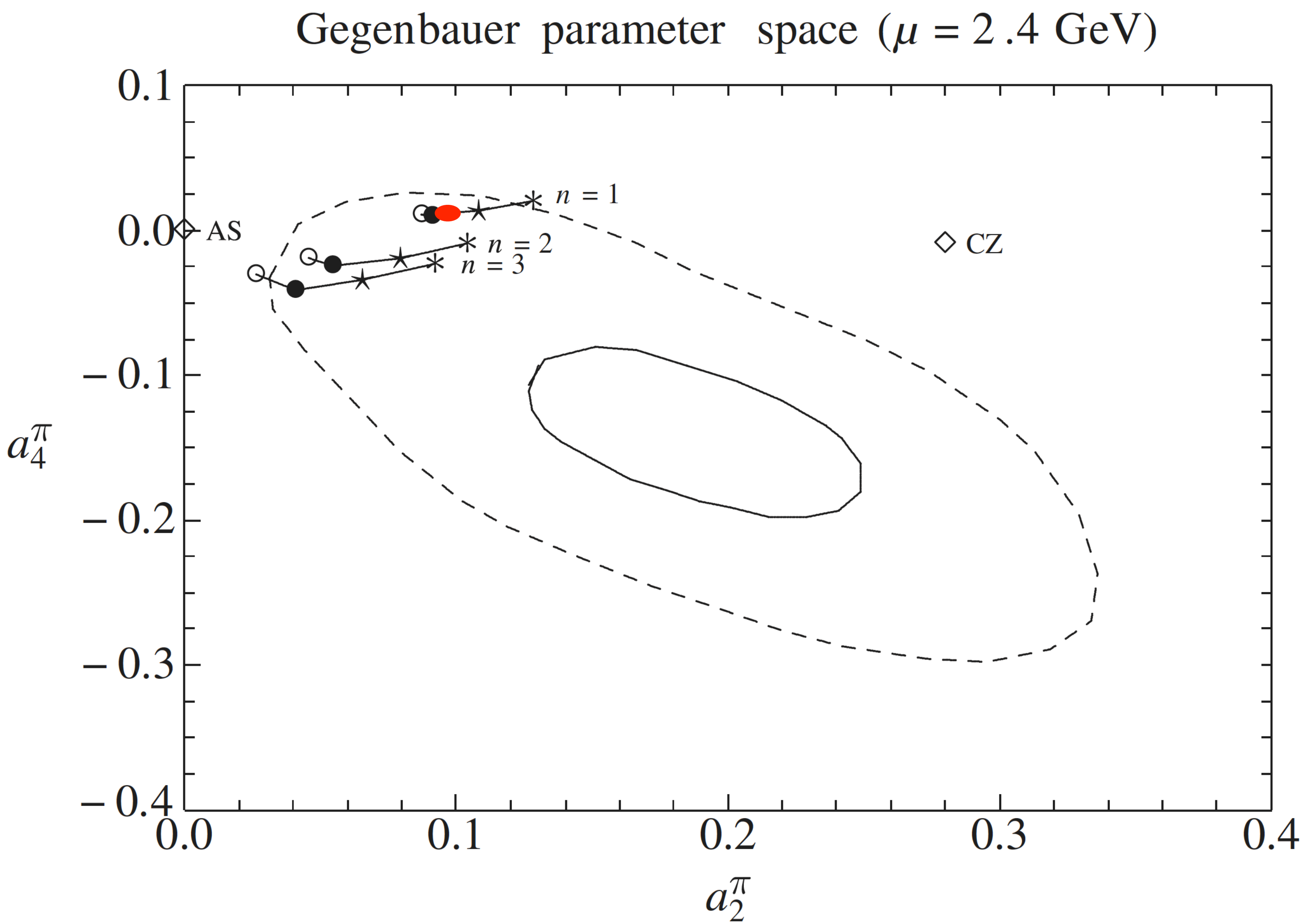
Pion QDA for p_3 [GeV] and y



Numerical results

J. Gronberg et al. (CLEO Collaboration), Phys. Rev. D 57, 33 (1998).
A. Schmedding and O. I. Yakovlev, Phys. Rev. D 62, 116002 (2000).

Gegenbauer coefficients from pion QDA for $p_3 \to \infty$



Numerical results

Moments from DA and QDA $\xi \equiv (y - \bar{y}) = 2x - 1$

$$\langle \xi^n \rangle_{\mathcal{M}}^{\rm DA} = \int_0^1 (2x-1)^n \phi_{\mathcal{M}}(x) dx\,, \quad \langle \xi^n \rangle_{\mathcal{M}}^{\rm QDA} = \lim_{p_3 \to \infty} \int_{-\infty}^{\infty} (2y-1)^n Q_{\mathcal{M}}(y, p_3) dy$$

Table 2. The moments from the pion and kaon DA and QDA for $\xi = 2x - 1$ and $2y - 1$, respectively.

	$n=1$	$n=2$	$n=3$	$n=4$		$n=1$	$n=2$	$n=3$	$n=4$
$\langle \xi^n \rangle_\pi^{\rm DA}$	–	0.2210	–	0.1002	$\langle \xi^n \rangle_K^{\rm DA}$	0.0277	0.2043	0.0122	0.0887
$\langle \xi^n \rangle_\pi^{\rm QDA}$ at $p_3 = 10\Lambda$	–	0.2287	–	0.1159	$\langle \xi^n \rangle_K^{\rm QDA}$ at $p_3 = 10\Lambda$	0.0277	0.2118	0.0120	0.1034
$\langle \xi^n \rangle_\pi^{\rm QDA}$ at $p_3 = 20\Lambda$	–	0.2229	–	0.1030	$\langle \xi^n \rangle_K^{\rm QDA}$ at $p_3 = 20\Lambda$	0.0277	0.2062	0.0121	0.0913
$\langle \xi^n \rangle_\pi^{\rm QDA}$ at $p_3 = 30\Lambda$	–	0.2218	–	0.1013	$\langle \xi^n \rangle_K^{\rm QDA}$ at $p_3 = 30\Lambda$	0.0277	0.2052	0.0122	0.0898

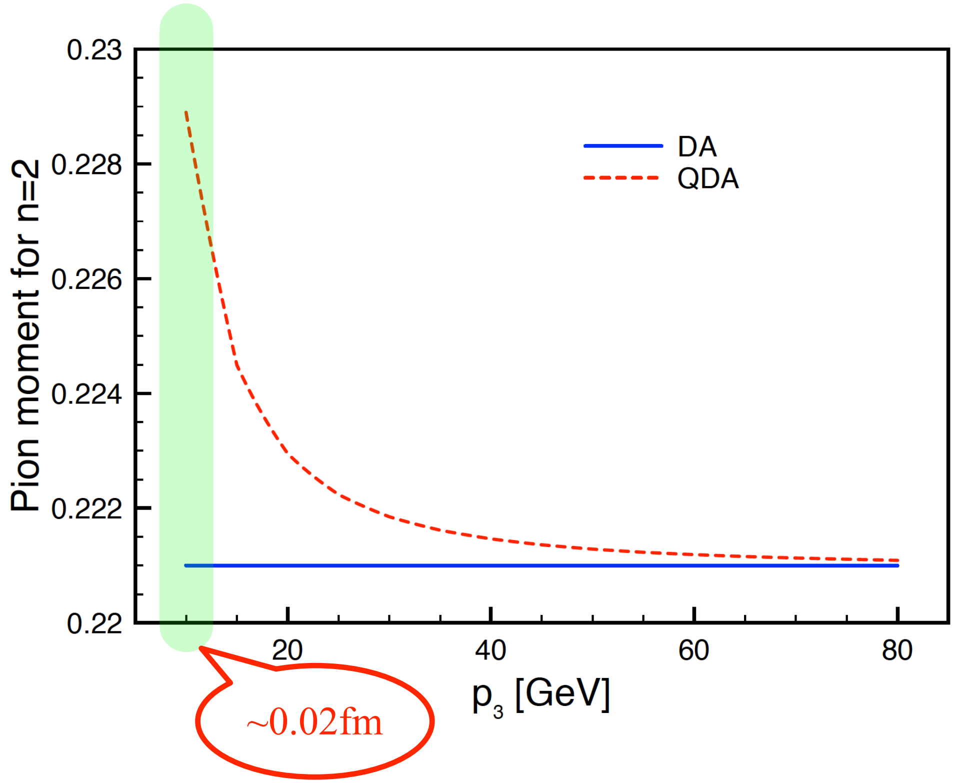
$$\langle \xi^2 \rangle^{\overline{\rm MS}} = \int_0^1 du\,(2u-1)^2 \phi_\pi(u,\mu) = 0.2361(41)(39)(?)$$

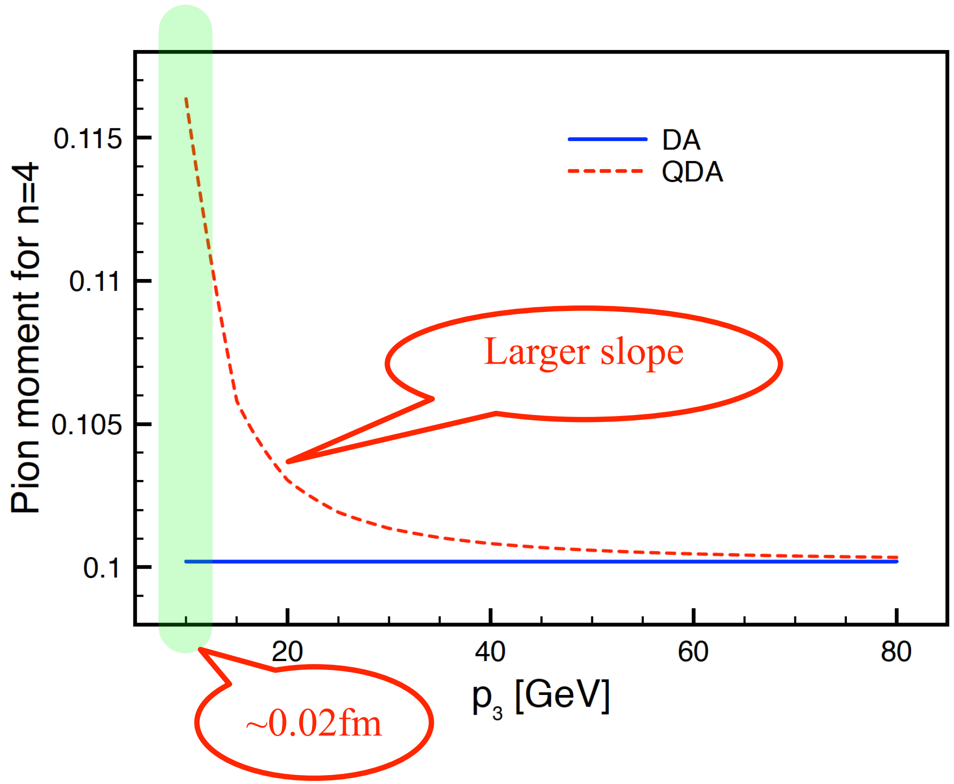
V.Braun et al. (RQCD Collaboration), PRD 92, 014504 (2015)

For p_3 > 30 GeV, differences reduced to ~ 10%

Numerical results

Behavior of moments for p_3





Higher moments depend much on p_3

Summary and perspectives

Verified that TMDA ~ LFWF at NP region

From LFWF from NLCHQM, analytic form for QDA derived for nonzero current-quark mass

Obtained QDA for pion and kaon successfully describe DAs for p3 → ∞, showing reproduction of exp. data

Higher moments are sensitive for p_3

Fragmentation func. (FragF) ↔ PDF via Drell-Levy-Yan
Then, QPDF ↔ QFragF ?!?! In progress!!

Thank you for your attention!

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NRF-2022R1A2C1003964, and NRF-2022K2A9A1A06091761.