Entanglement between Valence and Sea Quarks in Hadrons of 1+1 Dimensional QCD

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International Light Cone Advisory Committee

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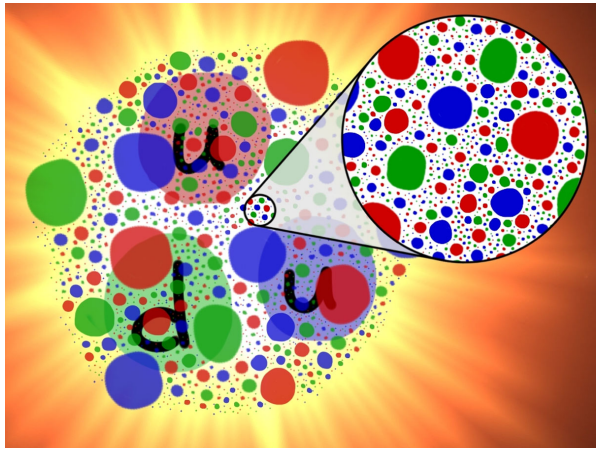
Intuition for QCD

The intuitive picture of the inside of a hadron has a small number of valence quarks amidst a sea of other quarks and gluons.

- Valence quarks determine properties of the hadron.



Intuition Difficulties

Intuitive picture does not explain certain properties of hadrons

- Results suggest that the valence spin content of the nucleon is only 30-40% of the total

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Intuition Difficulties

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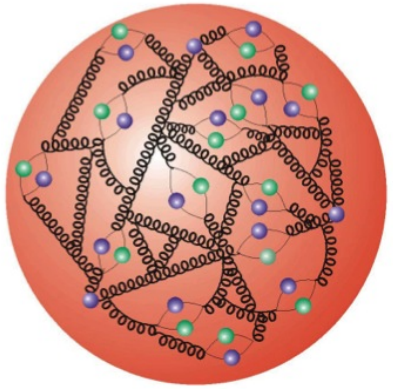
- Results suggest that the valence spin content of the nucleon is only 30-40% of the total

Valence content is obscured in QCD state vectors.

- Could be signature of strong entanglement between valence and sea content.



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Entanglement in QCD

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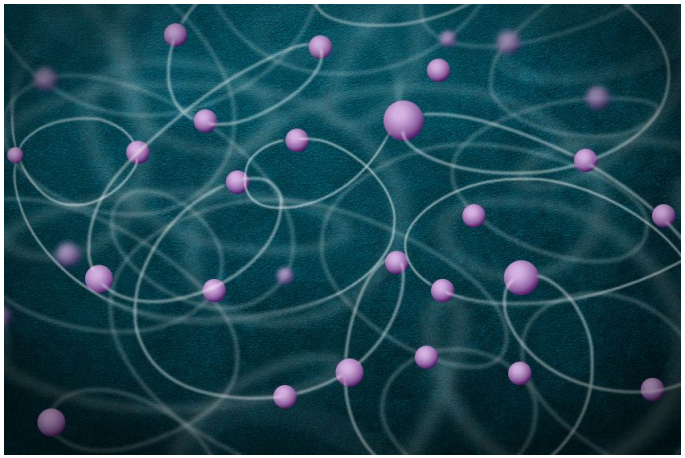
- Kharzeev & Levin posed the question "What is the entanglement entropy of deep inelastic scattering processes?"
- Could be useful for simulation on quantum computers.



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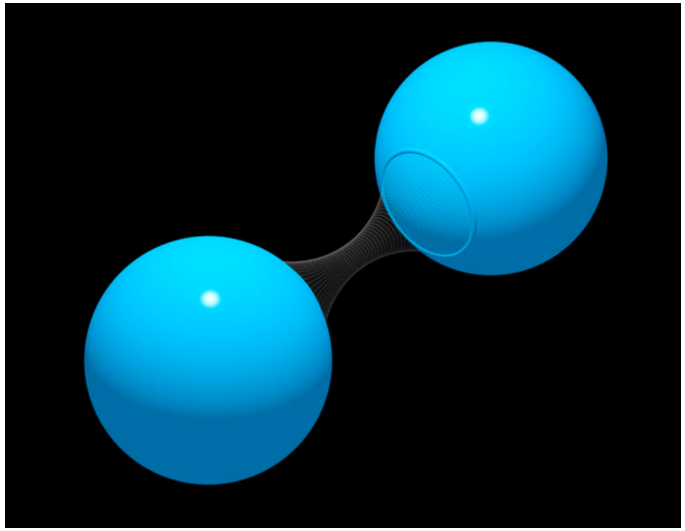
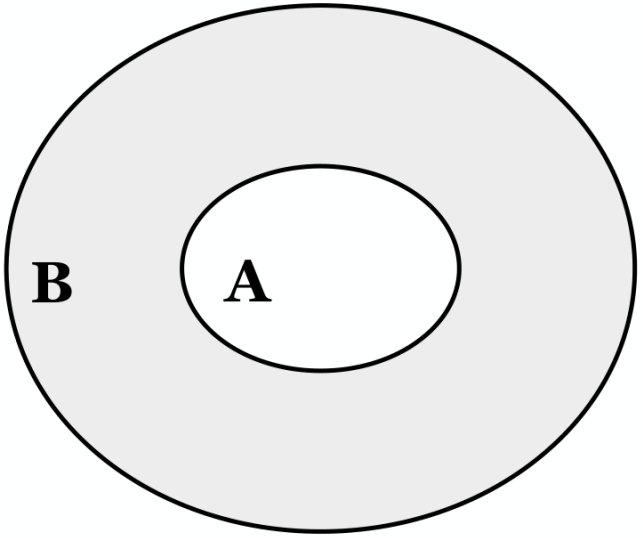
- Investigating the confined sector of QCD via 1+1d QCD
- Need a definition of valence-sea entanglement for QCD state vectors

Entanglement Entropy

Entropy is information to be gained from measurement.

- calculable for a state $|\psi\rangle$ in a Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$





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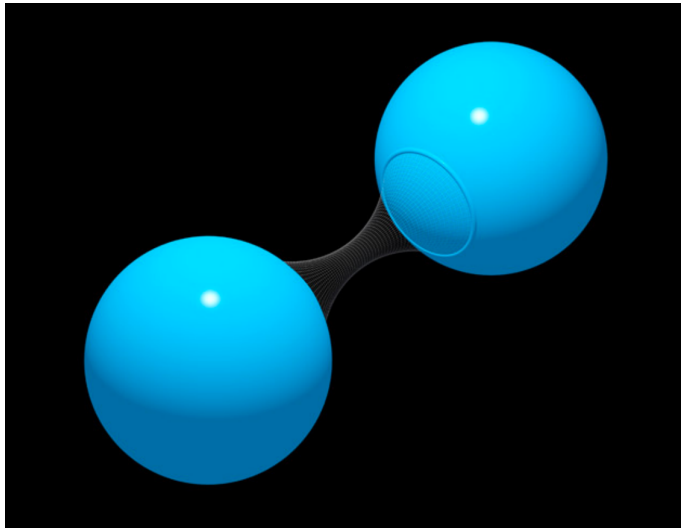

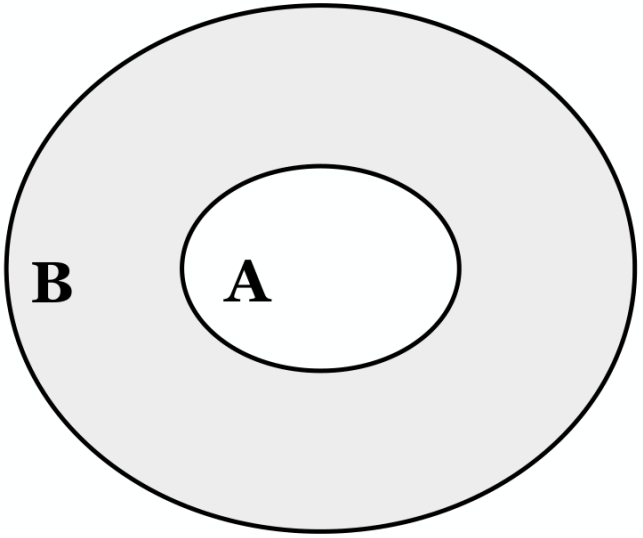

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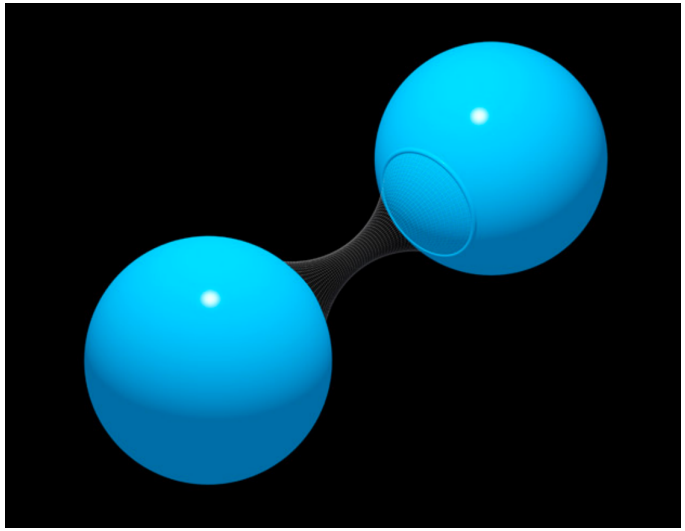
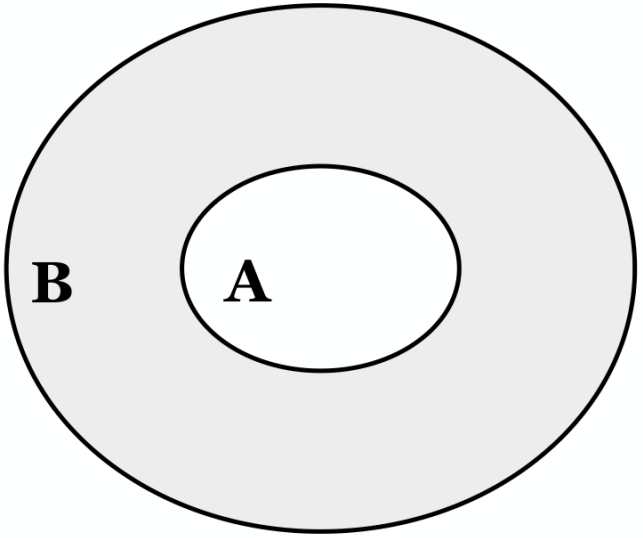
- Take $\rho_A = \mathrm{Tr}_B[|\psi\rangle\langle\psi|]$.
- Entanglement entropy is $S(\rho_A) = -\mathrm{Tr}_A[\rho_A \log(\rho_A)]$.





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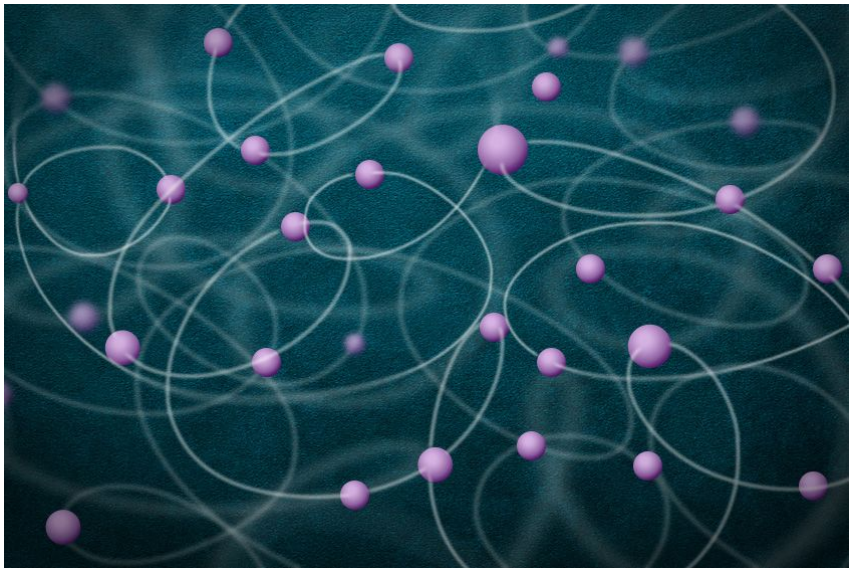

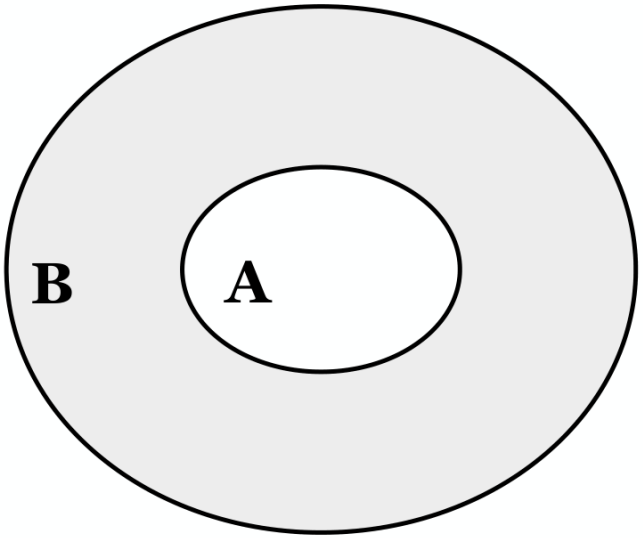

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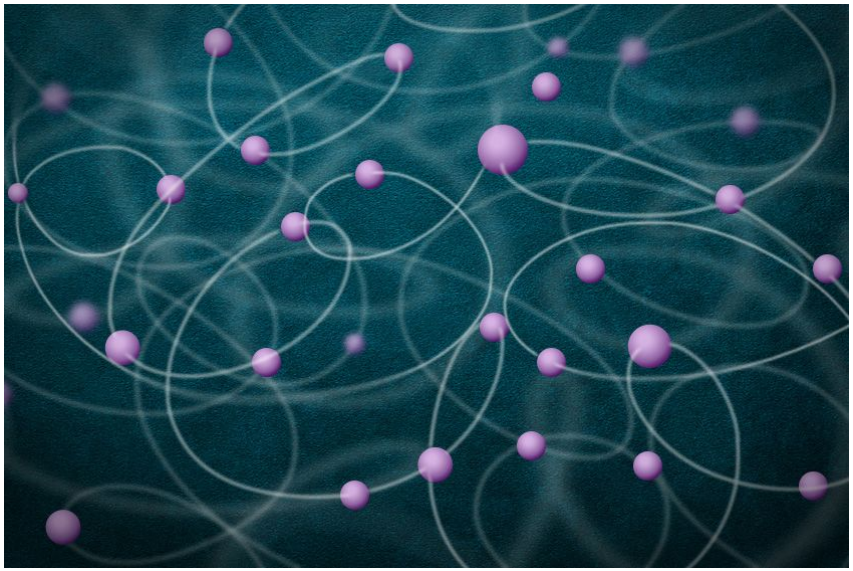
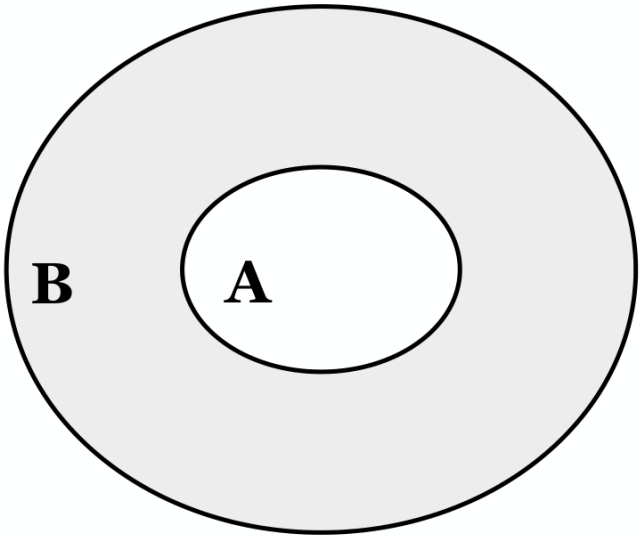
- Valence quarks and sea quarks are indistinguishable outside of flavor.





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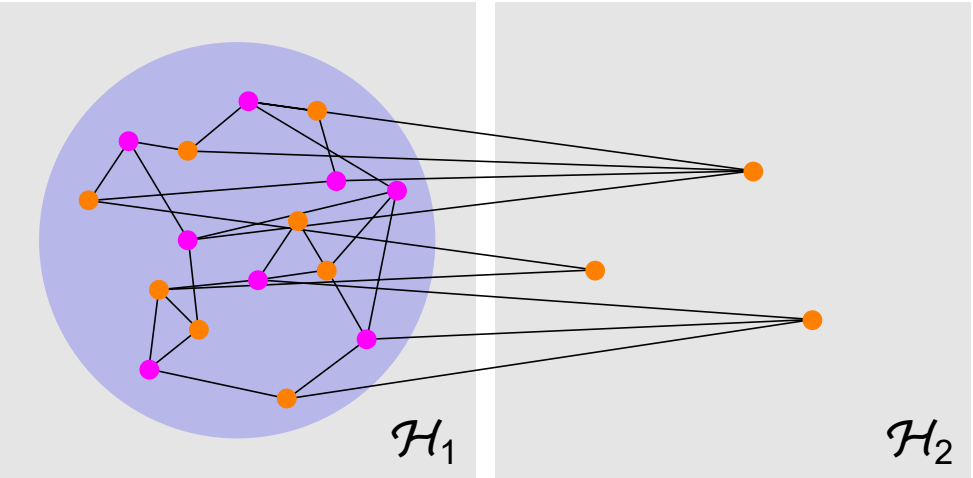
- Duplicate the Hilbert space $\mathcal{H} \to \mathcal{H}_1 \otimes \mathcal{H}_2$.



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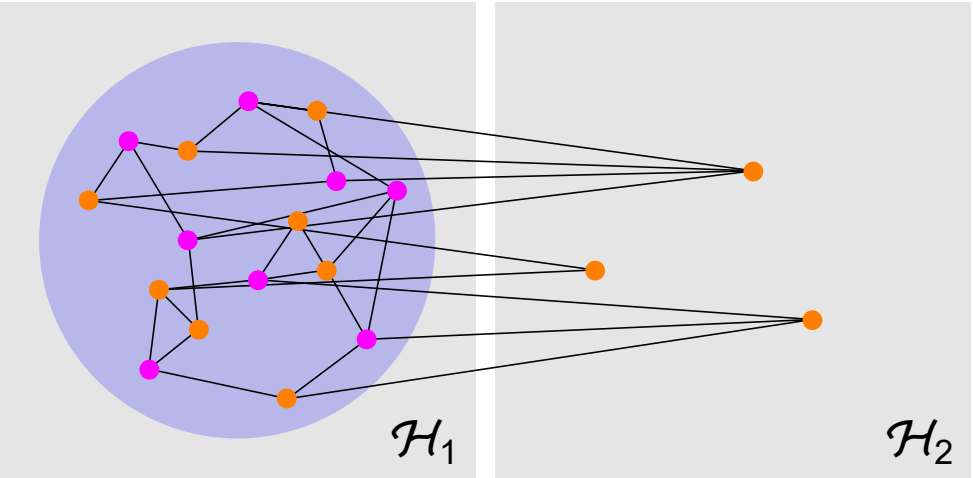

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- Duplicate the Hilbert space $\mathcal{H} \to \mathcal{H}_1 \otimes \mathcal{H}_2$.
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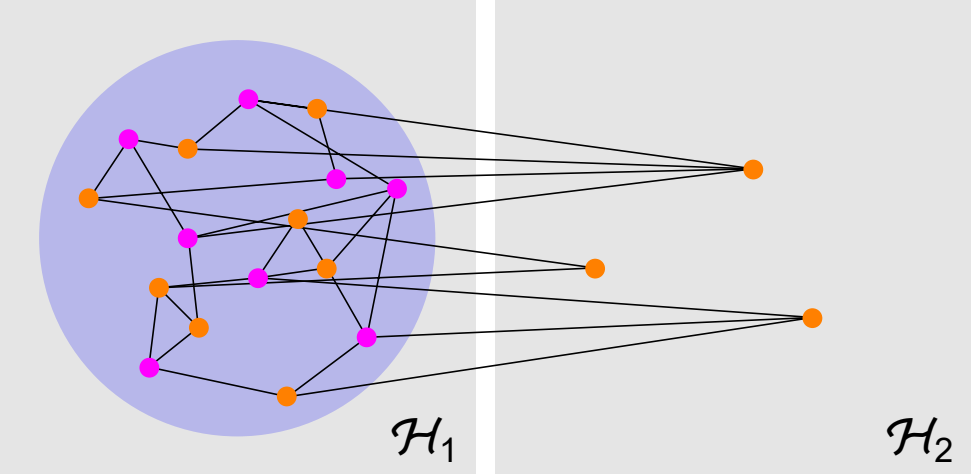
- Get the valence quark density matrix of the hadron ρ_h in $\mathcal{H}_2$.



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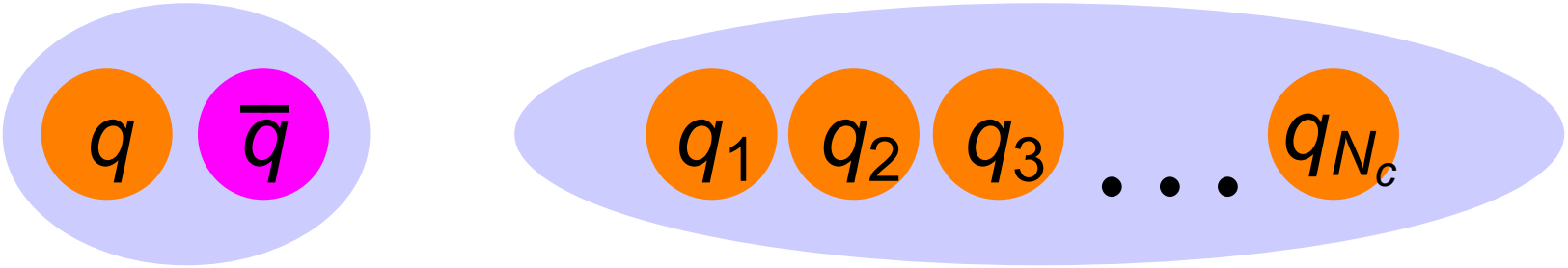

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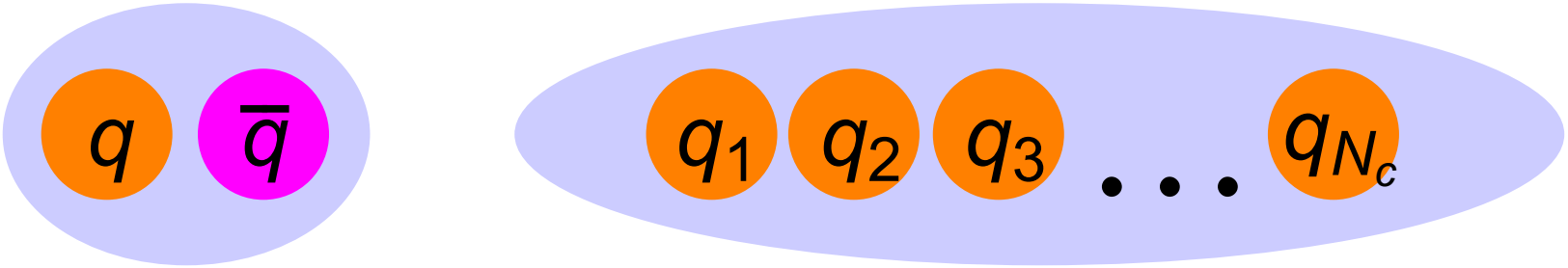
- Entanglement entropy vanishes when no sea quarks are present (Large-N_c limit).
- Standard definition of bipartite entanglement is a special case.



Spatial Entanglement Example 1

Consider a quantum system where particles can occupy two spatial sites.

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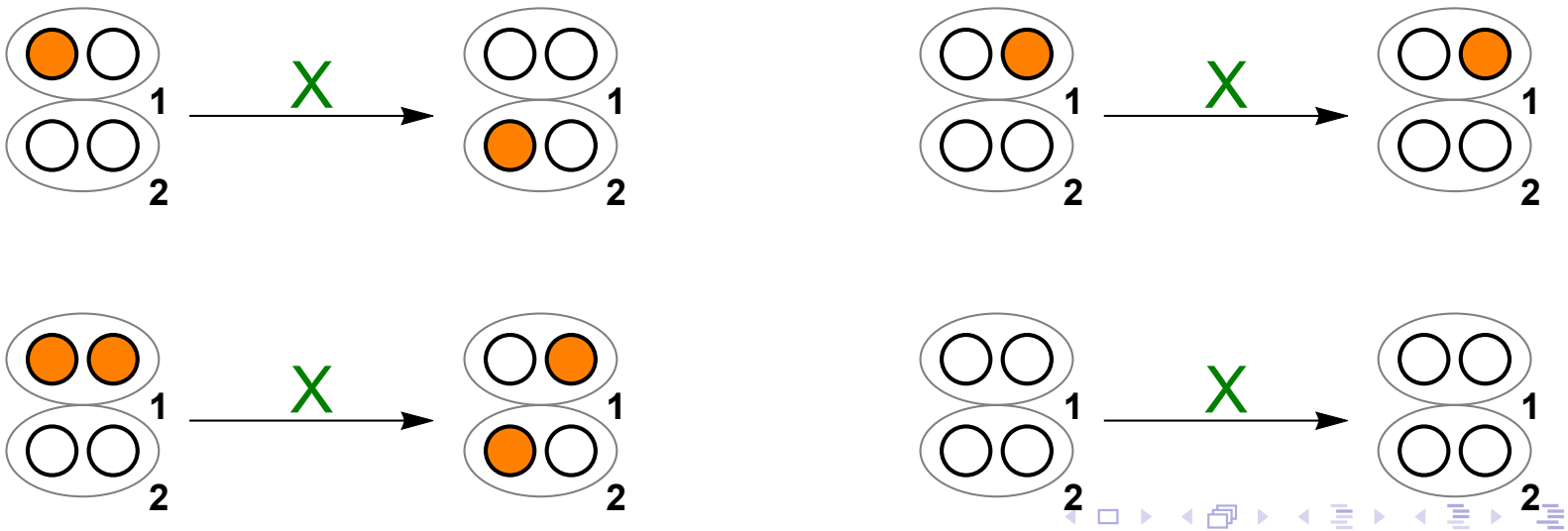
The reduced density matrix for spatial entanglement is

$$\rho_X = \mathcal{N}\mathrm{Tr}_1[X\,|\psi\rangle_1\,|\mathrm{OO}\rangle_2\,\langle\mathrm{OO}|_2\,\langle\psi|_1\,X]$$



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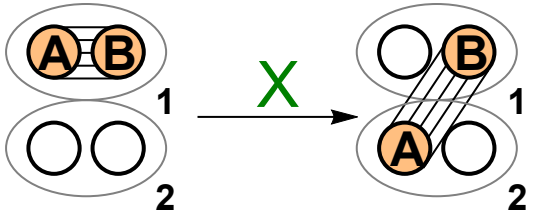
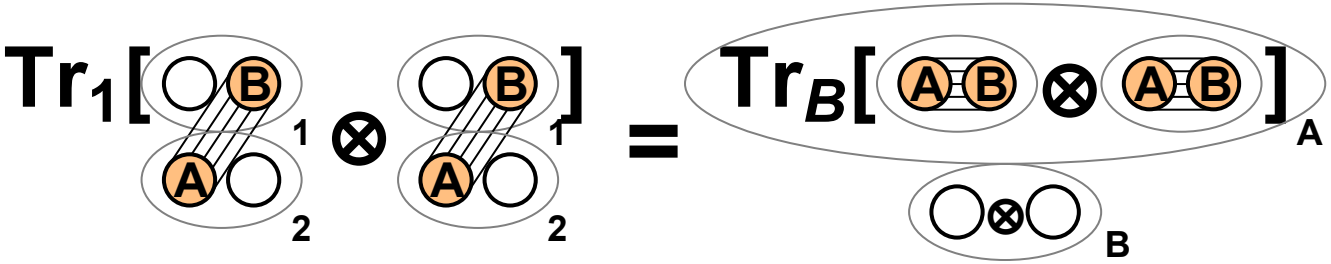
Density matrix is $\rho_A \otimes (|\mathrm{O}\rangle\langle\mathrm{O}|)_B$, where ρ_A is the usual bipartite density matrix.





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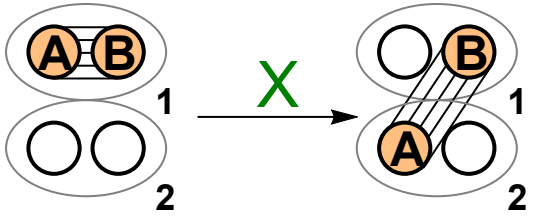

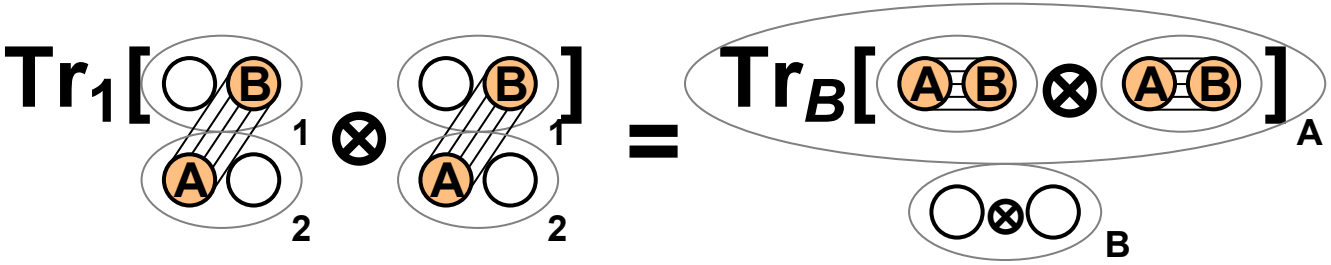

This logic can be applied to study any form of bipartite entanglement within this formalism.

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Consider the same quantum system and a state $|\phi\rangle$ which has one valence particle (no $|\mathrm{OO}\rangle$ overlap).

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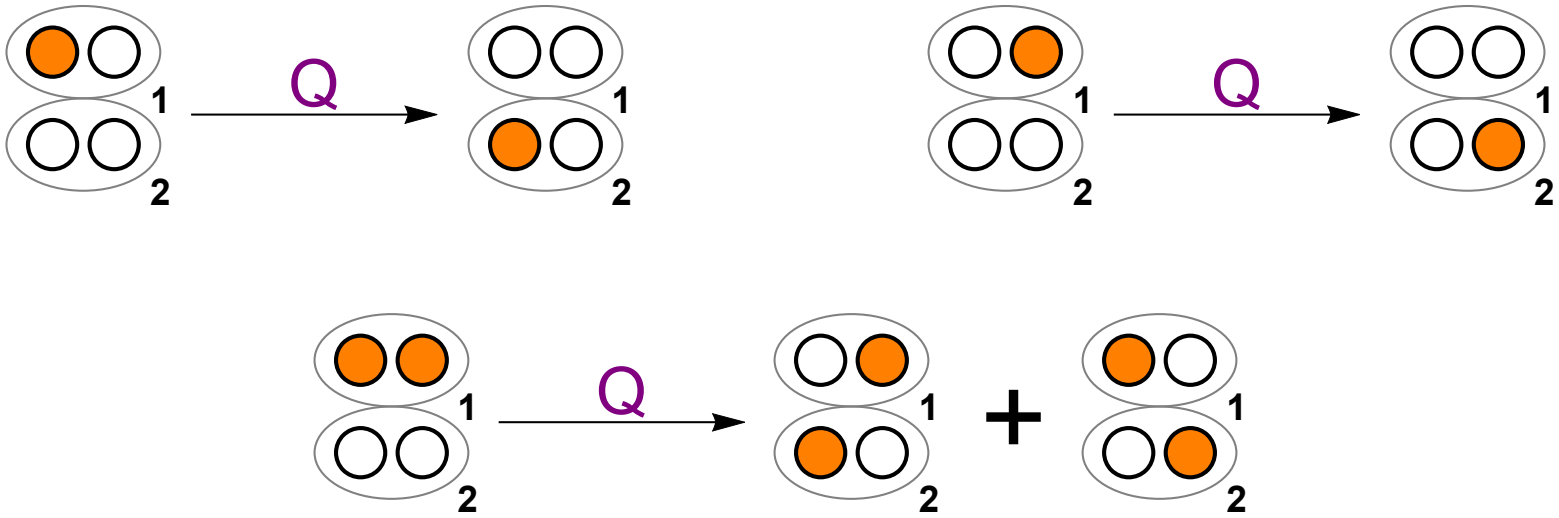
The reduced density matrix for valence-sea entanglement is

$$\rho_Q = \mathcal{N}\mathrm{Tr}_1[Q\,|\phi\rangle_1\,|\mathrm{OO}\rangle_2\,\langle\mathrm{OO}|_2\,\langle\phi|_1\,Q]$$



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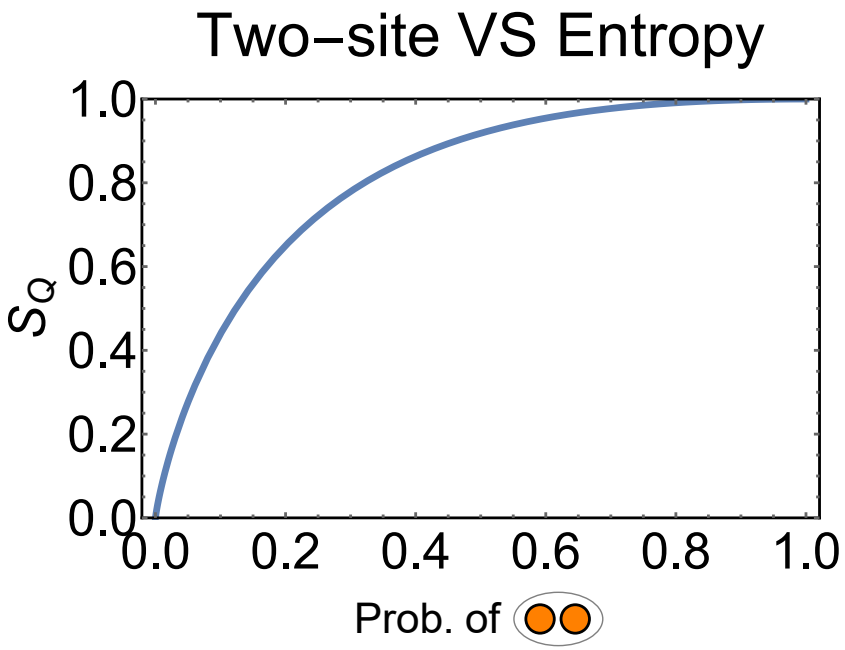

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- No entanglement with no sea particles.

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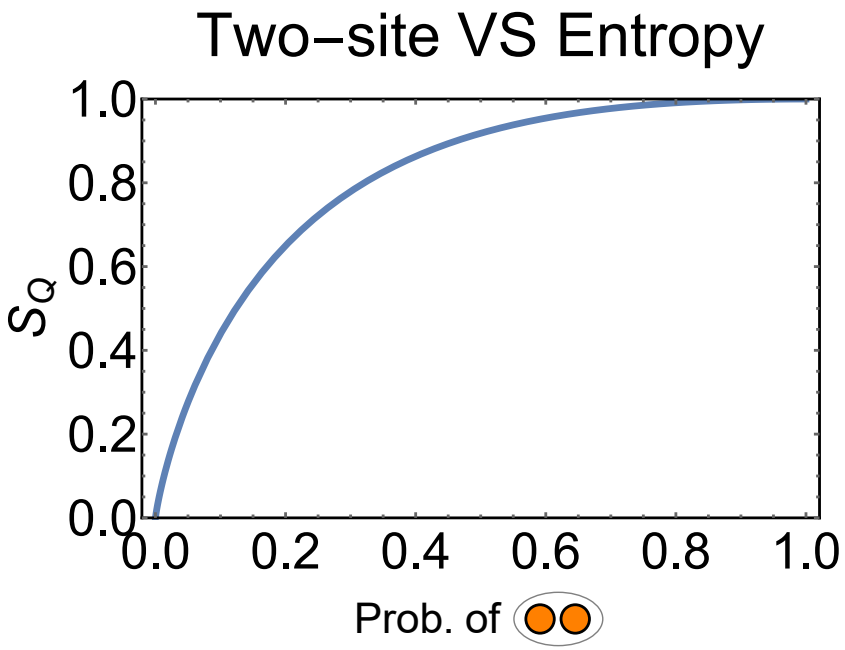

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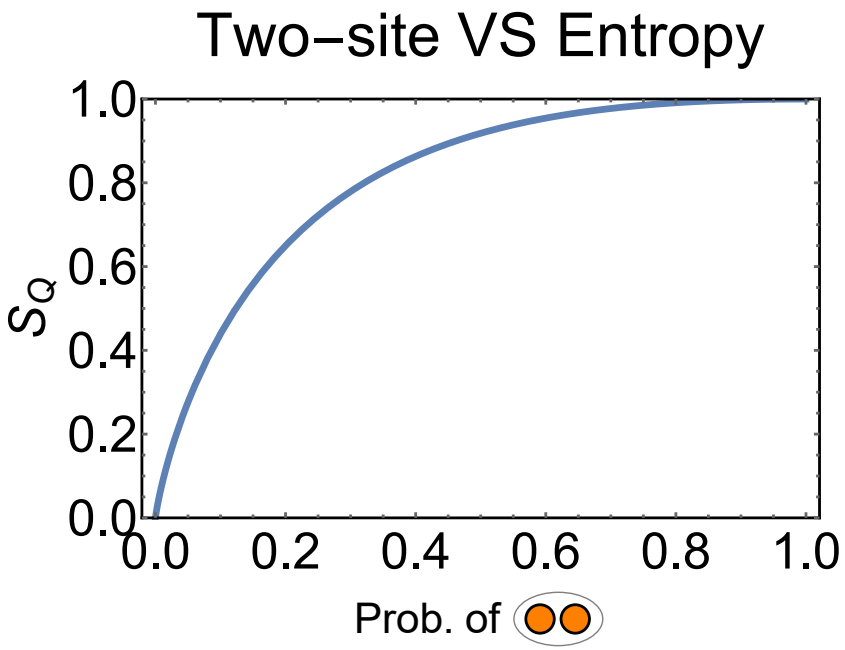

This definition is sensible for several reasons:

- No entanglement with no sea particles.
- Does not discriminate between spatial sites.
- Highly sensitive to the presence of the sea.

Quark Entanglement

A single quark operator of flavor i in the light-cone formalism is given by

$$Q_i = \int dx^- dx_\perp^2 \; \bar{q}_{i,2}(x^-, \vec{x}_\perp)\gamma^+ q_{i,1}(x^-, \vec{x}_\perp)$$

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with $x^- = \frac{1}{\sqrt{2}}(x^0 - x^3)$ and $\gamma^+ = \frac{1}{\sqrt{2}}(x^0 + x^3)$. (Kogut-Soper conventions)

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- Note that $Q_i \left|\Psi\right\rangle_1 \left|0\right\rangle_2 \sim \int dk^+ dk_\perp^2 (b_{i,2}^\dagger(k) \left|0\right\rangle_2)(b_{i,1}(k) \left|\Psi\right\rangle_1)$.
- Likewise for $\bar{Q}_i$ and antiquark operators $d^\dagger, d$.

Single Quark Entanglement

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The resulting density matrix is given by (in light-cone gauge)

$$\rho_i \approx \mathcal{N} \int_0^1 dx \int_{-\infty}^{\infty} d\vec{k}_\perp^2 \; f_i(x, \vec{k}_\perp) \left(b_i^\dagger(k^+, \vec{k}_\perp) \, |0\rangle \, \langle 0| \, b_i(k^+, \vec{k}_\perp) \right)$$

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where $f_i(x, \vec{k}_\perp)$ is the transverse momentum dependent PDF.

- Provides an interpretation of PDFs as a measure of quark entanglement within a hadron.

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For example, the VS density matrix for a π^+ meson is given by

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- In Large-N_c QCD, all quarks are moved into $\mathcal{H}_2$, so this becomes quark-gluon entanglement as expected.

Lagrangian and Hamiltonian

1+1d QCD Lagrangian density:

$$\mathcal{L} = \bar{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{2}\mathrm{Tr}[G^{\mu\nu}G_{\mu\nu}]$$

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Null plane Hamiltonian (in LC gauge):

$$H = \int_{-\infty}^{\infty} dx^- \left(q_+^\dagger \frac{m_q^2}{2i\partial^+} q_+ + \frac{g^2}{2} \left(\frac{1}{\partial^+} (q_+^\dagger T^a q_+) \right)^2 \right)$$

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- Gluons are integrated out completely.

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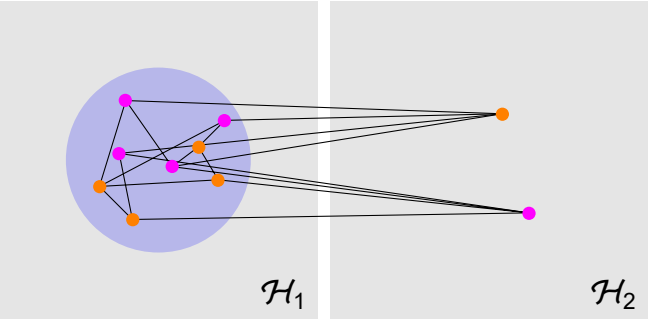
- Vacuum state in $\mathcal{H}_2$ eliminates $d_2 d_1^\dagger$ in Q_i and $b_2 b_1^\dagger$ in $\bar{Q}_i$.

π^+ VS Entanglement

For the π^+ meson in $1+1$d QCD, the reduced density matrix is

$$\rho_{\pi^+} = \mathcal{N}\ \mathrm{Tr}_1 \left[Q_u \bar{Q}_d \left| \pi^+, K \right\rangle_1 |0\rangle_2 \langle 0|_2 \left\langle \pi^+, K \right|_1 Q_d \bar{Q}_u \right]$$



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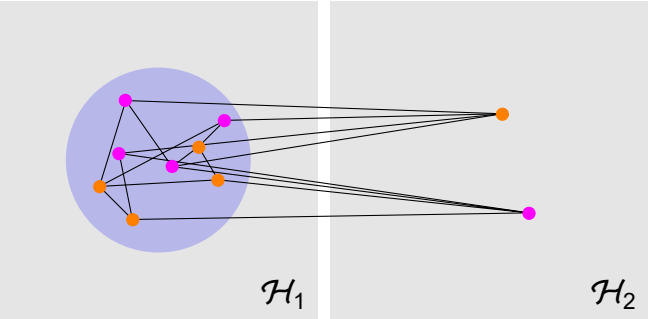

- $|\pi^+, K\rangle$ is the π^+ meson state with total momentum K.

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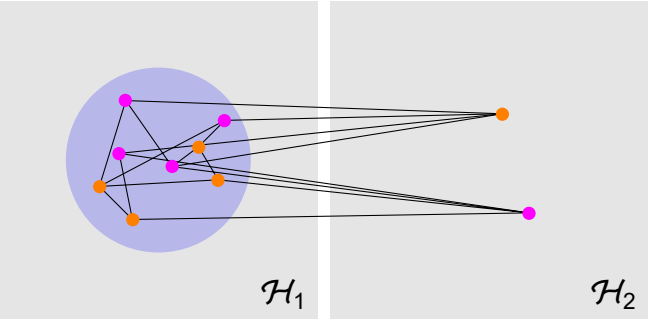
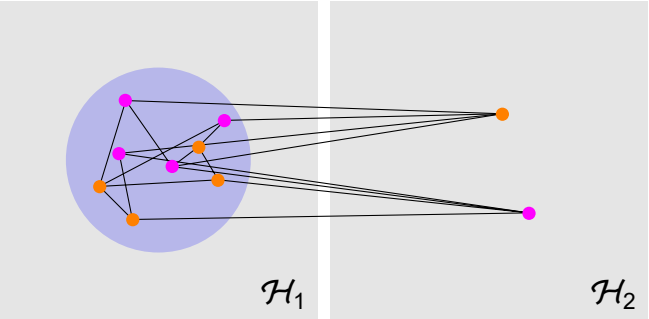


- $|\pi^+, K\rangle$ is the π^+ meson state with total momentum K.
- $|0\rangle$ is the light-cone vacuum.

π^+ VS Entanglement

For the π^+ meson in $1+1$d QCD, the reduced density matrix is

$$\rho_{\pi^+} = \mathcal{N}\ \mathrm{Tr}_1 \left[Q_u \bar{Q}_d \left| \pi^+, K \right\rangle_1 |0\rangle_2 \langle 0|_2 \left\langle \pi^+, K \right|_1 Q_d \bar{Q}_u \right]$$



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- $Q_i(\bar{Q}_i)$ moves a quark(antiquark) of flavor i from $\mathcal{H}_1$ to $\mathcal{H}_2$.

π^+ VS Density Matrix

The matrix elements of ρ_{π^+} are given by

$$\langle 0| \, d_{k_2,d,b} b_{k_1,u,a} \; \rho_{\pi^+} \; b^\dagger_{k_4,u,d} d^\dagger_{k_3,d,c} \, |0\rangle \;\; = \;\; \mathcal{N} \; f_{k_1 k_2 k_3 k_4, abcd},$$

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This turns out to be separable by the color representations of the valence quarks ($\mathbf{N_c} \otimes \bar{\mathbf{N}}_\mathbf{c} = \mathbf{1} \oplus (\mathbf{N_c^2} - \mathbf{1})$).

$$f_{k_1 k_2 k_3 k_4, abcd} = \frac{\delta_{ab}\delta_{cd}}{N_c} f^S_{k_1 k_2 k_3 k_4} + \frac{1}{N_c^2 - 1} \left(\delta_{ad}\delta_{bc} - \frac{1}{N_c}\delta_{ab}\delta_{cd} \right) f^A_{k_1 k_2 k_3 k_4}$$

π^+ VS Entropy

The density matrix thus separates by color and momentum into singlet and adjoint terms.

$$\rho_{\pi^+} = \omega^S(\Pi^S \otimes \rho_S) + \frac{1}{N_c^2 - 1}\omega^A(\Pi^A \otimes \rho_A)$$

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π^+ VS Entropy

This leads to an entanglement entropy that looks like

$$S_{\pi^+} = \boxed{\omega^S S_S + \omega^A S_A} + \boxed{\omega^A \log(N_c^2 - 1)} \boxed{-\omega^S \log \omega^S - \omega^A \log \omega^A}.$$

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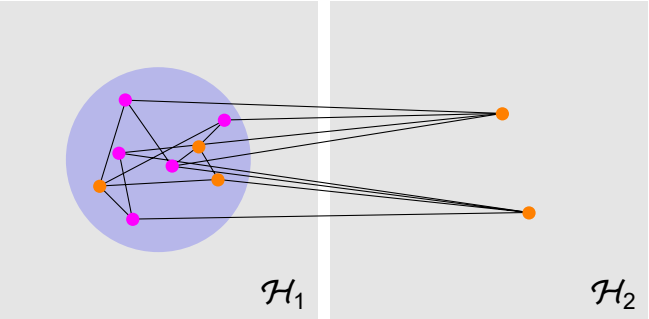
- Entropy of the ω distribution.

N^+ VS Entanglement

For the 2-color baryons ($N_c = 2$), the results are very similar to mesons.

$$\rho_{N^+} = \mathcal{N}\ \mathrm{Tr}_1 \left[Q_u Q_d \left| N^+, K \right\rangle_1 |0\rangle_2 \langle 0|_2 \left\langle N^+, K \right|_1 \bar{Q}_d \bar{Q}_u \right]$$



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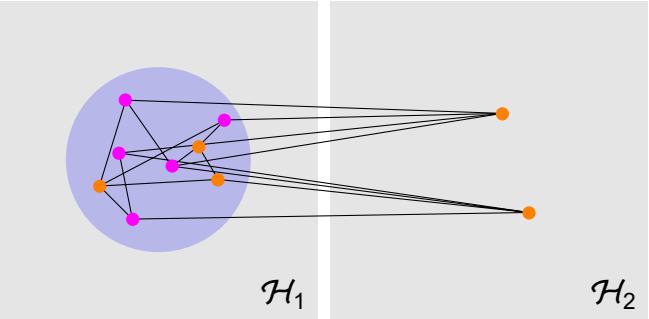

The density matrix and entropy also separate by color representations ($\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{3}$)

$$S_{N^+} = \boxed{\omega^S S_S + \omega^T S_T} + \boxed{\omega^T \log(3)}\boxed{-\omega^S \log \omega^S - \omega^T \log \omega^T}.$$

N^+ VS Entanglement

For the proton ($N_c = 3$), the results are similar.

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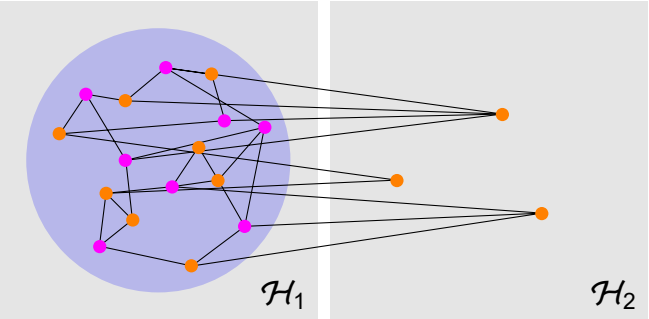

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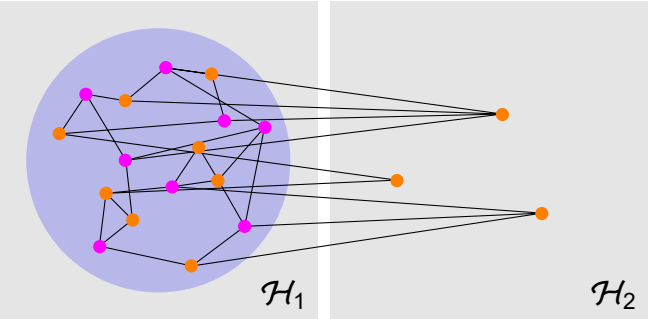

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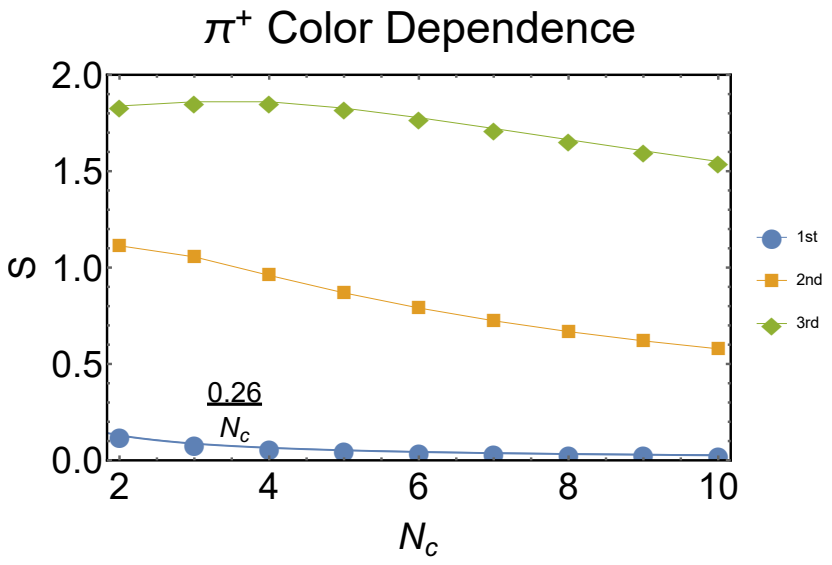
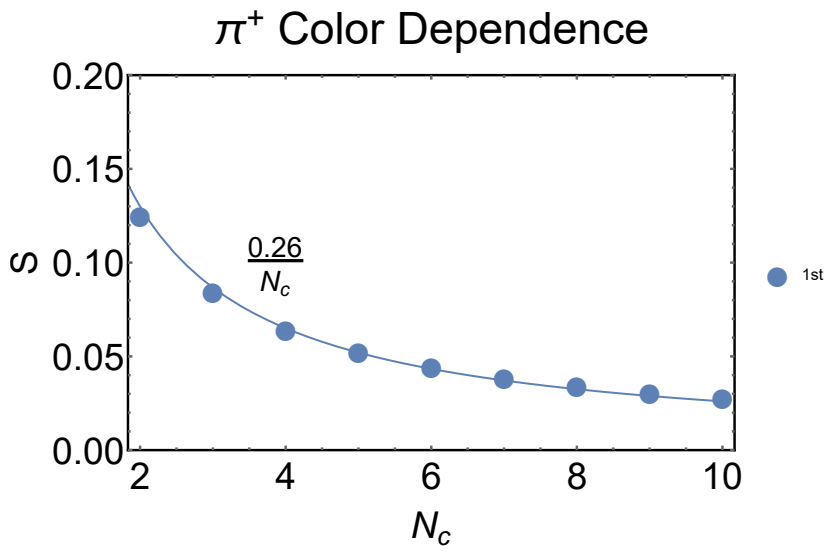
$$S_{N^+} = \boxed{\omega^{\mathcal{A}} S_{\mathcal{A}} + \omega^{M} S_{M} + \omega^{\mathcal{S}} S_{\mathcal{S}}} + \boxed{\omega^{M} \log(8) + \omega^{\mathcal{S}} \log(10)}$$

$$\boxed{-\omega^{\mathcal{A}} \log \omega^{\mathcal{A}} - \omega^{M} \log \omega^{M} - \omega^{\mathcal{S}} \log \omega^{\mathcal{S}}}.$$

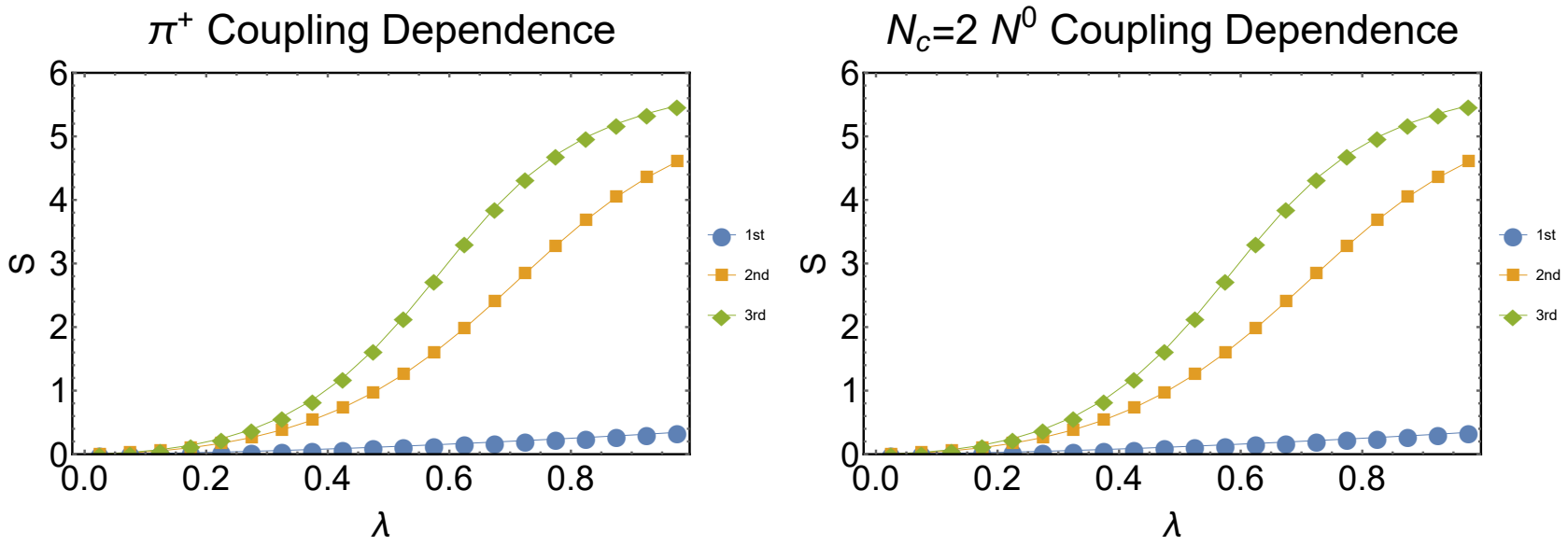
Plots





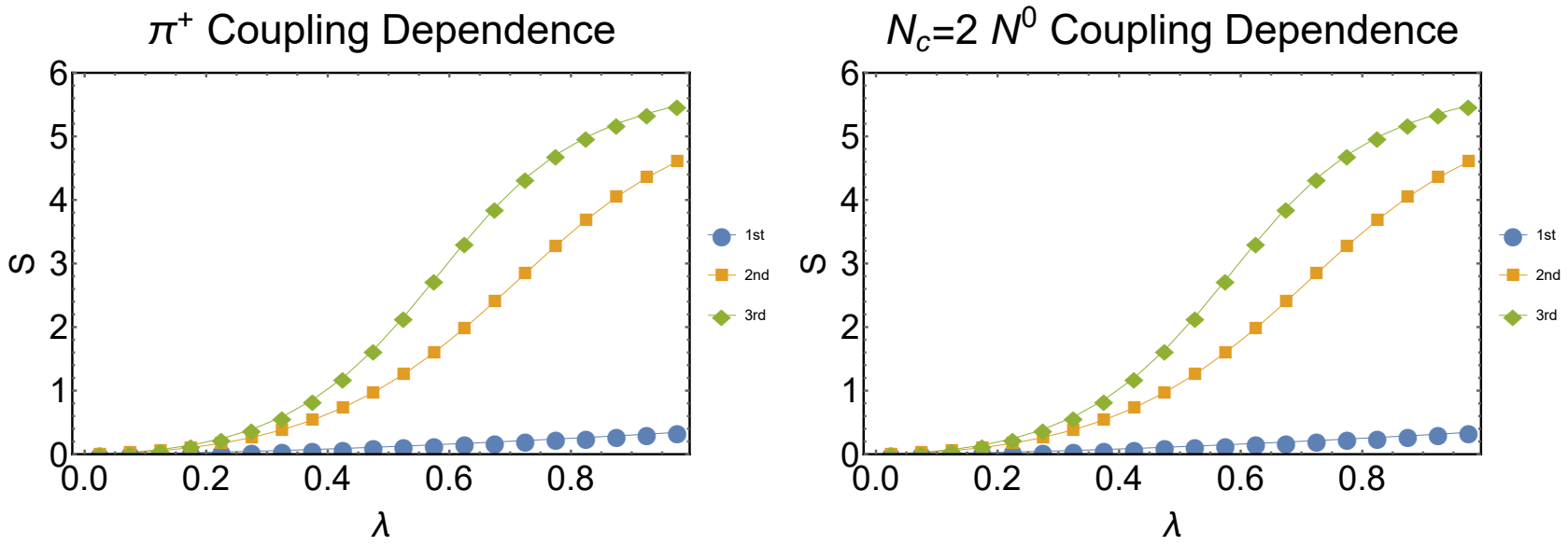
- Plotted with $N_f = 2$, $K_{tot} = 8$, $m^2 = \frac{g^2 N_c}{2\pi}$, and $g^2 \sim \frac{1}{N_c}$.
- Maximum entropy is $S_{max} = 2\log_2(6N_c)$ (above the plots).

Plots 2



- Plotted with $N_f = N_c = 2$, $K_{tot} = 11$, $\lambda = \frac{g^2}{\pi m^2 + g^2}$.
- Maximum entropy is $S_{max} \approx 8.04$.

Plots 2



- Plotted with $N_f = N_c = 2$, $K_{tot} = 11$, $\lambda = \frac{g^2}{\pi m^2 + g^2}$.
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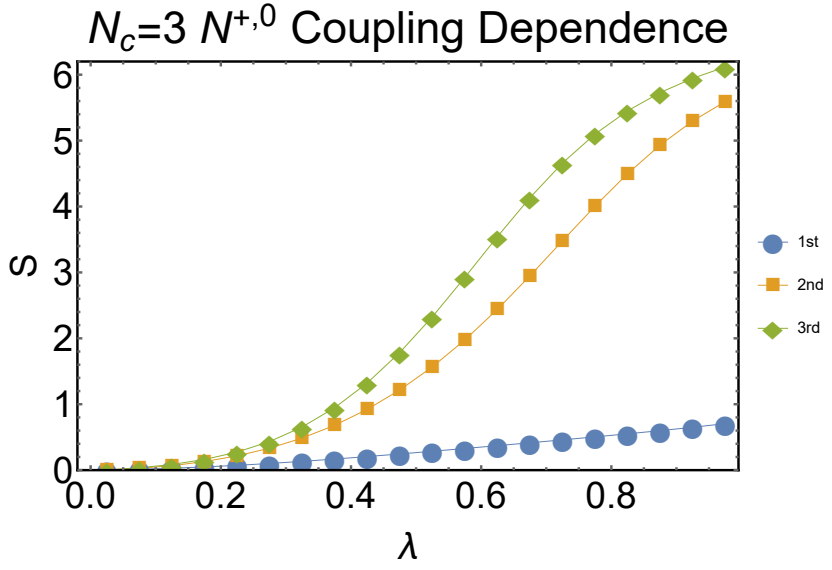
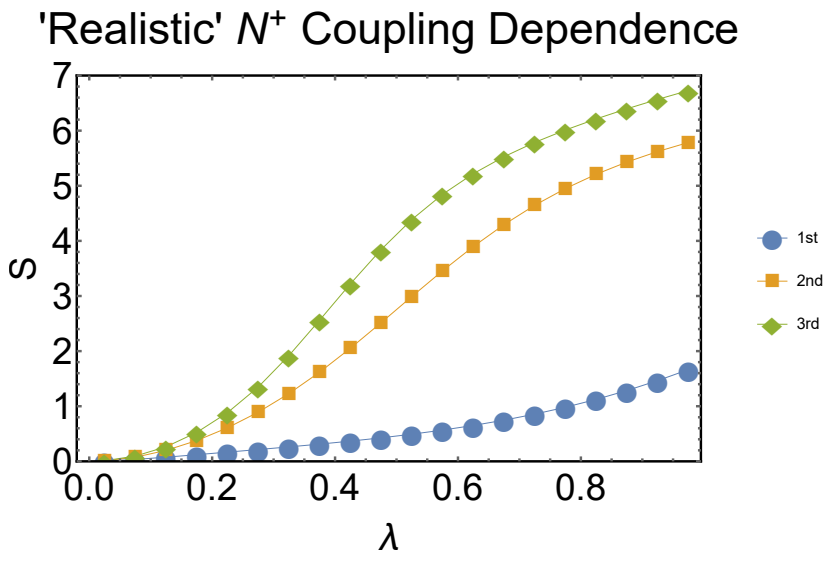
- These are exactly the same due to a $d_c \leftrightarrow \epsilon_{cc'} b_{c'}$ symmetry.

Plots 3

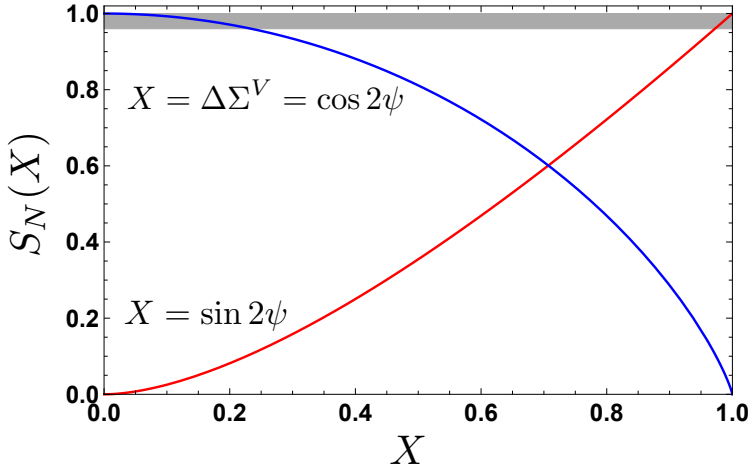




- First plot has $N_f = 2$, $N_c = 3$, $K_{tot} = \frac{21}{2}$, and $\lambda = \frac{3g^2}{2\pi m^2 + 3g^2}$.
- Second plot has $N_f = 3$, $N_c = 3$, $K_{tot} = \frac{17}{2}$, and $\lambda = \frac{3g^2}{2\pi \bar{m}^2 + 3g^2}$, where $\bar{m}^2 = \frac{2m_u^2 + m_d^2}{3}$.
- Maximum entropies are $S_{max} \approx 11.47$ and 10.58 respectively.

VS entropy in real QCD

Previous work modeled quark entanglement due to chiral symmetry breaking.



S. R. Beane and P. Ehlers, Mod. Phys. Lett. A 35, 2050048 (2019), arxiv: 1905.03295

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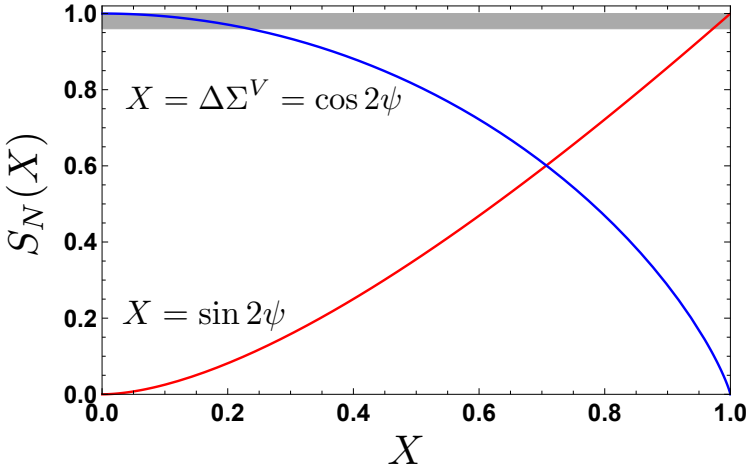

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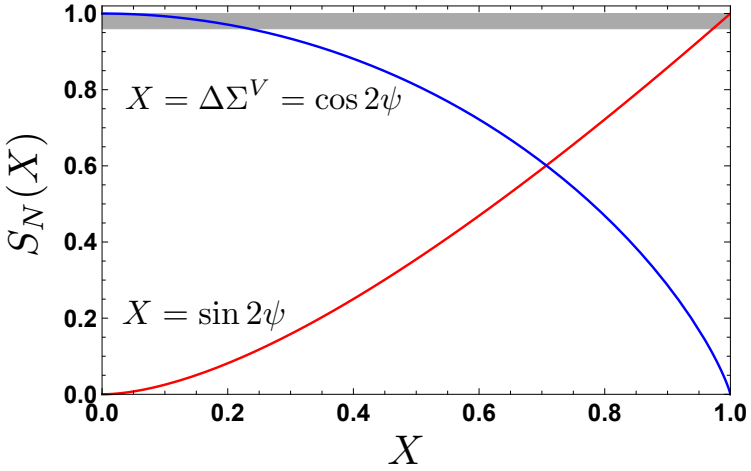

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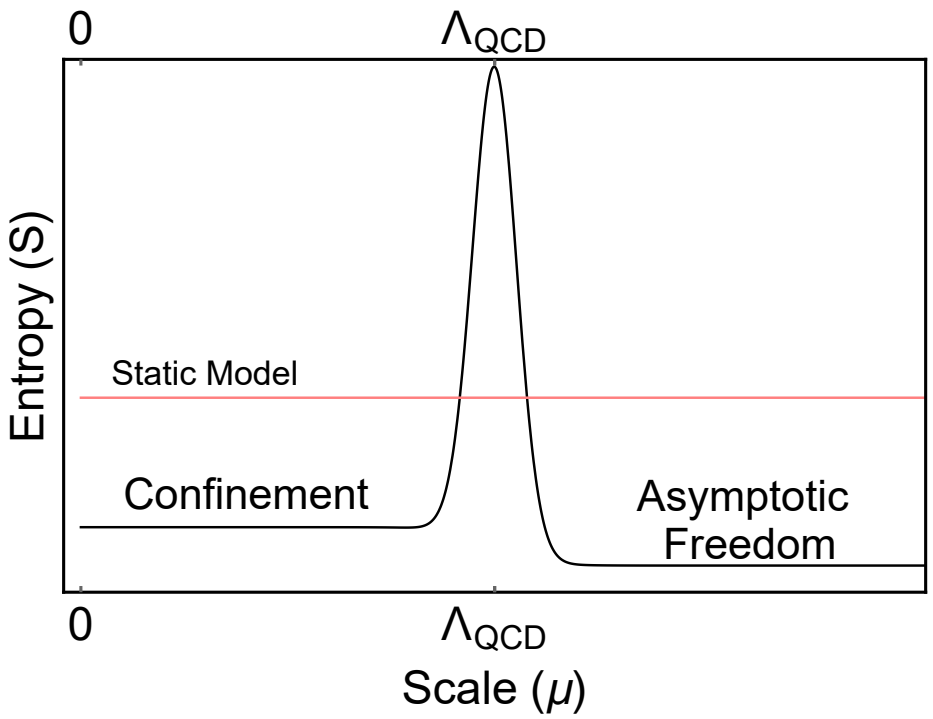

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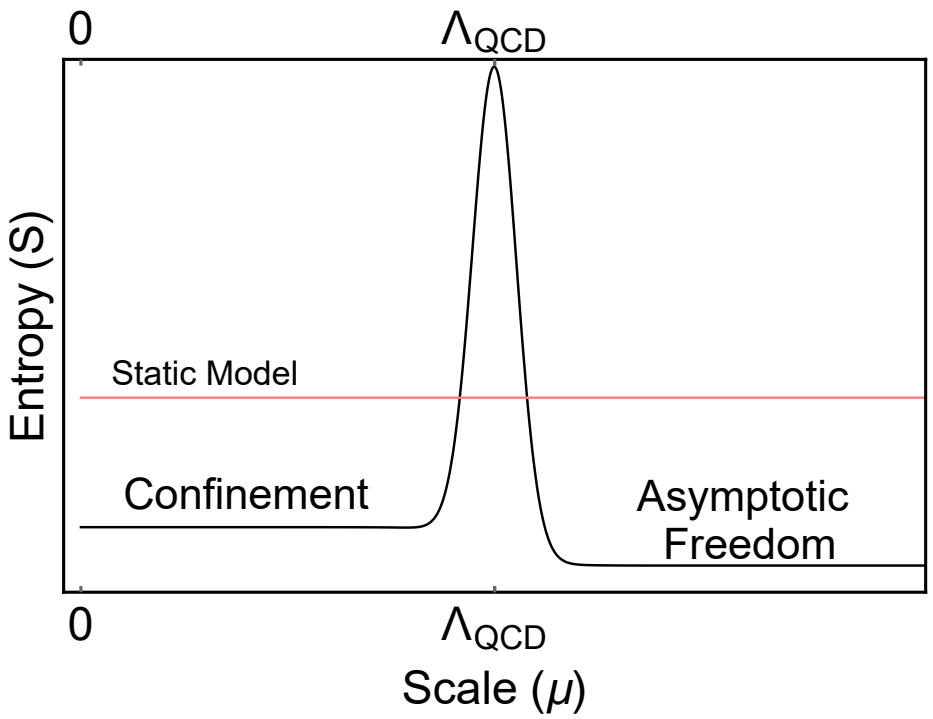

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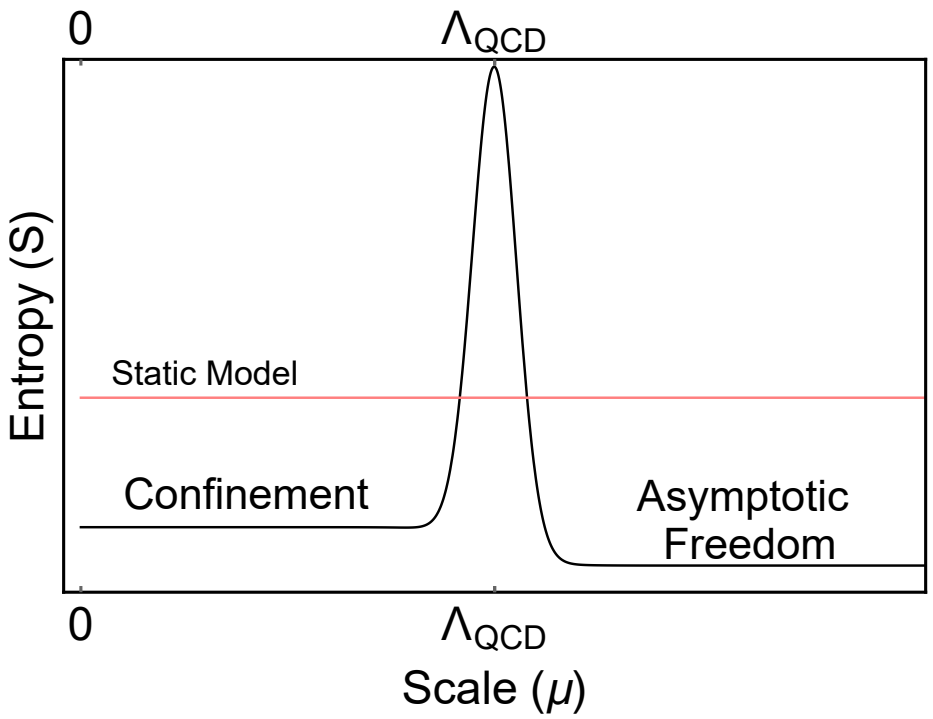
- Source should then be at scales near Λ_{QCD}.



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  - May hold for baryons and in real QCD.
- May signal the shift between perturbative & nonperturbative scales in 3+1d QCD.

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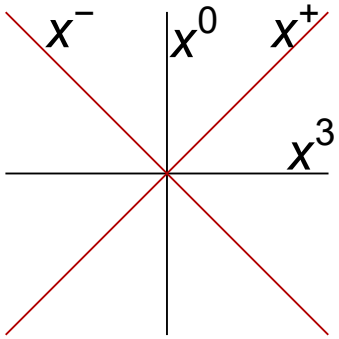
- Amenable to discrete light-cone quantization.

Light-cone Formalism

In 1+1d QCD it is especially useful to work in light-cone coordinates $x^+ = \frac{1}{\sqrt{2}}(x^0 + x^3)$ and $x^- = \frac{1}{\sqrt{2}}(x^0 - x^3)$. (Kogut-Soper conventions)



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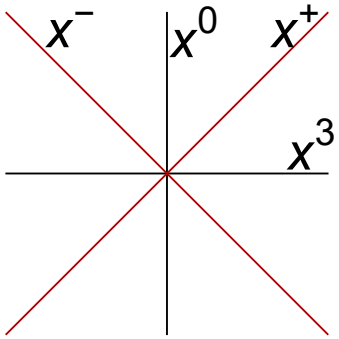

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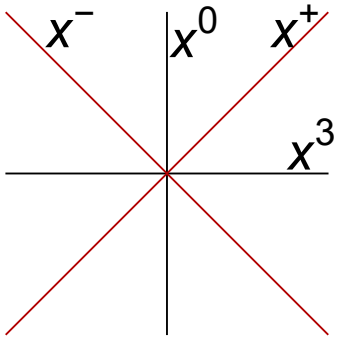

Construct the Hilbert space and along $x^+ = 0$ plane rather than $t = 0$.

- Hamiltonian $H = P^-$ translates along x^+ coordinate instead of t.

DLCQ

Use discrete light-cone quantization (DLCQ)

- Discretize momentum space to half-integers with spacing δp.

$$|\Psi(4)\rangle = \alpha\,|1,3\rangle + \beta\,|2,2\rangle + \gamma\,|3,1\rangle + \delta\,|1,1,1,1\rangle$$

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